Quantum Computer, Quantum Parallelism, and Quantum Electromagnetics

W.C. Chew

Distinguished Professor, School of Electrical and Computer Engineering
Purdue University, USA

PQSEI Seminar Series
Purdue University
Wednesday Dec 2, 2020

- This is an extension of a keynote talk given in IEEE ICCEM, Aug 2020, Singapore.

[IEEE Journal on Multiscale and Multiphysics Computational Techniques Special Section on the 2020 IEEE International Conference on Computational Electromagnetics (ICCEM 2020)]

Prof. Chew's Keynote talk can be downloaded: PDF Adobe NEW (*Updated on Aug. 29, 2020)

The Organizing Committee (OC) of ICCEM2020 would like to show our sincere appreciation to all the delegates who attended the ICCEM2020 conference physically or virtually during the pandemic.

Keynote & DL Speakers

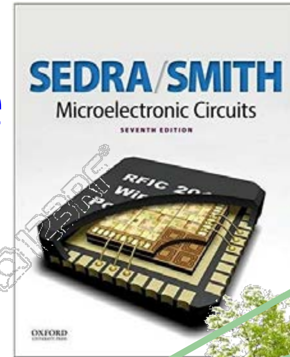
Professor Weng Cho Chew
Distinguished Professor of
ECE, Purdue University
Member of U.S. NAE
IEEE Fellow
View Details >>

Professor Yahya Rahmat-Samii
Distinguished Professor of
University of California
Member of U.S. NAE
IEEE Fellow
View Details >>

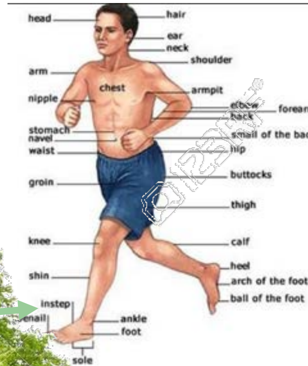
Professor Tie Jun Cui
Chief Professor of
Southeast University
Academician of CAS
IEEE Fellow
View Details >>

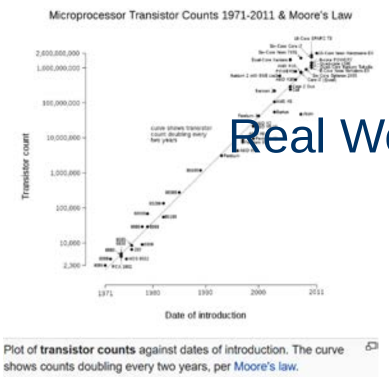
Professor David Davidson
Director, Engineering: ICRAR
Curtin University
AP-S Distinguished Lecturer
IEEE Fellow
View Details >>

Knowledge Grows Like a Tree



6-60 billion transistors on a chip.



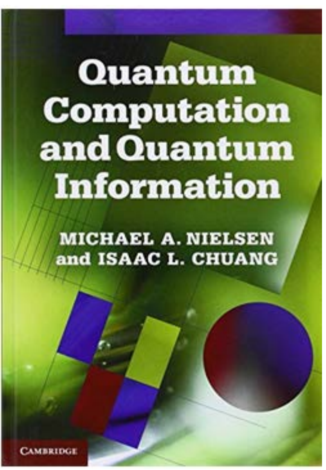


Real World Applications and Technologies

Application-Based Engineering

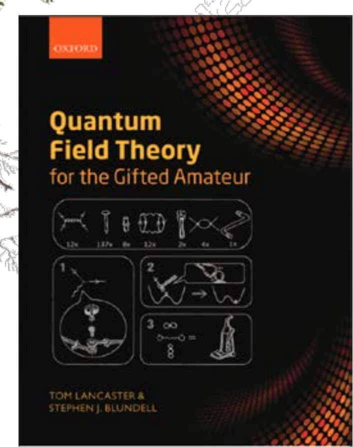
Verb.

Simplicity Rules!



Science-Based Engineering

Mathematics and Sciences

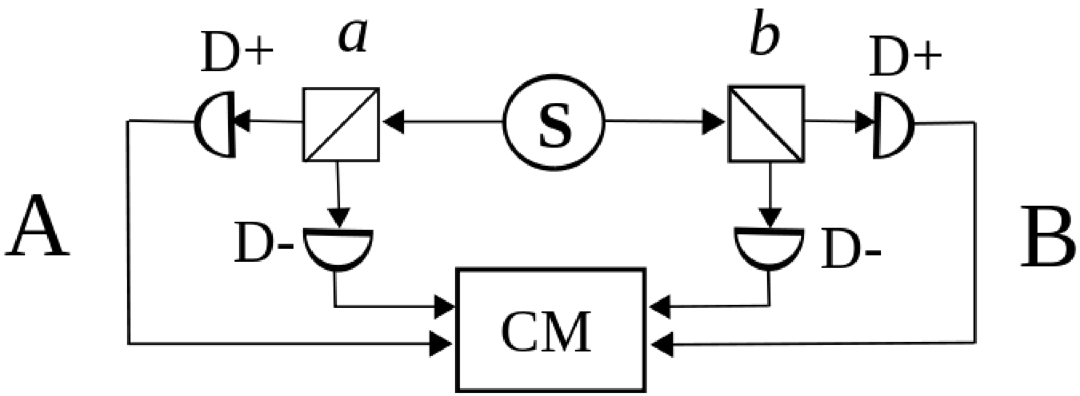


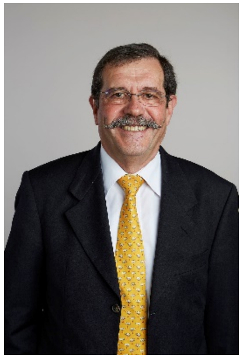
Important Milestones in Quantum Interpretation and Quantum Information

- Quantum measurements are random.
- Two prevailing schools of thoughts.
- Bell's theorem and inequality: John Stewart Bell (1928 – 1990).

$$|E(\mathbf{a},\mathbf{b}) - E(\mathbf{a},\mathbf{c})| \leq 1 - E(\mathbf{b},\mathbf{c})$$

- Test of Bell's theorem in 1982 by Alain Aspect.

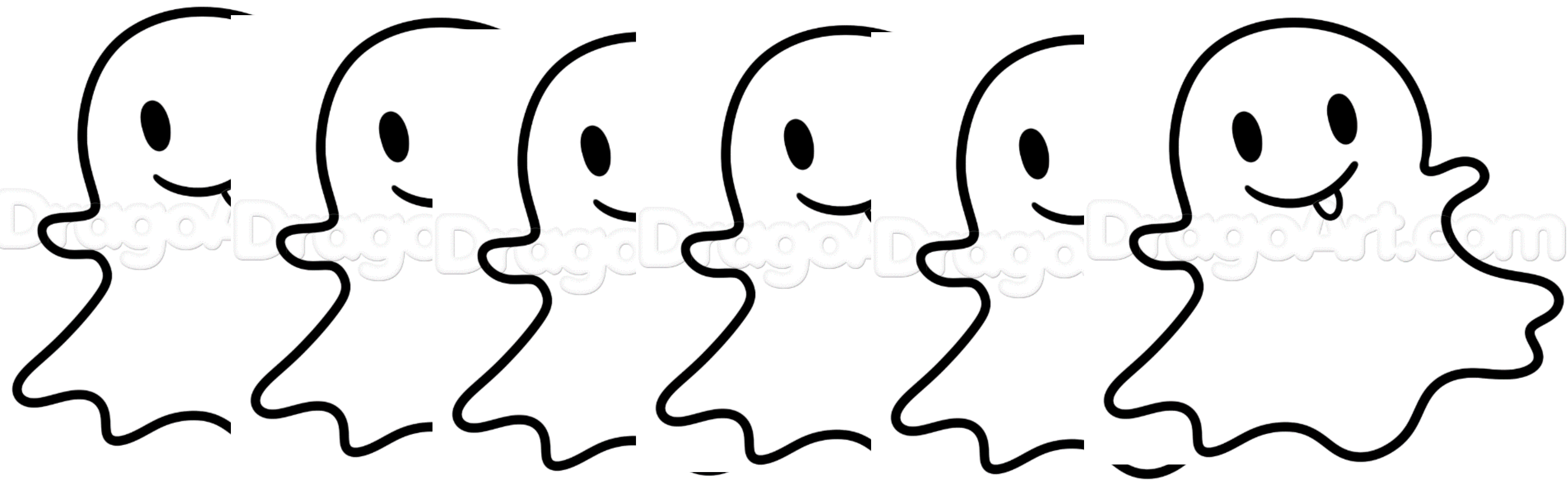


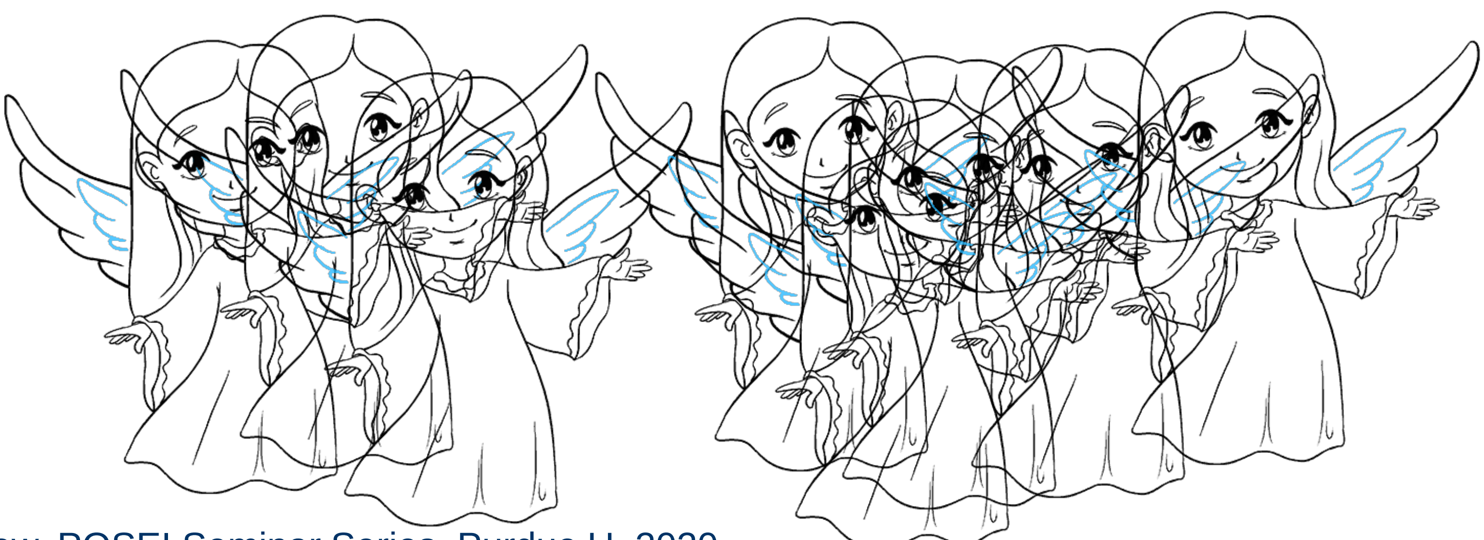


Our Karma is not written on our forehead when we were born!
Our future is in our hands!

A Quantum State is a Linear Superposition of States
--Quantum Weirdness

It is not real: only ghosts and angels can do that.





More on Quantum Linear Superposition:

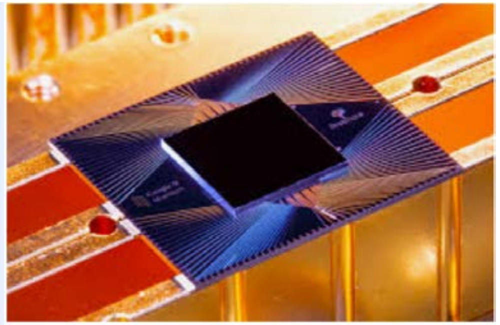
Proverbial Story of a Schrodinger Live Cat vs a Dead Cat!

Tiger, c1890. Lim Kheng Chye Collection ...
pinterest.com

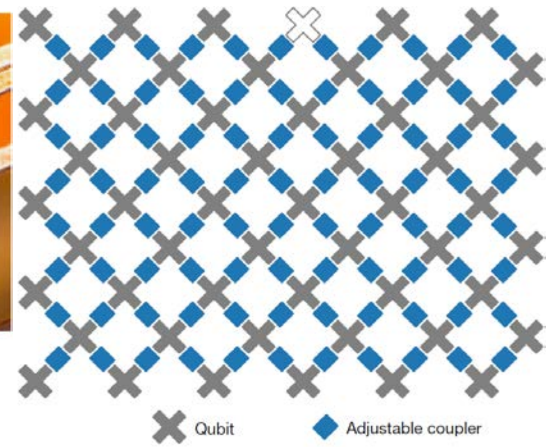
Erwin Schrodinger - Biography, Facts ...
famousscientists.org

Google's Quantum Computer:

Google's Sycamore Processor: n=53, and $2^{53} \approx 10^{16}$



Quantum Supremacy Milestone Matters ...
nytimes.com



Quantum linear superposition of 10^{16} quantum states!

