

**HYDRODYNAMIC PHENOMENA IN  
THERMAL TRANSPORT**  
**Universitat Autònoma de Barcelona**  
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Purdue, July 2022

# Universitat Autònoma de Barcelona UAB

# UAB

Universitat Autònoma de Barcelona



PURDUE, JULY 2022

# □ Summary

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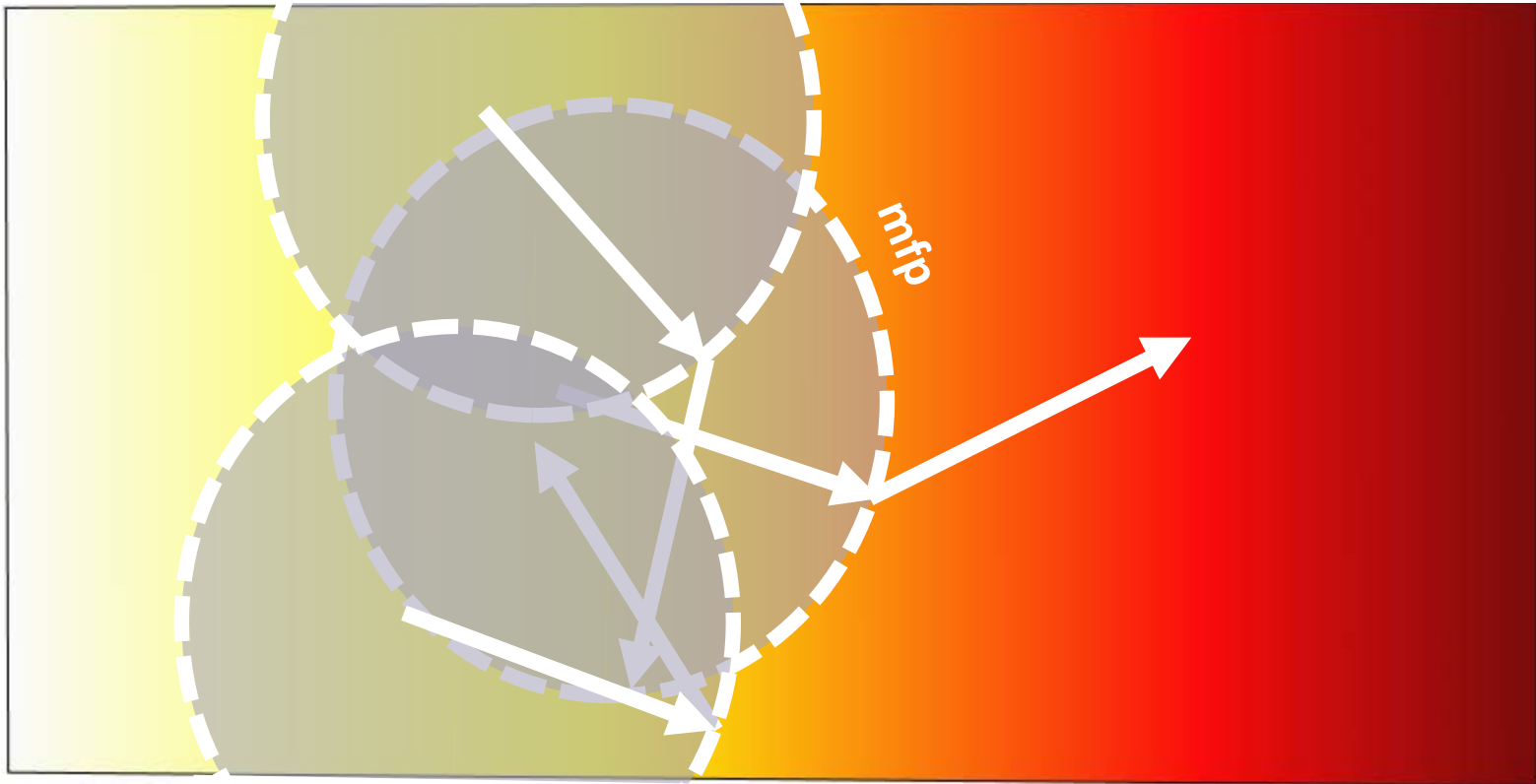
- Introduction.
- From the phonons to the moments basis
- Hydrodynamic behavior of semiconductors
- BTE calculations for hydrodynamic parameters
- New phenomena
- Conclusions



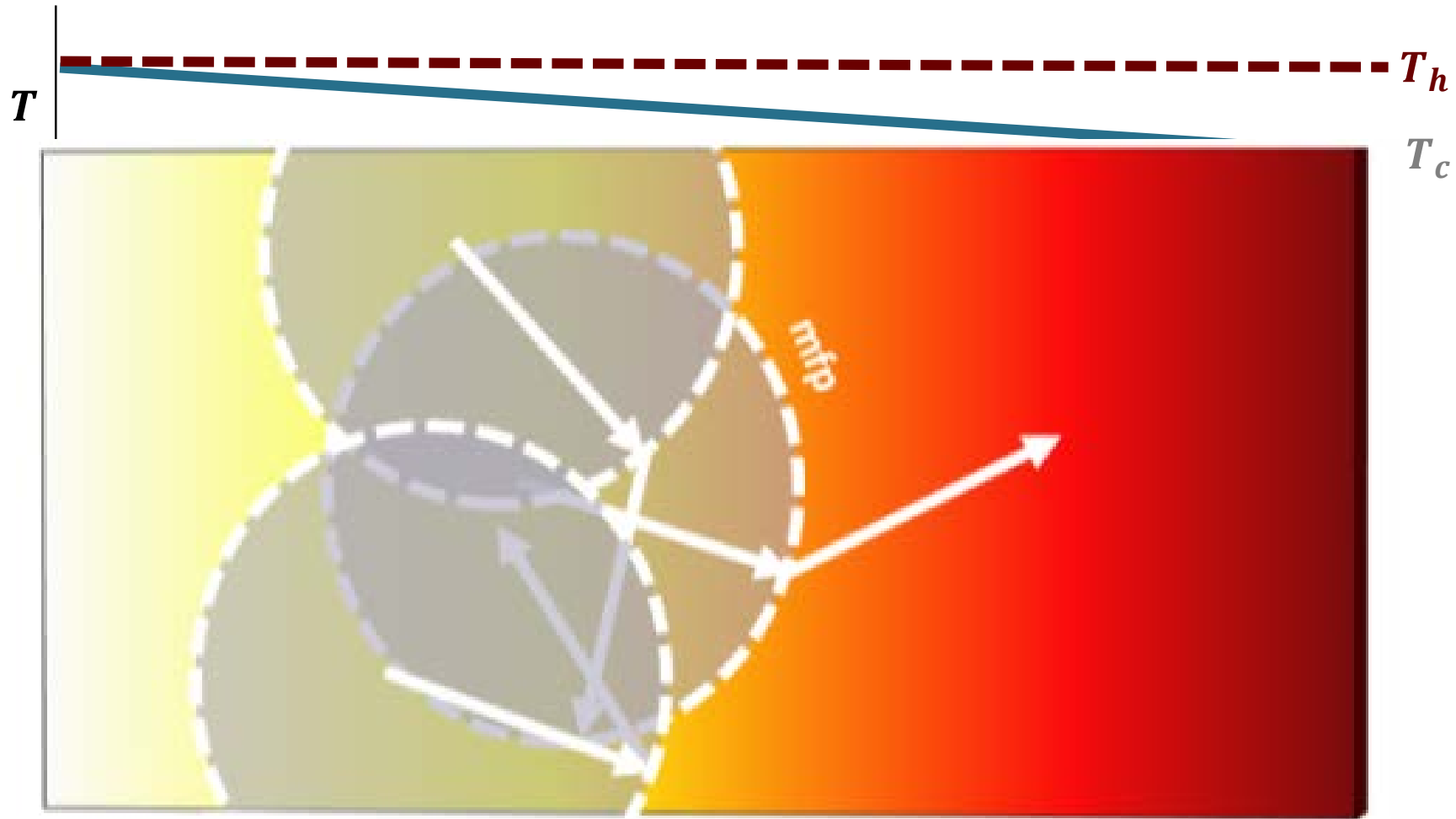
**INTRODUCTION:  
THERMAL TRANSPORT  
AT THE NANOSCALE**

# Fourier's law

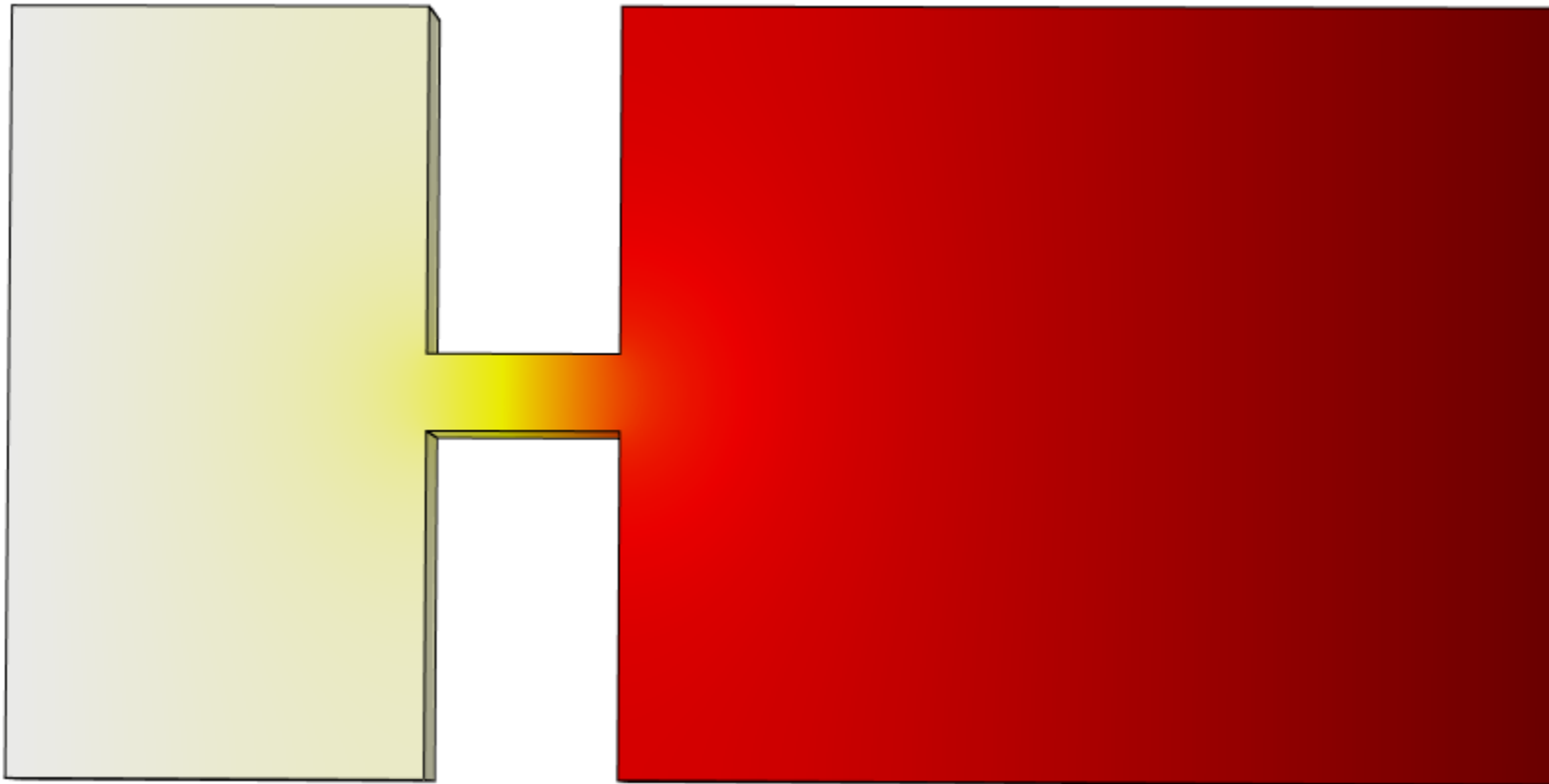
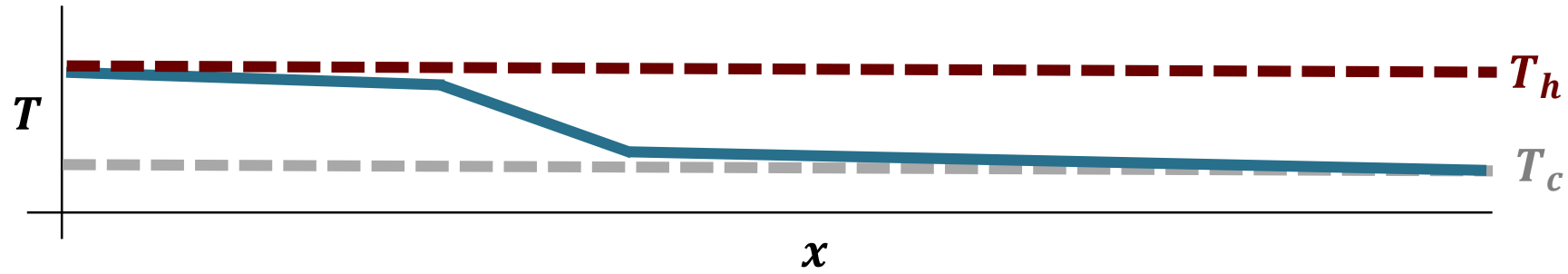
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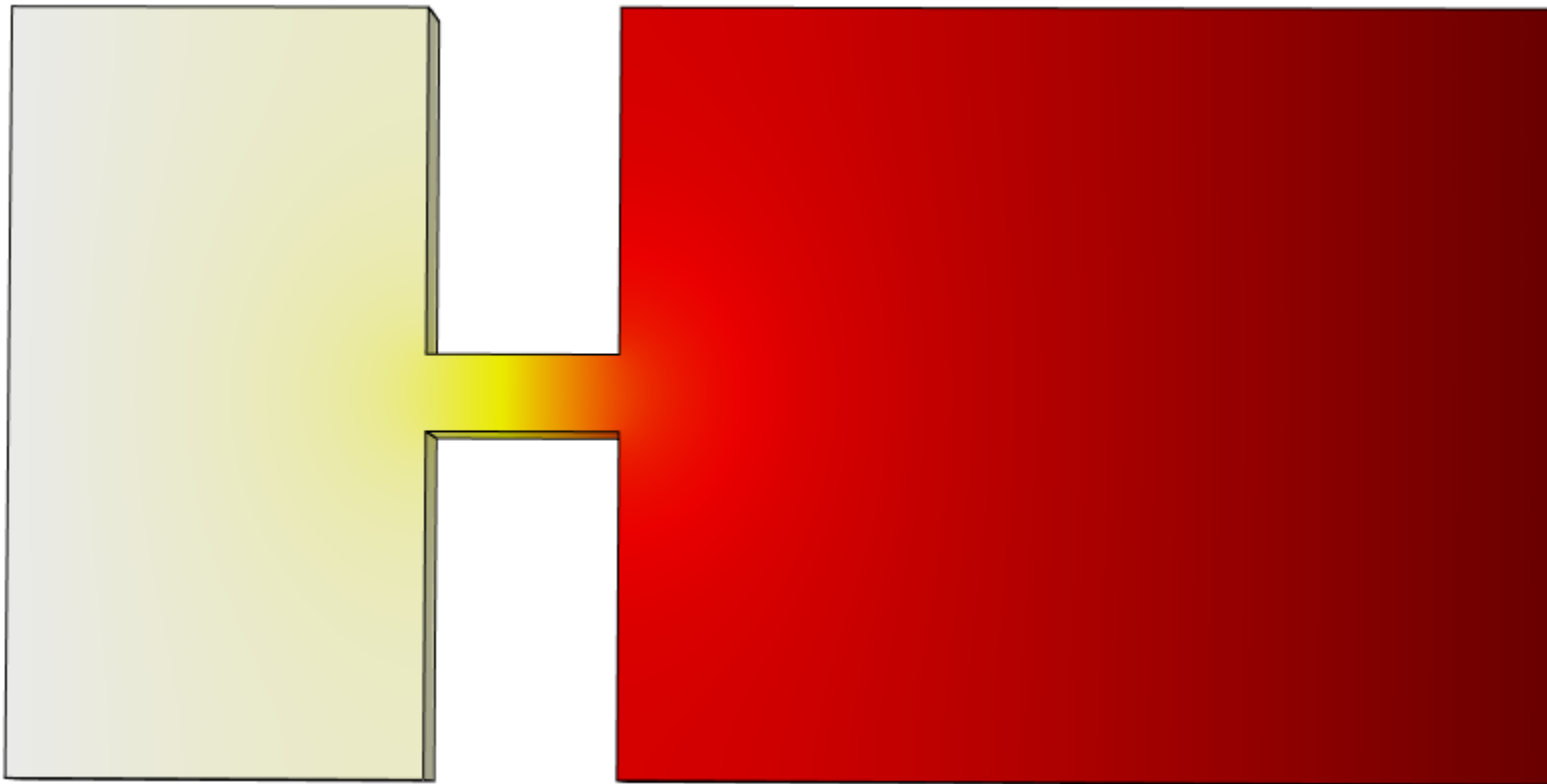
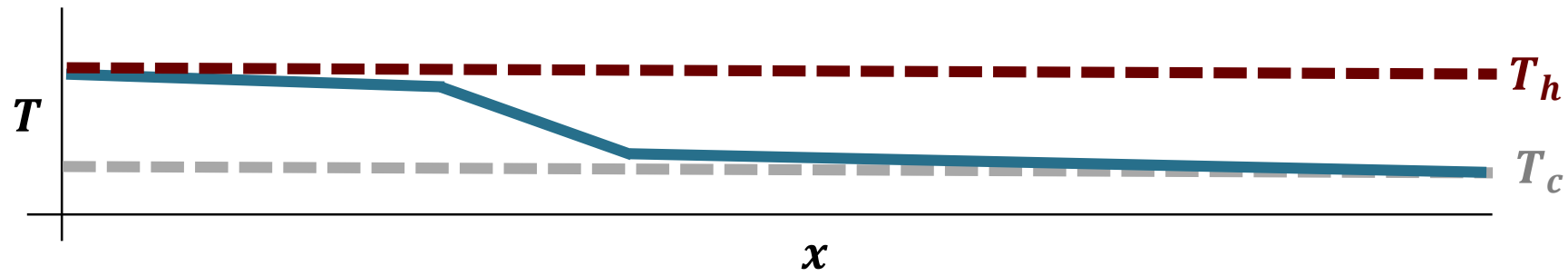
# Fourier's law



