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Designing a NISQ reservoir with maximal memory capacity for Volatility Forecasting

PQSEI

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Why Risk Management Matters

Motivation

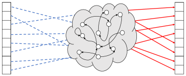
- Quantitative risk management, particularly volatility forecasting, is critically important to the economy.
- We applied quantum reservoir computing for forecasting VIX (the CBOE volatility index)
- VIX is a highly non-linear and memory intensive 'real-life' signal that is driven by market dynamics and trader psychology and cannot be expressed by a deterministic equation.

SOURCE: Google Images

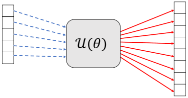
Using a NISQ reservoir as a computing engine

Introduction

Classical Reservoir



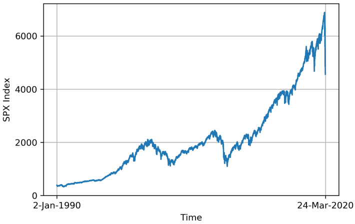
Quantum Reservoir

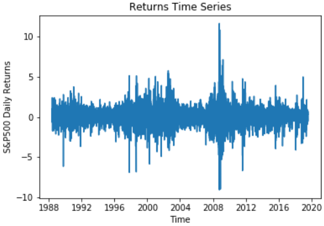


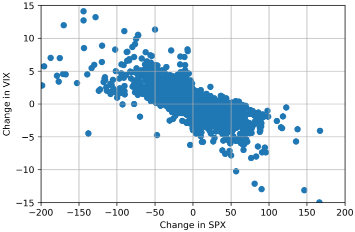
- Classical reservoir computing provides a road map towards using 'signal driven dynamical systems' to process information with non-von Neumann architectures.
- The connections within the reservoir are not trained; inputs are mapped to a high dimensional space and the output from the high dimensional state is trained to predict the desired function using a simple method like linear regression.
- Quantum Reservoir Computing (QRC) exploits quantum dynamics for machine learning.
- QRC does not require any sophisticated quantum gate (natural dynamics is enough).
- Numerical experiments show that quantum systems consisting of 5-7 qubits possess computational capabilities comparable to conventional recurrent neural networks of 100 to 500 nodes.

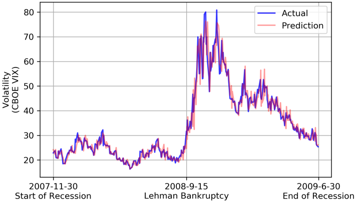
Forecasting VIX (the CBOE volatility index)

Problem Statement and Results









Forecast Approach

Mathematical Framing

$$r(t) = log\left[\frac{SPX(t)}{SPX(t-1)}\right] \tag{1}$$

$$u(t) = 1 - \exp[-(a_0 + I(\Delta r_t)a_1\Delta r_t)] \tag{2}$$

$$\begin{aligned} s &= Pr(1) - Pr(0) \\ \vec{s}_t &= [s_0(t), s_1(t), s_2(t), s_3(t), s_4(t), s_5(t)] \\ \vec{u}(t) &= [u_0(t), u_1(t), u_2(t), u_3(t), u_4(t), u_5(t)] \\ u_m(t) &= r'(t-m) \text{ where } m \in [0,5] \end{aligned} \tag{3}$$

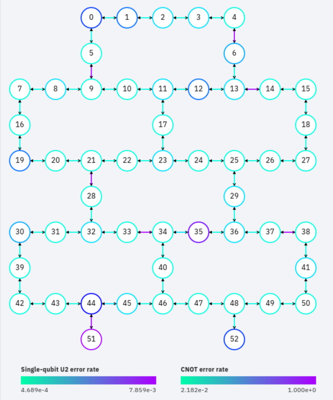
$$\theta_m(t+1) = \frac{\pi}{2}\left(\alpha * u_m(t) + \beta * \frac{s_m(t)+1}{2} + \gamma * e_t\right) \tag{4}$$

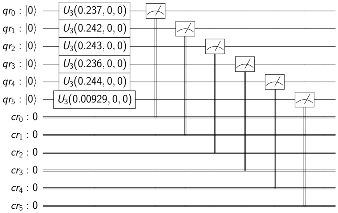
$$\begin{aligned} \hat{\sigma}_{t+1} &= \vec{w}(t) \cdot \vec{s}(t) \\ \varepsilon_{t+1} &= \sigma_{t+1} - \hat{\sigma}_{t+1} \end{aligned} \tag{5}$$

$$MSE = \frac{1}{T}\sum \varepsilon_t^2 \tag{6}$$

Device and Circuit Layout

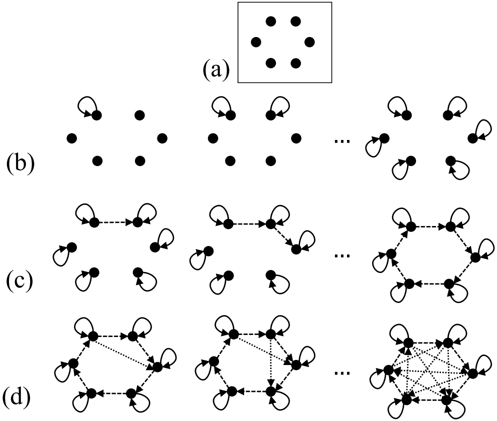
We used the 53-qubit IBM Rochester Device for our experiment.