Nature | Vol 574 | 24 OCTOBER 2019

Quantum Coherence Made Simple:

$$P = |\Psi(x,t)|^2.$$

$$\Psi(x,t) = \Psi_1(x,t) + \Psi_2(x,t).$$

incoherent if averages to 0.
<•••>=0

$$|\Psi(x,t)|^2 = |\Psi_1(x,t)|^2 + |\Psi_2(x,t)|^2 + 2\,\mathrm{Re}\left\{\Psi_1(x,t)\Psi_2^*(x,t)\right\}.$$

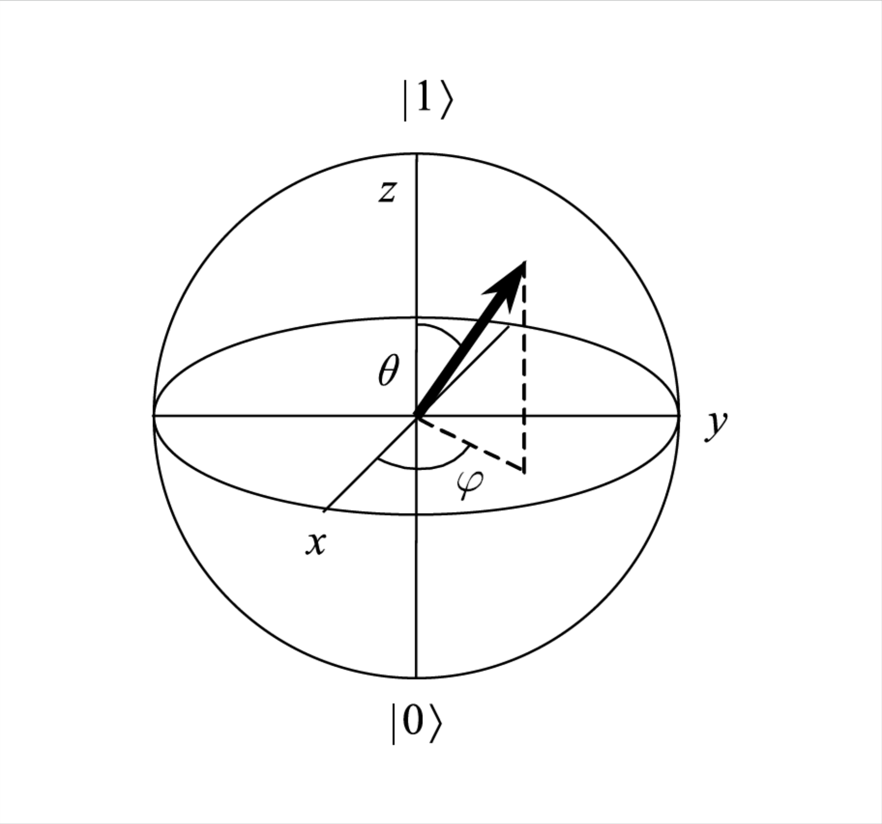
Dead Cat | Live Cat | Neither Dead Nor Alive Cat

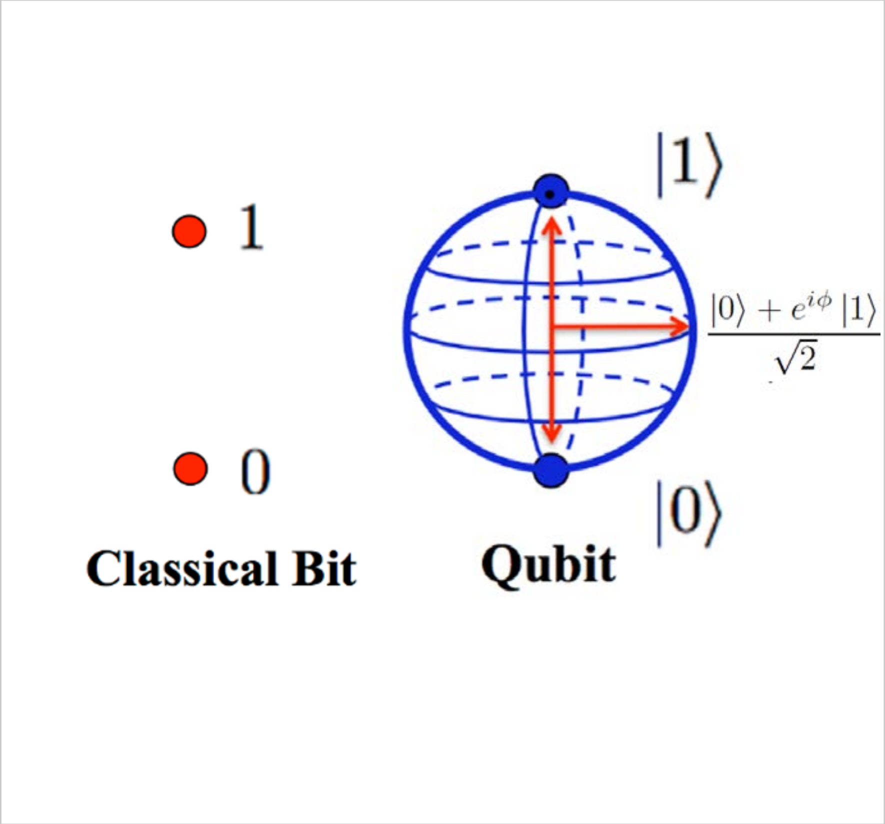
Bloch Sphere---Spin State

Spin is Unusual!

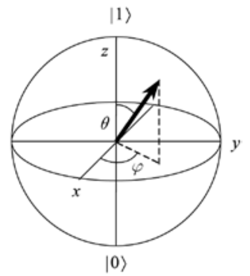
$$|\Psi\rangle = a_\uparrow |\uparrow\rangle + a_\downarrow |\downarrow\rangle = \cos\left(\frac{\theta}{2}\right)|\uparrow\rangle + e^{i\phi}\sin\left(\frac{\theta}{2}\right)|\downarrow\rangle$$

For $\theta = \frac{\pi}{2}$, we have $$|\Psi\rangle = \frac{|\uparrow\rangle + e^{i\phi}|\downarrow\rangle}{\sqrt{2}}$$





Quantum State Equation



$$\hat{H}|\Psi(t)\rangle = i\hbar \frac{d}{dt}|\Psi(t)\rangle$$

$$|\Psi(t)\rangle = a_{\uparrow}(t)\,|\uparrow\rangle + a_{\downarrow}(t)\,|\downarrow\rangle = a_1(t)\,|1\rangle + a_0(t)\,|0\rangle$$

$$|\Psi(t)\rangle = e^{-i\hat{H}t/\hbar}|\Psi(0)\rangle$$

State-variable approach in control theory:

$$\overline{\mathbf{H}} \cdot \mathbf{v}(t) = i\frac{d}{dt}\mathbf{v}(t)$$

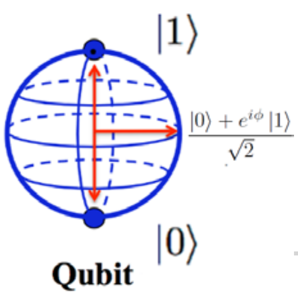
$$\mathbf{v}(t) = e^{-i\overline{\mathbf{H}}t}\mathbf{v}(0)$$

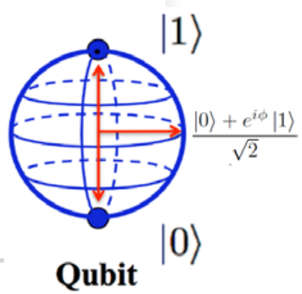
Aggregate State Vector

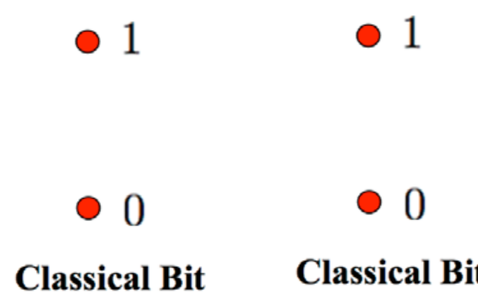
Two Factory Case:

$$\mathbf{v}_a(t) = \mathbf{v}_1(t) \otimes \mathbf{v}_2(t)$$
$$= \mathbf{v}_1(t)\mathbf{v}_2(t)$$

Two-qubit Register:



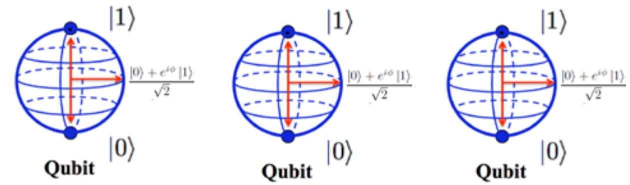




$$|\Psi_a\rangle = |\Psi\rangle \otimes |\Psi'\rangle = |\Psi\rangle|\Psi'\rangle = \{a_0\,|0\rangle + a_1\,|1\rangle\}\,\{a_{0'}\,|0'\rangle + a_{1'}\,|1'\rangle\}$$
$$= a_0 a_{0'}|0\rangle|0'\rangle + a_0 a_{1'}|0\rangle|1'\rangle + a_1 a_{0'}|1\rangle|0'\rangle + a_1 a_{1'}|1\rangle|1'\rangle$$
$$= a_{00}|00\rangle + a_{01}|01\rangle + a_{10}|10\rangle + a_{11}|11\rangle$$

Linear Superposition of 4 States.

N-Register Qubit:

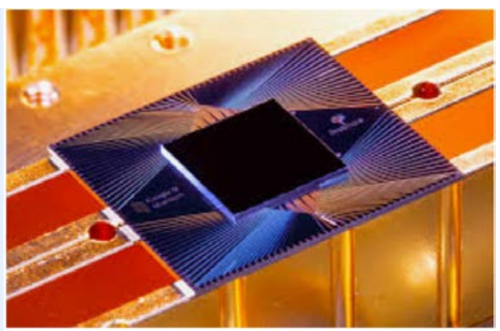


Three-qubit Register: Linear Superposition of 8 States.

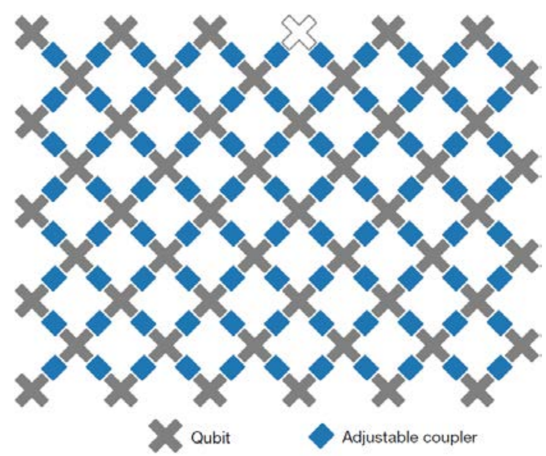
$$
\begin{aligned}
|\Psi_a\rangle &= |\Psi\rangle|\Psi'\rangle|\Psi''\rangle \\
&= a_{000}|001\rangle + a_{001}|001\rangle + a_{010}|010\rangle + a_{011}|011\rangle \\
&\quad + a_{100}|100\rangle + a_{101}|101\rangle + a_{110}|110\rangle + a_{111}|111\rangle \\
&= a_0|0\rangle + a_1|1\rangle + a_2|2\rangle + a_3|3\rangle + a_4|4\rangle + a_5|5\rangle + a_6|6\rangle + a_7|7\rangle
\end{aligned}
$$

n-qubit Register: Linear Superposition of 2^n States.

Google's Sycamore Processor: n=53, and $2^{53} \approx 10^{16}$

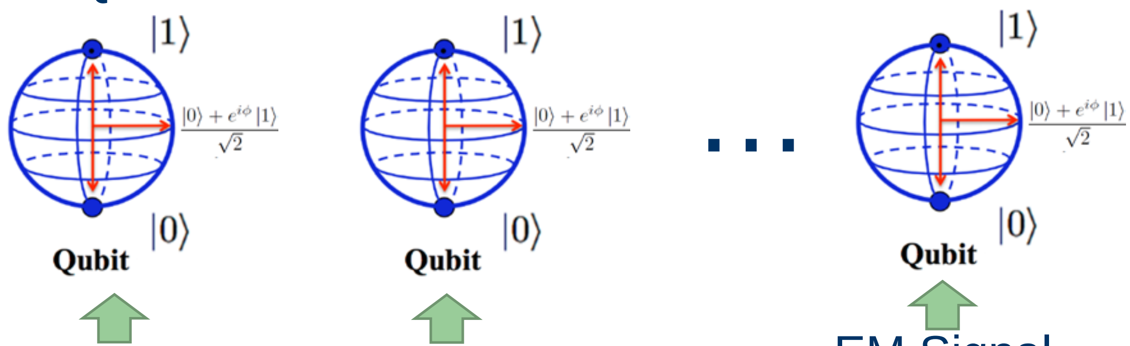


Quantum Supremacy Milestone Matters ...
nytimes.com



Quantum Fourier Transform: Power of Quantum Parallelism:

n Qubits



EM Signal EM Signal EM Signal

$$\frac{|0\rangle + e^{i\phi}|1\rangle}{\sqrt{2}}$$

Spin dynamics in NMR

$$|\Psi_a\rangle = \frac{1}{2^{n/2}} \overbrace{\left(|0\rangle + e^{i\phi_1}|1\rangle\right)\left(|0\rangle + e^{i\phi_2}|1\rangle\right)\cdots\left(|0\rangle + e^{i\phi_n}|1\rangle\right)}^{n-\text{qubits}}$$

$$= \frac{1}{\sqrt{N}} \underbrace{\left(e^{i\beta_0}|0\rangle + e^{i\beta_1}|1\rangle + \ldots + e^{i\beta_{N-1}}|N-1\rangle\right)}_{N=2^n\,\text{states}}$$

$$= \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} e^{2\pi i jk/N}|k\rangle$$

Quantum Fourier Transform, Contd:

n Unitary Operators

$\hat{U}$

$$|j\rangle \rightarrow \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} e^{2\pi i jk/N} |k\rangle$$

$$\sum_{k=0}^{N-1} x_j |j\rangle \rightarrow \sum_{k=0}^{N-1} y_k |k\rangle$$

$\hat{U}$

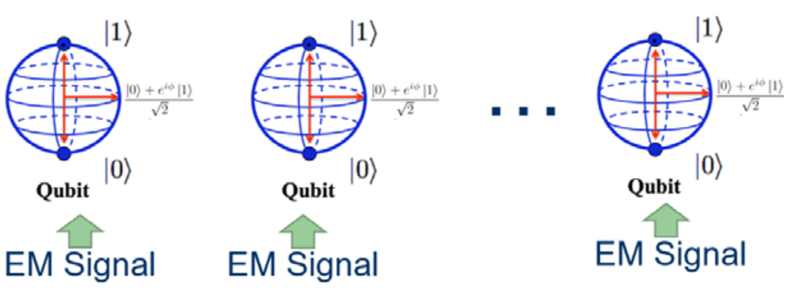
Order n unitary operations $\rightarrow$ linear superposition of 2^n states.

Computational complexity $\quad O(n^2) = O(\log^2 N) \ll N \log N.$

The above is an important component of Shor's algorithm, with order finding and period finding.

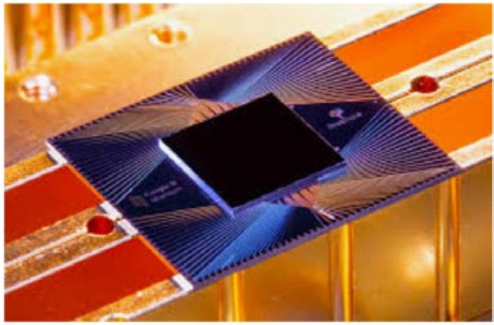
How can CEM help?

- Problem: Present day quantum computers are very noisy! (not enough knowledge base)
- Spins are mimicked with two-level atoms: artificial or real.
- Many of the spin dynamics or two-level systems are done with EM fields.
- Better math-physics modeling with CEM can reduce errors and noise, and improve precision engineering.



