ECE595 / STAT598: Machine Learning I
Lecture 26.2: Growth Function - Examples of mH(N)

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Today’s Lecture:
- Overcoming the $M$ Factor
  - Decisions based on Training Samples
  - Dichotomy
- Examples of $m_{H}(N)$
  - Finite 2D Set
  - Positive ray
  - Interval
  - Convex set
Examples of $m_\mathcal{H}(N)$

- $\mathcal{H} =$ linear models in 2D
- $N = 3$
- How many dichotomies can I generate by moving the three points?
- This gives you 8. Are we the best?
Examples of $m_{\mathcal{H}}(N)$

- $\mathcal{H} = \text{linear models in 2D}$
- $N = 3$
- How many dichotomies can I generate by moving the three points?
- This gives you 6. The previous is the best. So $m_{\mathcal{H}}(3) = 8$. 

![Diagram showing examples of $m_{\mathcal{H}}(N)$]
What about $m_H(4)$? Ans: 14.

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Another Example

- $\mathcal{H}$ = set of $h$: $\mathbb{R} \rightarrow \{+1, -1\}$
- $h(x) = \text{sign}(x - a)$
- Cut the line into two halves
- You can only move along the line
- $m_{\mathcal{H}}(N) = N + 1$
- The $N$ comes from the $N$ points
- The $+1$ comes from the two ends
Another Example

- $\mathcal{H}$ = set of $h$: $\mathbb{R} \rightarrow \{+1, -1\}$
- Put an interval
- Length of the interval is $N$ points

$$m_{\mathcal{H}}(N) = \binom{N + 1}{2} + 1 = \frac{N^2}{2} + \frac{N}{2} + 1$$

- Think of $N + 1$ balls, pick 2.
Another Example

- $\mathcal{H} = \text{set of } h: \mathbb{R}^2 \rightarrow \{+1, -1\}$
- $h(x) = +1$ is convex
- Here are some examples
Another Example

- How about this collection of data points?
- Can you find an \( h \) such that you get a convex set?
- Yes. Do convex hull.
- Does it give you the maximum number of dichotomies?
- No. All interior points do not contribute.
Another Example

- The best you can do is this.
- Put all the points on a circle.
- Then you can get at most $2^N$ different dichotomies
- So
  \[ m_{\mathcal{H}}(N) = 2^N \]
- That is the best you can ever get with $N$ points
Summary of the Examples

- $H$ is positive ray:
  \[ m_H(N) = N + 1 \]

- $H$ is positive interval:
  \[ m_H(N) = \binom{N+1}{2} + 1 = \frac{N^2}{2} + \frac{N}{2} + 1 \]

- $H$ is convex set:
  \[ m_H(N) = 2^N \]

So if we can replace $M$ by $m_H(N)$

And if $m_H(N)$ is a polynomial

Then we are good.
Reading List

- Yasar Abu-Mostafa, Learning from Data, chapter 2.1
- Mehrya Mohri, Foundations of Machine Learning, Chapter 3.2