# □ Geometry effects



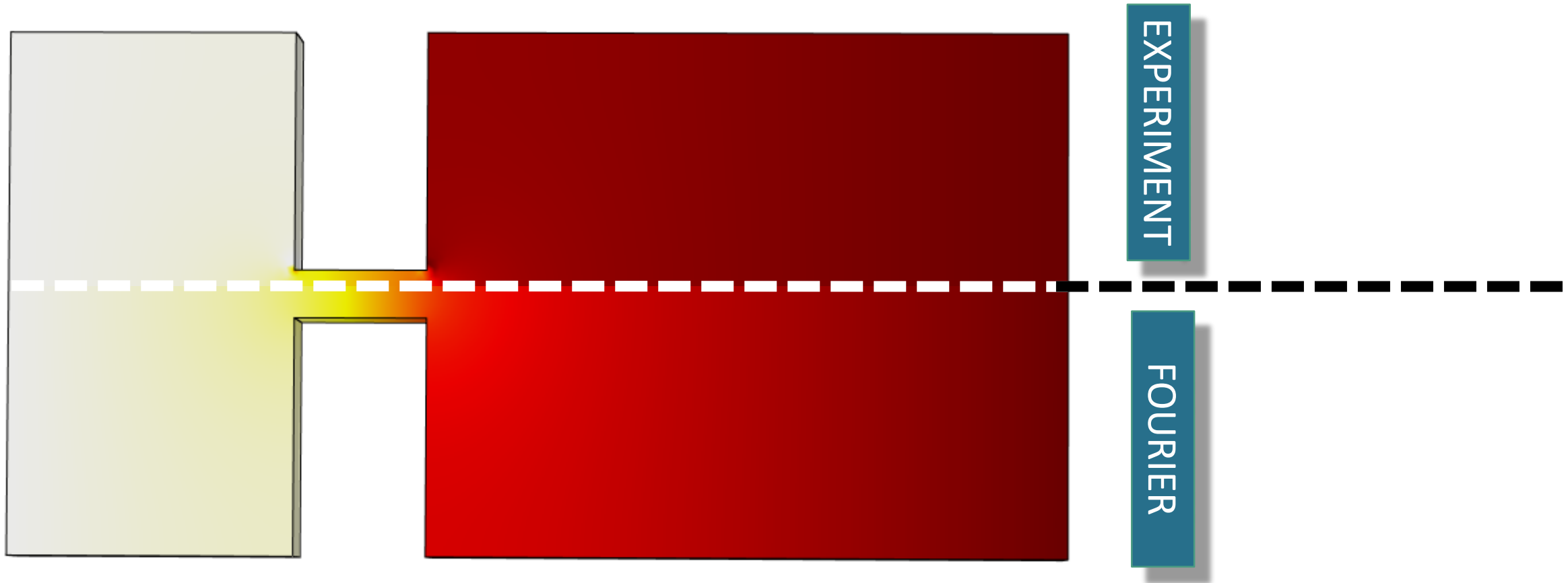
# □ Geometry effects



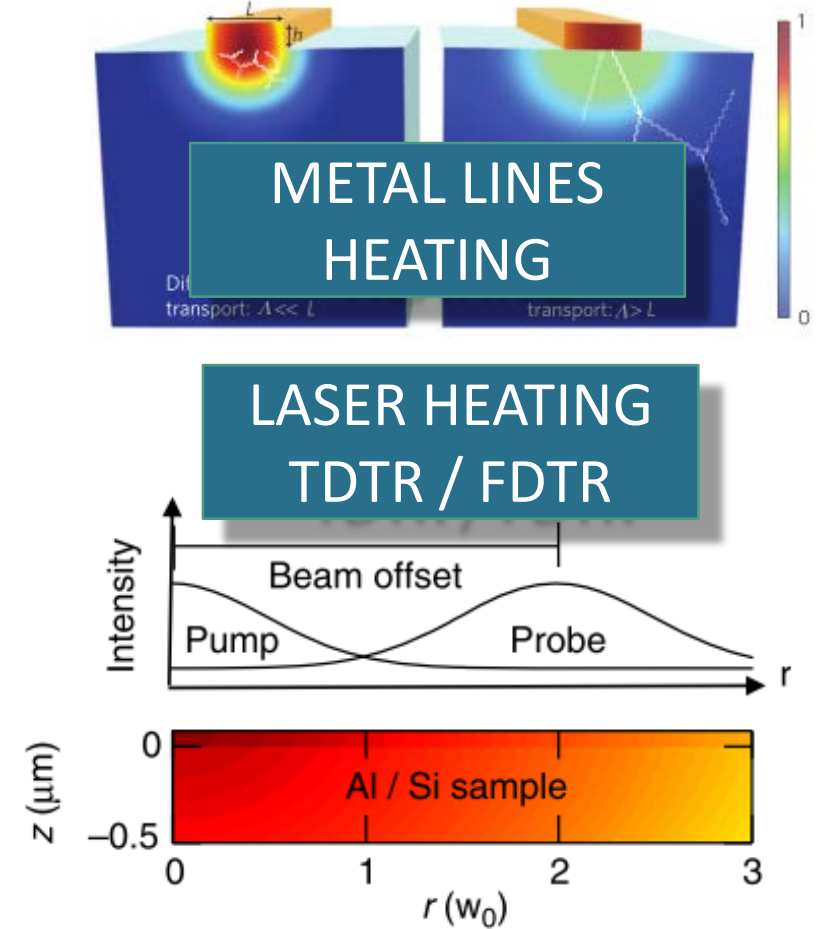
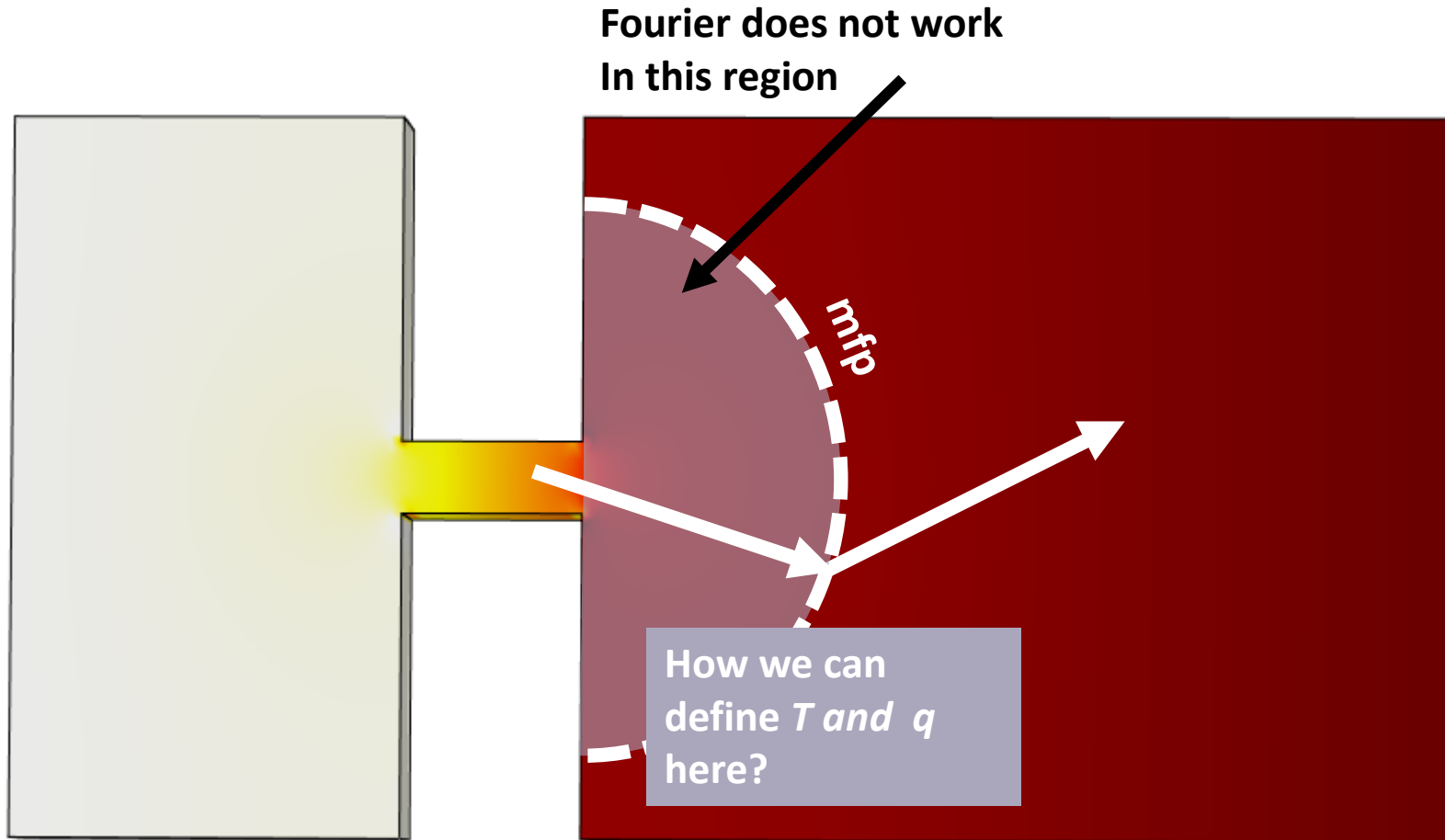


# □ Geometry effects

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# Geometry effects



# Mecanical (Hamiltonian) vs Entropic description



The main goal of the talk is to find contact points!

# **PHONONS VS MOMENTS BASIS**

# □ Boltzmann Transport Equation

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PHONONS

$$f_k(x, t)$$



BTE

$k$  equations for the  $k$  modes

$$\frac{\partial f_k}{\partial t} + v \cdot \frac{df_k}{dT} \nabla T = C(f_k)$$

# □ Boltzmann Transport Equation

PHONONS

$$f_k(x, t)$$



BTE

$k$  equations for the  $k$  modes

$$\frac{\partial f_k}{\partial t} + v \cdot \frac{df_k}{dT} \nabla T = \frac{f_k - f_k^0}{\tau_k}$$

# From phonons $f(\mathbf{k}, \mathbf{x}, t)$ to moments $Q^{(n)}(\mathbf{x}, t)$

Thermodynamic energy

Phonon energy

0<sup>th</sup> ORDER: Energy

$$f(\mathbf{k}, \mathbf{x}, t)$$

1<sup>st</sup> ORDER: Heat flux

$$q(\mathbf{x}, t) = \int \hbar \omega_{\mathbf{k}} \vec{v}_{\mathbf{k}} f(\mathbf{k}, \mathbf{x}, t) \frac{d^3 k}{(2\pi)^3}$$

2<sup>nd</sup> ORDER: Flux of the flux

$$Q^{(2)}(\mathbf{x}, t) = \int \hbar \omega_{\mathbf{k}} (\vec{v}_{\mathbf{k}} \cdot \vec{v}_{\mathbf{k}}) f(\mathbf{k}, \mathbf{x}, t) \frac{d^3 k}{(2\pi)^3}$$

n<sup>th</sup> ORDER

$$Q^{(n)}(\mathbf{x}, t) = \int \hbar \omega_{\mathbf{k}} (\vec{v}_{\mathbf{k}} \cdots \vec{v}_{\mathbf{k}}) f(\mathbf{k}, \mathbf{x}, t) \frac{d^3 k}{(2\pi)^3}$$

# From phonons $f(\mathbf{k}, \mathbf{x}, t)$ to moments $Q^{(n)}(\mathbf{x}, t)$

0<sup>th</sup> ORDER: Energy

$$e(\mathbf{x}, t) = \int \hbar \omega_{\mathbf{k}} f(\mathbf{k}, \mathbf{x}, t) \frac{d^3 k}{(2\pi)^3}$$

1<sup>st</sup> ORDER: Heat flux

$$q(\mathbf{x}, t) = \int \hbar \omega_{\mathbf{k}} \vec{v}_{\mathbf{k}} f(\mathbf{k}, \mathbf{x}, t) \frac{d^3 k}{(2\pi)^3}$$

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# From phonons $f(\mathbf{k}, \mathbf{x}, t)$ to moments $Q^{(n)}(\mathbf{x}, t)$

0<sup>th</sup> ORDER: Energy

$$e(\mathbf{x}, t) = \int \hbar \omega_{\mathbf{k}} f(\mathbf{k}, \mathbf{x}, t) \frac{d^3 k}{(2\pi)^3}$$

Thermodynamic Heat flux (left), Phonon Heat flux (right)

1<sup>st</sup> ORDER: Heat flux

$$q(\mathbf{x}, t) = \int \hbar \omega_{\mathbf{k}} \vec{v}_{\mathbf{k}} f(\mathbf{k}, \mathbf{x}, t) \frac{d^3 k}{(2\pi)^3}$$

Thermodynamic Flux of the flux (left), Phonon Stress T (right)

2<sup>nd</sup> ORDER: Flux of the flux

$$Q^{(2)}(\mathbf{x}, t) = \int \hbar \omega_{\mathbf{k}} (\vec{v}_{\mathbf{k}} \cdot \vec{v}_{\mathbf{k}}) f(\mathbf{k}, \mathbf{x}, t) \frac{d^3 k}{(2\pi)^3}$$

n<sup>th</sup> ORDER

$$Q^{(n)}(\mathbf{x}, t) = \int \hbar \omega_{\mathbf{k}} (\vec{v}_{\mathbf{k}} \cdots \vec{v}_{\mathbf{k}}) f(\mathbf{k}, \mathbf{x}, t) \frac{d^3 k}{(2\pi)^3}$$

# From phonons $f(\mathbf{k}, \mathbf{x}, t)$ to moments $Q^{(n)}(\mathbf{x}, t)$

0<sup>th</sup> ORDER: Energy

$$\epsilon(\mathbf{x}, t) = \int \hbar \omega_{\mathbf{k}} f(\mathbf{k}, \mathbf{x}, t)$$

1<sup>st</sup> ORDER: Heat flux

$$q(\mathbf{x}, t) = \int \hbar \omega_{\mathbf{k}} \vec{v}_{\mathbf{k}} f(\mathbf{k}, \mathbf{x}, t) \frac{d^3 k}{(2\pi)^3}$$

Thermodynamic  
Flux of the flux

Phonon  
Stress T

2<sup>nd</sup> ORDER: Flux of the flux

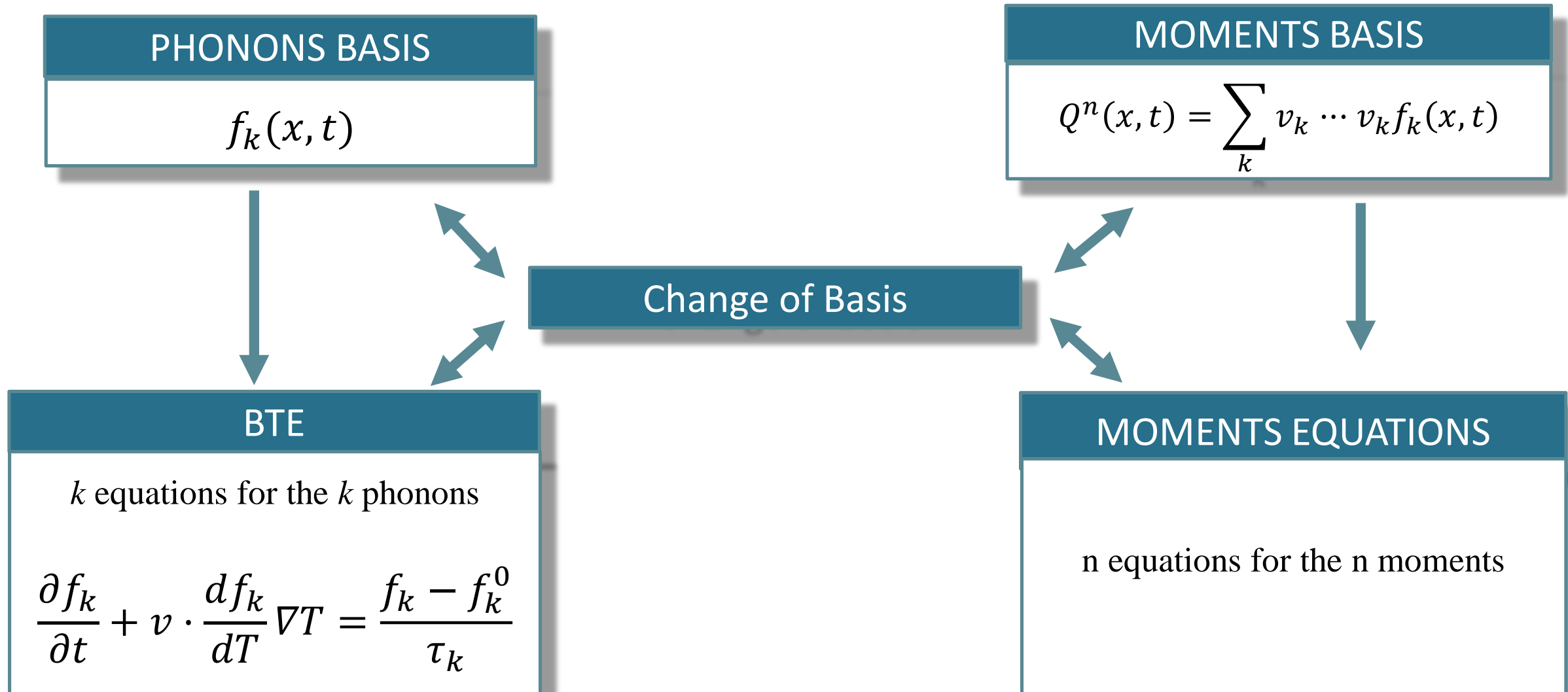
$$Q^{(2)}(\mathbf{x}, t) = \int \hbar \omega_{\mathbf{k}} (\vec{v}_{\mathbf{k}} \cdot \vec{v}_{\mathbf{k}}) f(\mathbf{k}, \mathbf{x}, t) \frac{d^3 k}{(2\pi)^3}$$

n<sup>th</sup> ORDER

$$Q^{(n)}(\mathbf{x}, t) = \int \hbar \omega_{\mathbf{k}} (\vec{v}_{\mathbf{k}} \cdots \vec{v}_{\mathbf{k}}) f(\mathbf{k}, \mathbf{x}, t) \frac{d^3 k}{(2\pi)^3}$$

**Moments are more easily to measure experimentally than phonon abundances: Temperature, fluxes, etc...**

# From phonons $f(k, x, t)$ to moments $Q^{(n)}(x, t)$



## □ From phonons $f(\mathbf{k}, \mathbf{x}, t)$ to moments $Q^{(n)}(\mathbf{x}, t)$

---

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \nabla f = - \frac{f - f_0}{\tau}$$

$$c_V \frac{\partial T}{\partial t} - \nabla \cdot \mathbf{q} = 0$$

0<sup>th</sup> ORDER: Energy conservation

## □ From phonons $f(\mathbf{k}, \mathbf{x}, t)$ to moments $Q^{(n)}(\mathbf{x}, t)$

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$$\int \frac{\partial f}{\partial t} + \mathbf{v} \cdot \nabla f = - \frac{f - f_0}{\tau}$$

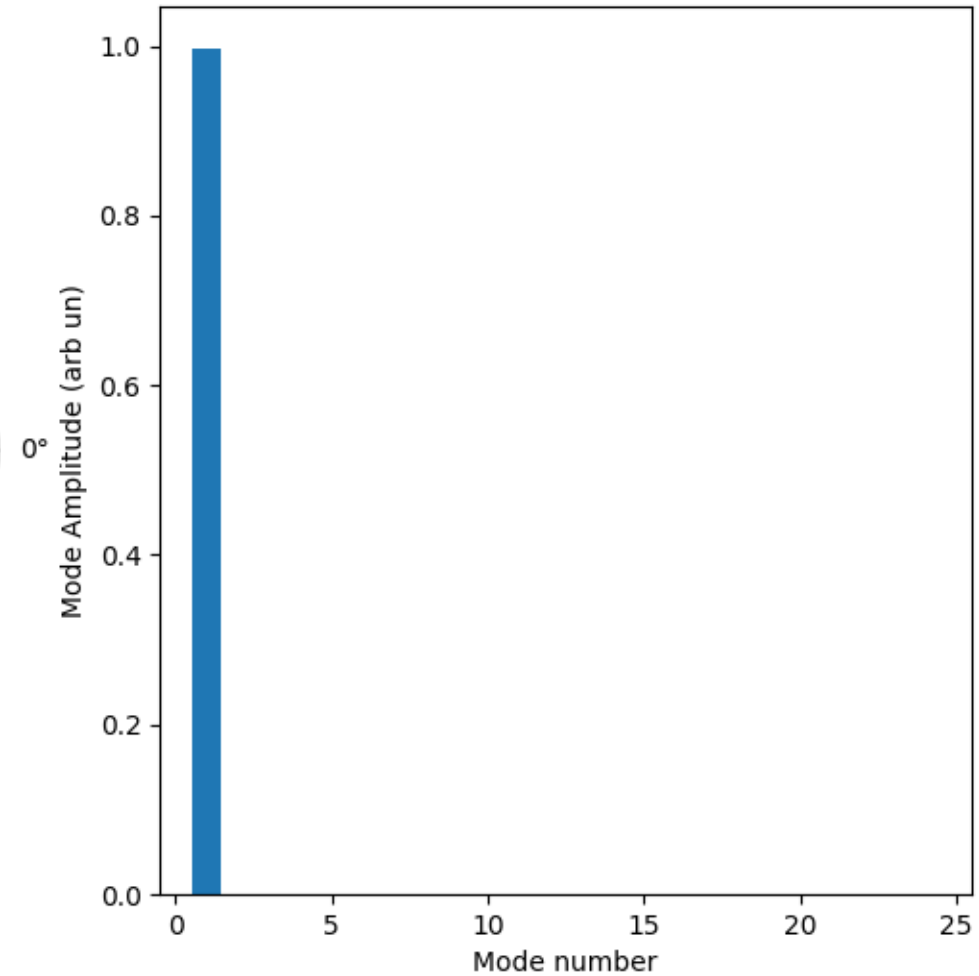
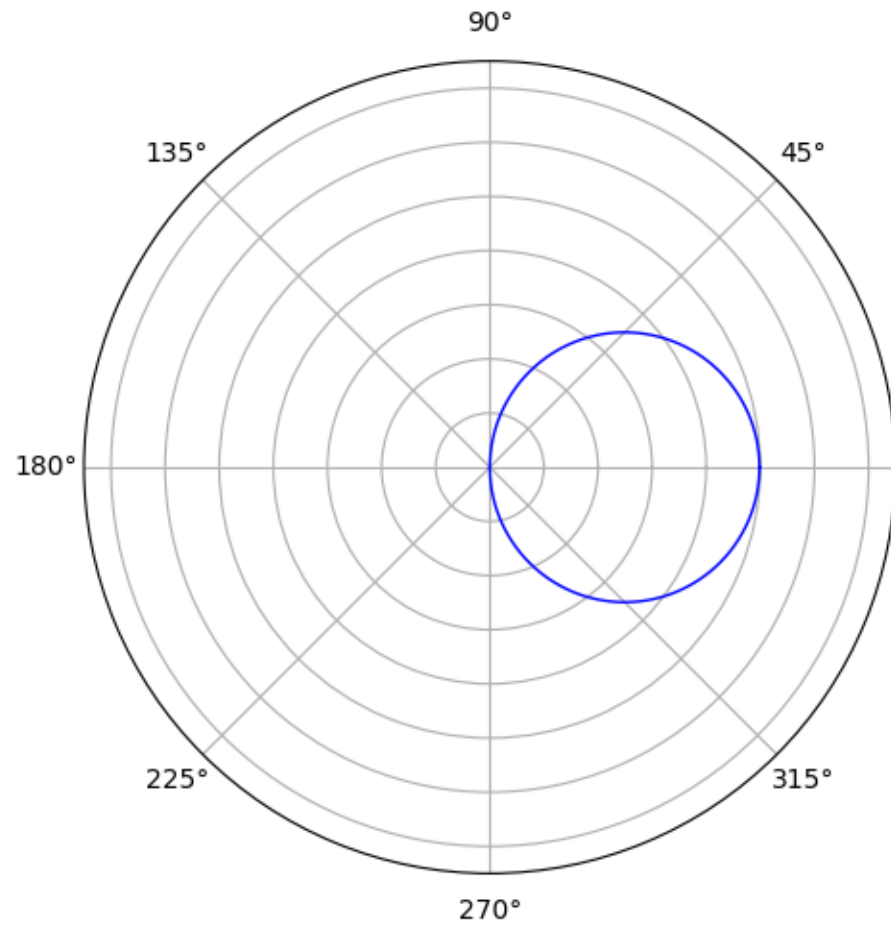
$$\frac{\partial q}{\partial t} - \nabla \cdot \mathbf{Q} = \frac{q}{\tau_q}$$

1<sup>st</sup> ORDER: Energy conservation

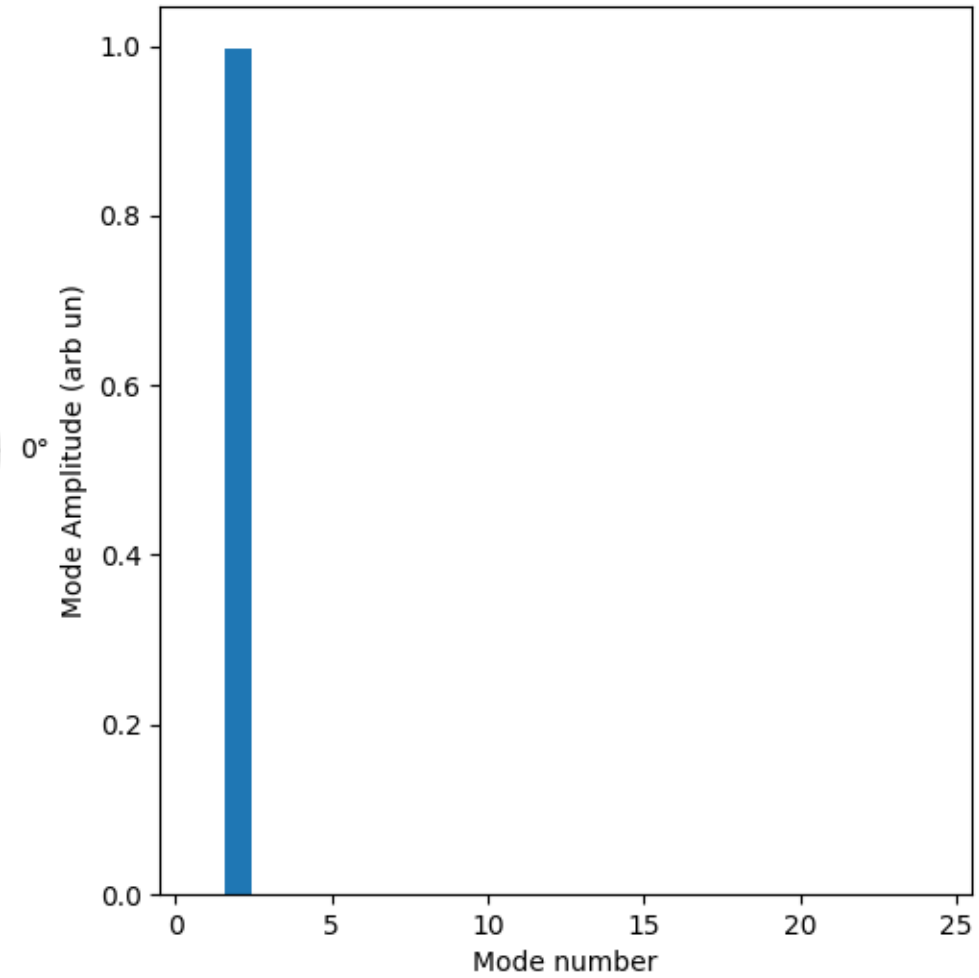
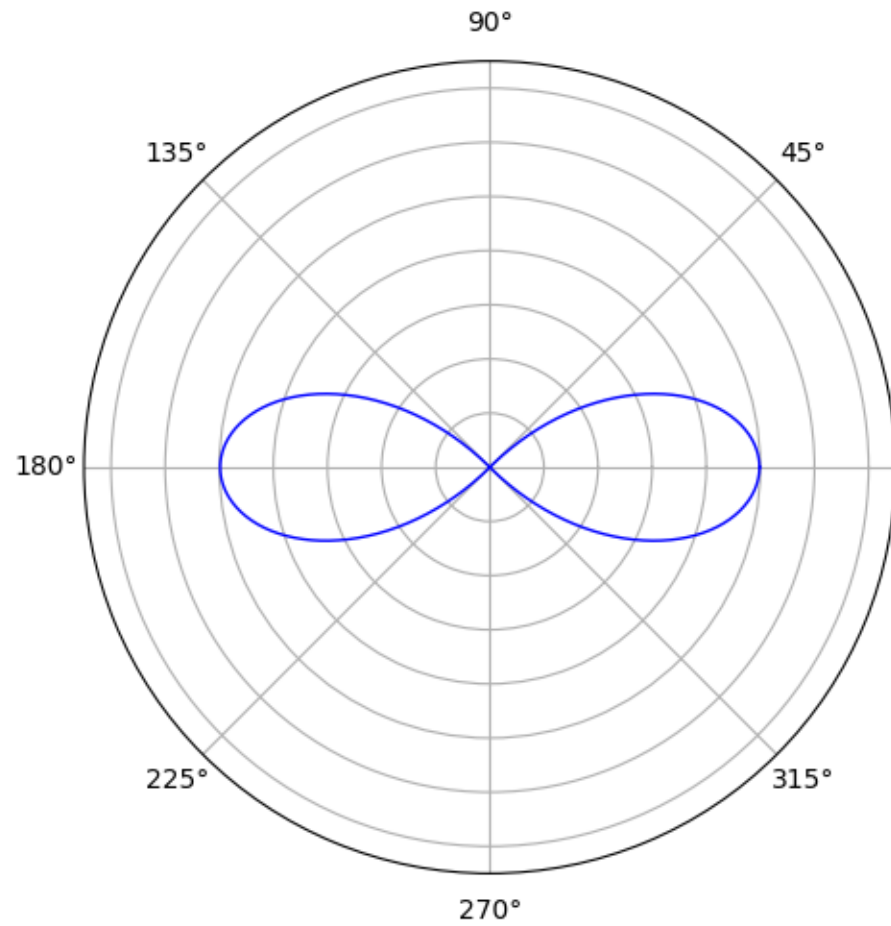
# From phonons $f(\mathbf{k}, \mathbf{x}, t)$ to moments $Q^{(n)}(\mathbf{x}, t)$