Quantum Maxwell's Equations (Heisenberg Picture)

- Derived using energy conservation
- Quantized in coordinate space



Quantum Supremacy Milestone Matters ...
nytimes.com

$$\nabla \times \hat{\mathbf{H}}(\mathbf{r},t) - \partial_t \hat{\mathbf{D}}(\mathbf{r},t) = \hat{\mathbf{J}}_{\text{ext}}(\mathbf{r},t),$$

$$\nabla \times \hat{\mathbf{E}}(\mathbf{r},t) + \partial_t \hat{\mathbf{B}}(\mathbf{r},t) = 0,$$

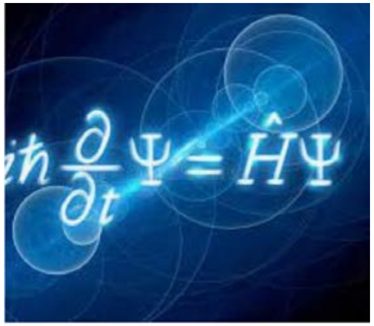
$$\nabla \cdot \hat{\mathbf{D}}(\mathbf{r},t) = \hat{\varrho}_{\text{ext}}(\mathbf{r},t), \quad \nabla \cdot \hat{\mathbf{B}}(\mathbf{r},t) = 0.$$

$$\hat{\mathbf{B}}|\varphi\rangle = |\chi\rangle$$

- Quantum State Equation for a Quantum System:

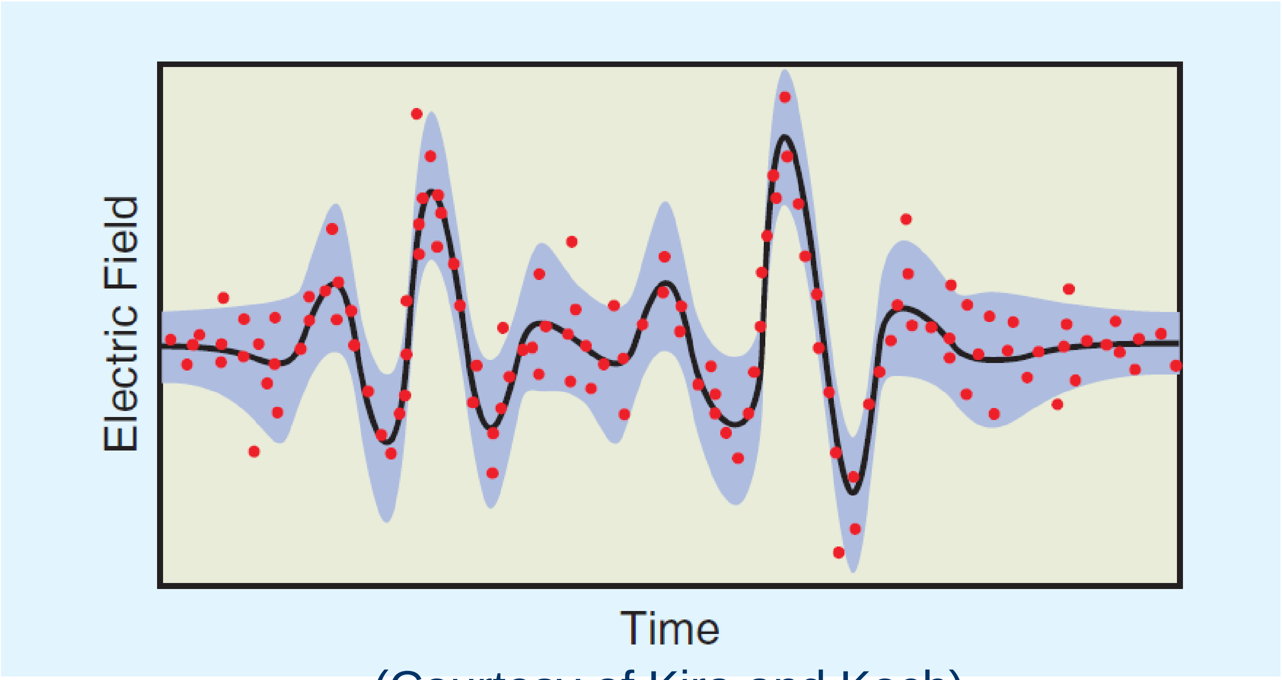
$$\hat{H}|\Psi\rangle = i\hbar\partial_t|\Psi\rangle.$$

$$|\Psi(t)\rangle = e^{-i\hat{H}t/\hbar}|\Psi(0)\rangle$$



Schrodinger Stock Illustrations – 5...
dreamstime.com

Quantum Field is a Random Variable



(Courtesy of Kira and Koch)

$$\langle q \rangle = \langle \Psi | \hat{q} | \Psi \rangle . \quad \{\hat{q}, |\Psi\rangle\} \text{ two-some}$$

$$\sigma_q^2 = \langle \Psi | (\hat{q} - \langle q \rangle)^2 | \Psi \rangle .$$

$$\langle q^n \rangle = \langle \Psi | (\hat{q})^n | \Psi \rangle .$$

Mode Decomposition Approach

$$\nabla \cdot \overline{\varepsilon}(\mathbf{r}) \cdot \nabla \Phi(\mathbf{r}, t) - \chi(\mathbf{r}) \partial_t^2 \Phi(\mathbf{r}, t) = 0.$$

$$\Phi(\mathbf{r}, t) = s_k(t) \Phi_k(\mathbf{r}).$$

$$\partial_t^2 s_k(t) = -\Omega_k^2 s_k(t)$$

$$\nabla \cdot \overline{\varepsilon} \cdot \nabla \Phi_k(\mathbf{r}) + \Omega_k^2 \chi(\mathbf{r}) \Phi_k(\mathbf{r}) = 0.$$

$$\int d\mathbf{r} \nabla \Phi(\mathbf{r}, t) \cdot \overline{\varepsilon}(\mathbf{r}) \cdot \nabla \Phi(\mathbf{r}, t)$$

$$= \sum_{k,k'} s_k(t) s_{k'}^*(t) \int d\mathbf{r} \nabla \Phi_k(\mathbf{r}) \cdot \overline{\varepsilon}(\mathbf{r}) \cdot \nabla \Phi_{k'}^*(\mathbf{r})$$

$$= \sum_k |s_k(t)|^2 \Omega_k^2.$$

$$\int d\mathbf{r} \chi(\mathbf{r}) (\partial_t \Phi(\mathbf{r}, t))^2 = \sum_k |\partial_t s_k(t)|^2$$

$$H_\Phi = \frac{1}{2} \sum_k \left[|P_{k,\Phi}(t)|^2 + |Q_{k,\Phi}(t)|^2 \right]$$

$$H_A = \frac{1}{2} \sum_k \left[|P_{k,\mathbf{A}}(t)|^2 + |Q_{k,\mathbf{A}}(t)|^2 \right]$$

$$H = H_A - H_\Phi = \frac{1}{2} \sum_k \left[|P_k(t)|^2 + |Q_k(t)|^2 \right]$$

Sycamore
quantum computer

Chew, PQSEI Seminar Series, Purdue U, 2020

More on Mode Decomposition:

$$H = \frac{1}{2}\sum_k \left[P_k^2(t) + Q_k^2(t)\right]$$
$$= \frac{1}{2}\sum_k \left[iP_k(t) + Q_k(t)\right]\left[-iP_k(t) + Q_k(t)\right]$$
$$= \sum_k B_k(t)B_k^*(t)$$
$$= \frac{1}{2}\sum_k \left(B_k(t)B_k^*(t) + B_k^*(t)B_k(t)\right)$$

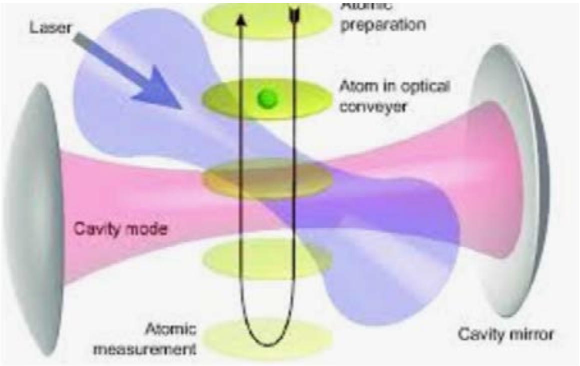
$$B_k(t) \rightarrow \sqrt{\hbar\Omega_k}\hat{a}_k(t) \quad B_k^*(t) \rightarrow \sqrt{\hbar\Omega_k}\hat{a}_k^\dagger(t)$$

$$\hat{H} = \frac{1}{2}\sum_k \hbar\Omega_k \left(\hat{a}_k(t)\hat{a}_k^\dagger(t) + \hat{a}_k^\dagger(t)\hat{a}_k(t)\right)$$

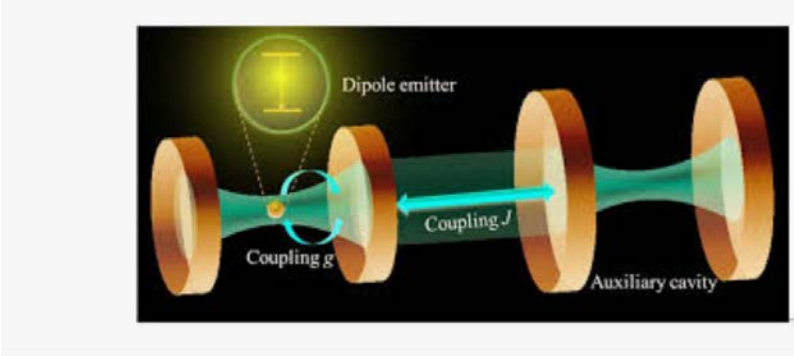
$$\hat{\mathbf{A}}\left(\mathbf{r}, t\right) = \hat{\mathbf{A}}^{(+)}\left(\mathbf{r}, t\right) + \hat{\mathbf{A}}^{(-)}\left(\mathbf{r}, t\right)$$

$$\hat{\mathbf{A}}^{(+)}\left(\mathbf{r}, t\right) = \sum_{\boldsymbol{\kappa}} \sqrt{\frac{\hbar}{2\omega_{\boldsymbol{\kappa}}}} \tilde{\mathbf{A}}_{\boldsymbol{\kappa}}\left(\mathbf{r}\right) e^{-i\omega_{\boldsymbol{\kappa}} t} \hat{a}_{\boldsymbol{\kappa}},$$

$$\hat{\mathbf{A}}^{(-)}\left(\mathbf{r}, t\right) = \sum_{\boldsymbol{\kappa}} \sqrt{\frac{\hbar}{2\omega_{\boldsymbol{\kappa}}}} \tilde{\mathbf{A}}_{\boldsymbol{\kappa}}^*\left(\mathbf{r}\right) e^{i\omega_{\boldsymbol{\kappa}} t} \hat{a}_{\boldsymbol{\kappa}}^\dagger,$$



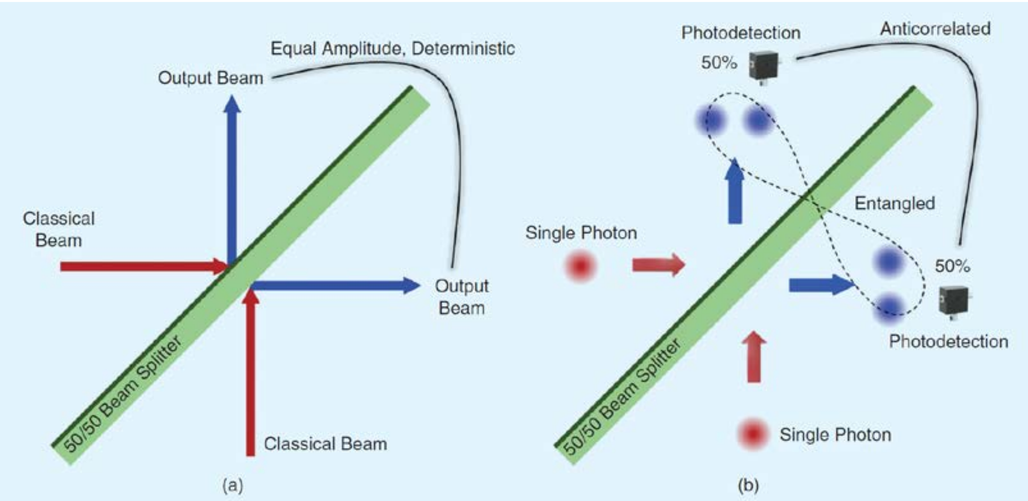
Cavity QED vacuum measurement. The ...
researchgate.net



New design surpasses the coherent ...
phys.org

A Quantum Beam Splitter Can Be Modeled Using Mode Decomposition (Bloch-Floquet Modes)

Dr Dong-Yeop Na

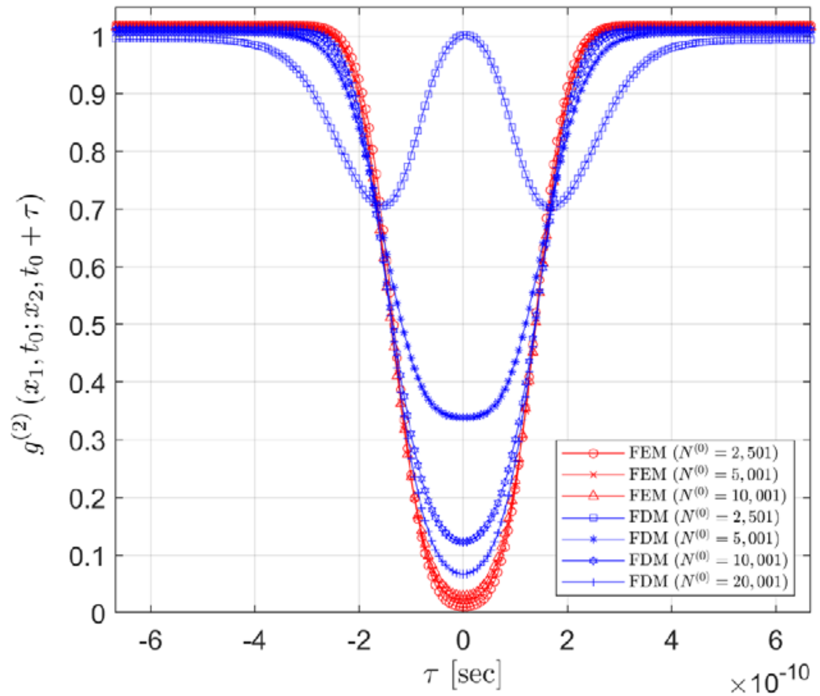


$$A = \left\langle \Psi^{(2)} \middle| \hat{A}^{(-)}(x_1, t_0)\, \hat{A}^{(-)}(x_2, t_0+\tau) \times \hat{A}^{(+)}(x_2, t_0+\tau)\, \hat{A}^{(+)}(x_1, t_0) \middle| \Psi^{(2)} \right\rangle,$$

$$B_1 = \left\langle \Psi^{(2)} \middle| \hat{A}^{(-)}(x_2, t_0)\, \hat{A}^{(+)}(x_2, t_0) \middle| \Psi^{(2)} \right\rangle,$$

$$B_2 = \left\langle \Psi^{(2)} \middle| \hat{A}^{(-)}(x_2, t_0+\tau)\, \hat{A}^{(+)}(x_2, t_0+\tau) \middle| \Psi^{(2)} \right\rangle,$$

$$g^{(2)}(x_1, t_0; x_2, t_0+\tau) = \frac{A}{B_1 B_2}$$



HOM Effect

Quantum FDTD for Solving Quantum Maxwell's Equations:

Dr Dong-Yeop Na

FDTD for the Field Operator

$$\left[\frac{\partial^2}{\partial x^2} - \epsilon(x)\,\mu_0 \frac{\partial^2}{\partial t^2}\right] \hat{A}^{(+)}(x,t) = 0. \qquad (4$$

Then, using finite difference method, (46) can be approximated as

$$\frac{\left[\hat{A}^{(+)}\right]_{i+1}^{n} - 2\left[\hat{A}^{(+)}\right]_{i}^{n} + \left[\hat{A}^{(+)}\right]_{i-1}^{n}}{\Delta x^2} - \epsilon_i \mu_0 \frac{\left[\hat{A}^{(+)}\right]_{i}^{n+1} - 2\left[\hat{A}^{(+)}\right]_{i}^{n} + \left[\hat{A}^{(+)}\right]_{i}^{n-1}}{\Delta t^2} = 0 \qquad (47)$$

