Surface Area to Volume Ratio

Center for nanotechnology Education

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Surface Area to Volume Ratio

Abstract

This simple activity conveys one of the most profound aspects of Nanoscience – that for a given volume of material the total surface area increases non-linearly as the volume is divided into smaller and smaller pieces. This concept is being applied to creating better batteries and catalytic converters. Students will measure the time it takes effervescent tablets, which have been broken into different number of pieces, to dissolve. Calculations of surface area and volume can also be performed along with graphing techniques.

Outcomes

Understanding of surface area to volume ratio concepts.

Prerequisites

- Geometry
- Algebra
- Rules of exponents.
- Linear and nonlinear conversions

Correlation

Science Concepts

Chemical reactivity

Nanoscience Concepts

Non-linear scaling

Background Information

Surface Area to Volume Ratio

It is well known that most chemical and biological processes depend on the atoms or molecules that are located on the surface of the entity. Our sense of taste is dependent on the amount of the surface area a grain of salt, for example, has in contact with our tongue. The chemical reactions in automobile catalytic converters are dependent upon surface area available for interaction. The effectiveness of a drug is dependent upon the ability of that drug to contact the appropriate segments of the protein and interact with them. If the critical protein portions are buried within the protein then the interactions will not occur and the drug will not be as effective. Of course, the Golgi apparatus and mitochondrion in cells are designed to increase their interactive surface area therefore making them more efficient.

Approach: First, it is important to remind students of the equations required to calculate surface area and volume. Start with a simple cube and remind students that volume is the width times the height times the depth. Also remind them that for a cube with the total surface area, which is what we want to talk about at the nanoscale is going to be equal to six times the area of one individual side. Often, a simple example will remind them of the math required.

After having reviewed volume and surface area calculations for cube and rectangular objects, we can move on to the aspect of spheres because many of the nanoscale objects created will be spherical or oval in nature. It is worth the time to make sure that students are comfortable with calculating volume, surface area and circumference of spherical objects. Using a sphere as an example will also allow students to become familiar and comfortable with the Greek letter pi.

For an example, as shown below, have the students begin with a spherical volume of a 1 cm diameter. To calculate the volume of the sphere we need to use the equation that the volume is equal to $4/3\ \pi\ r^3$. Students need to be reminded to be careful about distinguishing between diameter and radius. Placing the information in a table or using a spreadsheet will help keep the calculations straight.

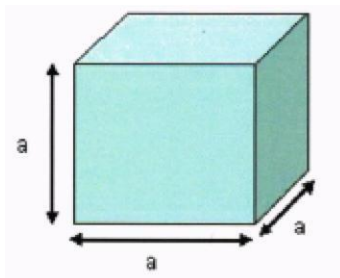
These calculations are also an opportunity for students to practice exponential multiplication, and application of basic exponential laws. It's a good idea to provide initial dimensions in one unit, for example centimeters, and ask the students to provide the answers in a different unit, for example millimeters.

Ratio of Surface Area to Volume:

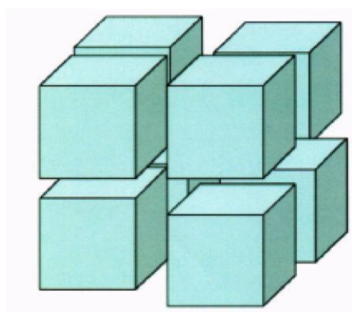
Volume = a^3

Surface area =6 a^2

Increasing the surface area (with respect to a constant volume) is one of the benefits of creating particles at the nanoscale that will have a large impact on chemical reactions. It's important that students understand the relationship between surface area and volume since this "squared" versus "cubed" relationship of mathematics will repeat itself in several Nanoscience concepts.