$\partial/\partial t$ (n)	(n)		Div (n+1)	Grad (n-1)	
$\alpha_1 \frac{\partial T}{\partial t}$		=	$-\nabla \cdot \mathbf{q}$		0 <sup>th</sup> ORDER
$\alpha_2 \frac{\partial \mathbf{q}}{\partial t}$	$-\frac{\mathbf{q}}{\tau_q}$	=	$-\nabla \cdot \mathbf{Q}^{(2)}$	$-\beta_1 \nabla T$	1 <sup>st</sup> ORDER
$\alpha_3 \frac{\partial \mathbf{Q}^{(2)}}{\partial t}$	$-\frac{\mathbf{Q}^{(2)}}{\tau_{Q^{(2)}}}$	=	$-\nabla \cdot \mathbf{Q}^{(3)}$	$-\beta_2 \nabla \mathbf{q}$	2 <sup>nd</sup> ORDER
$\alpha_n \frac{\partial \mathbf{Q}^{(n)}}{\partial t}$	$-\frac{\mathbf{Q}^{(n)}}{\tau_{Q^{(n)}}}$	=	$-\nabla \cdot \mathbf{Q}^{(n+1)}$	$-\beta_n \nabla \mathbf{Q}^{(n-1)}$	n <sup>th</sup> ORDER

# □ Moments of the distribution

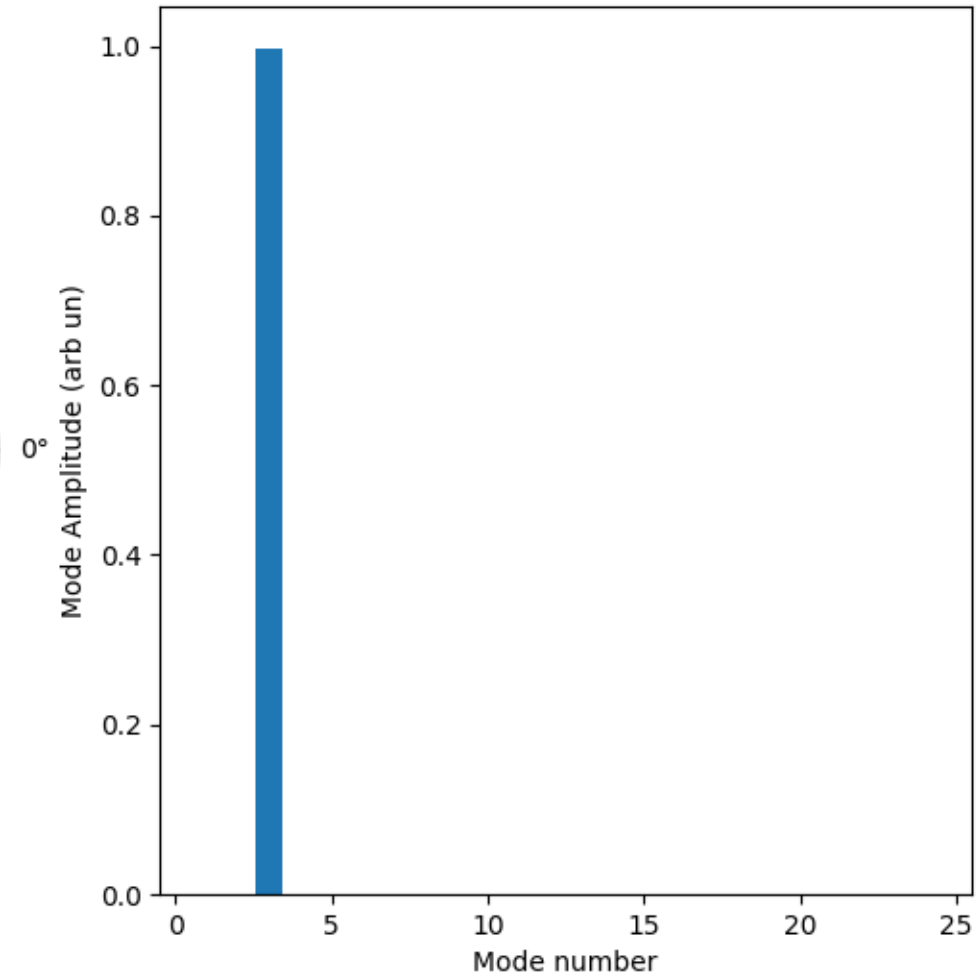
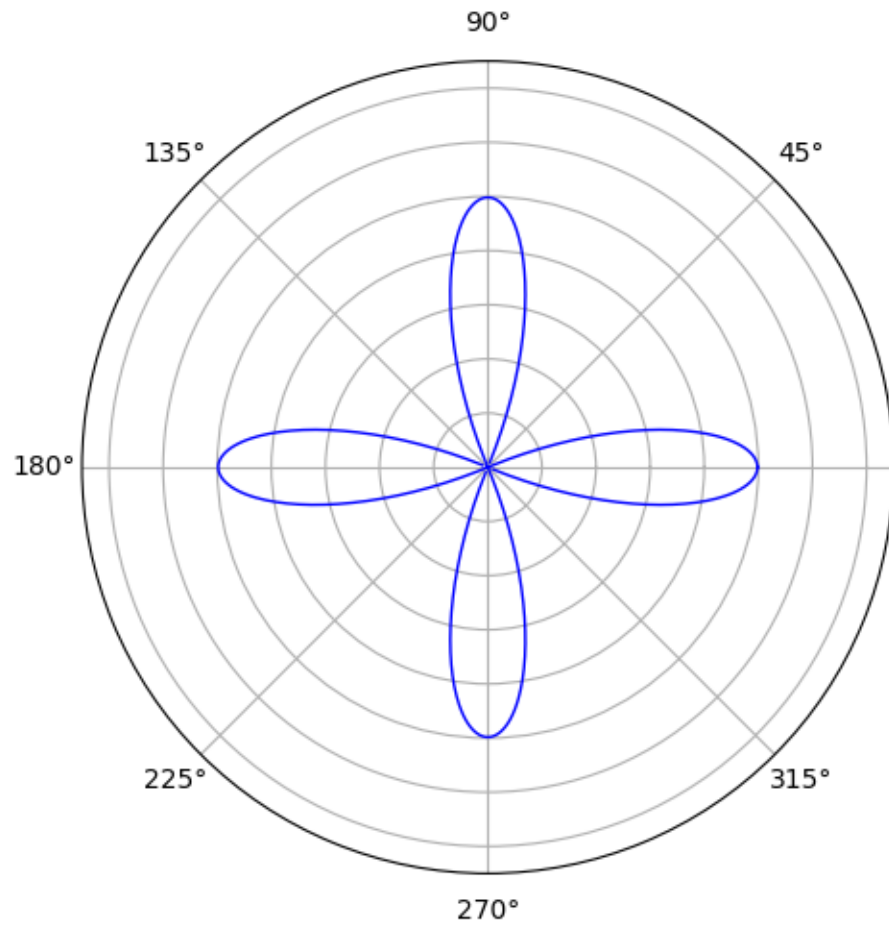


# □ Moments of the distribution

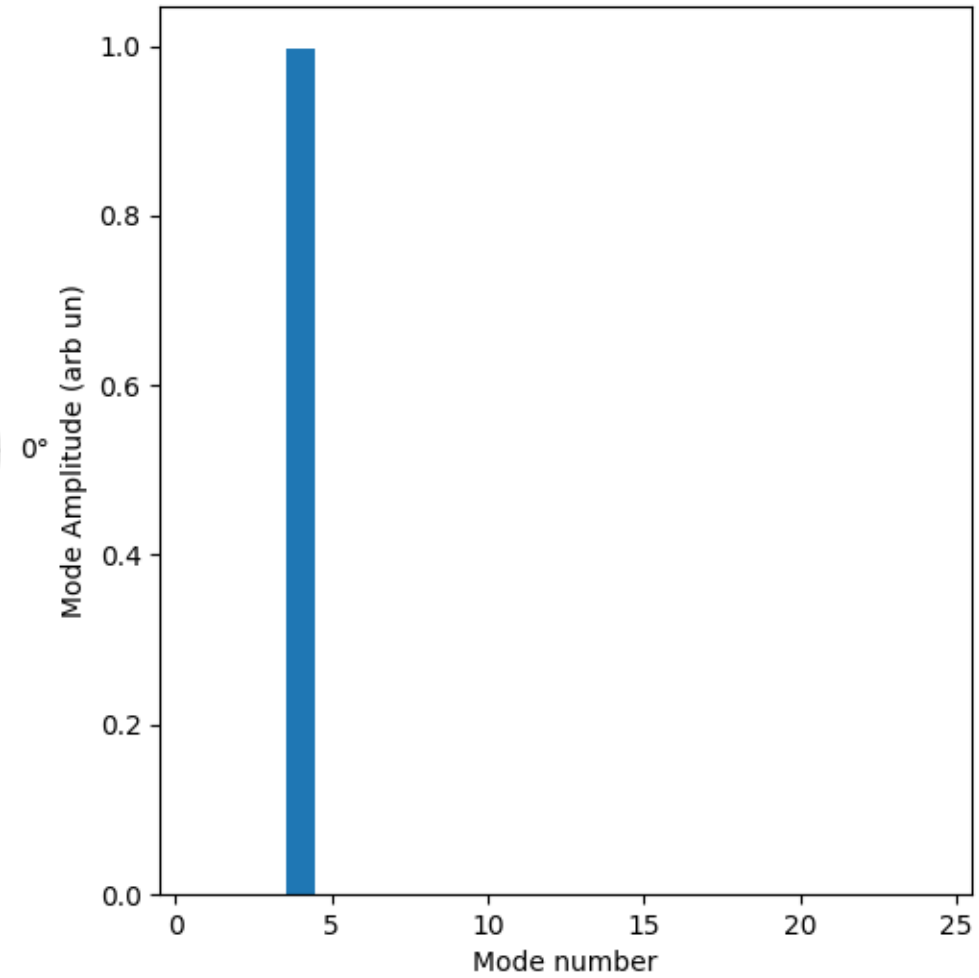
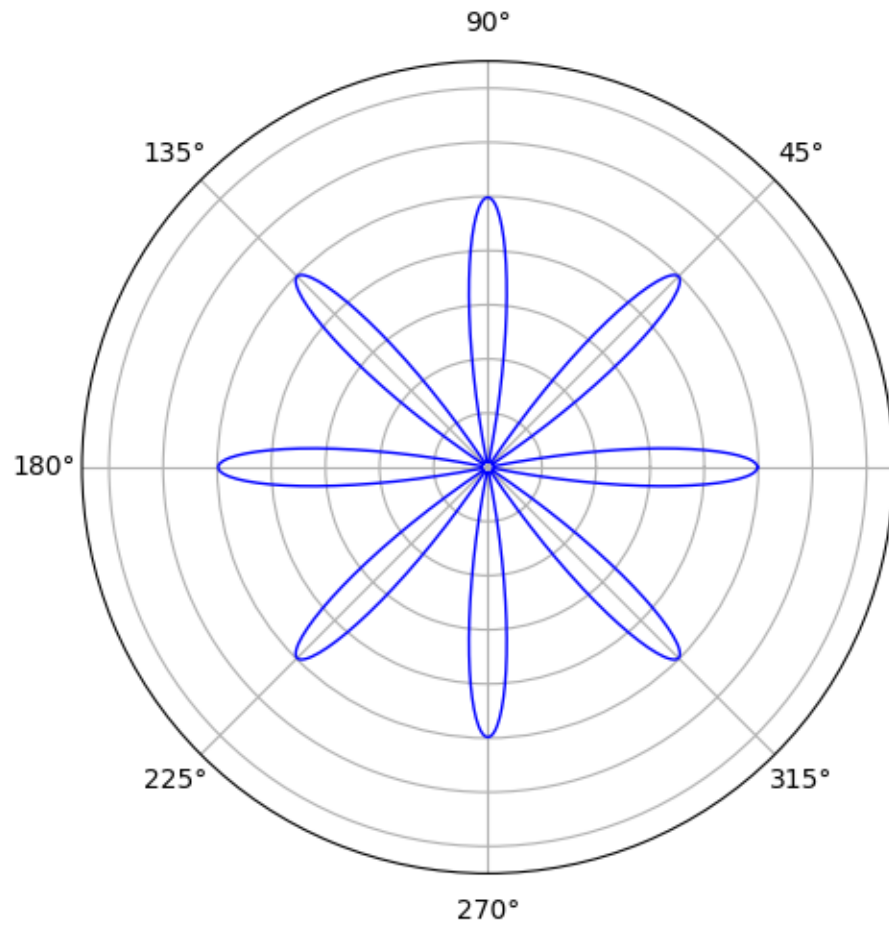




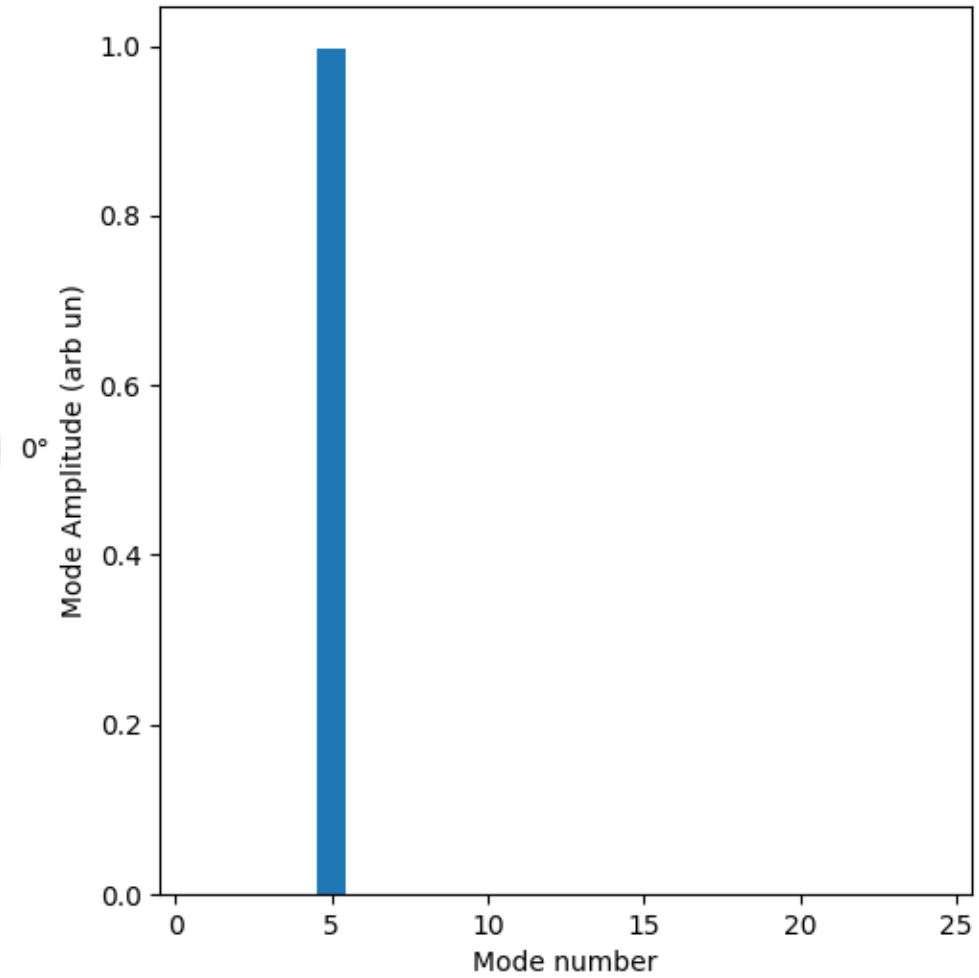
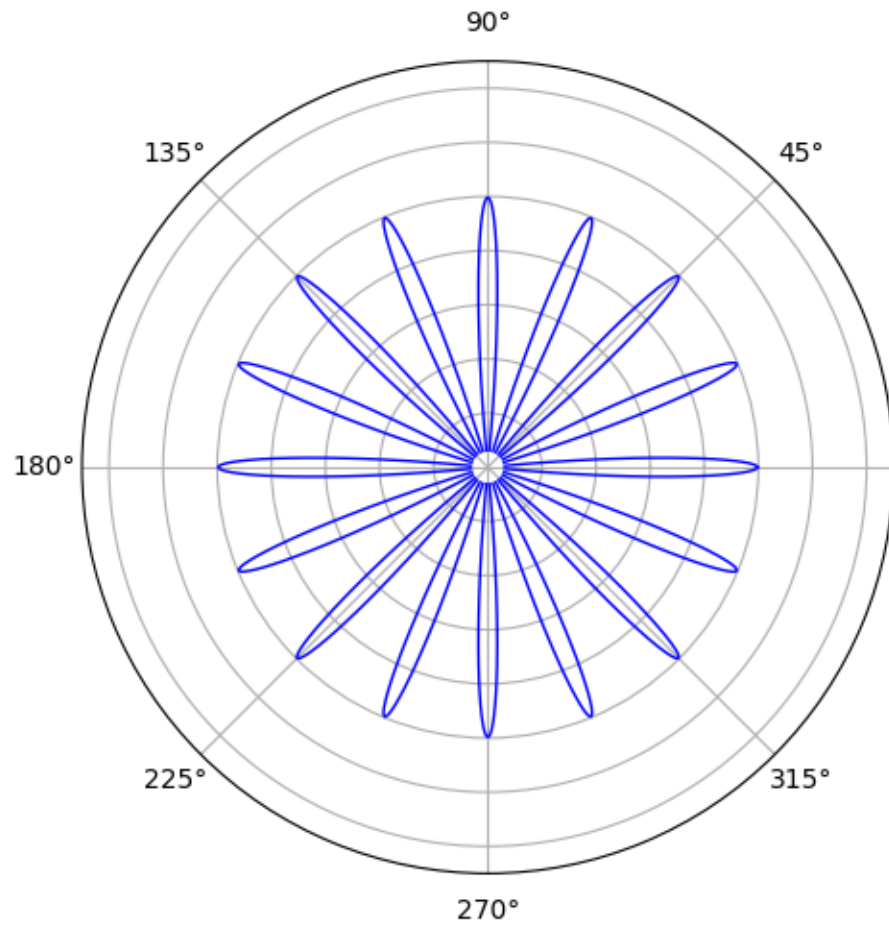
# □ Moments of the distribution



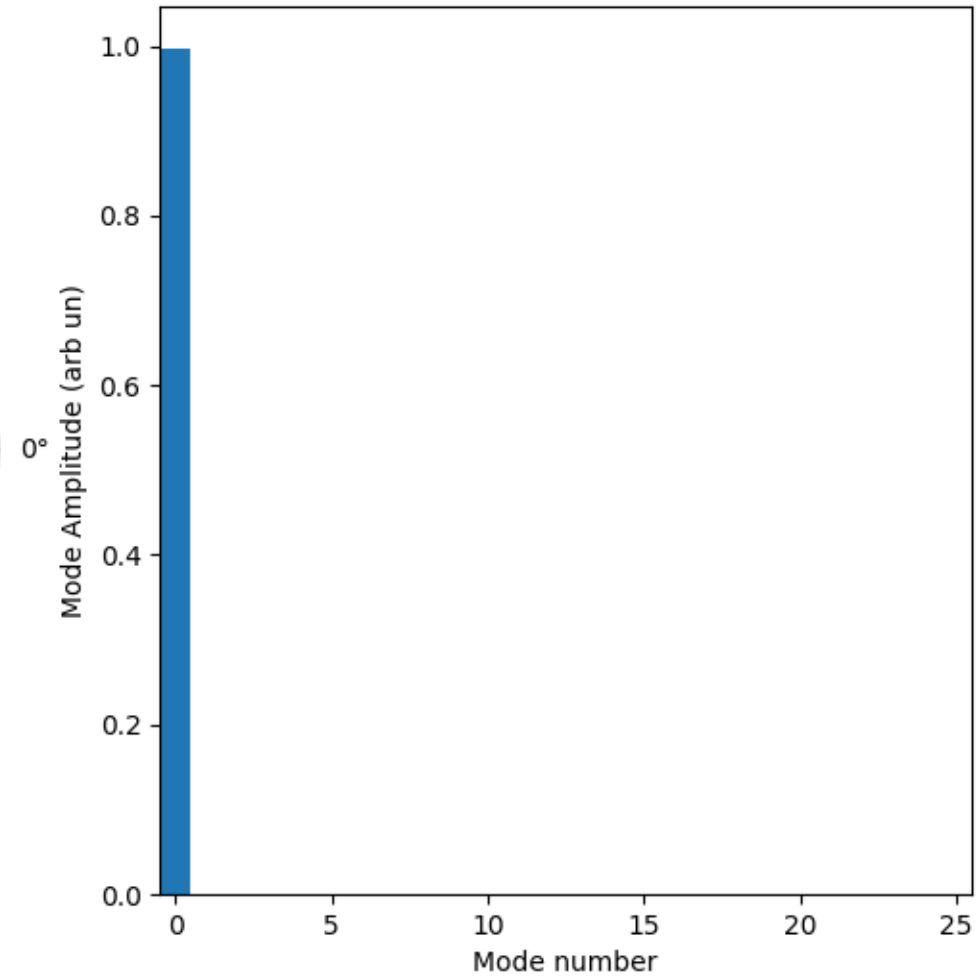
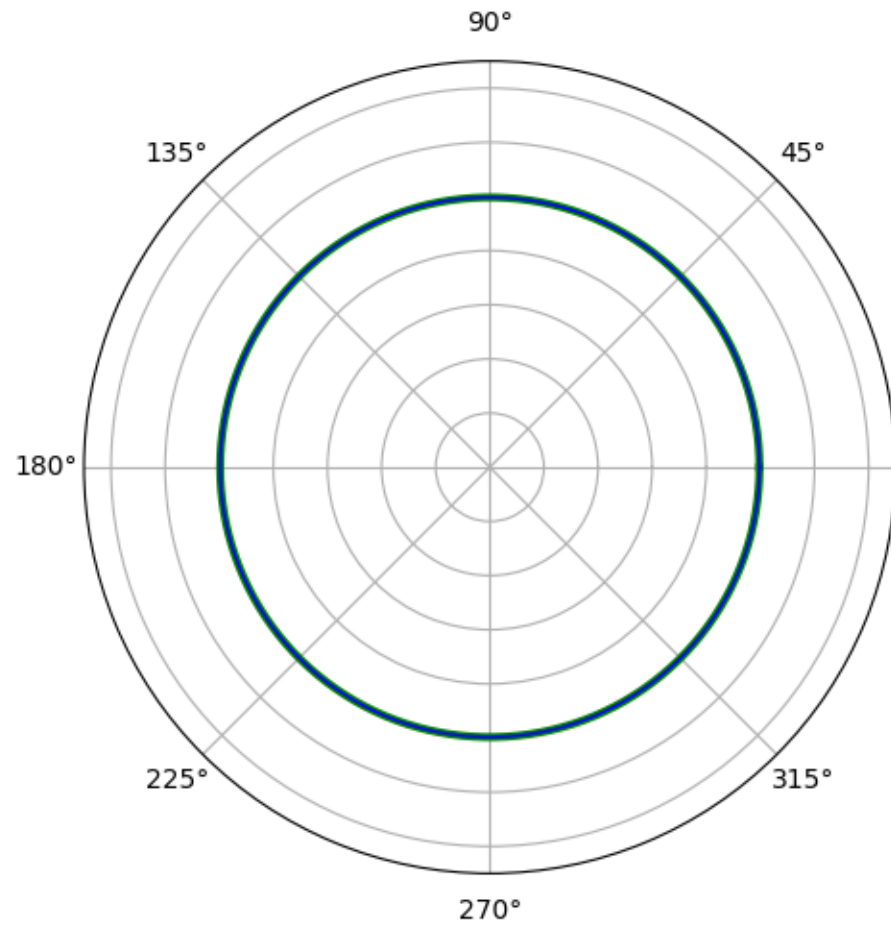
# □ Moments of the distribution