Define a relation between field operator and coordinate space operator Via the Vector Potential Hopping Function (VPHF) G

$$\left[\hat{A}^{(+)}\right]_{i}^{n} \equiv \hat{A}^{(+)}(x_i, t_n) = \sum_{j=1}^{N_0} \mathcal{G}(x_i, t_n; x_j, t_0 = 0)\, \hat{b}_{x_j} = \sum_{j=1}^{N_0} [\mathcal{G}]_{i,j}^{n}\, \hat{b}_{x_j}.$$

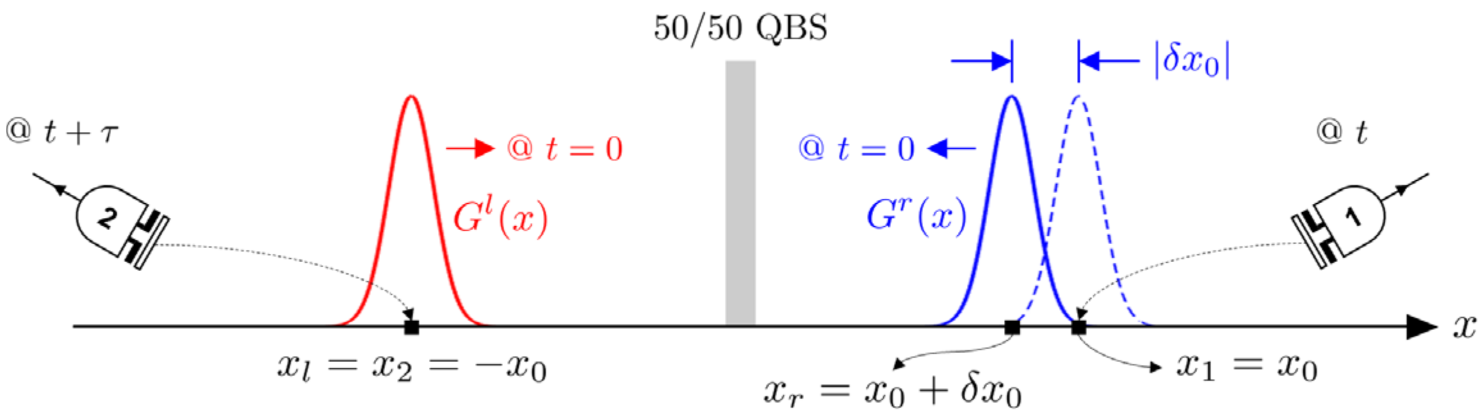
FDTD for the scalar hopping function G

$$\sum_{j=1}^{N_0}\left[\frac{[\mathcal{G}]_{i+1,j}^{n} - 2[\mathcal{G}]_{i,j}^{n} + [\mathcal{G}]_{i-1,j}^{n}}{\Delta x^2} - \epsilon_i \mu_0 \frac{[\mathcal{G}]_{i,j}^{n+1} - 2[\mathcal{G}]_{i,j}^{n} + [\mathcal{G}]_{i,j}^{n-1}}{\Delta t^2}\right]\hat{b}_{x_j} = 0 \qquad (50)$$

for $i = 1, 2, ..., N_0$. Since there is no coupling among VPHFs with different j, by solving

$$\frac{[\mathcal{G}]_{i+1,j}^{n} - 2[\mathcal{G}]_{i,j}^{n} + [\mathcal{G}]_{i-1,j}^{n}}{\Delta x^2} - \epsilon_i \mu_0 \frac{[\mathcal{G}]_{i,j}^{n+1} - 2[\mathcal{G}]_{i,j}^{n} + [\mathcal{G}]_{i,j}^{n-1}}{\Delta t^2} = 0 \qquad (51)$$

Quantum FDTD:



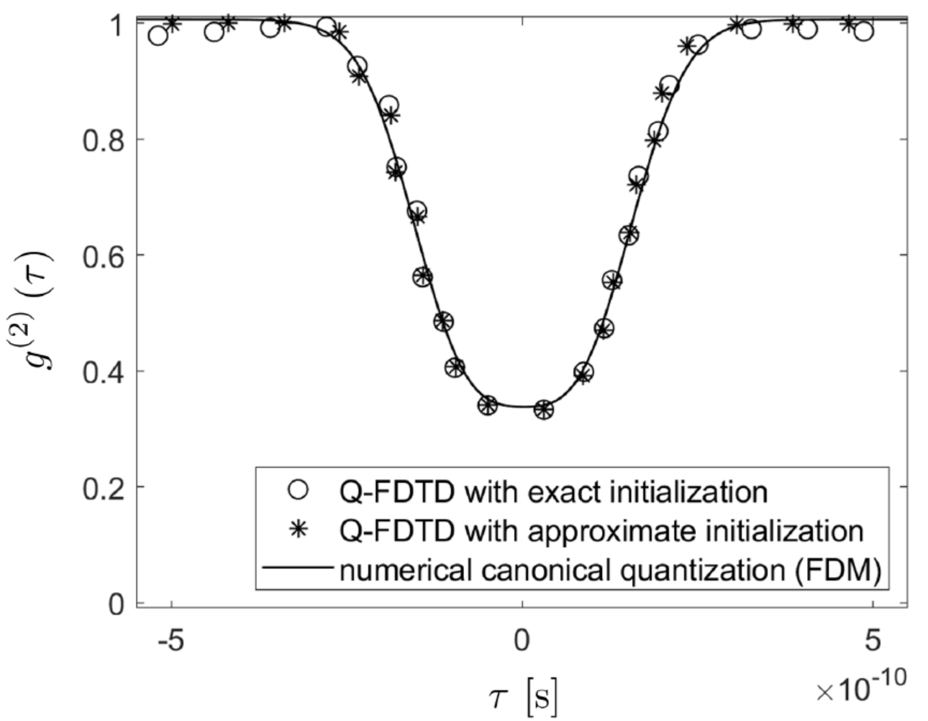
Dr. Dong-Yeop Na

$$A = \left\langle \Psi^{(2)} \middle| \hat{A}^{(-)}(x_1, t_0)\, \hat{A}^{(-)}(x_2, t_0+\tau) \times \hat{A}^{(+)}(x_2, t_0+\tau)\, \hat{A}^{(+)}(x_1, t_0) \middle| \Psi^{(2)} \right\rangle,$$

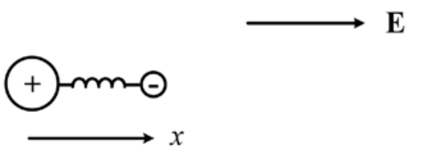
$$B_1 = \left\langle \Psi^{(2)} \middle| \hat{A}^{(-)}(x_2, t_0)\, \hat{A}^{(+)}(x_2, t_0) \middle| \Psi^{(2)} \right\rangle,$$

$$B_2 = \left\langle \Psi^{(2)} \middle| \hat{A}^{(-)}(x_2, t_0+\tau)\, \hat{A}^{(+)}(x_2, t_0+\tau) \middle| \Psi^{(2)} \right\rangle,$$

$$g^{(2)}(x_1, t_0; x_2, t_0+\tau) = \frac{A}{B_1 B_2}$$



Modeling of Dispersion in Quantized Field --Coupling of Field to Lorentz Oscilators



Fields — Lorentz Oscillator

$$H = \int dx^4 \, \frac{1}{2} \left[\mathbf{E}^2 + \mathbf{H}^2 + \beta \mathbf{V}^2 + f \mathbf{P}^2 \right]$$

$$\beta = 1/\omega_p^2, \; f = \omega_0^2/\omega_p^2.$$

Total energy of the system

$$\dot{\mathbf{P}}(\mathbf{r}, t) = \mathbf{V}(\mathbf{r}, t)$$

$$H = \int d\mathbf{r} \, \frac{1}{2} \Big[(\mathbf{\Pi}_{AP} + \mathbf{P})^2 + (\nabla \times \mathbf{A})^2 + (\nabla \cdot \mathbf{A})^2 - \Pi_\Phi^2 - (\nabla \Phi)^2 + \mathbf{\Pi}_P^2/\beta + f \mathbf{P}^2 + 2\mathbf{P} \cdot \nabla \Phi \Big]$$

Classical Hamiltonian with conjugate variables

$$\{\mathbf{\Pi}_{AP}, \mathbf{A}\}, \{\Pi_\Phi, \Phi\}, \{\Pi_P, \mathbf{P}\}$$

Energy conservation argument

$$\dot{\mathbf{H}}(\mathbf{r}, t) = -\nabla \times \mathbf{E}(\mathbf{r}, t)$$

$$\dot{\mathbf{E}}(\mathbf{r}, t) = \nabla \times \mathbf{H}(\mathbf{r}, t) - \mathbf{V}(\mathbf{r}, t)$$

$$\dot{\mathbf{P}}(\mathbf{r}, t) = \mathbf{V}(\mathbf{r}, t)$$

$$\dot{\mathbf{V}}(\mathbf{r}, t) + \omega_0^2 \mathbf{P}(\mathbf{r}, t) = \omega_p^2 \mathbf{E}(\mathbf{r}, t).$$

Classical Equations of Motion

Quantum Case:

Quantum Hamiltonian with conjugate variables

$$\hat{H} = \int d\mathbf{r}\, \frac{1}{2} \Big[\big(\hat{\mathbf{\Pi}}_{AP} + \hat{\mathbf{P}} \big)^2 + \big(\nabla \times \hat{\mathbf{A}} \big)^2 + \big(\nabla \cdot \hat{\mathbf{A}} \big)^2$$

$$- \hat{\Pi}_{\Phi}^2 - \big(\nabla \hat{\Phi} \big)^2 + \hat{\mathbf{\Pi}}_P^2 / \beta + f \hat{\mathbf{P}}^2 + 2 \hat{\mathbf{P}} \cdot \nabla \hat{\Phi} \Big] . \quad ($$

Energy conservation argument

$$\ddot{\hat{\mathbf{P}}}(\mathbf{r},t) + \omega_0^2 \hat{\mathbf{P}}(\mathbf{r},t) = \omega_p^2 \hat{\mathbf{E}}(\mathbf{r},t)$$

$$\dot{\hat{\mathbf{H}}}(\mathbf{r},t) = -\nabla \times \hat{\mathbf{E}}(\mathbf{r},t)$$

$$\dot{\hat{\mathbf{E}}}(\mathbf{r},t) = \nabla \times \hat{\mathbf{H}}(\mathbf{r},t) - \hat{\mathbf{V}}(\mathbf{r},t).$$

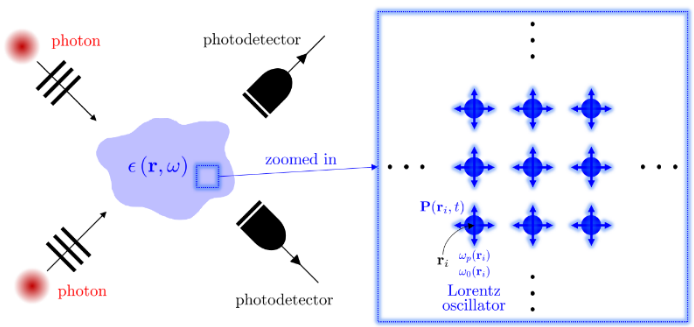
Quantum Equations of Motion

Dispersion Effect on Quantum Media

Potentially can be used for quantum plasmonics

Dr. Dong-Yeop Na

$$\ddot{\hat{\mathbf{P}}}(\mathbf{r},t) + \omega_0^2 \hat{\mathbf{P}}(\mathbf{r},t) = \omega_p^2 \hat{\mathbf{E}}(\mathbf{r},t)$$



Dispersion Effect on Quantum Beam Splitter

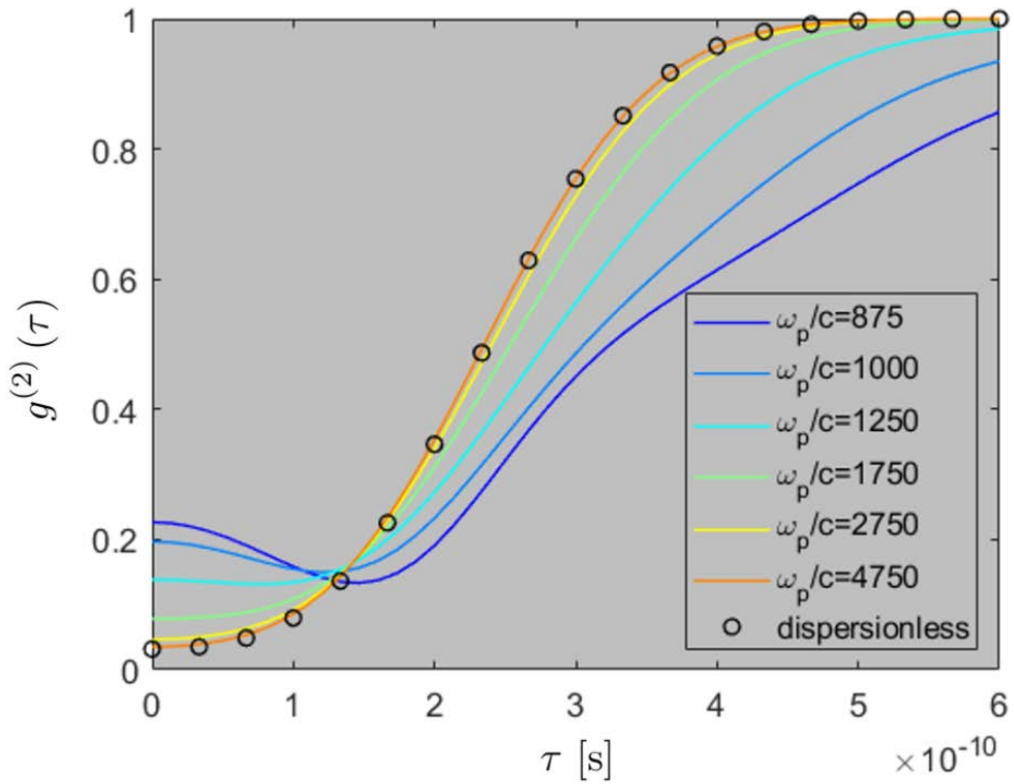


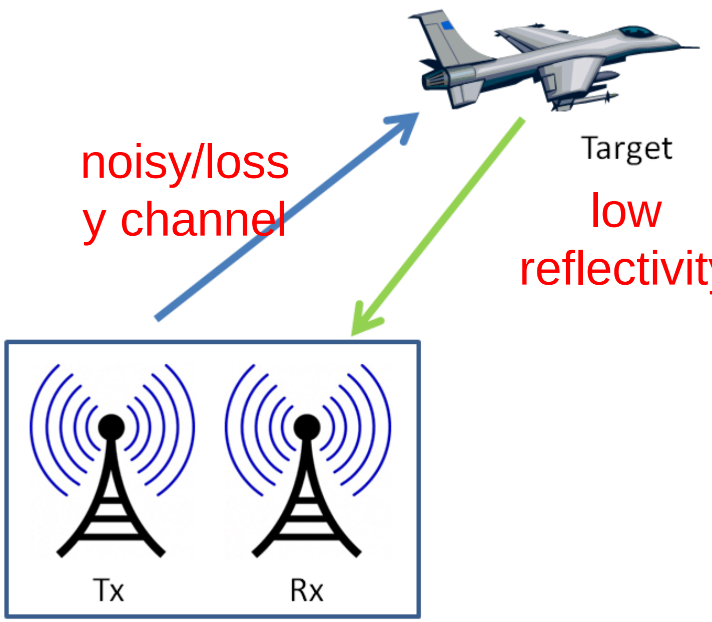
FIG. 5. Second order correlation versus time delay for various plasma frequencies.