Circuit space

Design Considerations

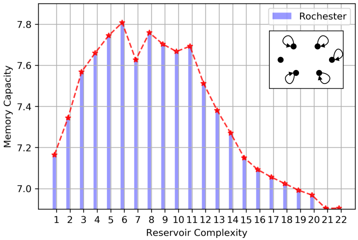


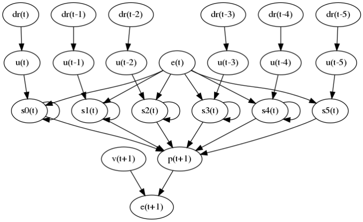
Design Considerations

- Synchronization
- Reservoir Dimensionality
- Adequate Memory
- Response Separability
- Adequate Non-linearity
- Edge Density
- Feedback Strength
- Noise induced regularization

Memory Capacity by Complexity

Reservoir with proper parameters can have memory of past inputs





- Suppose $u(t)$ and $\hat{u}(t)$ are two time series which are same everywhere except a small perturbation at $t = t_0 - 1$. This means:

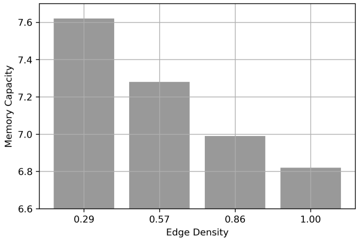
$$\hat{u}(t_0 - 1) = u(t_0 - 1) + \Delta, \text{ for } t = t_0 - 1$$
$$\hat{u}(t) = u(t), \text{ for all } t \neq t_0 - 1$$

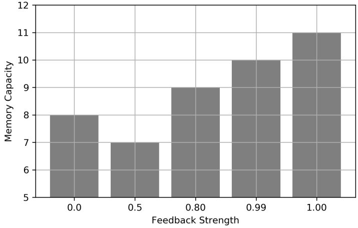
- When we feed $u(t)$ or $\hat{u}(t)$ into the NISQ reservoir, we get the spin time series $\{s(t)\}$ and $\{\hat{s}(t)\}$ respectively (let's consider a one qubit reservoir for simplicity). Let $\delta S(t)$ denote the difference between the outputs $s(t)$ and $\hat{s}(t)$ i.e.

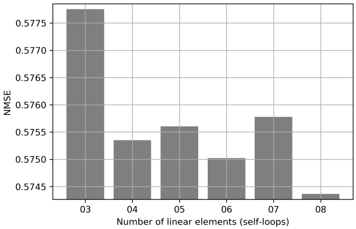
$$\delta s(t) = s(t) - \hat{s}(t)$$

- We say the reservoir has memory when $\delta s(t)$ and $\delta s(0)$ are related i.e. $\delta s(t)$ can provide information about $\delta s(0)$. The stronger the mutual information between $\delta s(t)$ and $\delta s(0)$, stronger is the memory capacity.

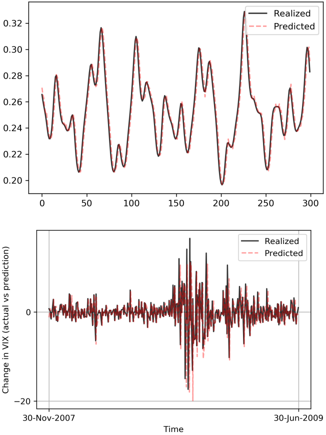
Empirically, three drivers explain the observed peak







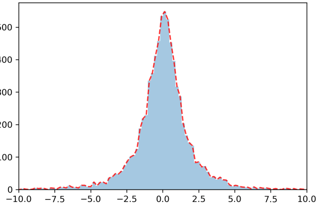
Benchmarking and Application Result



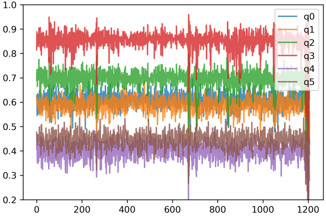
- Non-linear Auto-regressive Moving Average is a challenging machine learning task with high degree of non-linearity and significant memory requirements (i.e. dependence on long time lags).
- We compared the performance of our quantum reservoir construction to other published work. Our reservoir achieved an NMSE of 6×10^{-4}. Others research groups reported values in the range $[3 \times 10^{-3}, 7.6 \times 10^{-6}]$.

Error and Spin Evolution

The prediction error shows very little bias
i.e. it is centered around zero.



Steady state view of the average spin of the 6 qubits. These signals are linearly combined by an optimized weight vector to produce the forecast.



Quantum Computing in finance

Conclusion

- Better security, faster solution times and ability to solve classically intractable problems are all sought-after objectives in the world of empirical finance.
- Hence, quantum computing (which promises these advances) should be a focus area for researchers in finance.
- A fault-tolerant quantum computer could turbo-charge progress in several sub-fields that deal with computationally expensive optimization problems (often including big data) such as:
    1. Asset Management e.g. portfolio optimization
    2. Investment Banking e.g. option pricing
    3. Retail Banking e.g. mortgage securitization schemes
    4. Asset Liability Management e.g. liquidity optimization
    5. Volatility forecasting (e.g. this work)
    6. Financial crisis prediction
    7. Compliance e.g. optimal monitoring and surveillance
    8. Fraud Management e.g. credit card fraud detection
    9. Legal e.g. searching for key clauses in vast database of legal documents (potential Grover search application)
    10. Secure Communications e.g. building next generation of hacker resistant networks (potential quantum crypotgraphy application)