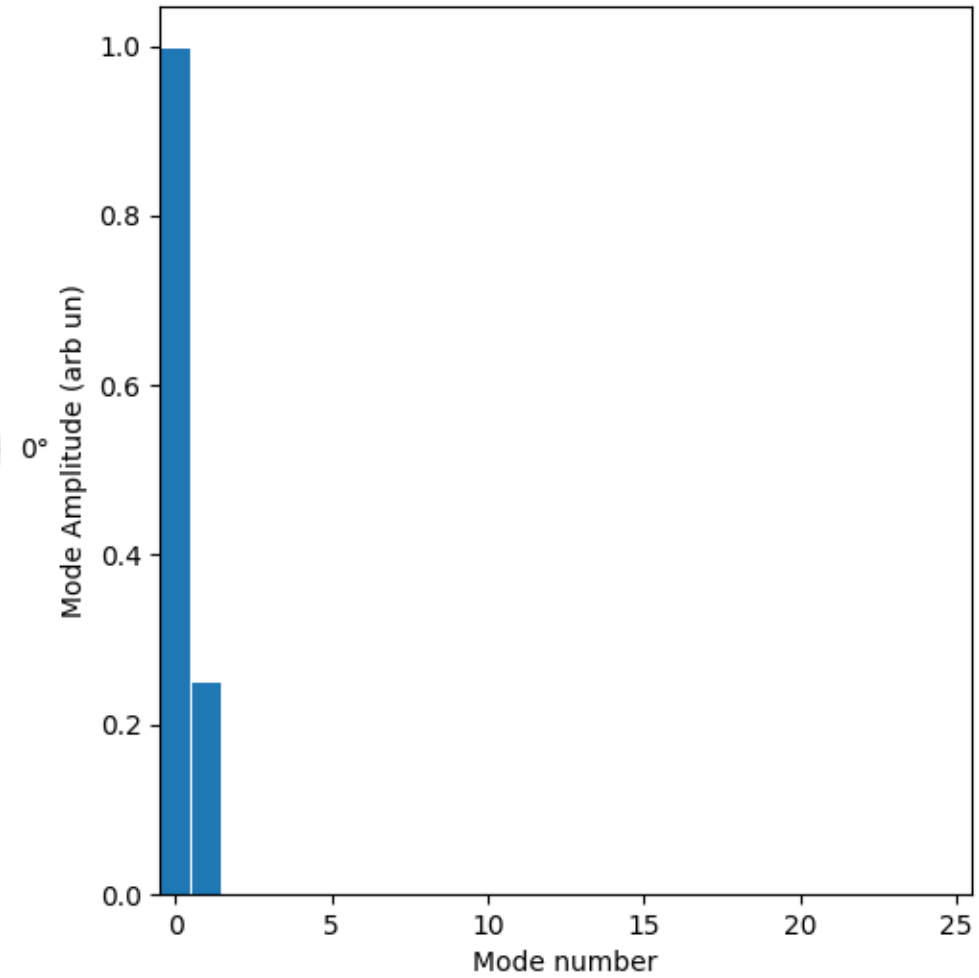
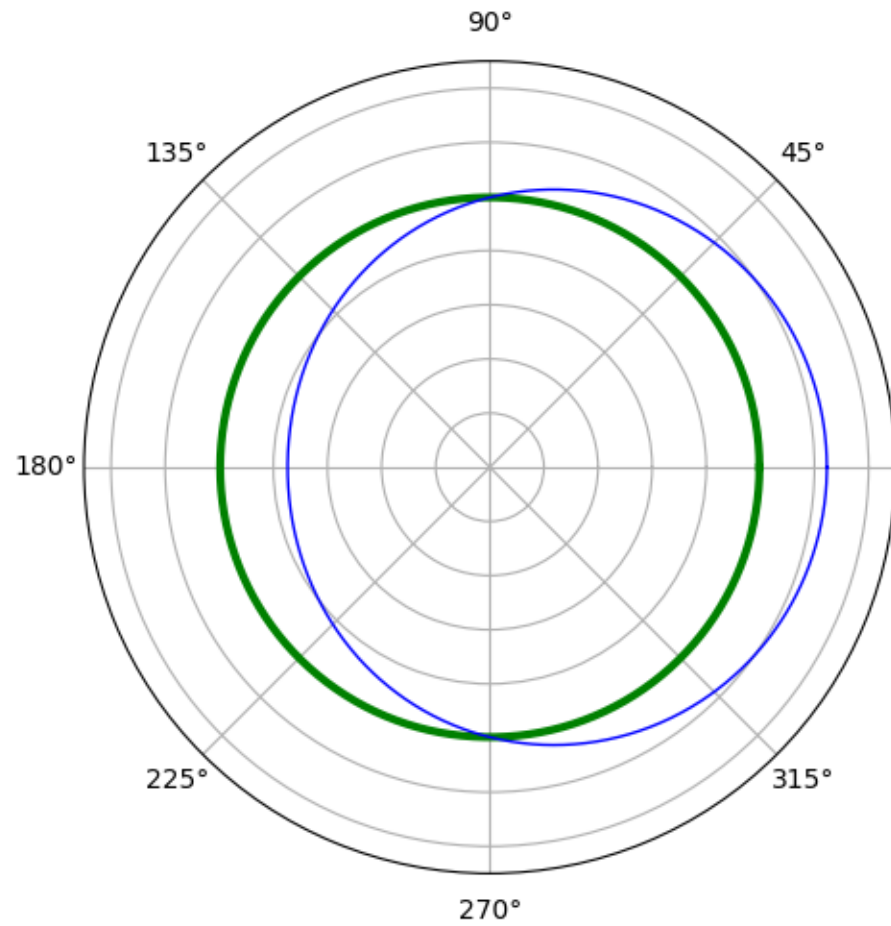
# □ Moments of the distribution



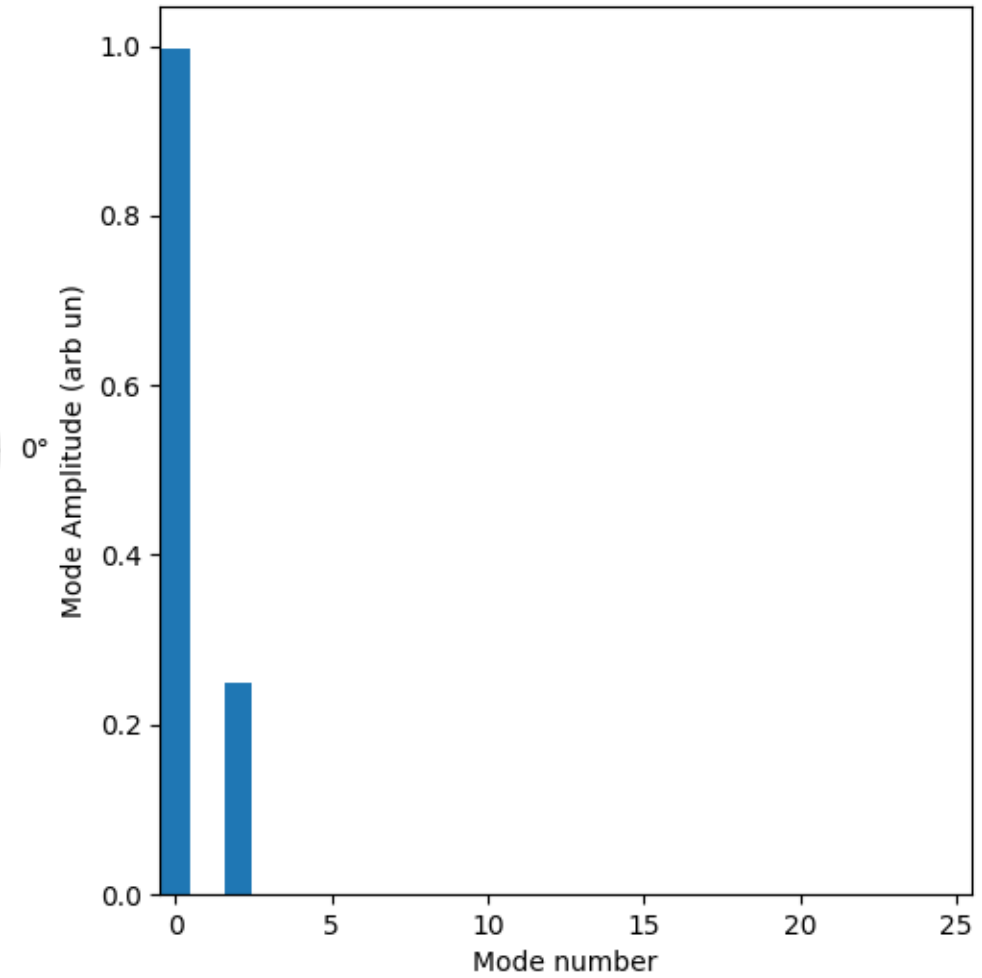
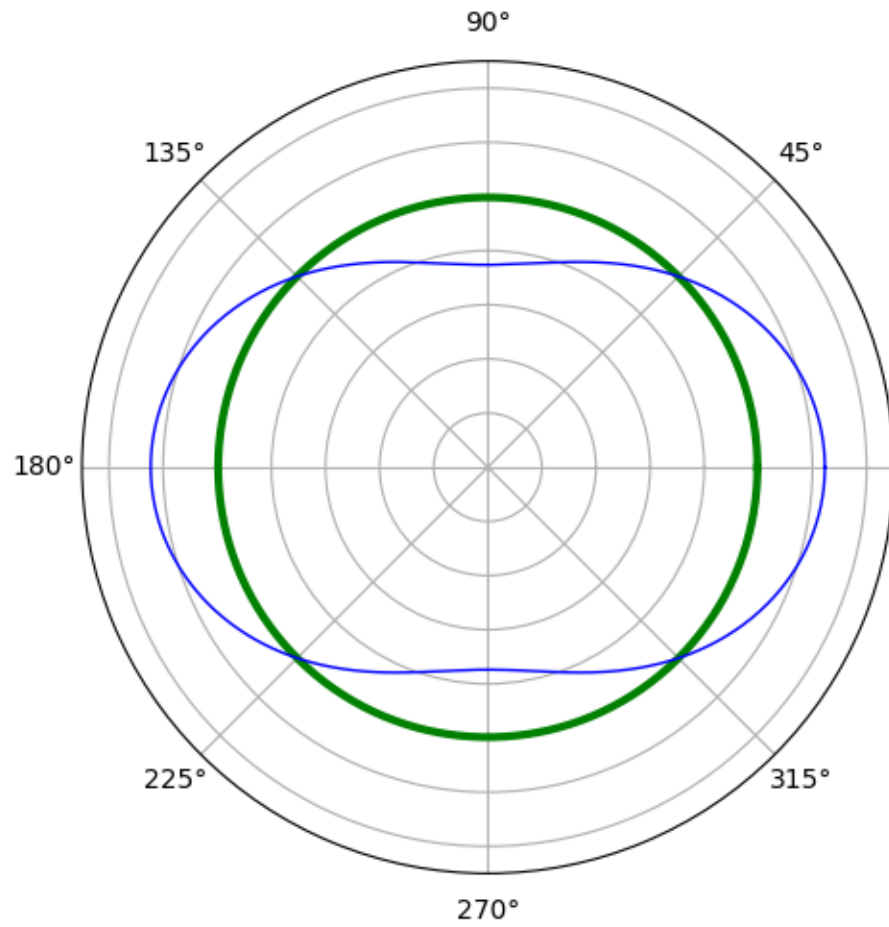
# □ Moments of the distribution



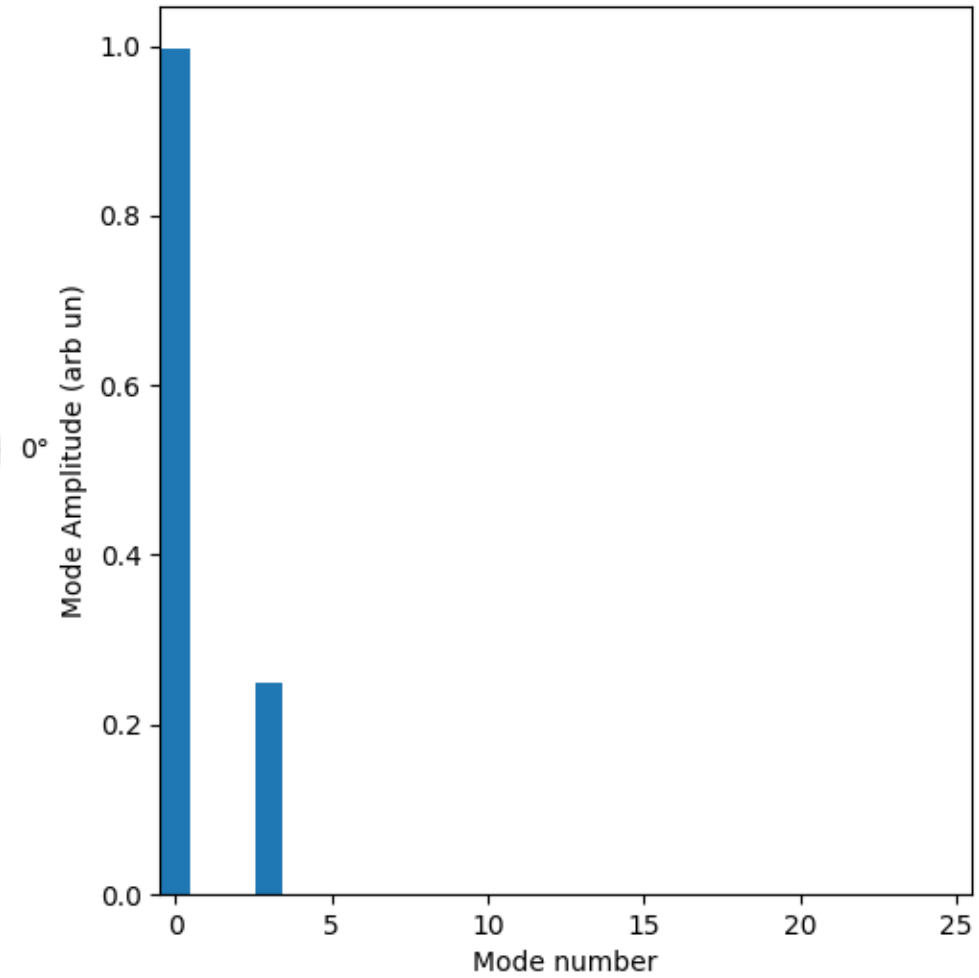
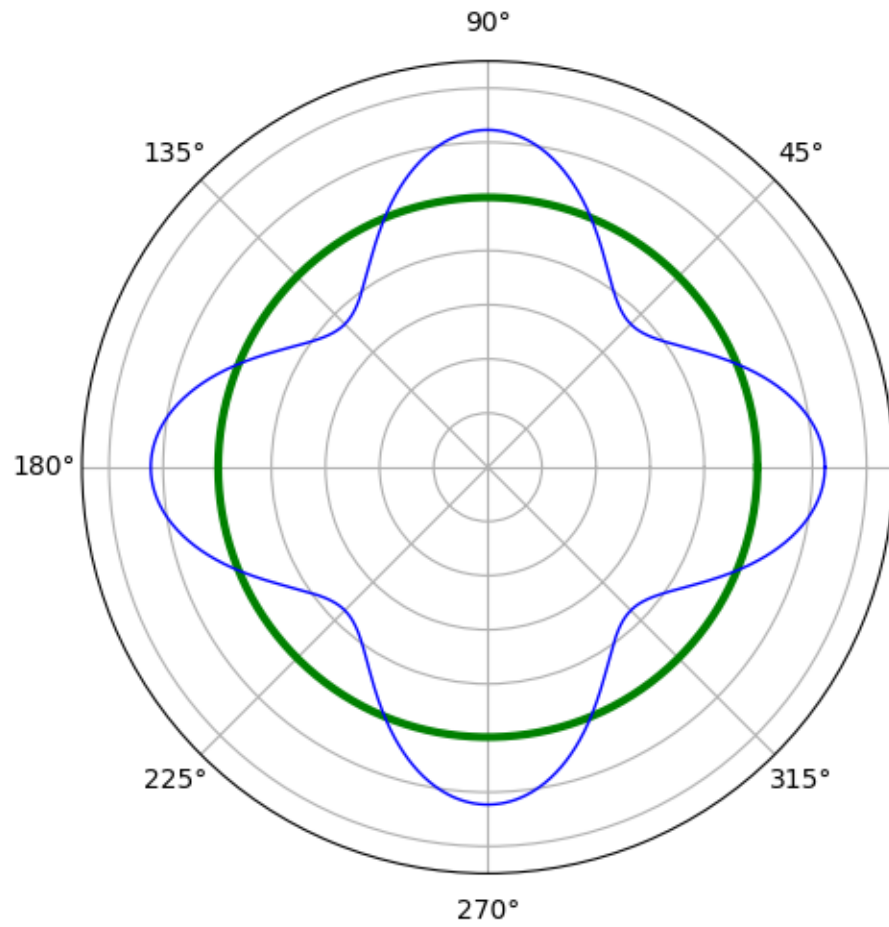
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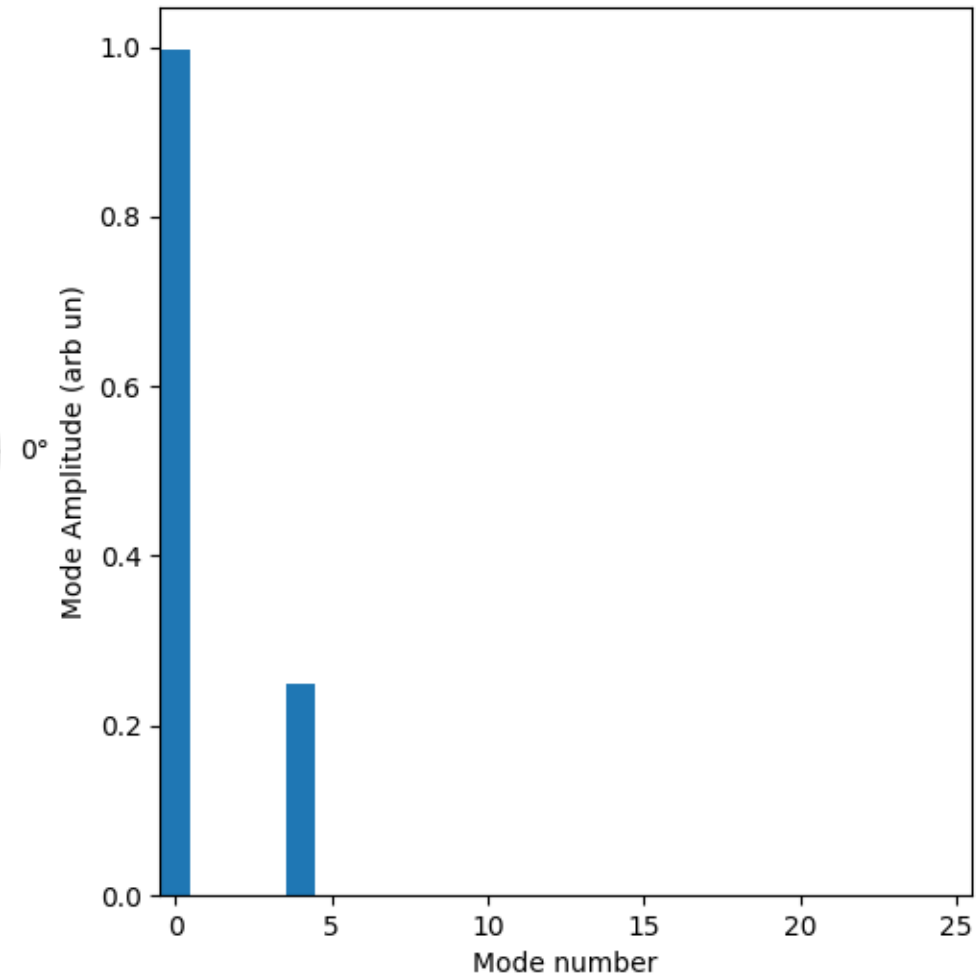
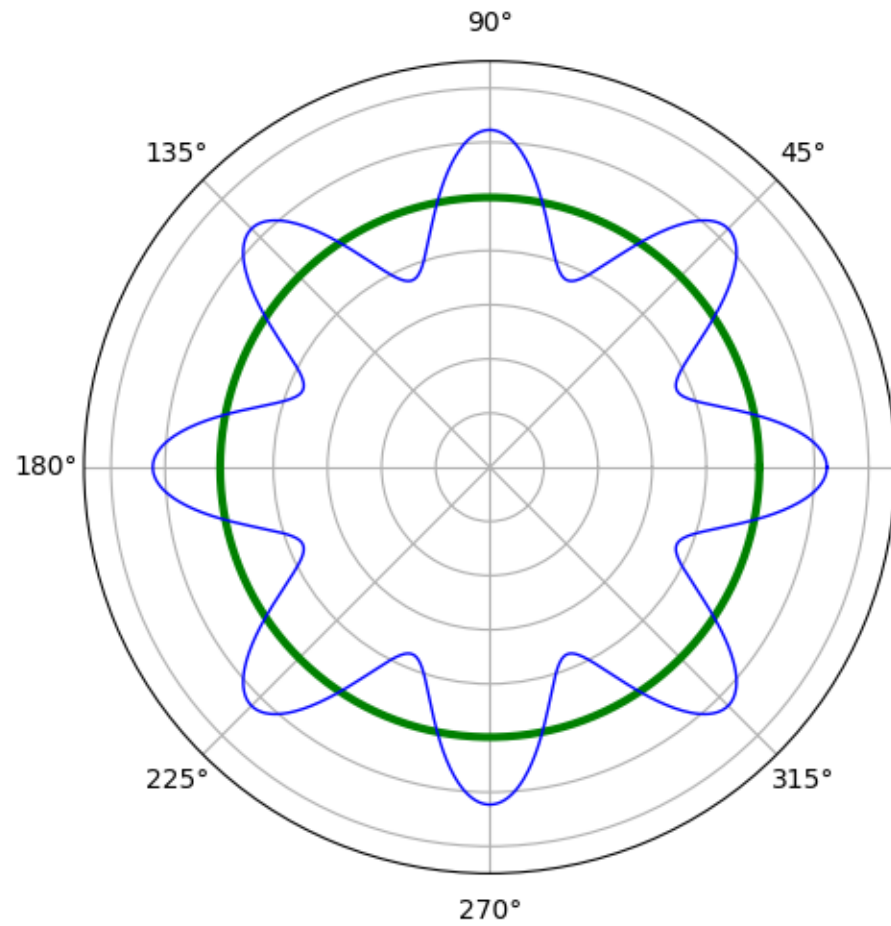
# □ Moments of the distribution