Chew, PQSEI Seminar Series, Purdue U, 2020

Quantum Sensing

Early alarming

Radar : Target detection (Yes/No), Target Ranging (Distance), Target Imaging

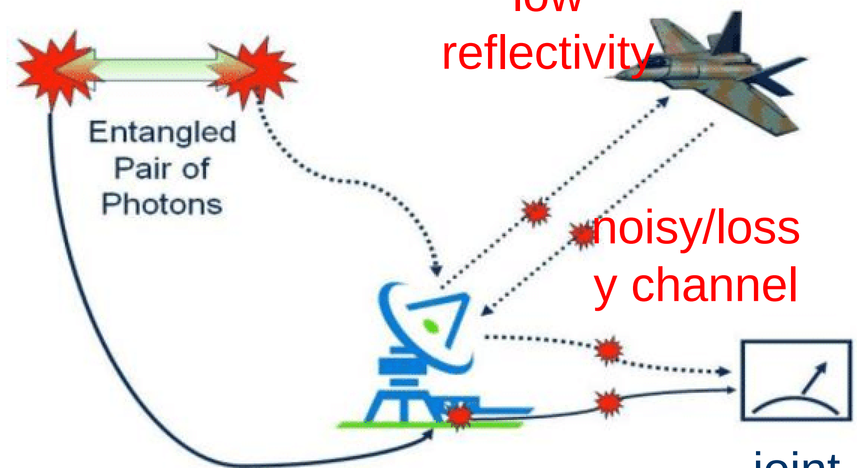


noisy/lossy channel

low reflectivity

Classical Radar

$$\mathrm{SNR} = \frac{P_{\mathrm{signal}}}{P_{\mathrm{noise}}}$$



low reflectivity

noisy/lossy channel

joint measurement

Quantum Radar (Quantum illumination)

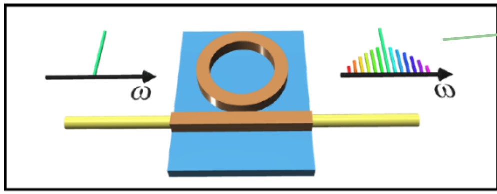
Higher sensitivity upon background noisy channel

1. Entangled photon pairs (signal/idler)
2. Joint measurement (strong correlation)

Possible collaboration with A. Weiner's group.

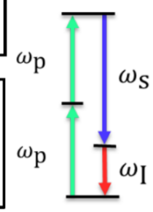
Frequency (energy)-time Entangled Photon Pairs*

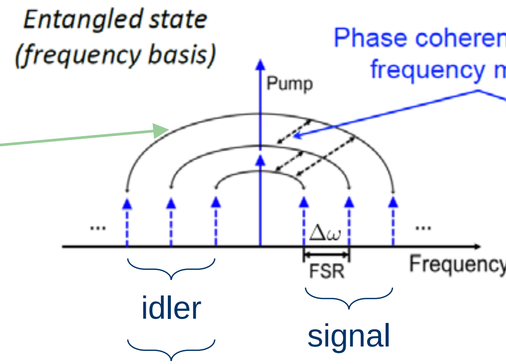
Spontaneous four-wave mixing (SFWM), in a microresonator

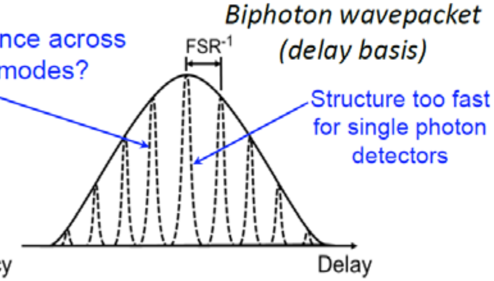


$$|\Psi\rangle = \sum_{k=1}^{N} \alpha_k |k,k\rangle_{SI}$$

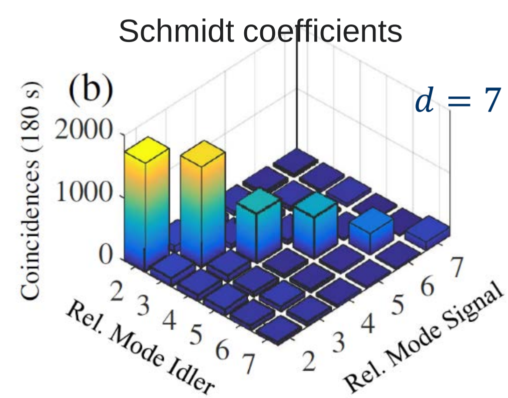
$$|k,k\rangle_{SI} = \int d\Omega\, \Phi(\Omega - k\Delta\omega)\, |\omega_p + \Omega, \omega_p - \Omega\rangle_{SI}$$







\# of modes = d



$$|\Psi\rangle = \alpha_1 |\omega_p + \Delta\omega\rangle_s |\omega_p - \Delta\omega\rangle_i + \alpha_2 |\omega_p + 2\Delta\omega\rangle_s |\omega_p - 2\Delta\omega\rangle_i$$
$$+\alpha_3 |\omega_p + 3\Delta\omega\rangle_s |\omega_p - 3\Delta\omega\rangle_i$$
$$+\cdots + \alpha_d |\omega_p + d\Delta\omega\rangle_s |\omega_p - d\Delta\omega\rangle_i$$

$$= \sum_{k=1}^{d} \alpha_k |\omega_p + k\Delta\omega\rangle_s |\omega_p - k\Delta\omega\rangle_i \equiv \sum_{k=1}^{d} \alpha_k |k\rangle_s |k\rangle_i$$

Entanglement dimension = d

has the **higher dimensional entanglement** than

$$|\psi\rangle = \alpha |\uparrow\rangle_1 |\downarrow\rangle_2 + \beta |\downarrow\rangle_1 |\uparrow\rangle_2$$

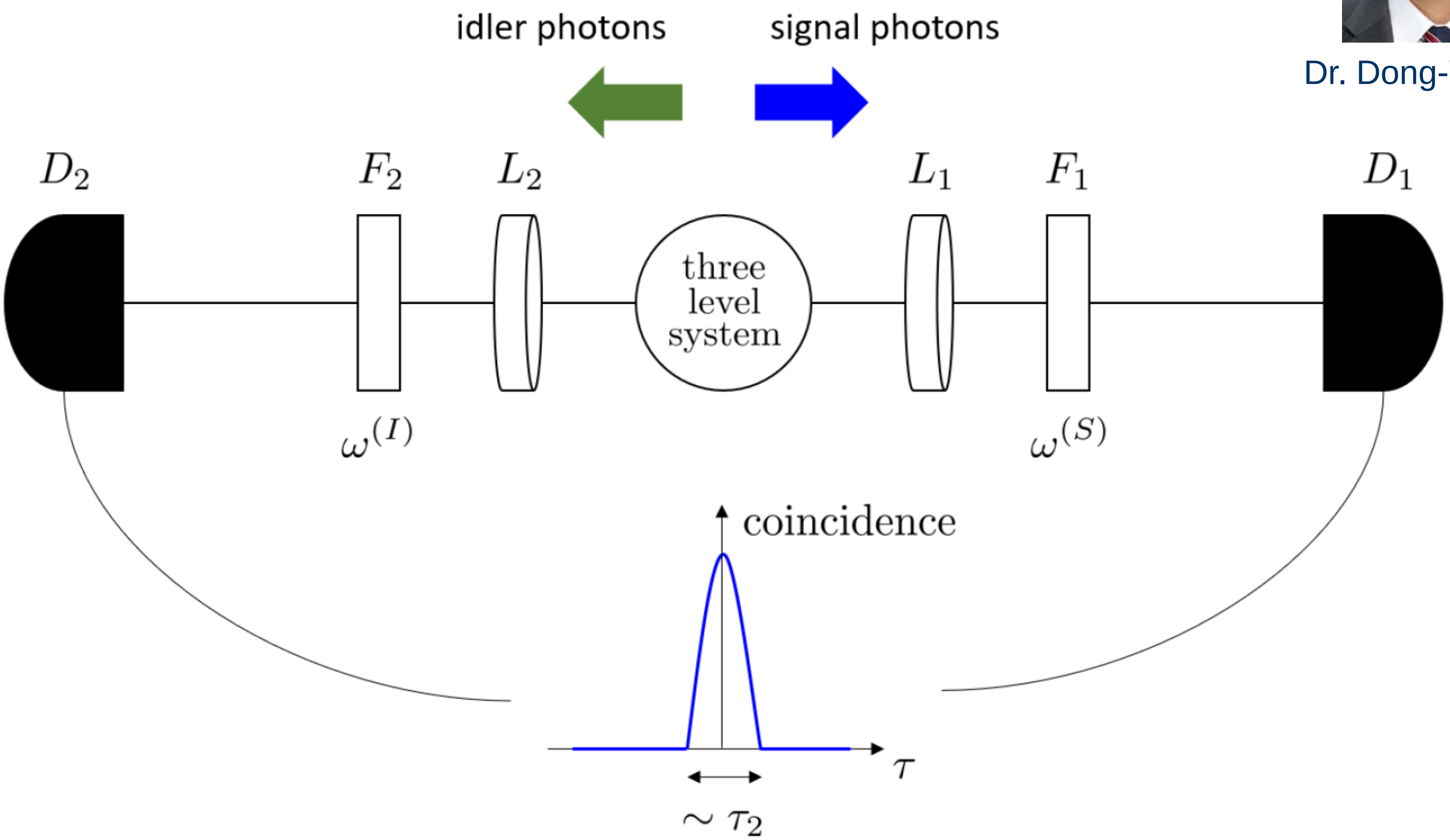
* J. A. Jaramillo-Villegas et al. (PI: **A. M. Weiner**), "Persistent energy-time entanglement covering multiple resonances of an on-chip biphoton frequency comb," Optica, vol. 4, pp. 655-658, 2017.

Performance Comparison

	Unentangled single photon	Entangled photons
Good regime	$\frac{\eta}{\bar{n}} > 1$	$\frac{\eta d}{\bar{n}} > 1$
# of trial to detect the presence of a target	$\mathcal{O}(1/\eta)$	$\mathcal{O}(1/\eta)$
Bad regime	$\frac{\eta}{\bar{n}} > 1$	$\frac{\eta d}{\bar{n}} < 1$
# of trial to detect the presence of a target	$\mathcal{O}(8\bar{n}/\eta^2)$	$\mathcal{O}(8\bar{n}/\eta^2 d)$

Time-Frequency Entanglement Modeling

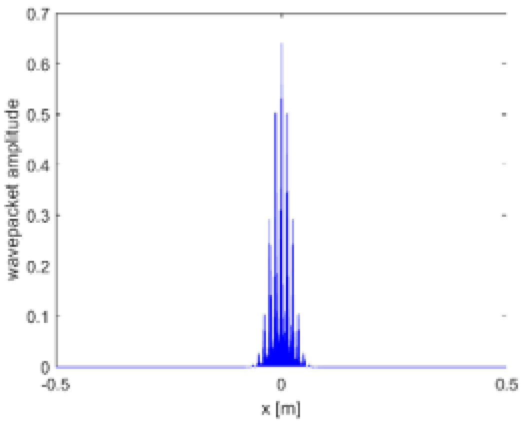
Dr. Dong-Yeop Na

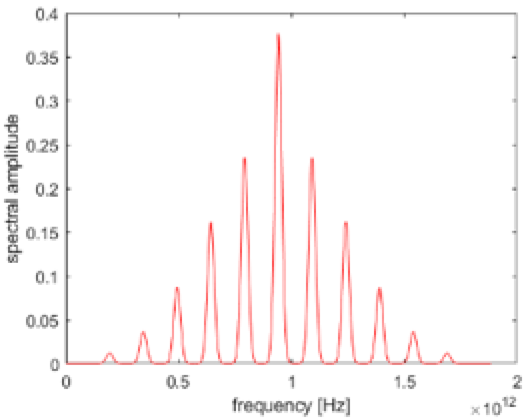


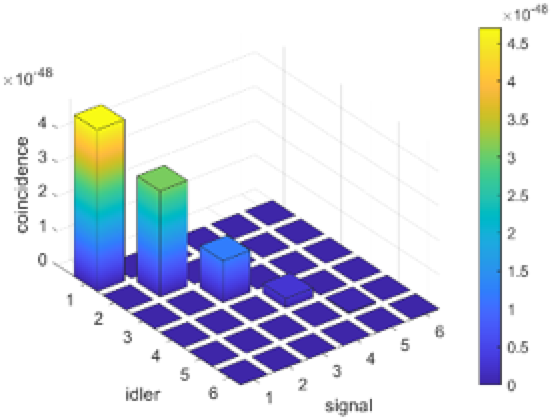
Correlation Tomogram (Using Synthetic Data)

Dr. Dong-Yeop Na

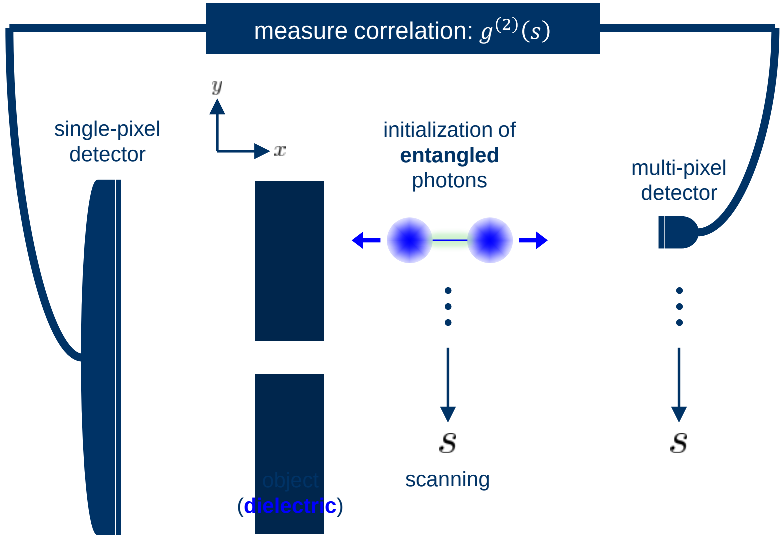
- Frequency bin entanglement

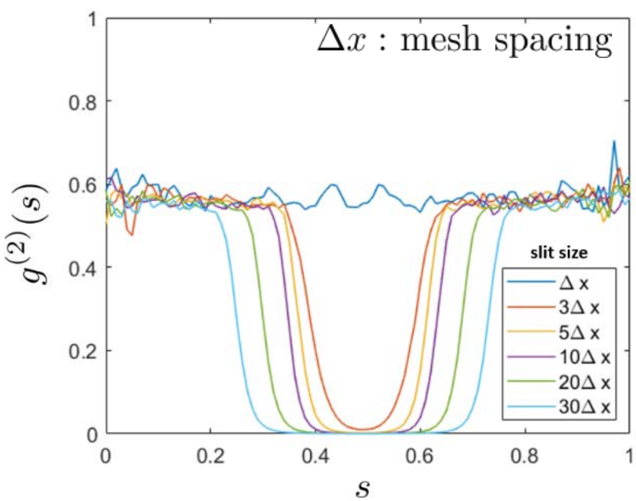






Quantum Ghost Imaging Experiment (Synthetic)