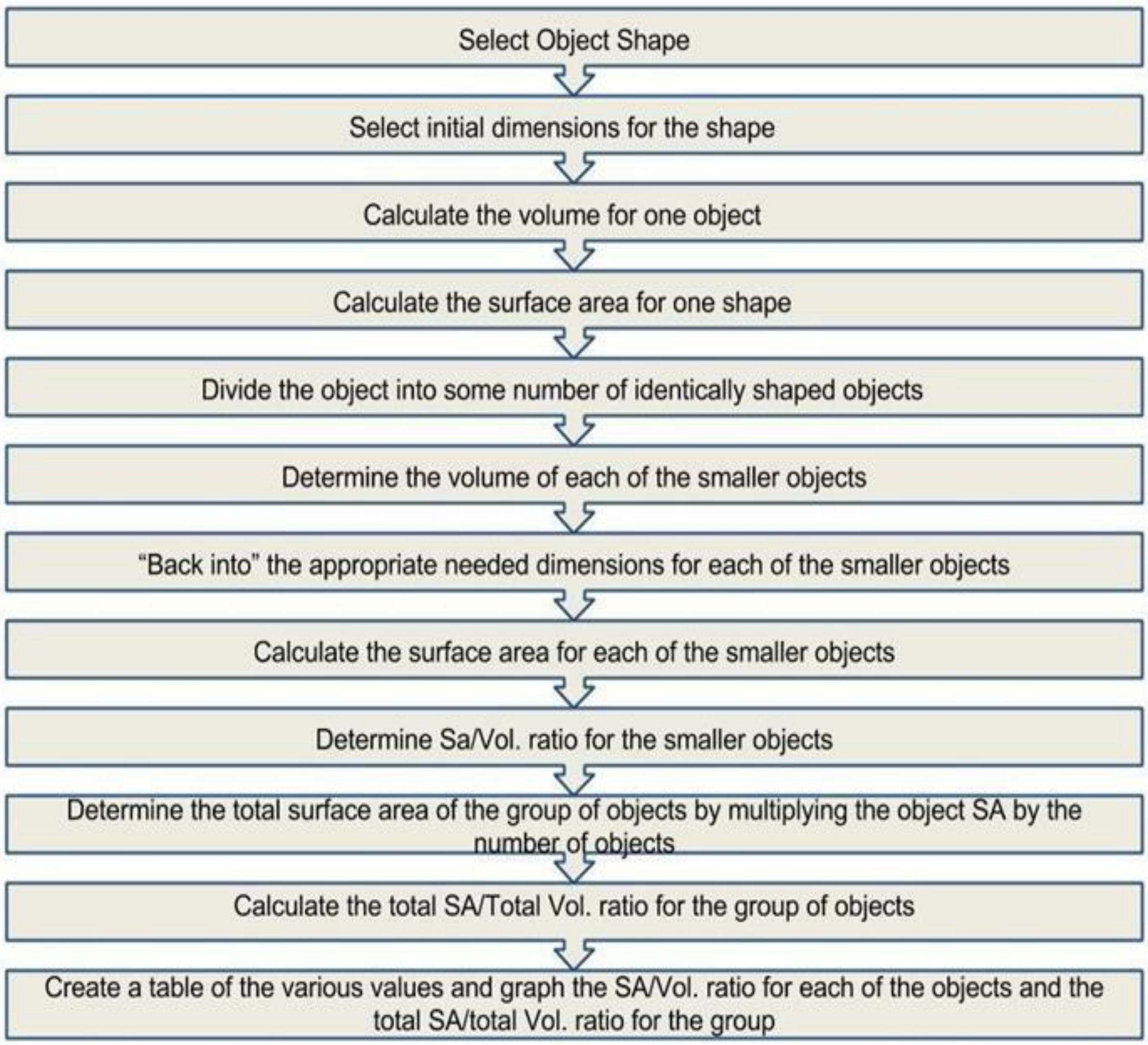
Because of the power inequality, as the same volume is broken down into smaller pieces the surface area increases non-linearly. For example, consider the cube above that has a length "a" on each side. Let us select a value of 6 cm for a. The volume of the cube then is 216 cm^3 and the total surface area is 216 cm^2. Now let us cut each dimension in half and create 8 smaller cubes as shown below.



The dimensions for each of the smaller cubes will be 3 cm x 3 cm x 3 cm. Therefore the volume of each of the smaller cubes is 27 cm^3 with the total volume still being equal to 216 cm^2. The surface area for each of the smaller cubes is 6 (3 cm x 3 cm) which is equal to 54cm^2. The total surface area then for all eight cubes is 8 x 54 cm^2 or 432 cm^2. By repeating this process and breaking the same initial volume into larger numbers of subunits students can observe the nonlinear relationship.

It is easy to see the potential benefit of creating smaller and smaller particles where the desired interaction is dependent on surface area. For example, in the catalytic converter used in automobiles, platinum (a very expensive element) is used as part of the chemical reaction - which is dependent on surface area. Therefore, if a specific amount of surface area is required for the chemical reaction, a smaller total amount of platinum will be required if the created particles can be made as small as possible.

Activity Flow Chart



Learning Activity

Alka-Seltzer Activity

Background: Almost all chemical reactions and interactions are dependent upon the amount of surface area available for the reaction. Thermal transfer, capacitance, and electrical conduction are also properties that are dependent on surface area.

One of the results of breaking materials down into smaller and smaller particles is that you increase the amount of surface area for say, chemical reactions using the same volume of starting material. For example, catalytic converters in automobiles use platinum - a very expensive material - in the exhaust system. The platinum interacts with the vapors created by the engine to remove pollutants, this interaction and the efficiency of the converter is dependent on the surface area of platinum available for the reaction. By breaking down a given volume of platinum into smaller particles, we can increase the surface area for the same amount (volume) of material

The same is true in ion batteries. Nanotechnology is being used in batteries to increase the lifetime, shorten the recharge cycles, and still use the same amount (volume) of the material.