# □ Moments of the distribution

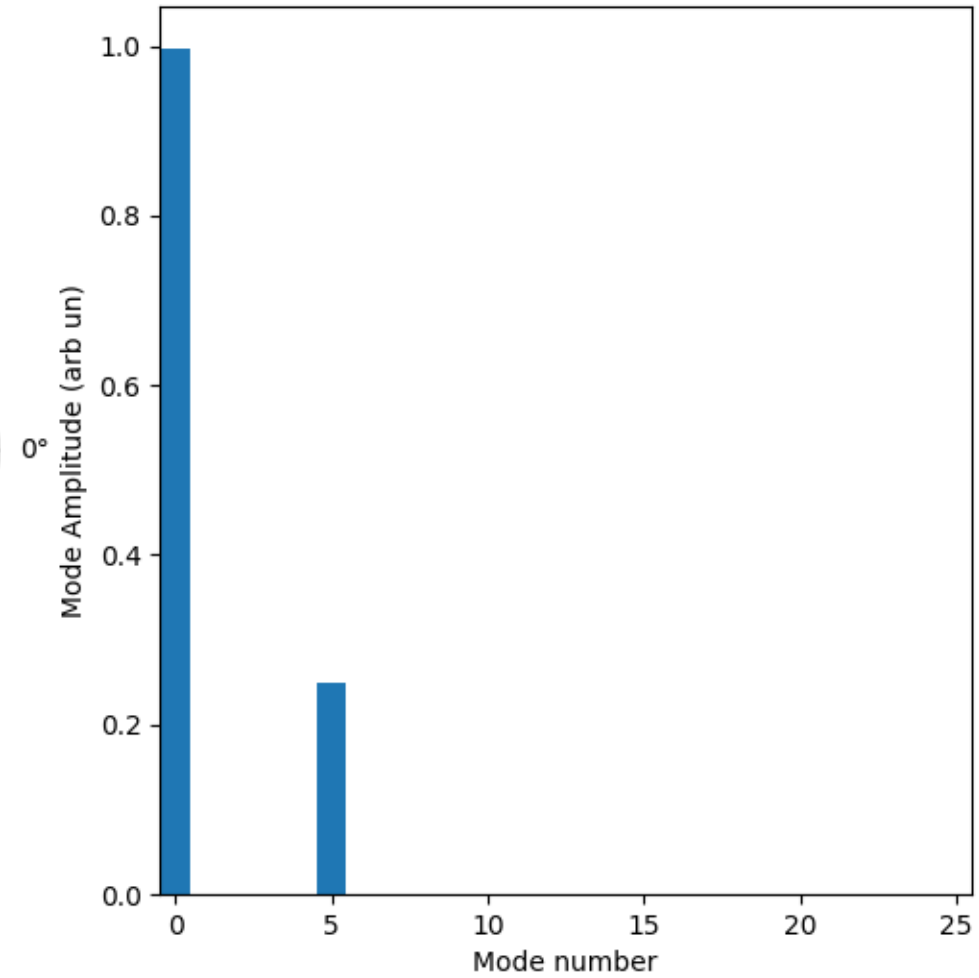
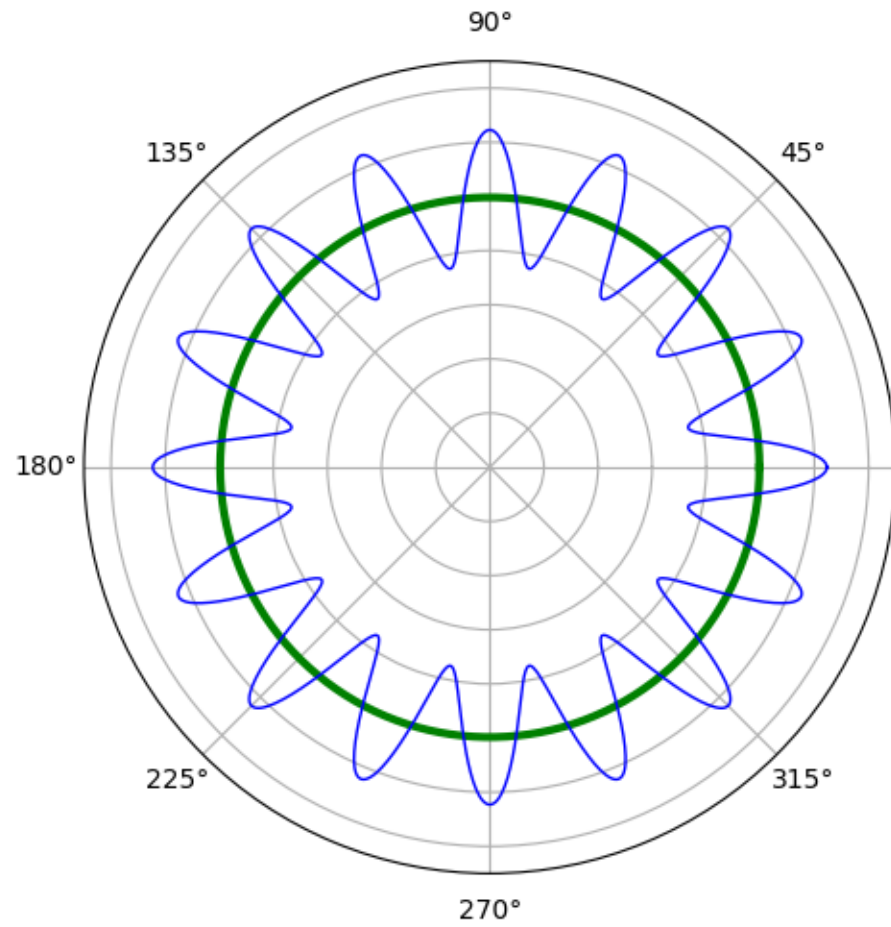


# □ Moments of the distribution

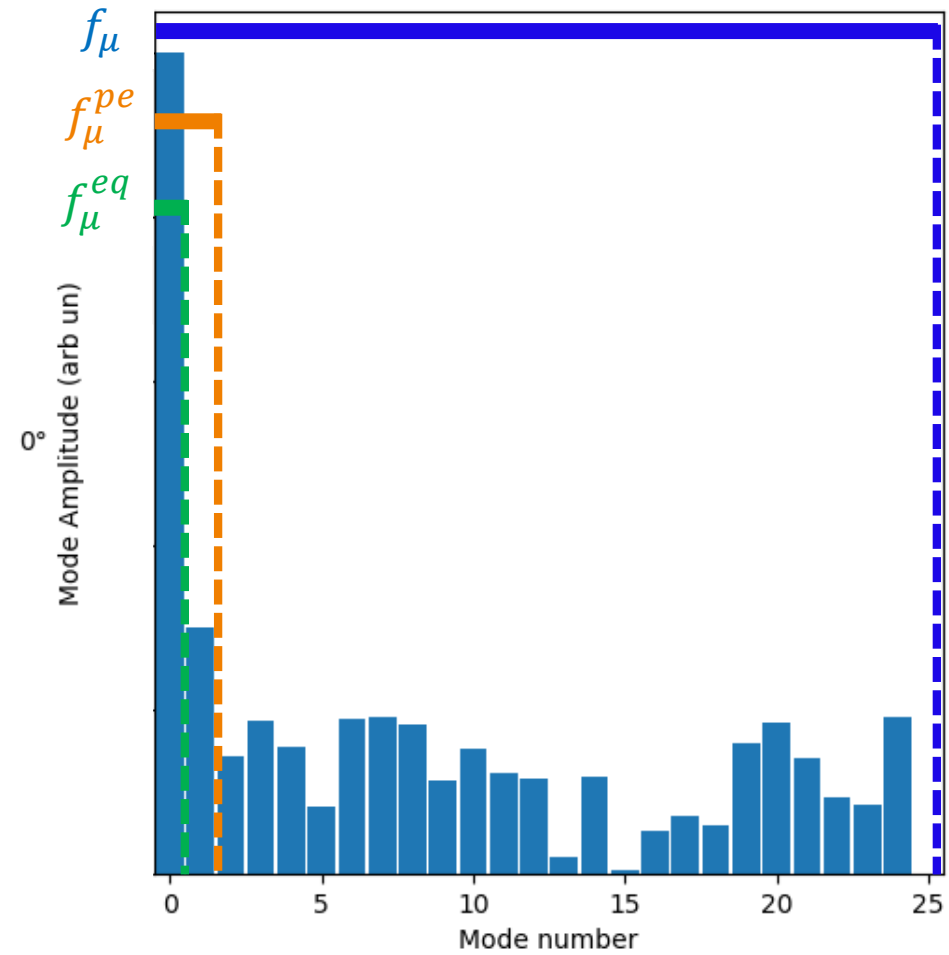
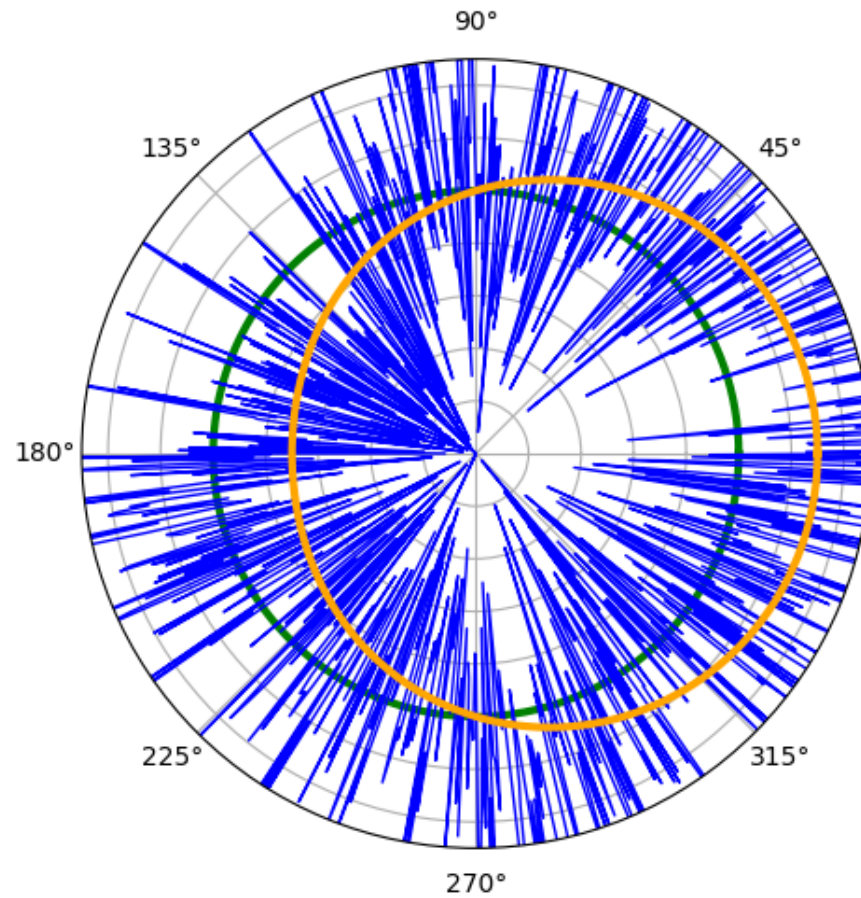




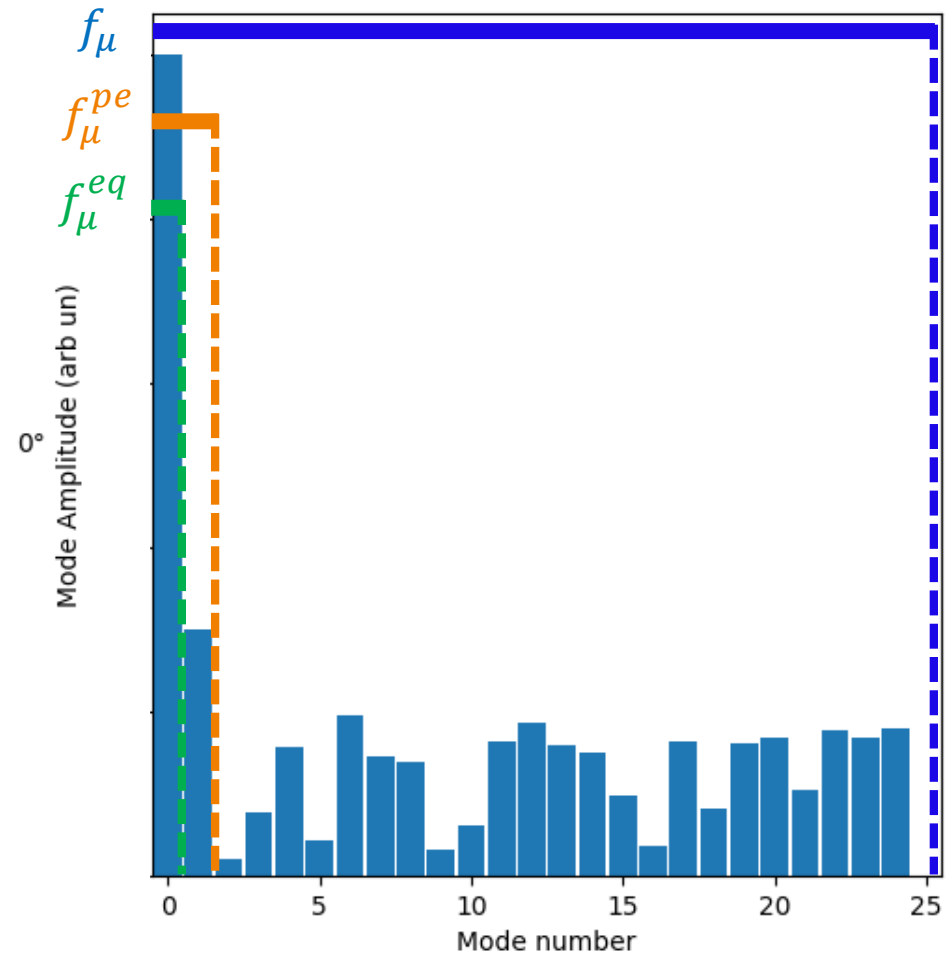
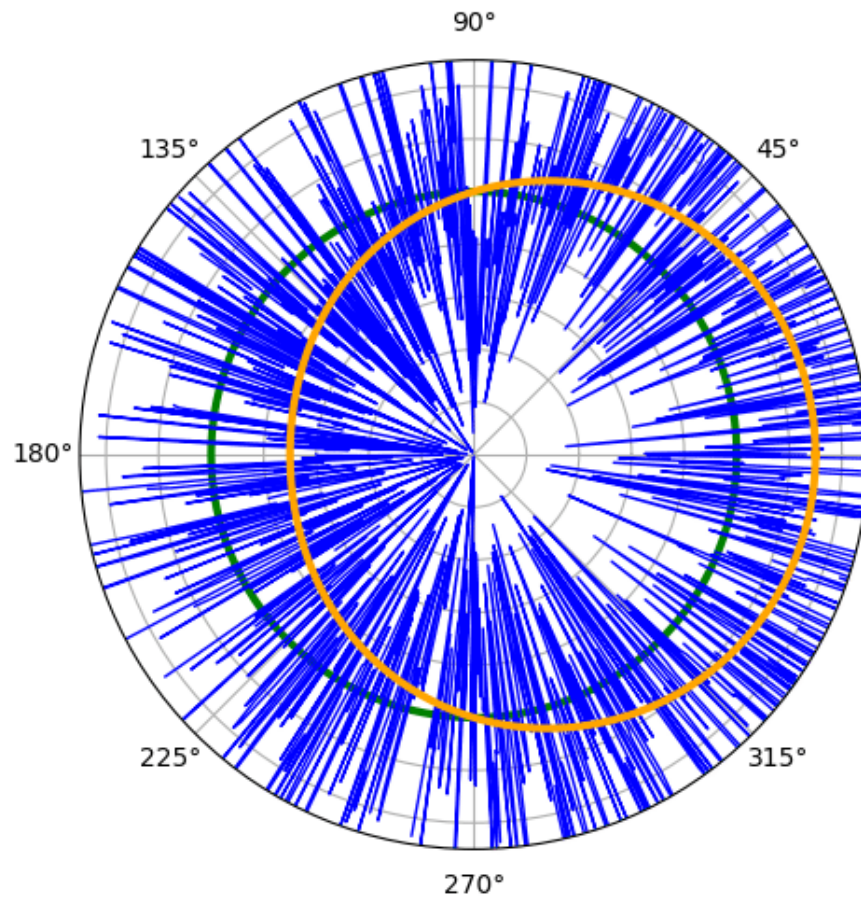
# □ Moments of the distribution



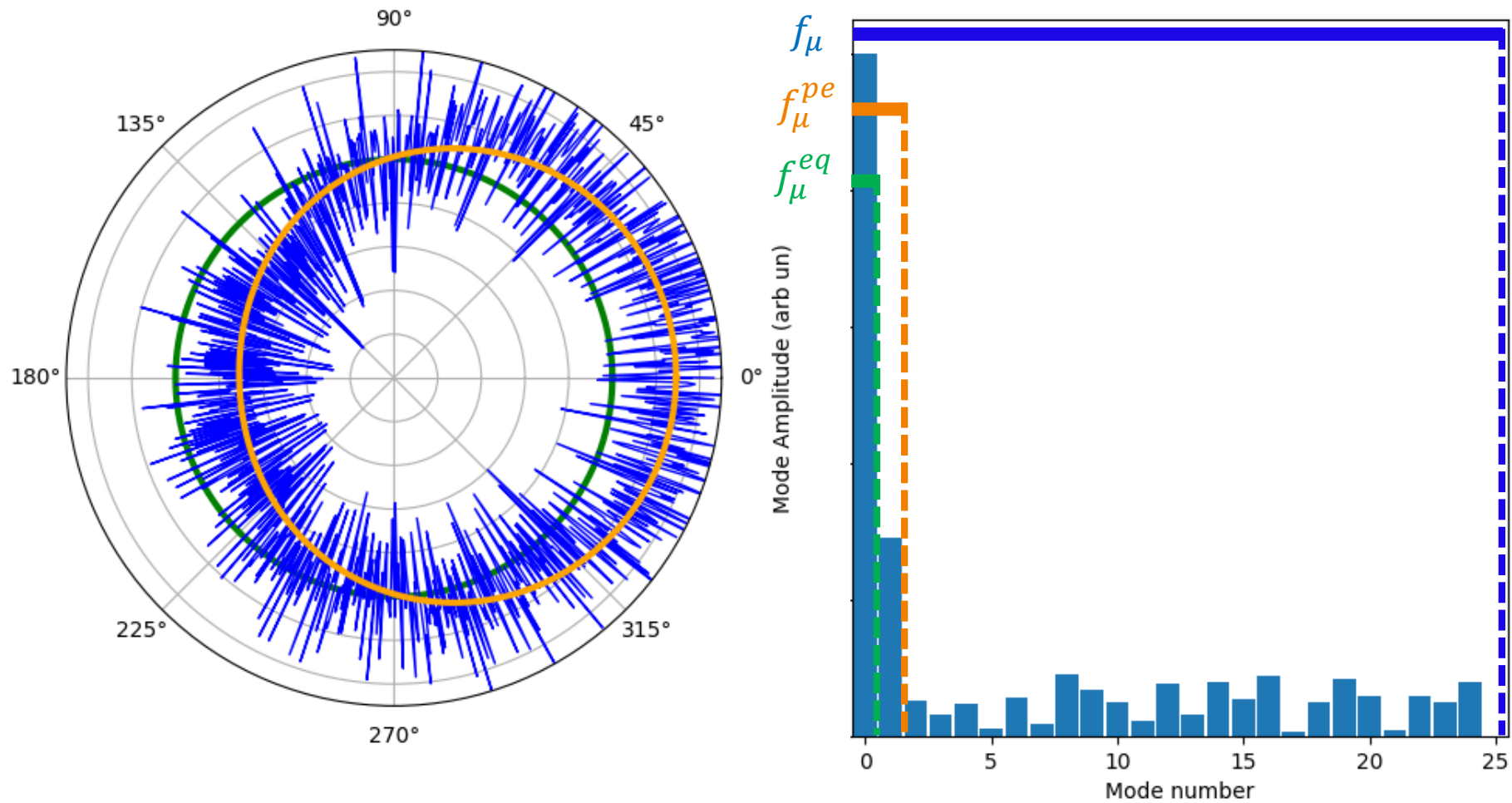
# □ (Pseudo)conserved magnitudes



# □ (Pseudo)conserved magnitudes



# □ (Pseudo)conserved magnitudes



# Fourier's law

$$\alpha_1 \frac{\partial T}{\partial t} = -\nabla \cdot \mathbf{q}$$

$$\alpha_2 \frac{\partial \mathbf{q}}{\partial t} - \frac{\mathbf{q}}{\tau_q} = -\nabla \cdot \mathbf{Q}^{(2)} - \beta_1 \nabla T$$

$$\alpha_3 \frac{\partial \mathbf{Q}^{(2)}}{\partial t} - \frac{\mathbf{Q}^{(2)}}{\tau_{Q^{(2)}}} = -\nabla \cdot \mathbf{Q}^{(3)} - \beta_2 \nabla \mathbf{q}$$

## THERMODYNAMIC EQUATIONS

$$c_v \frac{\partial T}{\partial t} + \nabla \cdot \mathbf{q} = 0$$

$$\mathbf{q} = -\lambda \nabla T$$

## BTE DISTRIBUTION FUNCT.

$$f = f_{eq} - \frac{3}{c_v v^2} \mathbf{q} \cdot \mathbf{v}_g \frac{\partial f_{eq}}{\partial T}$$

# □ Guyer and Krumhansl equation

$$\alpha_1 \frac{\partial T}{\partial t} = -\nabla \cdot \mathbf{q}$$

$$\alpha_2 \frac{\partial \mathbf{q}}{\partial t} - \frac{\mathbf{q}}{\tau_q} = -\nabla \cdot \mathbf{Q}^{(2)} - \beta_1 \nabla T$$

$$\alpha_3 \frac{\partial \mathbf{Q}^{(2)}}{\partial t} - \frac{\mathbf{Q}^{(2)}}{\tau_{Q^{(2)}}} = -\nabla \cdot \mathbf{Q}^{(3)} - \beta_2 \nabla \mathbf{q}$$