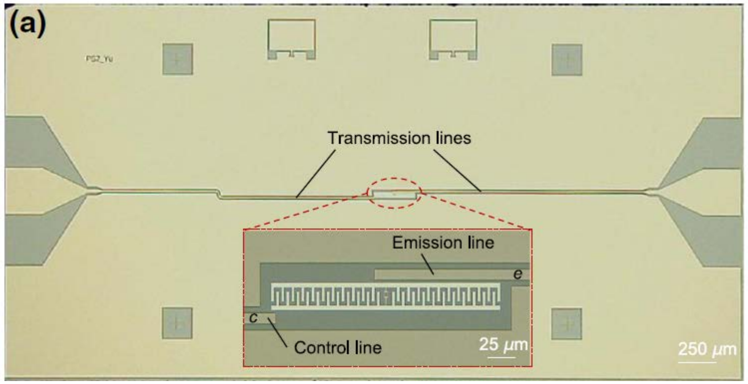
Dr. Dong-Yeop Na

Chew, PQSEI Seminar Series, Purdue U, 2020

Full-Wave Modeling of a Single Photon Source

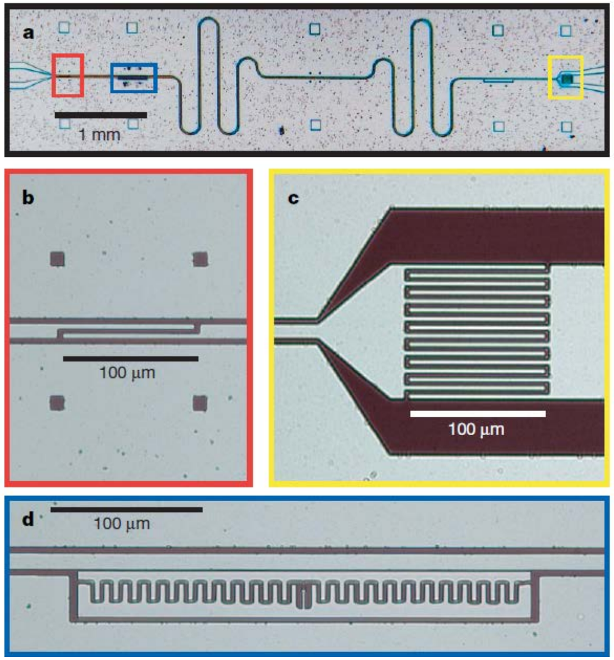
Dr. Thomas E Roth

- Single photon sources (SPSs) are important devices in various quantum information systems
- Current modeling methods do not incorporate photon propagation effects into estimations of photon coherence
- Will analyze a circuit QED SPS that uses a *transmon qubit* as a quantum emitter



$f_0 = 5.75\,\mathrm{GHz},\ \lambda_0 = 5.2\,\mathrm{cm}$

J. S. Tsai *et al.*, DOI: 10.1103/PhysRevApplied.13.034007



$f_0 = 4.68\,\mathrm{GHz},\ \lambda_0 = 6.4\,\mathrm{cm}$

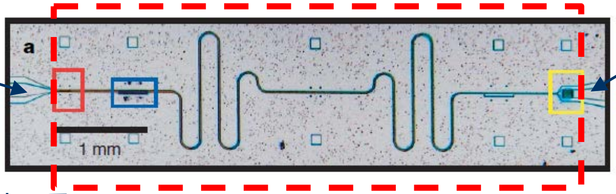
M. Devoret *et al.*, DOI: 10.1038/nature06126

Modeling Process Development

Dr. Thomas E Roth

1. Quantization of the Electromagnetic Field

Port Regions (analytically accounted for)



Port Regions (analytically accounted for)

Simulation Domain (explicitly modeled)

Modified from M. Devoret *et al.*, DOI: 10.1038/nature06126

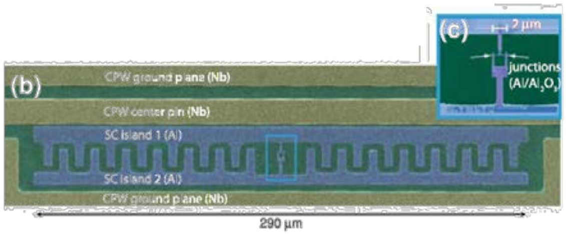
2. Formulate Coupled Field-Transmon System

Interaction Hamiltonian

$$\hat{H}_I = \int d\mathbf{r}\hat{\mathbf{A}}(\mathbf{r}) \cdot \hat{n}_{\mathrm{d}}(\mathbf{r}) \underbrace{\sum_j \left[2e\beta\langle j|\hat{n}|j+1\rangle \partial_t \hat{\sigma}_{j,j+1} + \mathrm{H.c.}\right]}_{-\hat{\mathbf{J}}_t}$$

Transmon Current Operator → $-\hat{\mathbf{J}}_t$

M. Devoret *et al.*, DOI: 10.1007/s11128-009-0100-6



3. Derive Coupled Equations of Motion

$$\frac{d\hat{n}}{dt} = -E_J \sin\hat{\varphi}$$

$$\frac{d\hat{\varphi}}{dt} = 8E_C(\hat{n} - n_g) + 2e\beta \iiint \hat{\mathbf{E}}(\mathbf{r}) \cdot \hat{n}_{\mathrm{d}}(\mathbf{r}) d\mathbf{r}$$

$$\nabla \times \nabla \times \hat{\mathbf{E}} + \mu\epsilon\partial_t^2 \hat{\mathbf{E}} = -\mu\partial_t \hat{\mathbf{J}}_t$$

Solution Procedure

Dr. Thomas E Roth

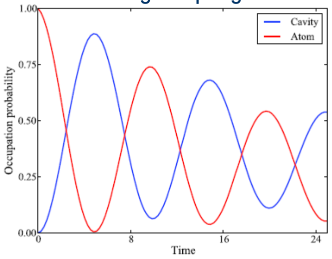
1. Weak Coupling Approximation – Linearizes System

$$\frac{d\hat{n}}{dt} = -E_J \sin\hat{\varphi}$$

$$\frac{d\hat{\varphi}}{dt} = 8E_C(\hat{n} - n_g) + 2e\beta \iiint \hat{\mathbf{E}}(\mathbf{r}) \cdot \hat{n}_d(\mathbf{r}) d\mathbf{r}$$

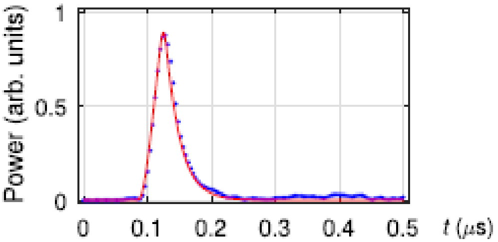
$$\nabla \times \nabla \times \hat{\mathbf{E}} + \mu\epsilon\partial_t^2 \hat{\mathbf{E}} = -\mu\partial_t \hat{\mathbf{J}}_t$$

Strong Coupling



F. Nori *et al.*, DOI: 10.1016/j.cpc.2012.02.021

Weak Coupling



J. S. Tsai *et al.*, DOI: 10.1103/PhysRevApplied.13.034007

2. Transmon Operator Time Evolution with Lindblad Master Equation

$$\frac{d}{dt}\hat{X}(t) = \frac{i}{\hbar}[\hat{H}, \hat{X}] + \sum_j \gamma_j \left(\hat{C}_j^\dagger \hat{X} \hat{C}_j - \frac{1}{2}\{\hat{C}_j^\dagger \hat{C}_j, \hat{X}\} \right)$$

$$\hat{H} = \hbar\left[\delta_1\hat{\sigma}_{11} + \delta_2\hat{\sigma}_{22} + \frac{1}{2}\mathcal{E}^x(t)(\hat{\sigma}_{10} + \hat{\sigma}_{01}) + \frac{\sqrt{2}}{2}\mathcal{E}^x(t)(\hat{\sigma}_{21} + \hat{\sigma}_{12})\right]$$

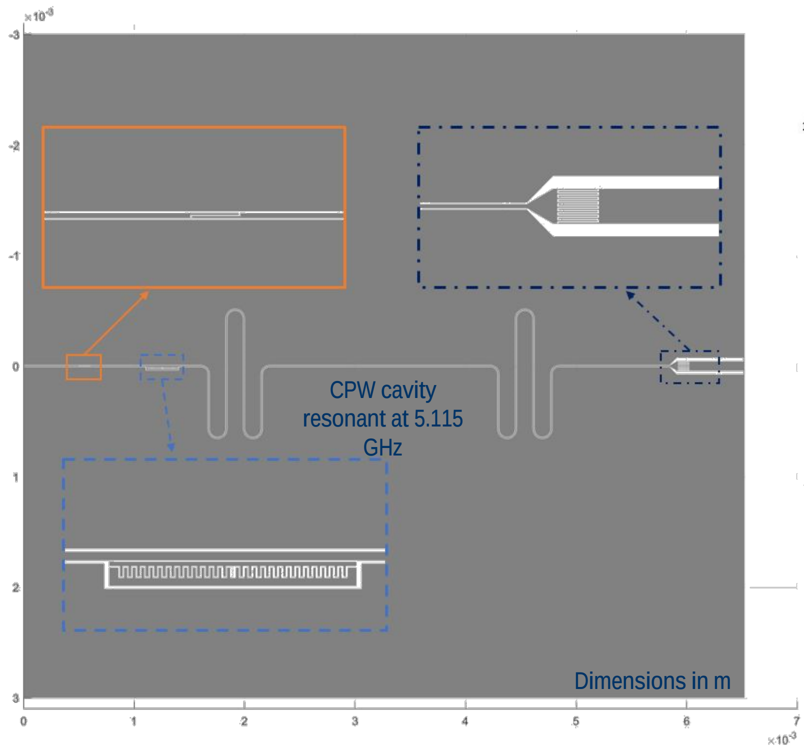
3. Dyadic Green's Function Photon Propagation Model

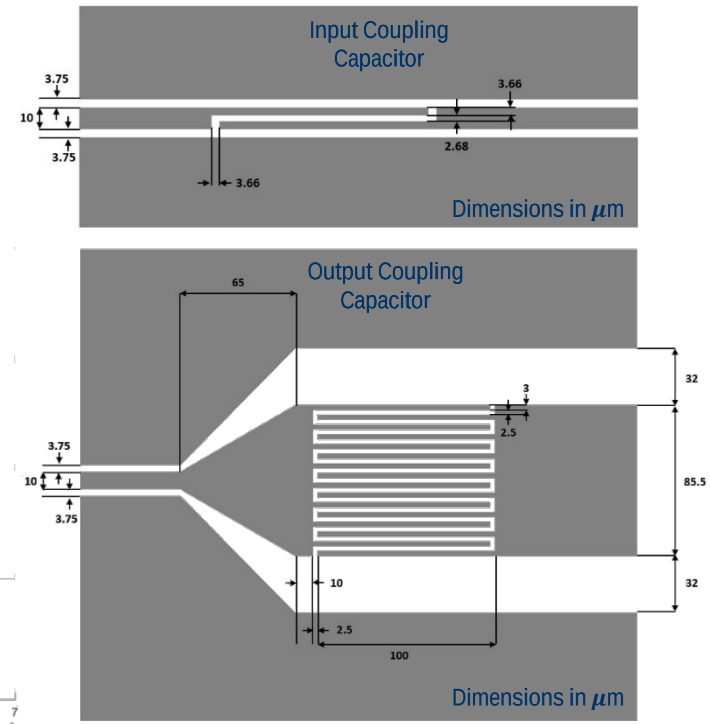
$$\hat{\mathbf{E}}(\mathbf{r}, t) = -\mu\partial_t \iint \overline{\mathbf{G}}_E(\mathbf{r}, \mathbf{r}', t - t') \cdot \hat{\mathbf{J}}_t(\mathbf{r}', t') dt' d\mathbf{r}'$$

$$\hat{\mathbf{H}}(\mathbf{r}, t) = \iint \overline{\mathbf{G}}_H(\mathbf{r}, \mathbf{r}', t - t') \cdot \hat{\mathbf{J}}_t(\mathbf{r}', t') dt' d\mathbf{r}'$$

Single Photon Source Geometry

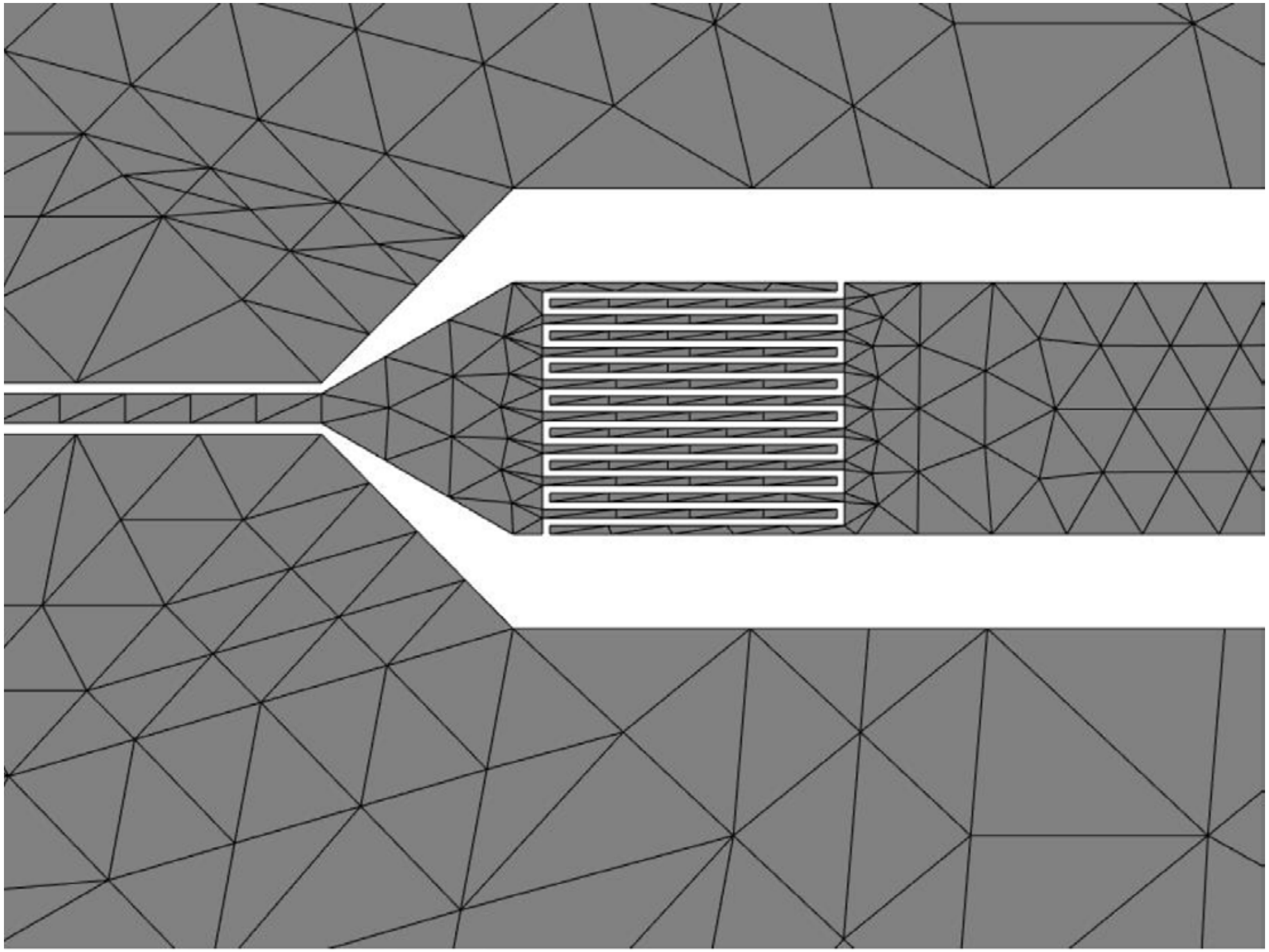
Dr. Thomas E Roth





Microwave single photon source is an excellent example of a multiscale structure typically encountered in circuit QED systems

Actual Mesh Used!