This activity is a very simple visual representation of the impact of surface area to volume.

<u>Materials:</u>

- 6 plastic cups
- 3 Alka-Seltzer tablets
- mortar and pestle
- stop watch
- water

Process:

1. Fill 3 of the plastic cups about halfway full of water. Try to have about the same amount of water in each cup
2. Place a whole Alka-Seltzer tablet in one of the empty cups.
3. Take the second tablet and gently break it into 3-5 fairly large pieces and place the pieces into the second empty cup. Record the number of pieces.
4. Take the third tablet and grind it up into a fine powder using the mortar and pestle. Place the powder in the third empty cup.
5. Take one cup at a time and pour water into it. Time how long each tablet takes to dissolve fully.

Result: By grinding the tablet into a fine powder, we significantly increase the surface area available for interaction and reaction with the water. This is similar to the reason we use small pieces of wood to start a fire and not a giant log.

Discussion Questions

- How does surface area affect the interaction?
- What do different interactions tell us about the molecular structure of the materials involved?
- What are typical strengths of different interaction forces?
- What are units of these forces?
- What environmental considerations may impact the interactions? By how much?
- What are some of the applications of this technology?

Current and Future Applications

The chemical reactions in automobile catalytic converters are dependent upon surface area available for interaction. The effectiveness of a drug is dependent upon the ability of that drug to contact the appropriate segments of the protein and interact with them. Lf the critical protein portions are buried within the protein then the interactions will not occur and the drug will not be as effective. Of course the Golgi apparatus and mitochondria in cells are designed to increase their interactive surface area therefore making them more efficient.

Contributors

This module was written by Deb Newberry and Billie Copley of Dakota County Technical College.

Multimedia Resources

Videos

- https://www.youtube.com/watch?v=ffS1VlS3KyU

Simulations

- www.nanohub.org
- mw.concord.org

Alignment of Surface Area to Volume Ratio Module to the Next Generation Science Standards

The Next Generation Science Standards (NGSS) were published in April 2013. They consist of statements that convey the performance expectations for students. Each performance expectation is a single statement that is built from three parts: science and engineering practices (Practices), disciplinary core ideas (DCI) and crosscutting concepts.

Since the Surface Area to Volume Ratio Module was created prior to the release of these standards one would expect that it aligns most readily to the individual statements that articulate the practices, DCIs, and crosscutting concepts. The background material, reading, and the slides from the module address the aspects of the NGSS shown in Table 1.

TABLE 1. ALIGNED PRACTICES, DISCIPLINARY CORE IDEAS, AND CROSSCUTTING CONCEPTS

PRACTICE	***DCI***	***CROSSCUTTING CONCEPT***
No alignments	*No alignments*	*HS: Scale, Proportion, and Quantity:* The significance of a phenomenon is dependent on the scale, proportion, and quantity at which it occurs. *Strong in teacher and student materials*

Alignment of Surface Area to Volume Ratio Module to the Common Core State Standards in English Language Arts/Literacy and Mathematics

The Common Core State Standards (CCSS) were published in June 2010. They articulate student skills for English language arts/literacy and mathematics. The content of the module addresses the concepts and skills shown in Tables 3 and 4.

For English language arts/literacy, the CCSS is organized around College and Career Anchor Standards (CCR) that articulate the over-arching skills that students need to be prepared for college and career. There are grade level versions of each Anchor Standard, as well as versions for science and social studies classrooms (literacy standards). Alignments in Table 3 were made to the Anchor Standards, unless a more specific version of the standard was a closer fit to the skills in the module. Additional alignments may be warranted, depending on the use of associated videos that are provided as links in the module and whether students engage in discussions.

TABLE 3. ALIGNED COMMON CORE STANDARDS FOR ENGLISH LANGUAGE ARTS & LITERACY
CCR.L.6: Acquire and use accurately a range of general academic and domain-specific words and phrases sufficient for reading, writing, speaking, and listening at the college and career readiness level; demonstrate independence in gathering vocabulary knowledge when encountering an unknown term important to comprehension or expression. *Partial in teacher and student materials*

For mathematics, Table 4 shows alignments to standards found in the 8th through 12th grade levels.

TABLE 4. ALIGNED COMMON CORE MATHEMATICS STANDARDS
8.G.9 Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems. *Strong in teacher and student materials*
HS.G-GMD.3 Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems. *Strong in teacher and student materials*
HS.G-MG.1 Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder) *Strong in student materials*