## THERMODYNAMIC EQUATIONS

$$c_v \frac{\partial T}{\partial t} + \nabla \cdot \mathbf{q} = 0$$

$$\mathbf{q} = -\lambda \nabla T - A_1 \nabla \cdot \mathbf{Q}^{(2)}$$

$$\mathbf{Q}^{(2)} = A_2 \nabla \mathbf{q}$$

## BTE DISTRIBUTION FUNCT.

$$f \simeq f_{eq} - \frac{3}{c_v v_g^2} \frac{\partial f_{eq}}{\partial T} q_i v_{gi} + \frac{\tau}{c_v} \frac{\partial q_i}{\partial x_i} \frac{\partial f_{eq}}{\partial T}$$

# □ Guyer and Krumhansl equation

$$\alpha_1 \frac{\partial T}{\partial t} = -\nabla \cdot \mathbf{q}$$

$$\alpha_2 \frac{\partial \mathbf{q}}{\partial t} - \frac{\mathbf{q}}{\tau_q} = -\nabla \cdot \mathbf{Q}^{(2)}$$

$$\alpha_3 \frac{\partial \mathbf{Q}^{(2)}}{\partial t} - \frac{\mathbf{Q}^{(2)}}{\tau_{Q^{(2)}}} = -\nabla \cdot \mathbf{Q}^{(3)}$$

## NEW BOUNDARY CONDITION

GK is a second order PDE, a new boundary condition is needed in order to be solvable

## THERMODYNAMIC EQUATIONS

$$c_v \frac{\partial T}{\partial t} + \nabla \cdot \mathbf{q} = 0$$

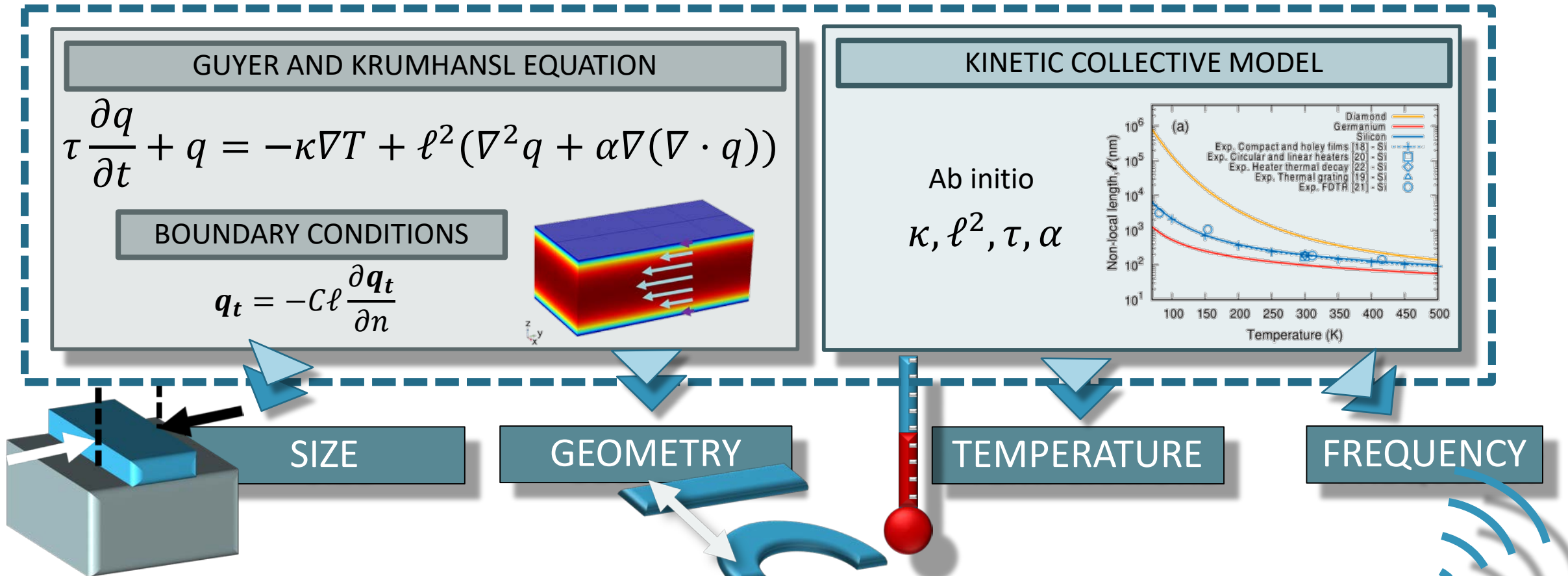
$$\mathbf{q} = -\lambda \nabla T - \ell^2 (\nabla^2 \mathbf{q} + 2\nabla \nabla \cdot \mathbf{q})$$

## BTE DISTRIBUTION FUNCT.

$$f \simeq f_{eq} - \frac{3}{c_v v_g^2} \frac{\partial f_{eq}}{\partial T} q_i v_{gi} + \frac{\tau}{c_v} \frac{\partial q_i}{\partial x_i} \frac{\partial f_{eq}}{\partial T}$$

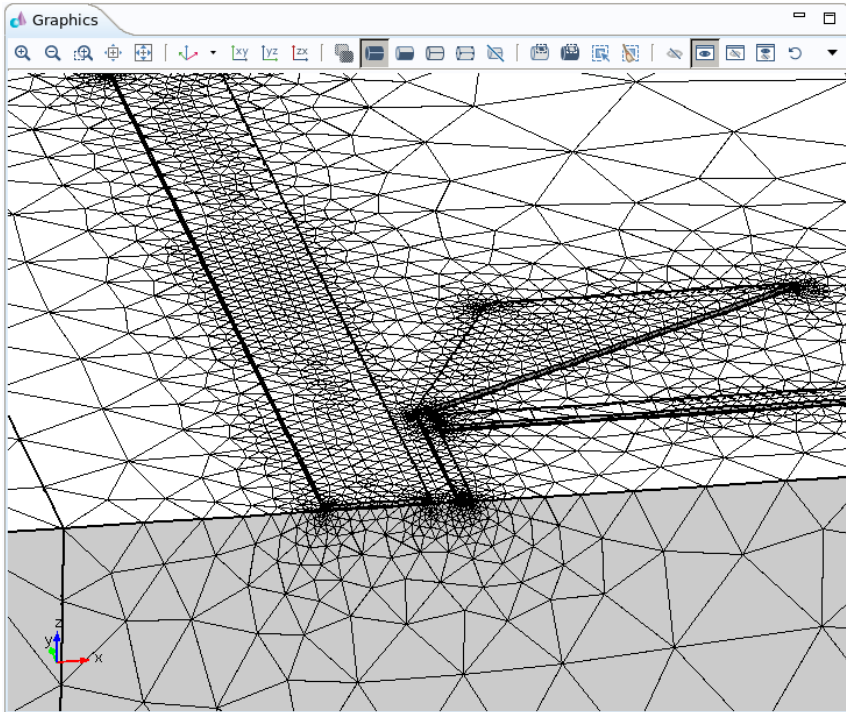
# □ GK-ab initio formalism

Combination of the **Guyer and Krumhansl** equation with ab initio calculations for the parameters in the framework of **Kinetic Collective Model** offers a **full predictive** model for materials like silicon





# COMSOL module



- graph\_phon\_crys4.mph (root)
  - Global Definitions
    - Parameters
    - Materials
  - Component 1 (comp1)
    - Definitions
    - Geometry 1
    - Materials
    - Nanoscale Heat Transfer - Kinetic Collective Model (kcm)
      - Initial Values 1
      - Hydrodynamic Heat Transfer 1**
      - Slip Boundary Condition 1
      - Periodic Heat Flux Condition 1
      - Pointwise Constraint 1
      - Temperature BC 1
      - Energy Source 1
      - Equation View
      - Heat Transfer in Solids (ht)
        - Mesh 1
      - Study KCM
      - Study Fourier
    - Results
      - Data Sets
      - Derived Values
      - Tables
      - Temperature (ht)
      - Isothermal Contours (ht)

Label: Hydrodynamic Heat Transfer 1

**Domain Selection**

Selection: Manual

<input checked="" type="checkbox"/>	1
<input type="checkbox"/>	2

Active

**Override and Contribution**

**Equation**

Show equation assuming:  
Study KCM, Time Dependent

$$C_v \frac{\partial T}{\partial t} + \nabla \cdot \mathbf{q} = Q$$

$$\mathbf{q} + \tau \frac{\partial \mathbf{q}}{\partial t} + k \nabla T = l^2 (\nabla^2 \mathbf{q} + 2 \nabla \nabla \cdot \mathbf{q})$$

**Heat Transfer Parameters**

Volumetric heat capacity:  $C_v$   J/(m<sup>3</sup>·K)

Thermal conductivity:  $k$   W/(m·K)

Hydrodynamic Length:  $l$   m

Hydrodynamic Time:  $\tau$   s

**EFFECTS OF THE  
GUYER AND KRUMHANSL  
TERMS**

# Fourier's law

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## TRANSPORT EQUATIONS

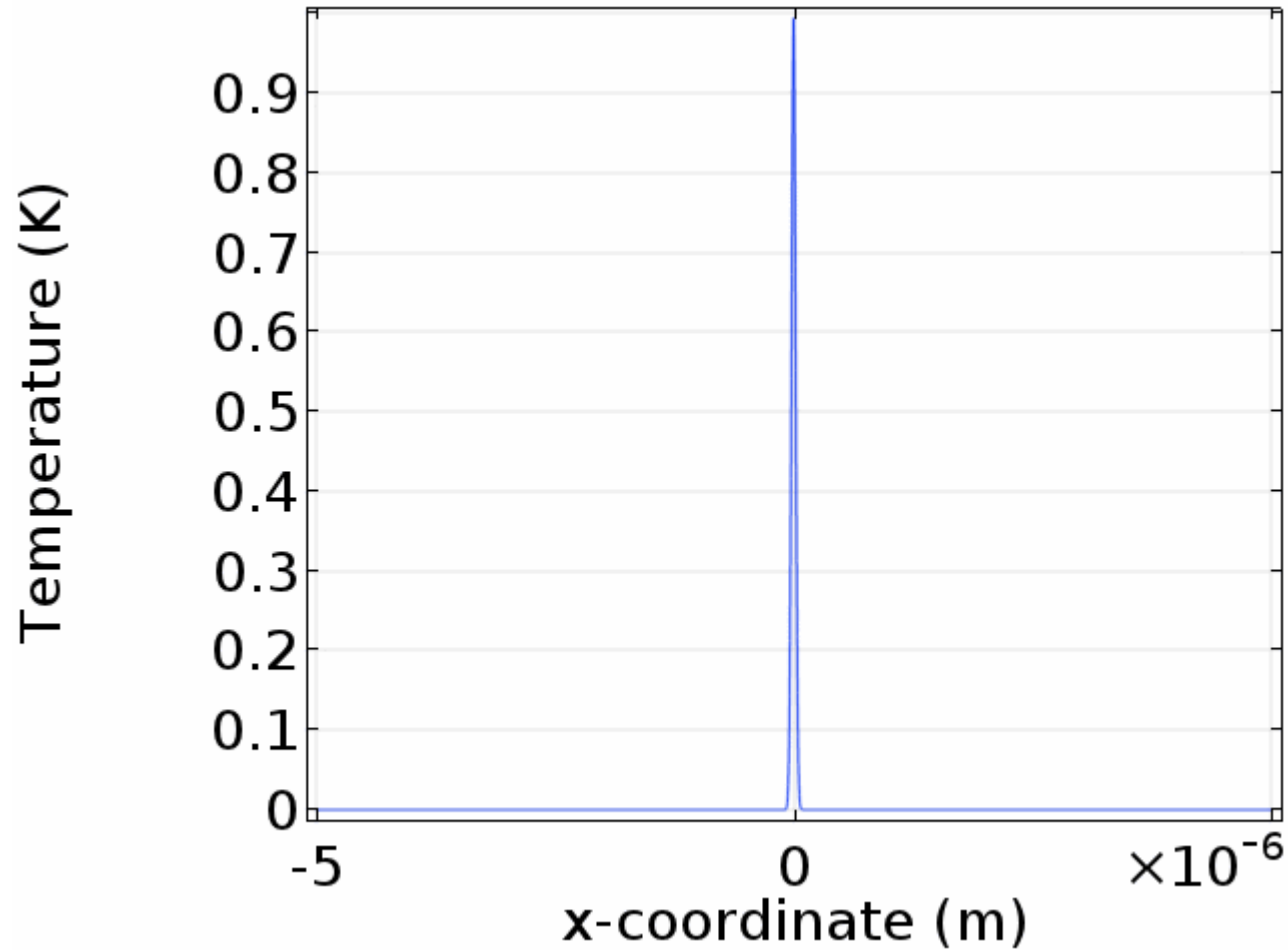
$$c_v \frac{\partial T}{\partial t} + \nabla \cdot \mathbf{q} = 0$$

$$\mathbf{q} = -\lambda \nabla T$$

## HEAT-DIFFUSION EQUATION

$$\frac{\partial T}{\partial t} = \chi \nabla^2 T$$

# Fourier's law



HEAT-DIFFUSION  
EQUATION

$$\frac{\partial T}{\partial t} = \chi \nabla^2 T$$

# Memory term

## TRANSPORT EQUATIONS

$$c_v \frac{\partial T}{\partial t} + \nabla \cdot \mathbf{q} = 0$$

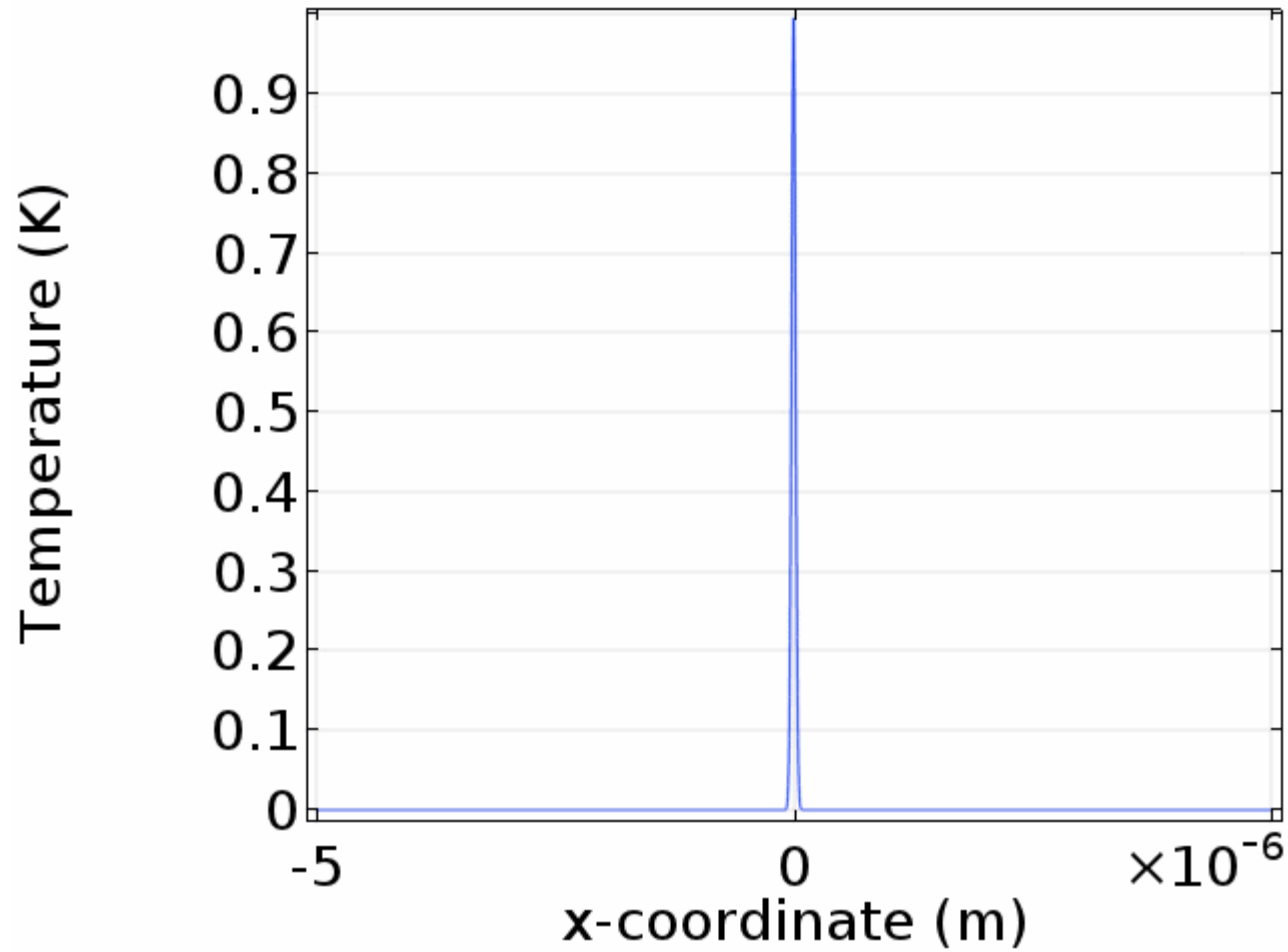
$$\tau \frac{\partial \mathbf{q}}{\partial t} + \mathbf{q} = -\lambda \nabla T$$

Fast excitation changes  $T \ll \tau$

## MAXWELL-CATTANEO EQUATION

$$\tau \frac{\partial^2 T}{\partial t^2} + \frac{\partial T}{\partial t} = \chi \nabla^2 T$$

## Memory term



MAXWELL-CATTANEO  
EQUATION

$$\tau \frac{\partial^2 T}{\partial t^2} + \frac{\partial T}{\partial t} = \chi \nabla^2 T$$

## □ Nonlocal term

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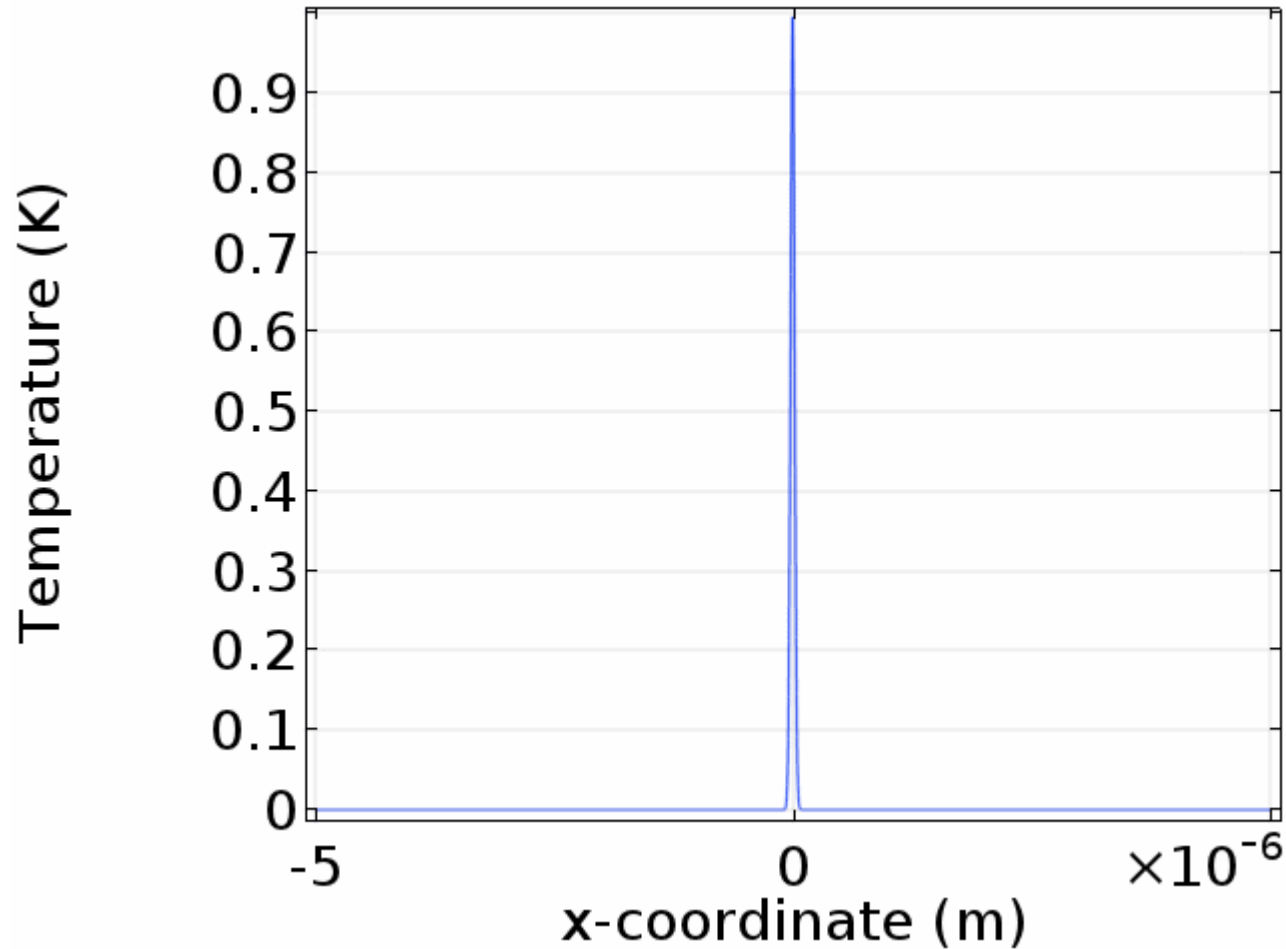
Steep spatial variations  $L \ll \ell$

TRANSPORT EQUATIONS

$$c_v \frac{\partial T}{\partial t} + \nabla \cdot \mathbf{q} = 0$$

$$\mathbf{q} = -\lambda \nabla T + \ell \nabla^2 \mathbf{q}$$

## □ Nonlocal term



### TRANSPORT EQUATIONS

$$c_v \frac{\partial T}{\partial t} + \nabla \cdot \mathbf{q} = 0$$

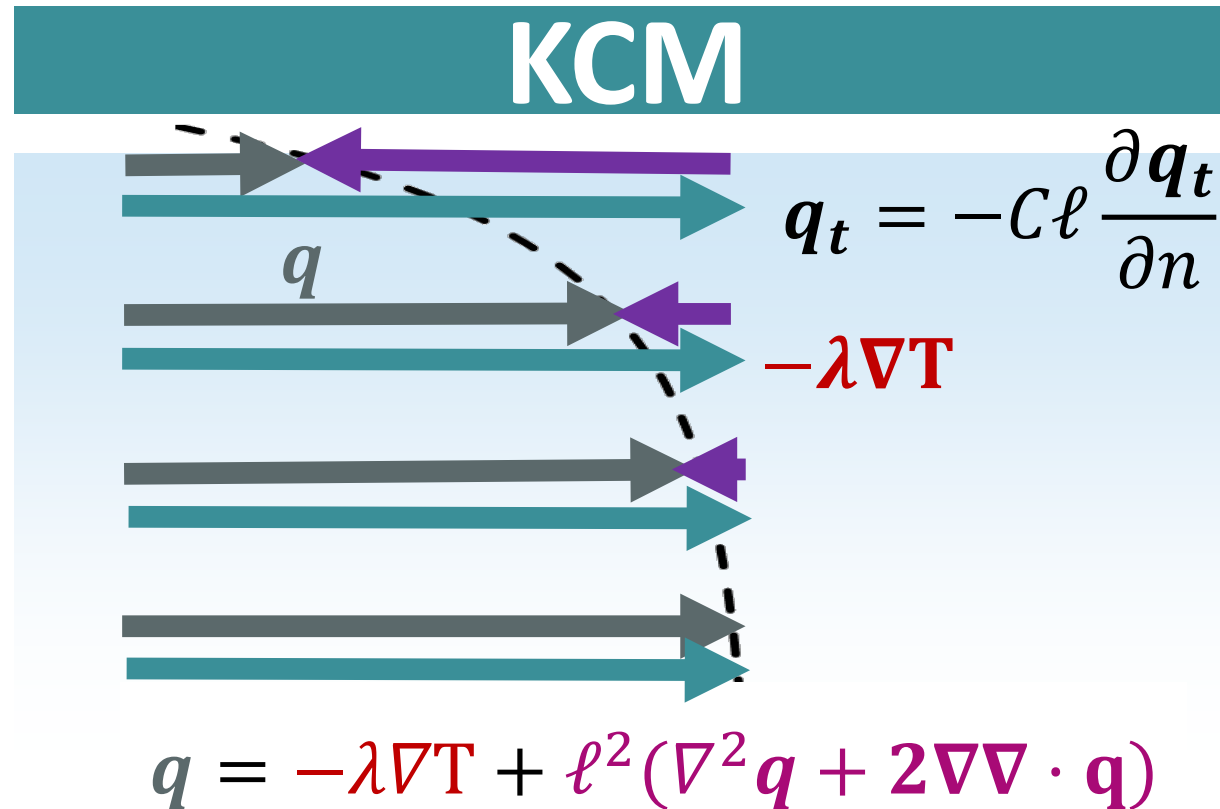
$$\mathbf{q} = -\lambda \nabla T + \ell \nabla^2 \mathbf{q}$$



