ECE595 / STAT598: Machine Learning I

Lecture 26.2: Growth Function - Examples of mH(N)

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Outline

- Lecture 25 Generalization
- Lecture 26 Growth Function
- Lecture 27 VC Dimension

Today's Lecture:

- Overcoming the M Factor
  - Decisions based on Training Samples
  - Dichotomy
- Examples of $m_{\mathcal{H}}(N)$
  - Finite 2D Set
  - Positive ray
  - Interval
  - Convex set

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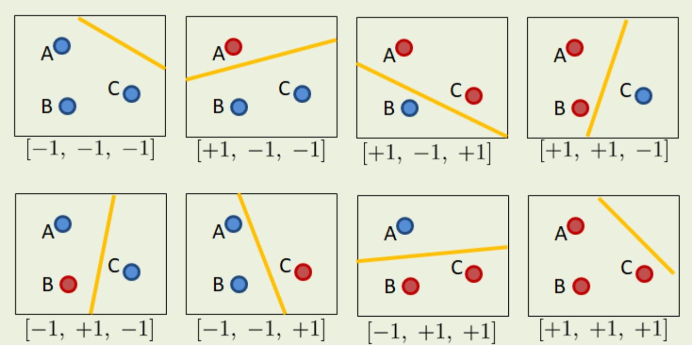
Examples of $m_{\mathcal{H}}(N)$

- $\mathcal{H}$ = linear models in 2D
- $N = 3$
- How many dichotomies can I generate by moving the three points?
- This gives you 8. Are we the best?



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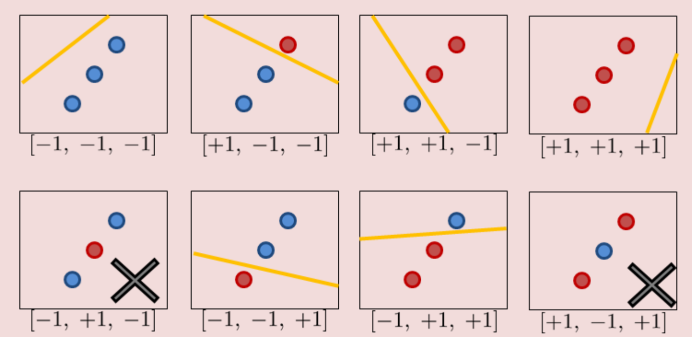
Examples of $m_{\mathcal{H}}(N)$

- $\mathcal{H}$ = linear models in 2D
- $N = 3$
- How many dichotomies can I generate by moving the three points?
- This gives you 6. The previous is the best. So $m_{\mathcal{H}}(3) = 8$.



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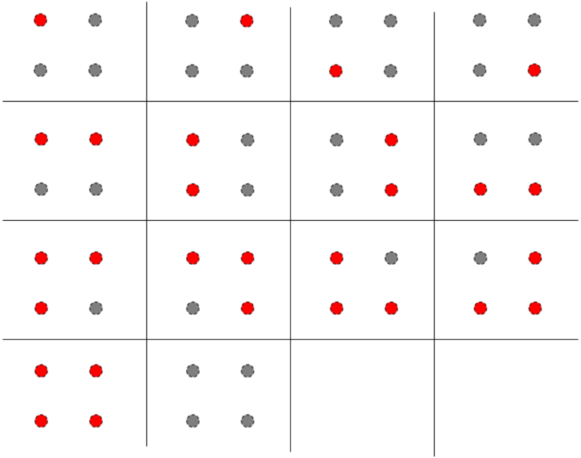
What about $m_{\mathcal{H}}(4)$? Ans: 14.