Dr. Thomas E Roth

Decay Rates

Dr. Thomas E Roth

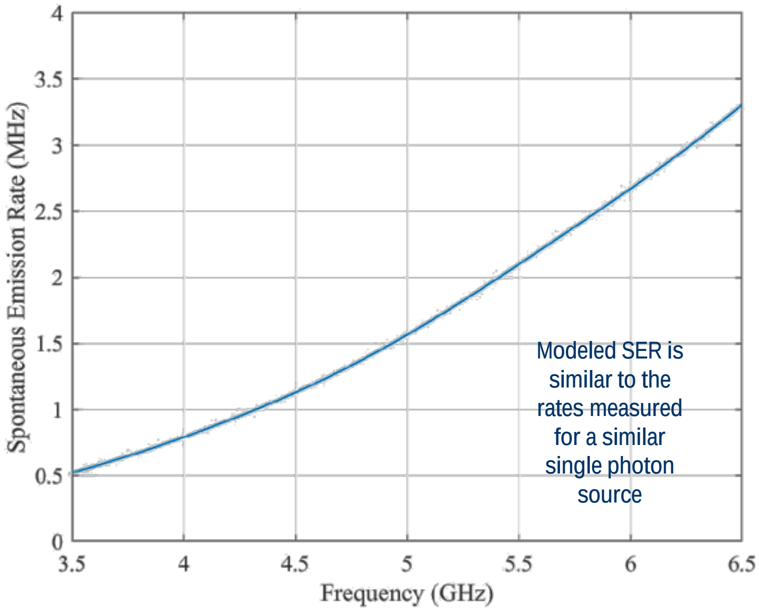
- Decay rates must be included in Lindblad master equation to correctly model system
  - Dephasing rate very difficult to calculate – used state of the art experimental parameters in modeling
    - Current state of the art is ~30 kHz
  - Spontaneous emission rate can be computed using potential-based TDIEs
    - Note: field-based method was unstable for this system

Spontaneous Emission Rate Computation

$$\gamma_{(f,i)}(\mathbf{r}_0,\omega_0) = \frac{2\omega_0^2}{\hbar\epsilon_0 c^2}(2e\beta)^2|\langle f|\hat{n}|i\rangle|^2 \underbrace{\left[\hat{n}_\mathrm{d}\cdot \mathrm{Im}\left\{\overline{\mathbf{G}}_E(\mathbf{r}_0,\mathbf{r}_0,\omega_0)\right\}\cdot\hat{n}_\mathrm{d}\right]}_{\text{Computed with potential-based TDIE}}$$

$$\gamma_{(0,1)}(\mathbf{r}_0, 2\pi\times 4.32\,\mathrm{GHz}) = 2\pi\times 1.0\,\mathrm{MHz}$$

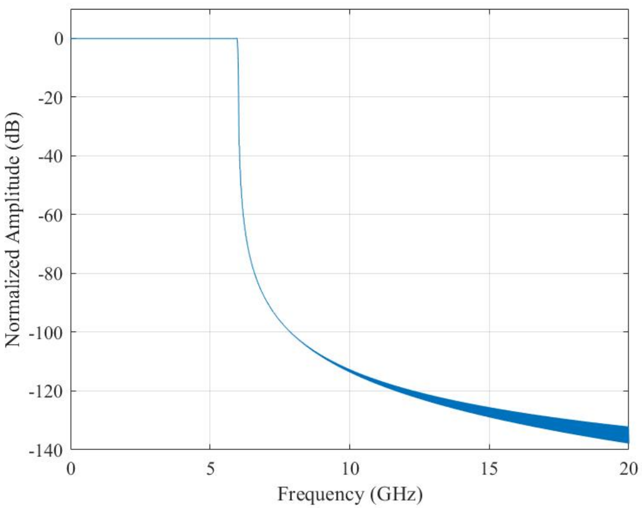
$$\gamma_{(1,2)}(\mathbf{r}_0, 2\pi\times 3.95\,\mathrm{GHz}) = 2\pi\times 1.52\,\mathrm{MHz}$$



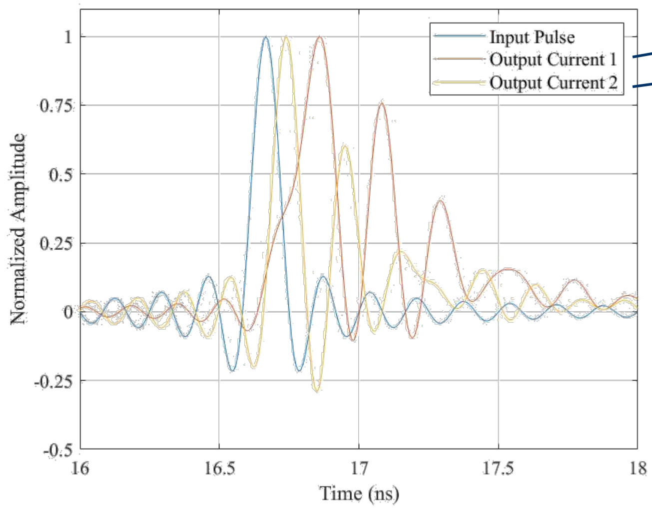
Photon Propagation Results

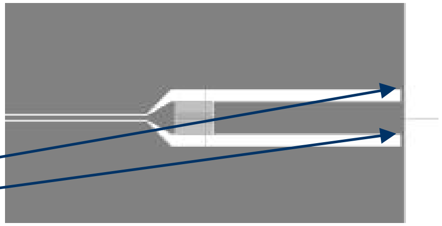
Dr. Thomas E Roth

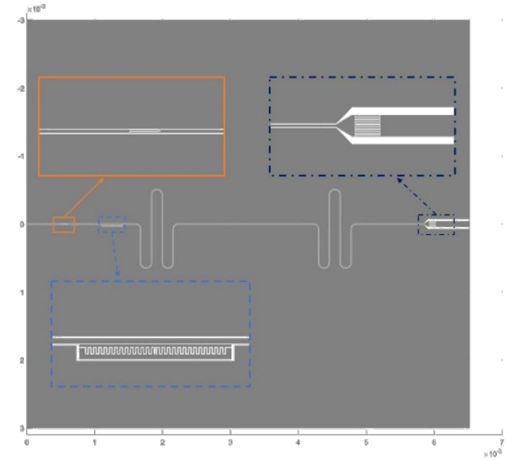
Approximately Bandlimited Impulse



Impulse Response to Output Resistors



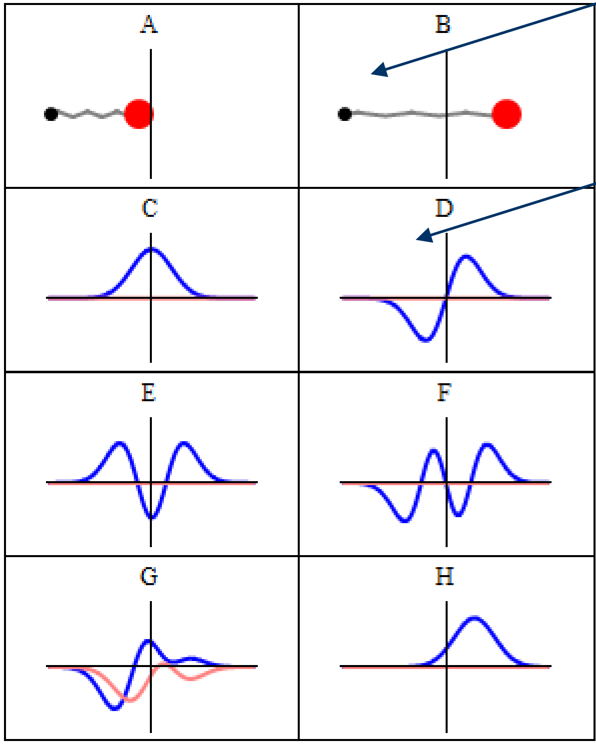




Transmon coupling scheme used in this single photon source leads to significant excitation of slotline modes as opposed to CPW modes

Casimir Force Calculation:

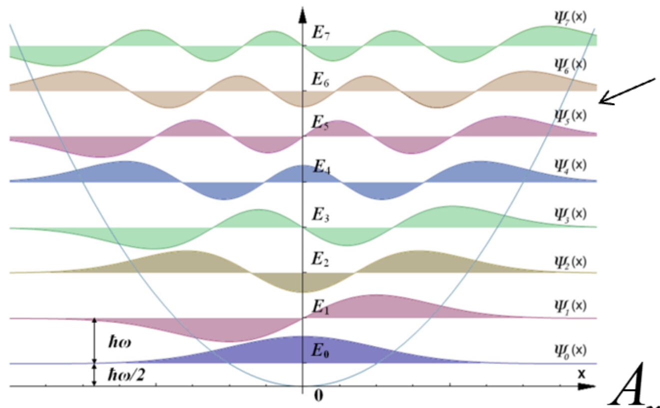
Courtesy of Wiki
Wave-Particle Duality



Classical harmonic oscillator

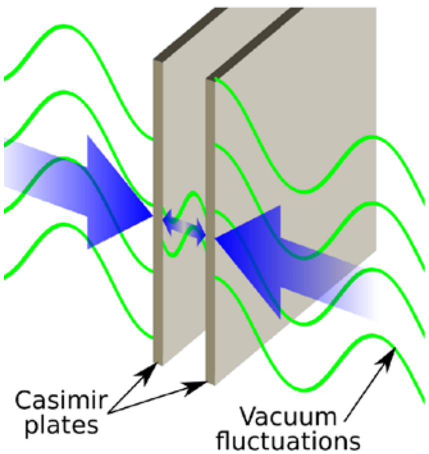
Quantum harmonic oscillator

x^2potential



A_x

Photon-Number State



ıe Casimir force between two metal plates exists at $T = 0°K$, due
ıence of vacuum fluctuating field (courtesy of Wikipedia).

Quantum State Eigenequation

$$\hat{H}\,|\psi\rangle = E\,|\psi\rangle \qquad E_n = \hbar\omega\left(n + \frac{1}{2}\right)$$

$$\hat{H} = \frac{\hat{p}^2}{2m} + \frac{1}{2}m\omega^2\hat{x}^2 \qquad \hat{p} = -i\hbar\frac{\partial}{\partial x}$$

Potential Well

Wave mechanics

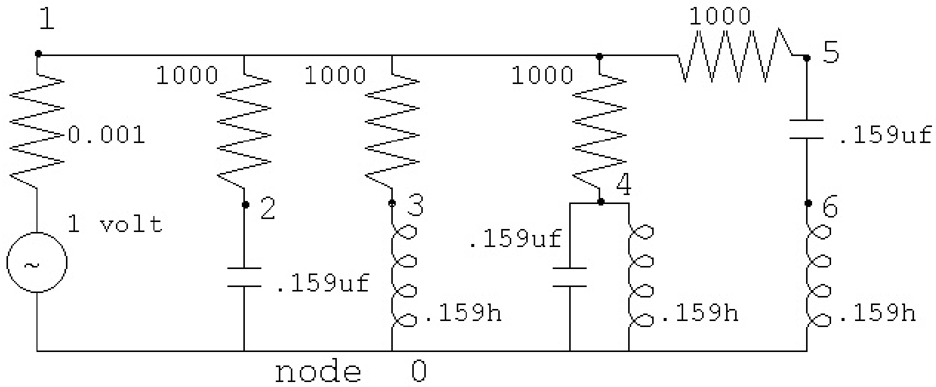
Zero point energy

$$E_0 = \frac{1}{2}\hbar\omega$$

Vacuum state

Finding resonant frequencies of complex systems

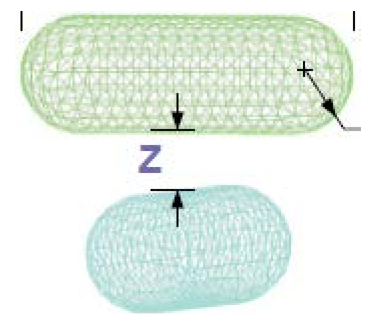
- Resonant frequencies of complex circuits.



$$\overline{\mathbf{Z}} \cdot \mathbf{I} = \mathbf{V}$$ KCL, KVL

$$\overline{\mathbf{Z}} \cdot \mathbf{I} = 0 \rightarrow \det[\overline{\mathbf{Z}}(\omega)] = 0 \rightarrow f(\omega) = 0$$

- Resonant frequencies of complex structures.
  - **Integral equation of scattering (EFIE)**

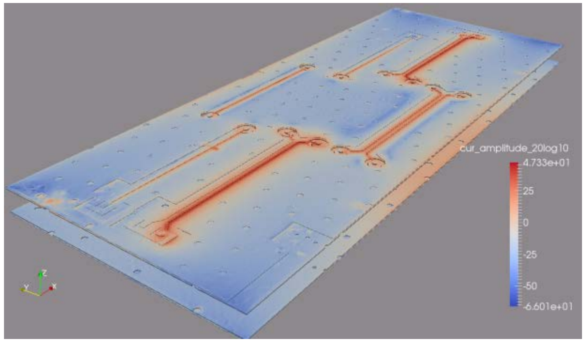


$$-\hat{n} \times \mathbf{E}^s(\mathbf{r}) = \hat{n} \times \mathbf{E}^i(\mathbf{r}) = \hat{n} \times i\omega\mu \int d\mathbf{r}' \overline{\mathbf{G}}(\mathbf{r}, \mathbf{r}') \cdot \mathbf{J}(\mathbf{r}')$$

$$\overline{\mathbf{Z}} \cdot \mathbf{J} = \mathbf{V}$$ Matrix representation

$$\overline{\mathbf{Z}} \cdot \mathbf{J} = 0 \rightarrow \det[\overline{\mathbf{Z}}(\omega)] = 0 \rightarrow f(\omega) = 0$$

- Host of CEM methods available.