# **APPLICATIONS**

**KCM VS KINETIC FORMALISM 1:  
SIZE EFFECTS**

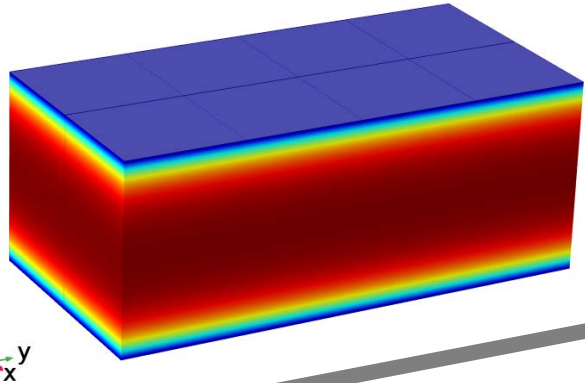
# Hydrodynamic effects I: Boundaries



The boundary condition is applied directly to the heat flux in a consistent way with respect to the transport equation

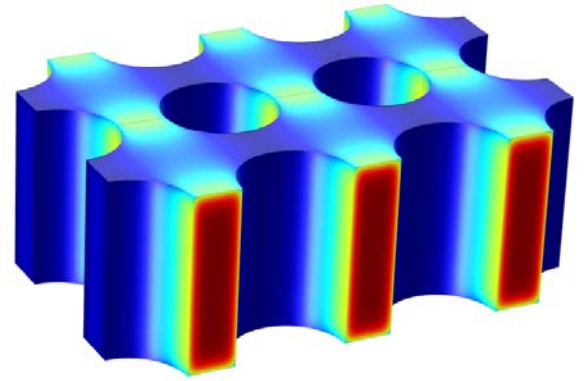
# Applicability of hydrodynamic ab initio model

## Thin films

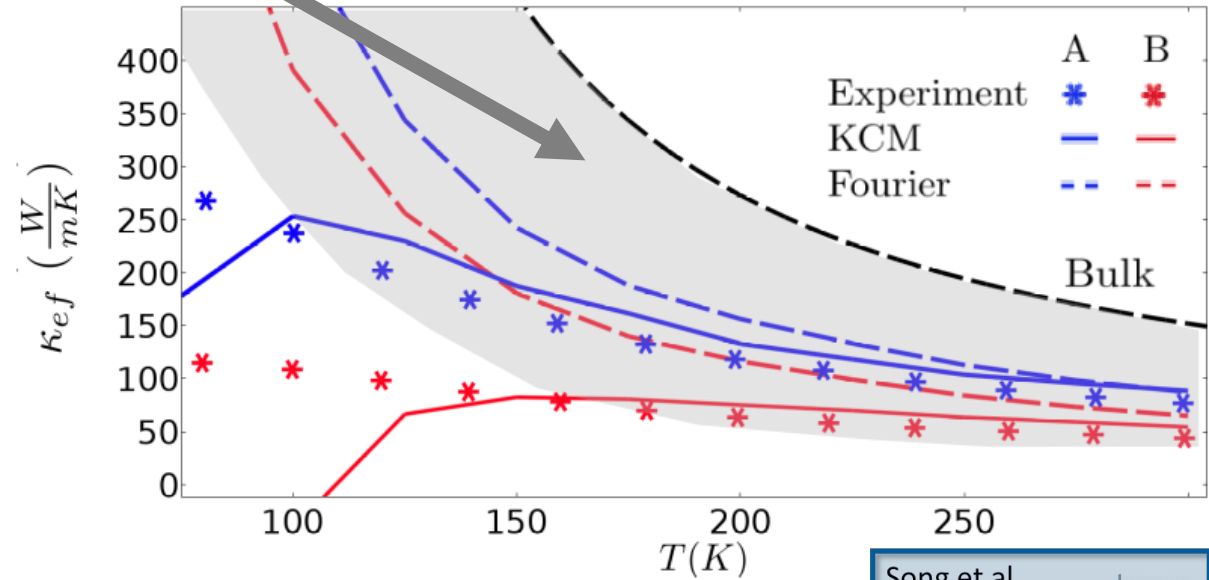
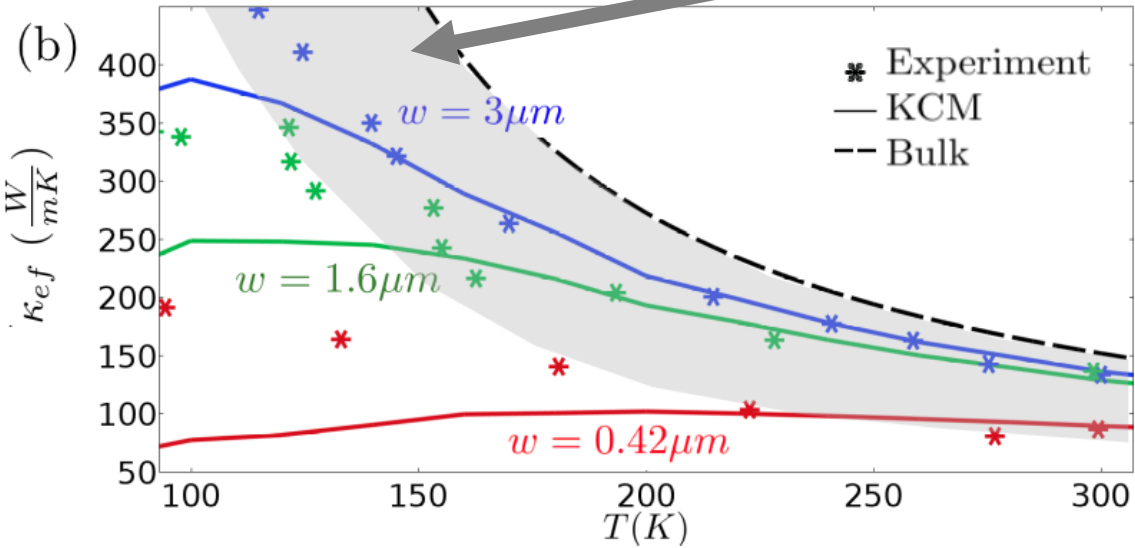


Asheghi et al.  
*J. Appl. Phys.*,  
**91**, 5079 (2002)

## Holey films



Region of predictability  
 $L < 2\ell$



Silicon  $\ell = 180 \text{ nm}$

Song et al.  
*Appl. Phys. Lett.*,  
**84**, 687 (2004)

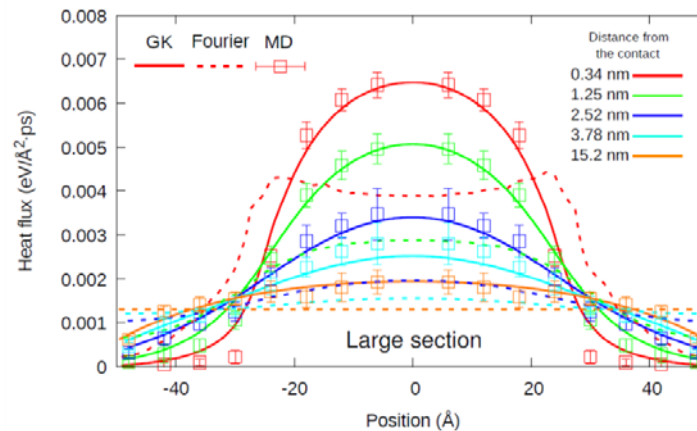
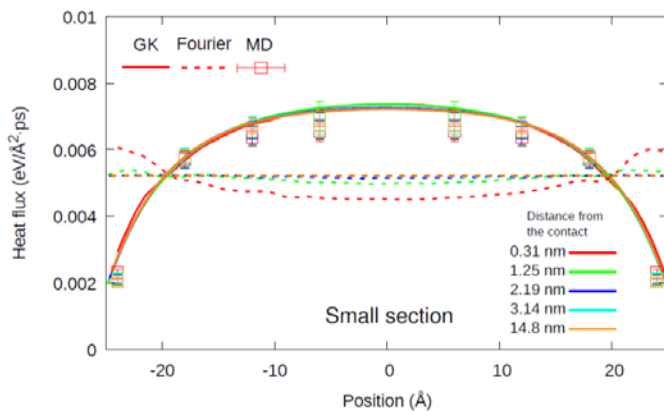
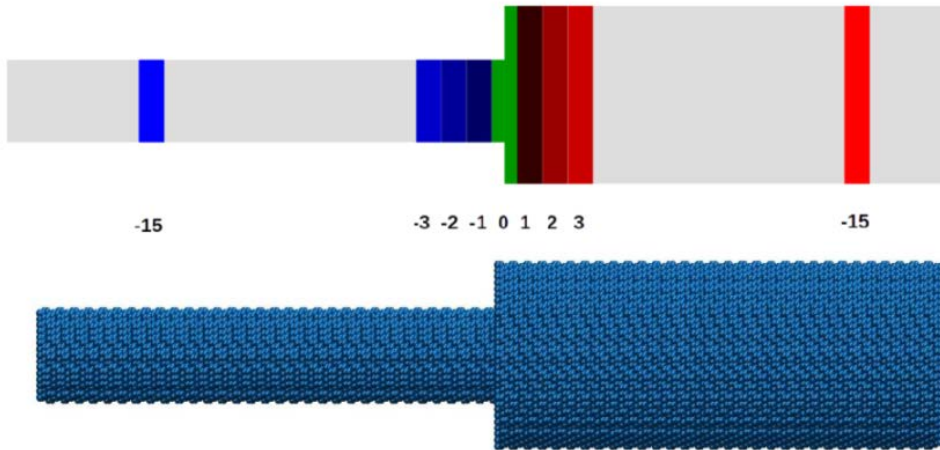
# Curved heat flow in MC, MD and FE



Melis et al.

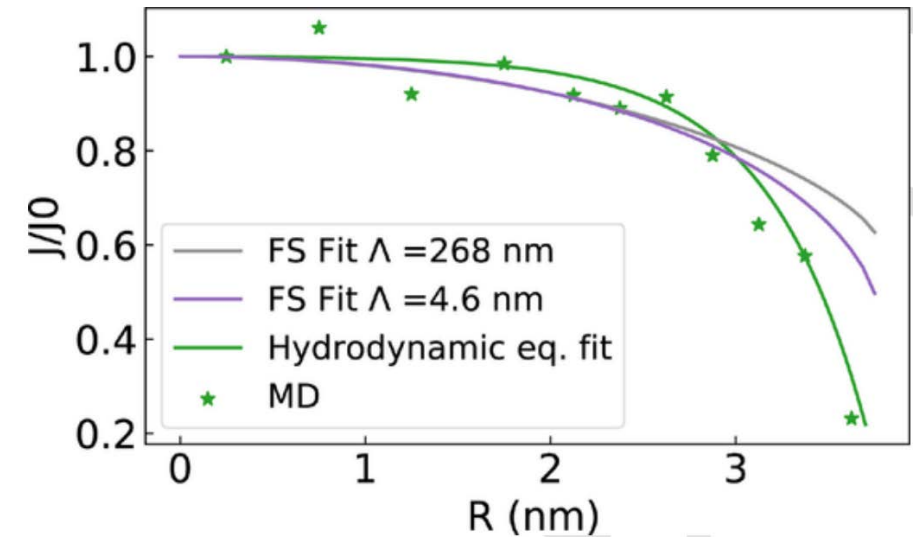
*Phys. Rev. Appl.*, **11**, 054059 (2019)

Positions where the flux is calculated



Desmarchelier et al.

*IJHMT*, **194** 123003 (2022)



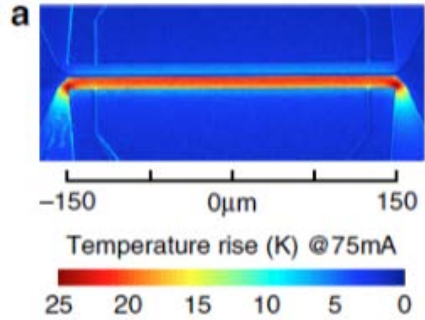
**KCM VS KINETIC FORMALISM 2:  
THERMAL BOUNDARY RESISTANCE**

# Thermoreflectance Imaging (TRI)

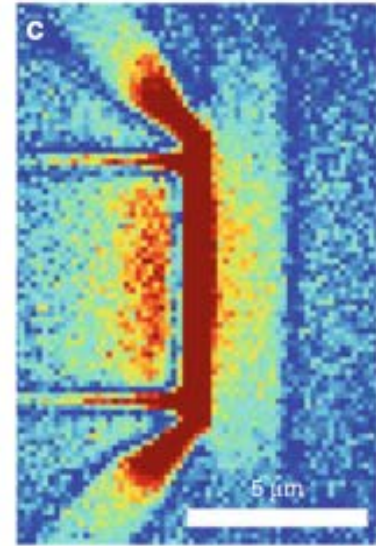
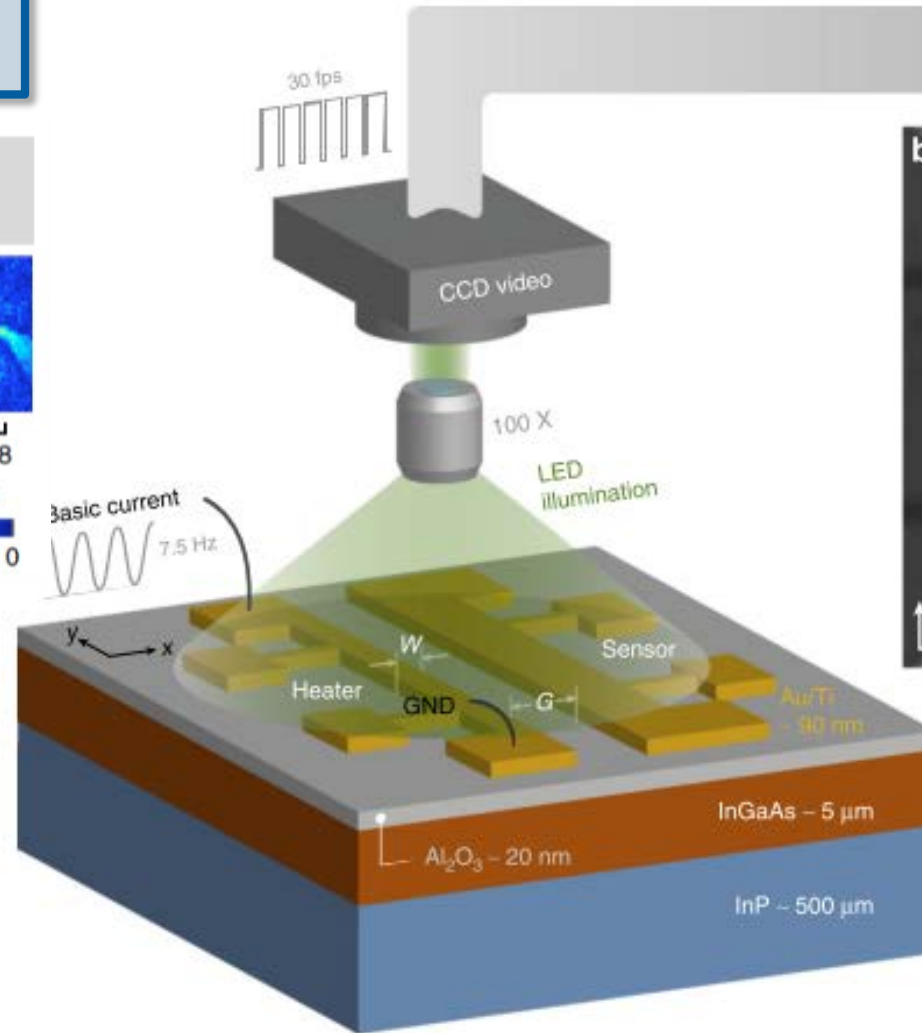
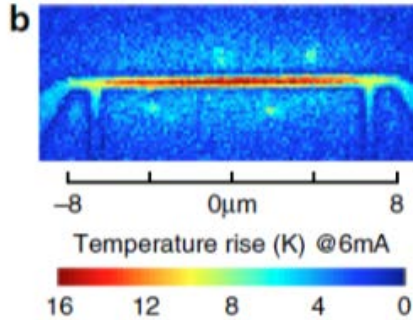


Ziabari et al.  
Nat. Commun. **9**, 255 (2018)

"Large" device  
 $W = 10\mu\text{m}$



"Small" device  
 $W = 400\text{nm}$



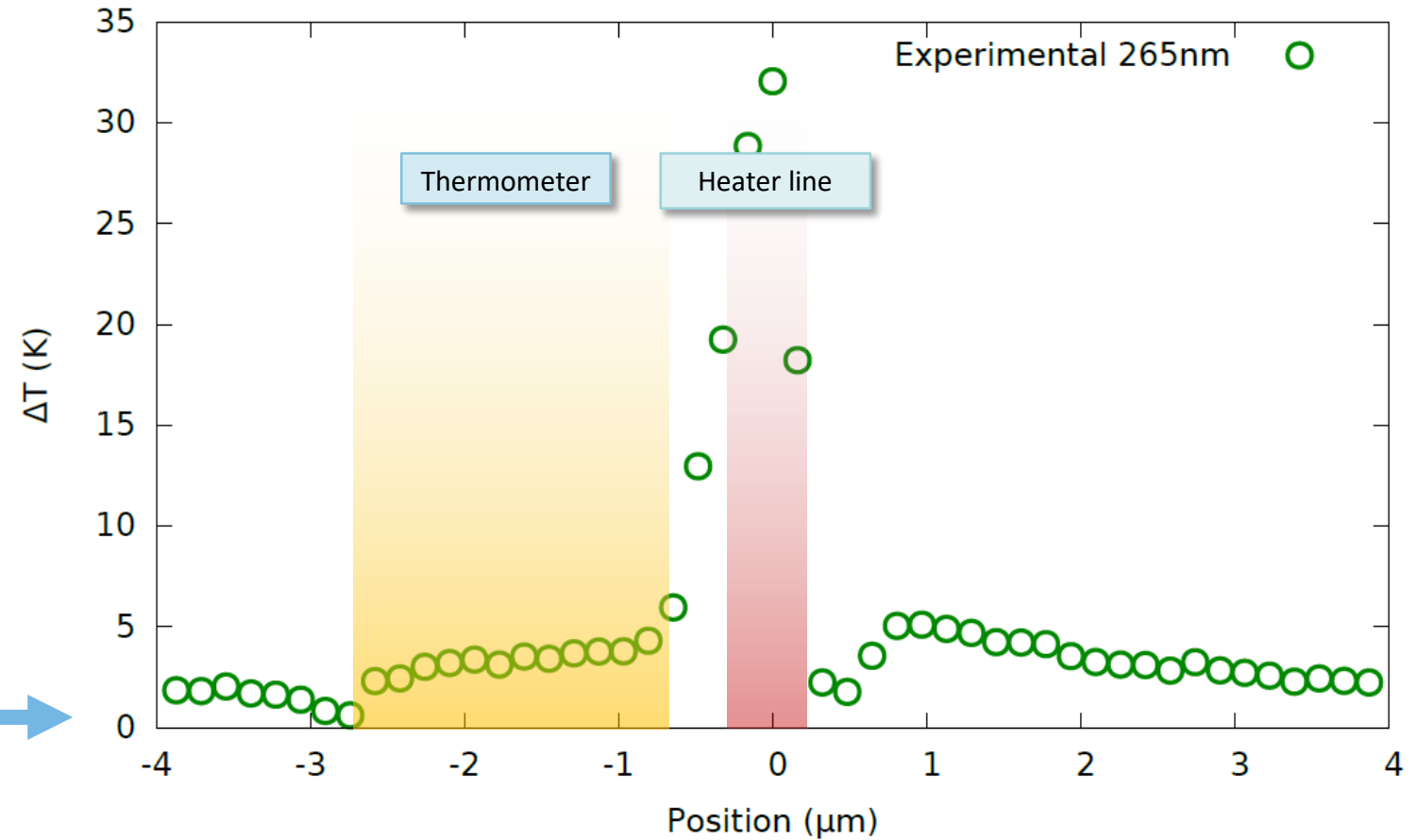
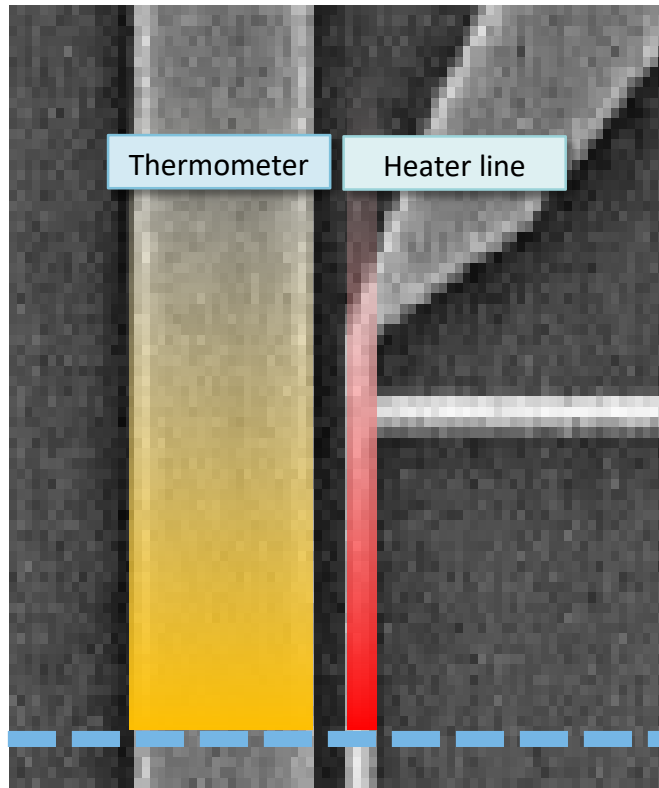
**P**  
PURDUE  
UNIVERSITY