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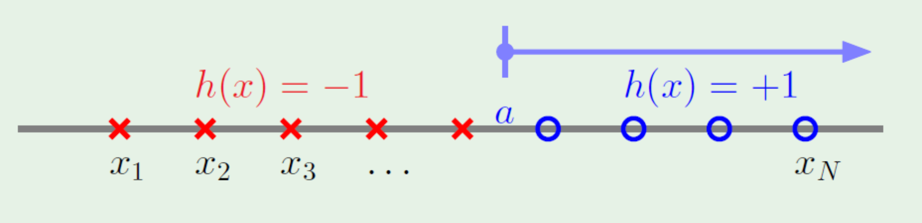
Another Example



- $\mathcal{H}$ = set of h: $\mathbb{R} \to \{+1, -1\}$
- $h(x) = \text{sign}(x - a)$
- Cut the line into two halves
- You can only move along the line
- $m_{\mathcal{H}}(N) = N + 1$
- The N comes from the N points
- The $+1$ comes from the two ends

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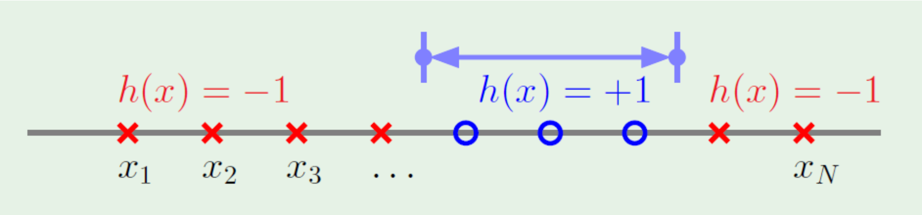
Another Example



- $\mathcal{H}$ = set of h: $\mathbb{R} \to \{+1, -1\}$
- Put an interval
- Length of the interval is N points
- $$m_{\mathcal{H}}(N) = \binom{N+1}{2} + 1 = \frac{N^2}{2} + \frac{N}{2} + 1$$
- Think of $N + 1$ balls, pick 2.

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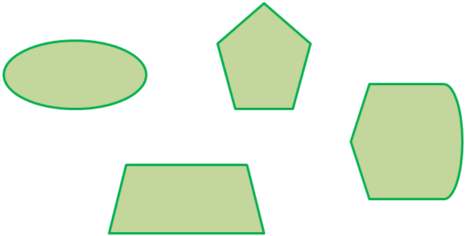
Another Example

- $\mathcal{H}$ = set of h: $\mathbb{R}^2 \rightarrow \{+1, -1\}$
- $h(\boldsymbol{x}) = +1$ is convex
- Here are some examples



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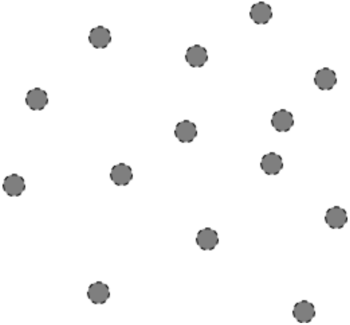
Another Example

- How about this collection of data points?
- Can you find an h such that you get a convex set?
- Yes. Do convex hull.
- Does it give you the maximum number of dichotomies?
- No. All interior points do not contribute.



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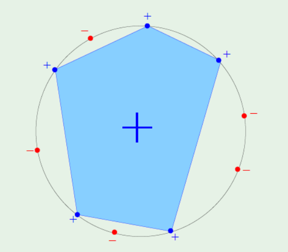
Another Example



- The best you can do is this.
- Put all the points on a circle.
- Then you can get at most 2^N different dichotomies
- So

$$m_{\mathcal{H}}(N) = 2^N$$

- That is the best you can ever get with N points

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Summary of the Examples

- $\mathcal{H}$ is positive ray:

$$m_{\mathcal{H}}(N) = N + 1$$

- $\mathcal{H}$ is positive interval:

$$m_{\mathcal{H}}(N) = \binom{N+1}{2} + 1 = \frac{N^2}{2} + \frac{N}{2} + 1$$

- $\mathcal{H}$ is convex set:

$$m_{\mathcal{H}}(N) = 2^N$$

- So if we can replace M by $m_{\mathcal{H}}(N)$
- And if $m_{\mathcal{H}}(N)$ is a polynomial
- Then we are good.

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Reading List

- Yasar Abu-Mostafa, Learning from Data, chapter 2.1
- Mehrya Mohri, Foundations of Machine Learning, Chapter 3.2
- Stanford Note http://cs229.stanford.edu/notes/cs229-notes4.pdf

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