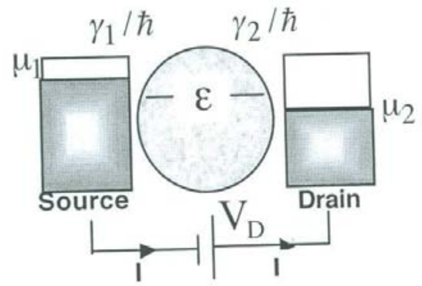
Problem 1: We have seen in class that the current-voltage (I-V) characteristics of a nanoscale device can be calculated from

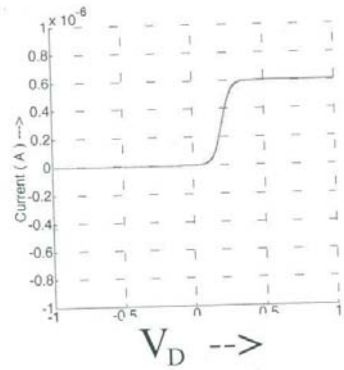
$$I = \frac{2q}{\hbar} \int dE\, D(E-U) \frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2} [f_1(E) - f_2(E)]$$

where $U = U_L + U_0(N - N_0)$, where the Laplace potential is given by a fraction α of the drain potential (there is no gate in this structure):

$U_L = -q\,\alpha V_D$, α being a constant between 0 and 1.

For a device having one energy level ε located above the equilibrium electrochemical potential μ isomeone has calculated the current versus voltage shown below:





Assume negligible charging energy: $U_0 = 0$
and equal escape rates: $\gamma_1/\hbar = \gamma_2/\hbar$.

(a) Estimate $\gamma_1/\hbar$ (same as $\gamma_2/\hbar$). Please be sure to mention units.
(b) What did the person doing the calculation assume for the constant α ?

(a) $I_{max} = .6 \times 10^{-6}\,A = \frac{2q}{\hbar}\frac{\gamma_1}{2}$

$= 1.6 \times 10^{-19}$ coul. $* (\gamma_1/\hbar)$

$\gamma_1/\hbar = .6 \times 10^{-6} / 1.6 \times 10^{-19} = 3.75 \times 10^{12}$ /sec.

(b) No current flows for negative V_D when μ_2 is raised.

Hence level must be fixed.

$\Rightarrow \quad \alpha = 0$

Problem 2: We have seen in class that free electrons in the absence of any external potential are described by (in one dimension) $i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2}$ (1)

whose solutions can be written in the form $\psi(x,t) = \underbrace{A}_{constant} \; e^{+ikx}\, e^{-iEt/\hbar}$ (2)

with E and k related by the dispersion relation: $E = \hbar^2 k^2/2m$ (3)

We have also seen that if the electrons are confined in a box of length L, the energy levels become discrete with the lowest energy given by $E_1 = \hbar^2\pi^2/2mL^2$ (4) (4)

(a) Can you suggest a suitable differential equation to replace (1) if you wanted the dispersion relation to look like $E = (\hbar^2 k^2/2m) + \alpha k^4$ (3')
(α being a constant) instead of (3) ?

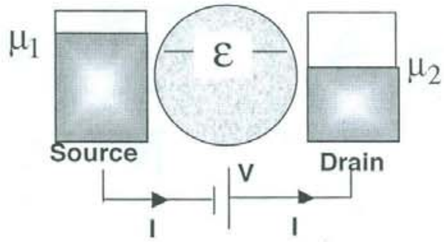
(b) If a system of electrons with a dispersion relation given by (3') were confined in a box of length L, how would the expression for the lowest energy given in (4) be modified?

(a) $$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2} + \alpha \frac{\partial^4 \psi}{\partial x^4}$$

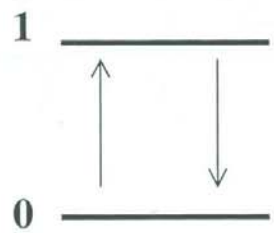
(b) $$E_1 = \frac{\hbar^2}{2m}\left(\frac{\pi}{L}\right)^2 + \alpha\left(\frac{\pi}{L}\right)^4$$

Problem 3: We wish to calculate the current, I through a single discrete energy level ε and the average number of electrons, N using the ***multielectron*** picture where we have two levels '0' and '1' corresponding to the one-electron level being empty or full respectively.

One-electron picture



Multi-electron picture



Your answers to all questions below should be in terms of the Fermi functions in the two contacts and the couplings γ_1 and γ_2 for the two contacts.

(a) Equate the rate of transition from '0' to '1'and that from '1' to '0' to obtain an expression relating the probabilities P_0 and P_1.

(b) From your result in (a), find an expression for the average number of electrons, N

(c) Obtain an expression for the current I from the rates of transition from '1' to '0' and from '0' to '1'.

(a) $P_0(\gamma_1 f_1 + \gamma_2 f_2) = P_1(\gamma_1 \bar{f}_1 + \gamma_2 \bar{f}_2)$

$$\frac{P_1}{P_0} = \frac{\gamma_1 f_1 + \gamma_2 f_2}{\gamma_1 \bar{f}_1 + \gamma_2 \bar{f}_2}$$

(b) $$\frac{P_1}{P_1 + P_0} = N = \frac{\gamma_1 f_1 + \gamma_2 f_2}{\gamma_1 + \gamma_2}$$

(c) $I = q/\hbar \cdot (P_0 \gamma_1 f_1 - P_1 \gamma_1 \bar{f}_1)$

$$= \frac{q\gamma_1}{\hbar}\left(P_0 f_1 + P_1 f_1 - P_1 \right)$$

$$= \frac{q\gamma_1}{\hbar}\left(f_1 - \frac{\gamma_1 f_1 + \gamma_2 f_2}{\gamma_1 + \gamma_2} \right)$$

$$= \frac{q}{\hbar}\gamma_1 \quad \frac{\gamma_2 (f_1 - f_2)}{\gamma_1 + \gamma_2}$$

$$= \frac{q}{\hbar}\frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2}(f_1 - f_2)$$