A very complex geometry


Jie XIONG


Wei SHA


Qi DAI


Phil ATKINS

Argument Principle

$$E_{vac} = \sum_i \frac{1}{2} \hbar \omega_i$$

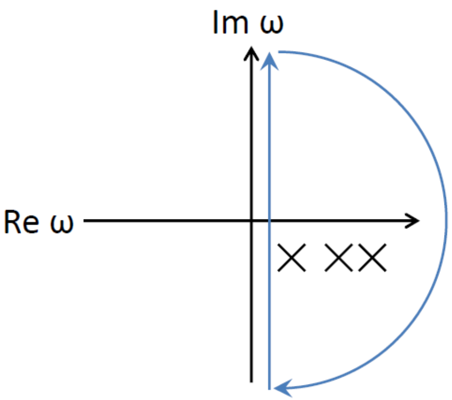
- The above sum is divergent! Renormalize below.

$$\mathcal{E} = E_{vac} - E_{norm} = \sum_{i,j} \frac{1}{2} \hbar \left[\omega_i - \omega_{j,norm} \right]$$

- Renormalized sum can be evaluated using argument principle.

$$\mathcal{E} = \frac{\hbar c}{2\pi} \int_0^\infty d\kappa \ln \frac{\det \overline{\mathbf{Z}}(\kappa)}{\det \overline{\mathbf{Z}}_\infty(\kappa)}$$

$$\mathbf{F} = -\frac{\hbar c}{2\pi} \int_0^\infty d\kappa \nabla_i \ln \det \overline{\mathbf{Z}}(\kappa)$$



where **Z** is a method of moments matrix.

Lots of math-physics, CEM training!

Repulsive Casimir Force:


Tian XIA

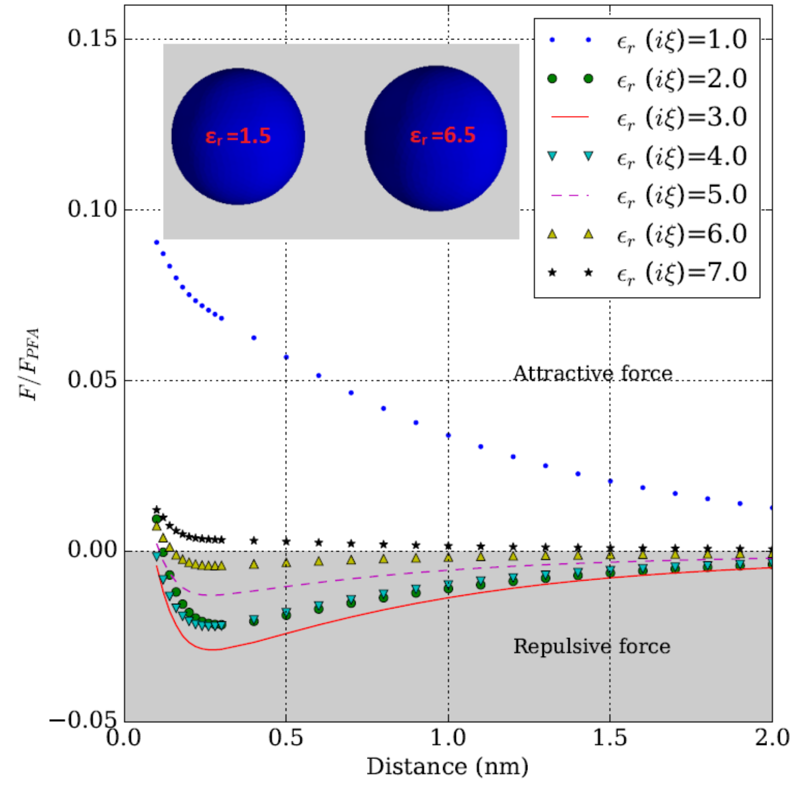


Fig. 6. Attractive and repulsive forces between dielectric objects at different background permittivities.

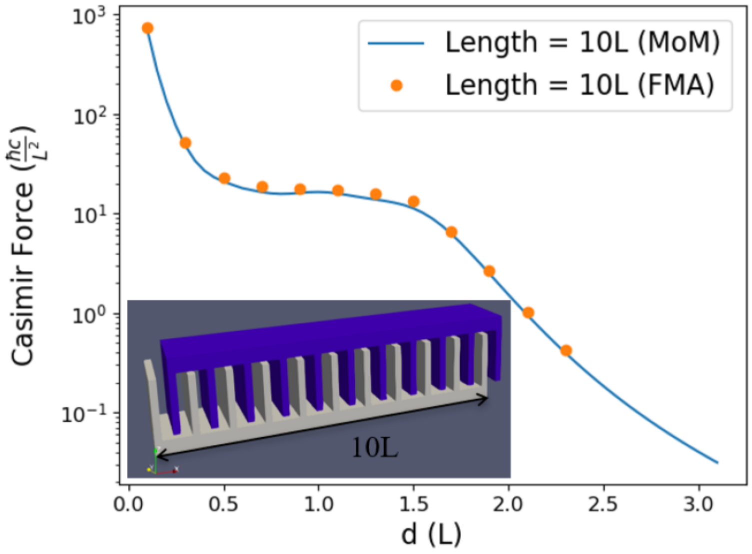


Fig. 8. Repulse force between the two U-shape PEC structures.

More Repulsive Casimir Force:

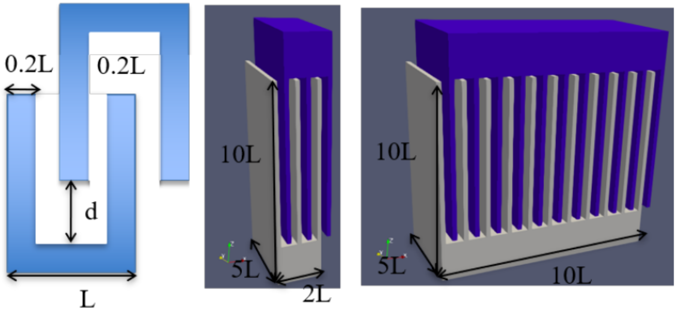


Fig. 9. The geometry and dimensions of the two tall U-shap PEC structures.

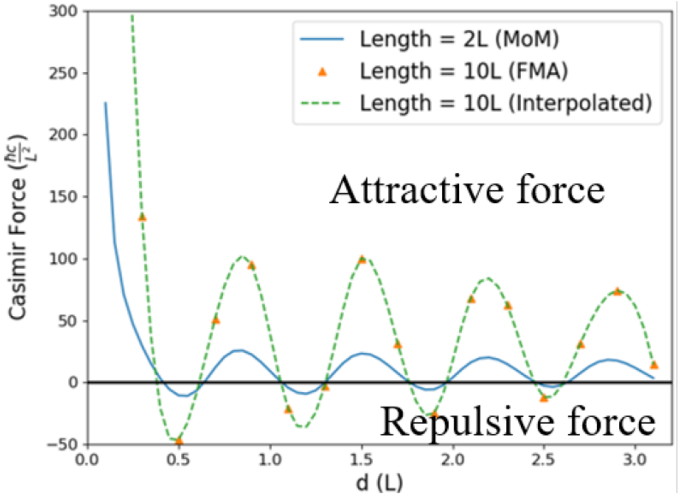


Fig. 10. Attractive and repulse force as the displacement varies between the two tall U-shape PEC structures as shown in Figure 9.

Possible Collaboration with Shalaev and Boltasseva's Group on quantum plasmonics.

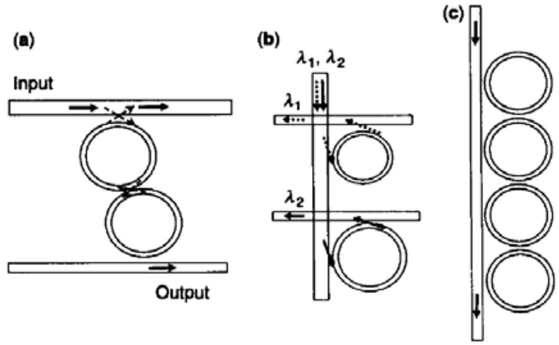
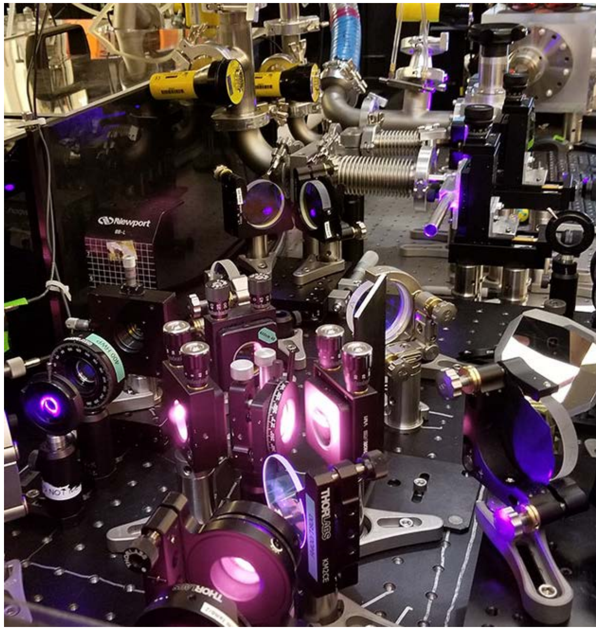


Figure 8.22 (a) A double-bus double-ring architecture. (b) A two-wavelength drop filter. (c) A single-bus periodic ring structure as a broad band filter or slow light device.

Questions to ask.

- Should future quantum computers work with optical photons or microwave photons?
- First attempt at optical computers failed in 1980's because of large optical components.
- Why're microwave components much smaller than optical components?
- Is the difference in mode confinement?

Typical optical table:



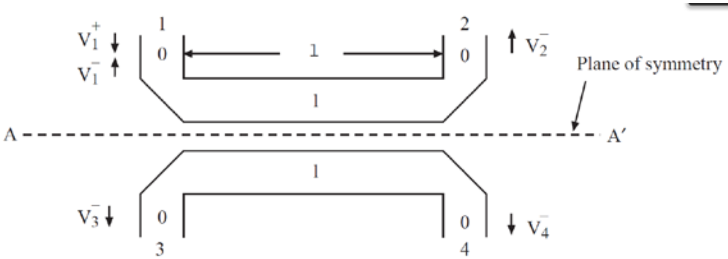


Figure 7.23: A directional coupler made of microstrip lines.

Typical microwave components: $\lambda / 4$

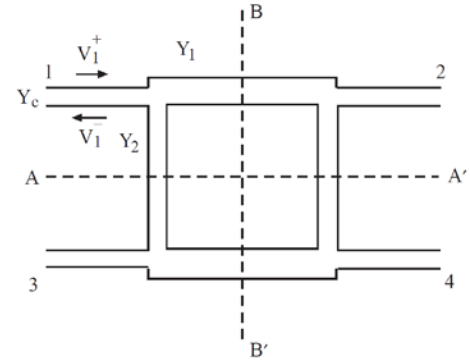
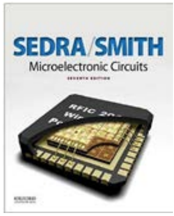


Figure 7.24: A branch line directional coupler.

Conclusions

- **Give an introduction on quantum parallelism and its power.**
- **Use the quantum Fourier transform as an illustration.**
- **Quantum computer has high payoffs but engineering a quantum coherent system is difficult.**
- **Recently, we have developed CEM methods to solve quantum Maxwell's equations. (Mode decomposition and quantum FDTD) (Dong-Yeop NA).**
- **Transmon modeling in circuit QED and Time Domain Integral Equations (TDIE) (Thomas E Roth).**
- **Report on recent progress on using CEM for Casimir force.**
- **Better math and full physics modeling through CEM can help improve the design of quantum computers. Math logic and computer codes don't lie.**
- **It is important to find the simplest approach to explain things, in order for knowledge transfer between disciplines and the development of advanced technologies.**

7 billion transistors on a chip.



Thank you!

- Thanks to colleagues at Purdue for interesting discussions and support!

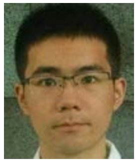
PURDUE UNIVERSITY

Neil Armstrong

Diversity and Inclusion

Members of the Group and Collaborators


Dong-Yeop NA

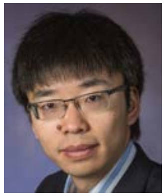

Jie ZHU

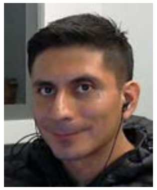

Lingling MENG


Chris J. RYU


Thomas E ROTH


Tian XIA


Hui GAN


Carlos SALAZAR


Shu CHEN


Mert HIDAYETOGLU


Boyuan ZHANG


Wei SHA


Aiyin LIU


Qi DAI


Dan JIAO


Erhan KUDEKI


Phil ATKINS


Wen-Mei HWU


Peter BERMEL


Lijun JIANG


Luis GOMEZ


Xiaoyan XIONG


Jie XIONG


Qin LIU


Ivan OKHMATOVSKII

Recent Papers Related to Quantum Technologies

- D.-Y. Na and W. C. Chew, “Classical and Quantum Electromagnetic Interferences: What Is The Difference?” PIER Journal, Vol. 168, 1-13, 2020.
- T. Xia, P. Atkins, W.E.I. Sha, and W. C. Chew “Casimir Force: Vacuum Fluctuation, Zero-Point Energy, and Computational Electromagnetics,” IEEE Antennas and Propagation Magazine, in press.
- W. C. Chew, D.-Y. Na, T. E. Roth, C. J. Ryu, and E. Kudeki, “Quantum Maxwell's Equations Made Simple,” IEEE Antennas and Propagation Magazine, scheduled for Feb. 2020.
- W. C. Chew, A. Y. Liu, C. Salazar-Lazaro, D.-Y. Na, and W.E.I. Sha, “Hamilton Equations, Commutator, and Energy Conservation,” *Quantum Reports*, vol. 1, pp. 295-303, 2019.
- D.-Y. Na and W. C. Chew, “Quantum Electromagnetic Finite-Difference Time Domain Solver,” *Quantum Reports*, vol. 2, pp. 253-265, 2020.
- D.-Y. Na, J. Zhu, W. C. Chew, and F. L. Teixeira. "Quantum information preserving computational electromagnetics." *Physical Review A* 102, no. 1 (2020): 013711.
- W. C. Chew, A.Y. Liu, C. Salazar-Lazaro, W.E.I. Sha, "Quantum electromagnetics: A new look—Parts I & II." *IEEE Journal on Multiscale and Multiphysics Computational Techniques* 1 (2016): 85-97.
- W. E. I. Sha, A. Y. Liu, and W. C. Chew, Dissipative quantum electromagnetics, *J. Multiscale and Multiphys. Comput. Techn.* 3, 198 (2018).
- P.R. Atkins, W.C. Chew, M.K. Li, L.E. Sun, Z.H. Ma, and L.J. Jiang. "Casimir force for complex objects using domain decomposition techniques." *Progress In Electromagnetics Research* 149 (2014): 275-280.
- P. R. Atkins, Q. I. Dai, W.E.I. Sha, and W. C. Chew, "Casimir Force for Arbitrary Objects Using the Argument Principle and Boundary Element Methods," Progress In Electromagnetics Research, vol. 142, pp. 615-624, Sep. 2013.
- W. C. Chew, “Quantum mechanics made simple: Lecture notes UIUC,” http://wcchew.ece.illinois.edu/chew/course/QMAll20161206.pdf, 2016.