Ali  
Shakouri

BOULDER, JULY 2022

# Experimental Data

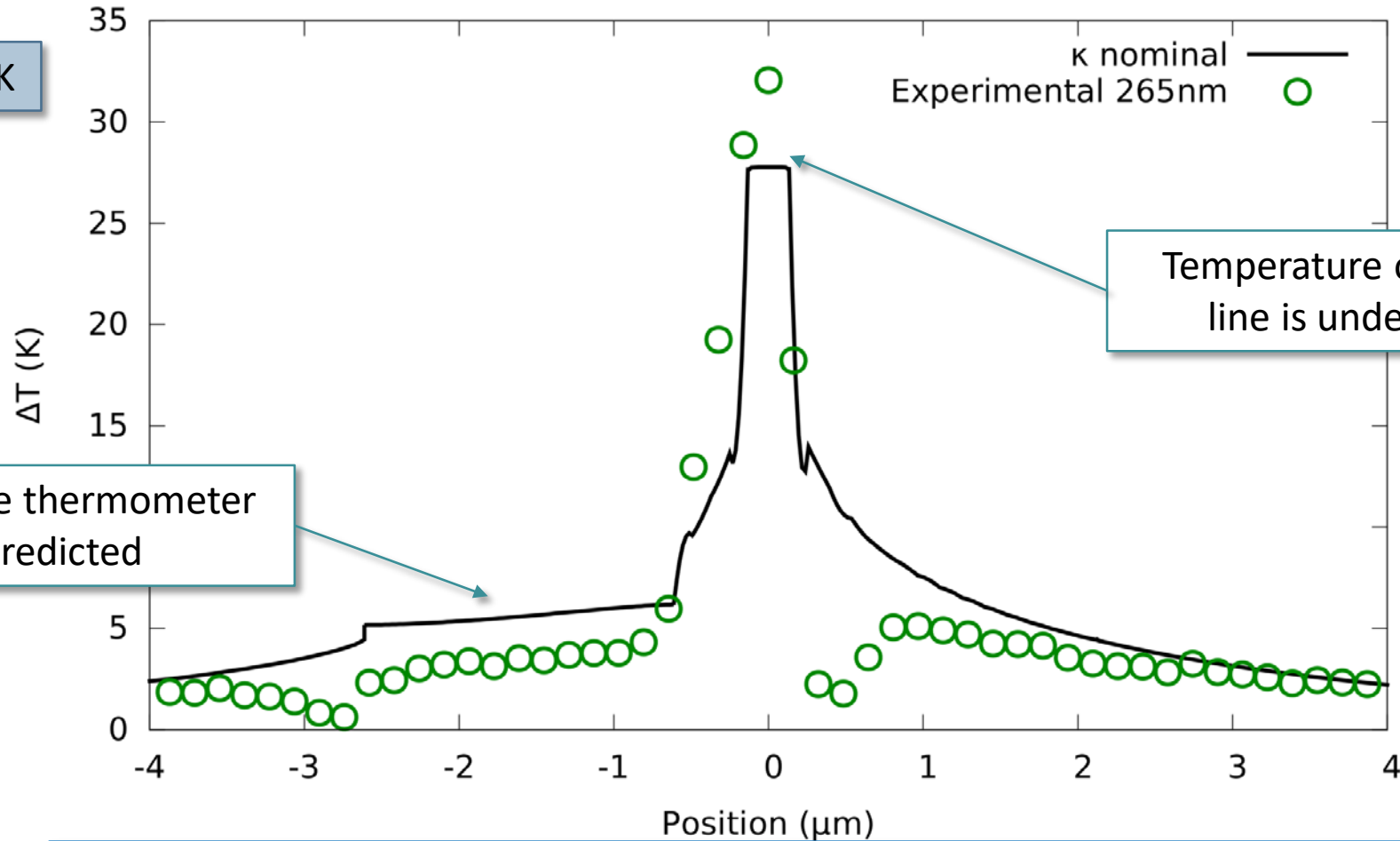


We obtain a thermal map of the surface of the sample using the optical setup. Heater line and thermometer are also obtained using electrical measurements.



# Fourier's law test (I)

$$\kappa_{\text{InGaAs}} = 5.4 \text{ W/m}\cdot\text{K}$$



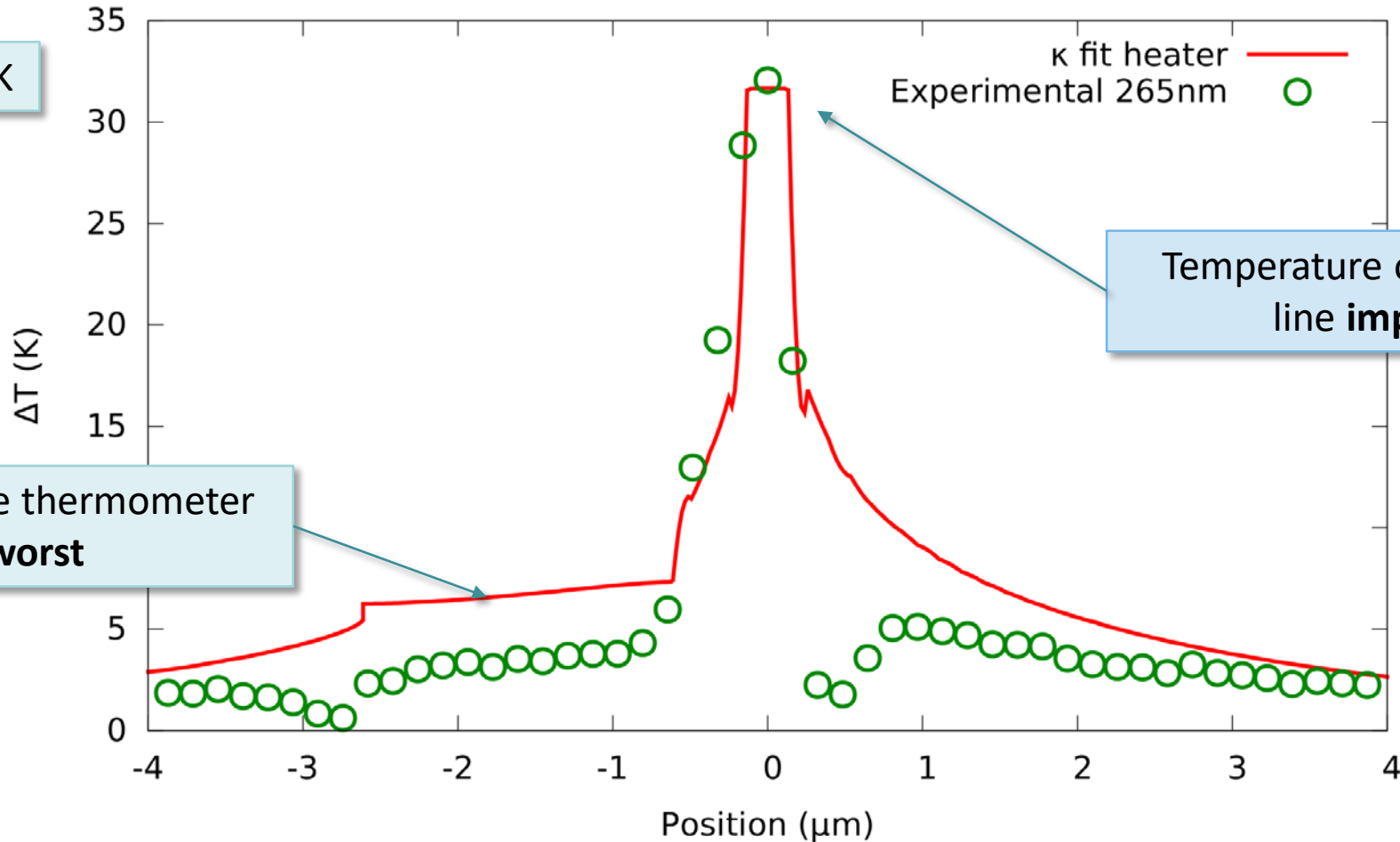
Temperature of the thermometer line is overpredicted

Temperature of the heater line is underpredicted

Using a finite element software (COMSOL) we test the validity of the Fourier's law using the nominal value of the thermal conductivity of InGaAs

# Fourier's law test (II)

$$\kappa_{\text{InGaAs}} = 4.4 \text{ W/m}\cdot\text{K}$$



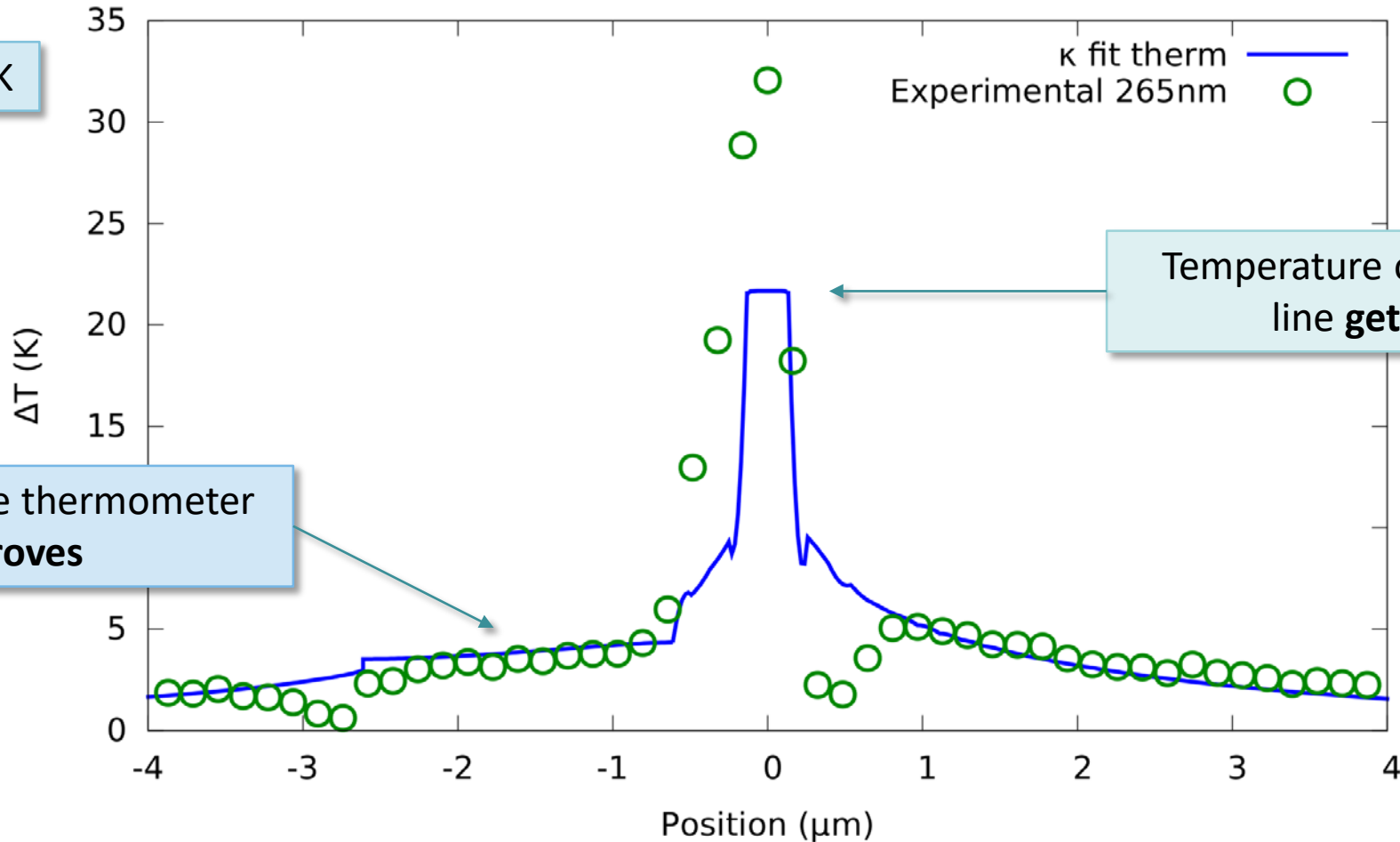
Temperature of the thermometer line get worst

Temperature of the heater line improves

Using a finite element software (COMSOL) we test the validity of the Fourier's law using a fitted value of the thermal conductivity of InGaAs to fit the heating line

# Fourier's law test (III)

$$\kappa_{\text{InGaAs}} = 6.4 \text{ W/m}\cdot\text{K}$$

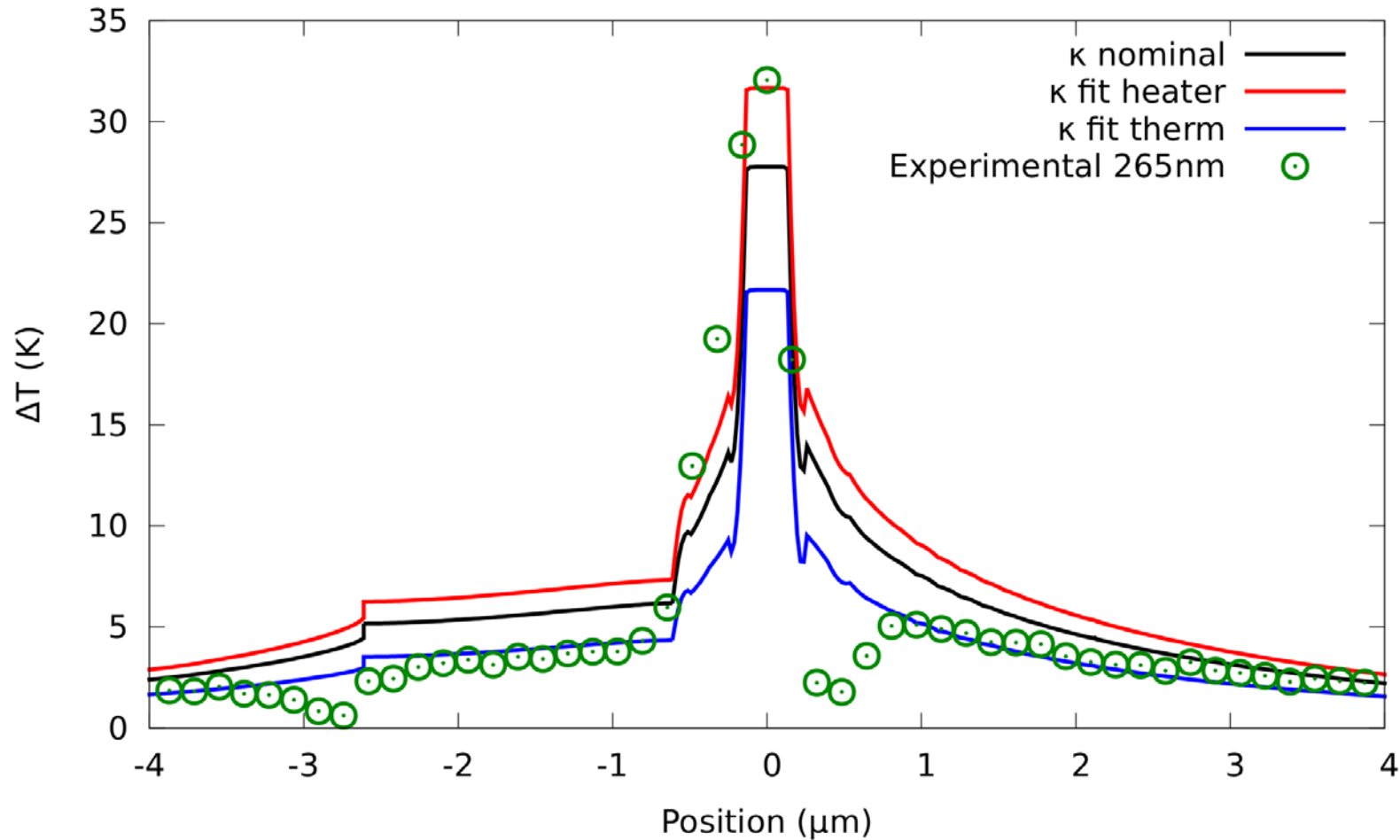


Temperature of the thermometer line **improves**

Temperature of the heater line **get worst**

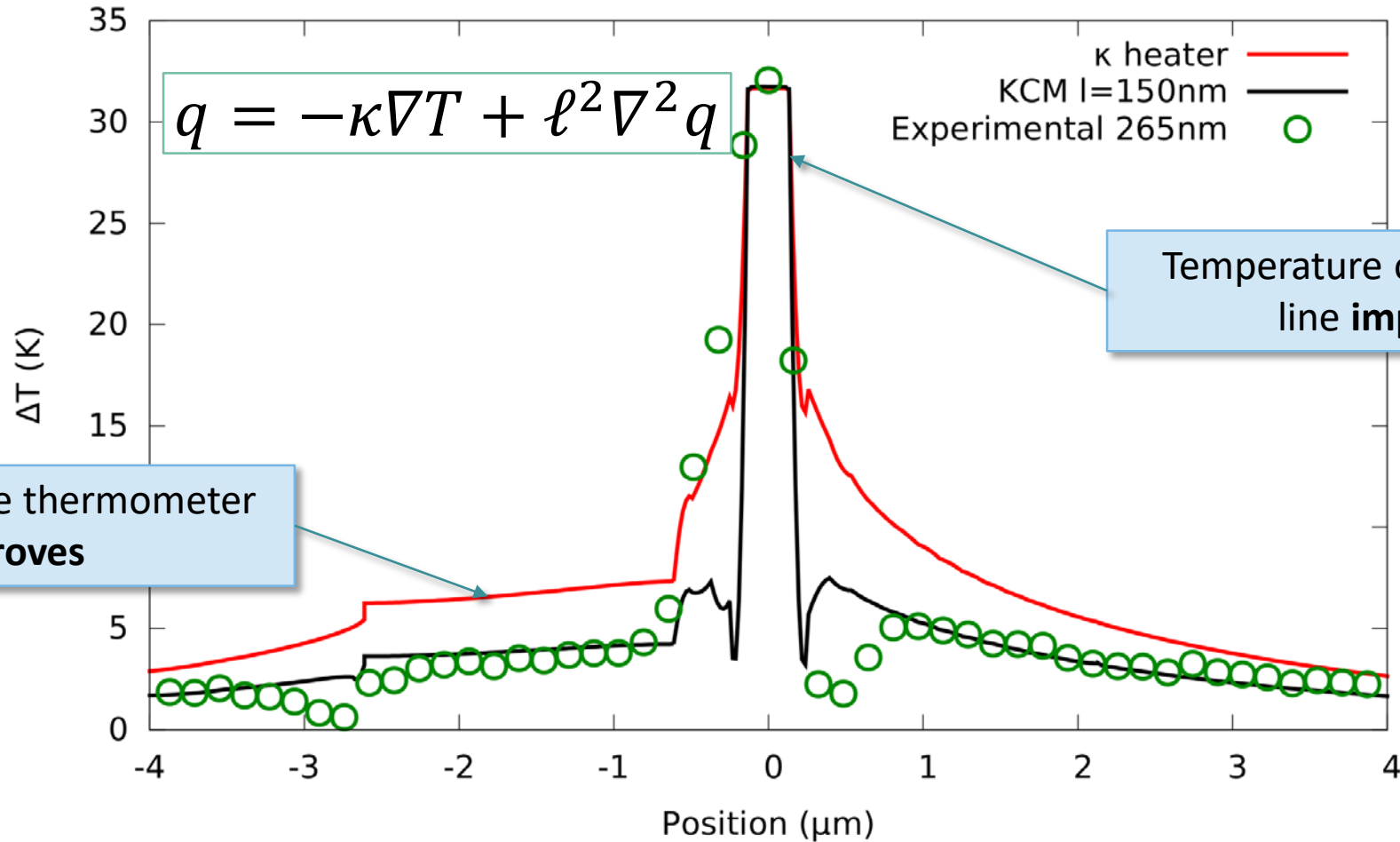
Using a finite element software (COMSOL) we test the validity of the Fourier's law using a fitted value of the thermal conductivity of InGaAs to fit the thermometer line

# Fourier's law summary



**Conclusion:** Fourier's law cannot describe thermal transport in this setup. New equation is needed.

# GK equation

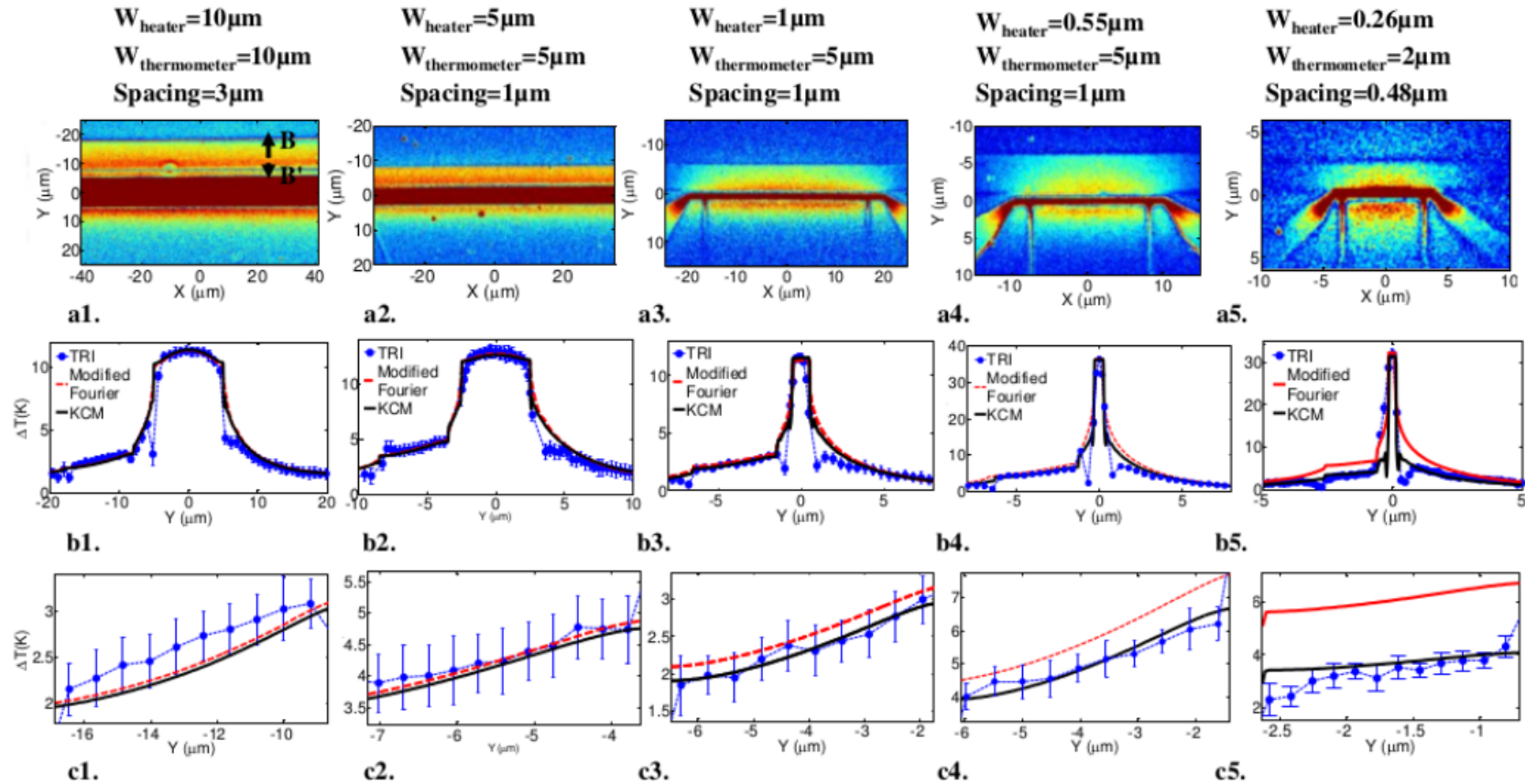


Kinetic Collective Model fits better to experimental data

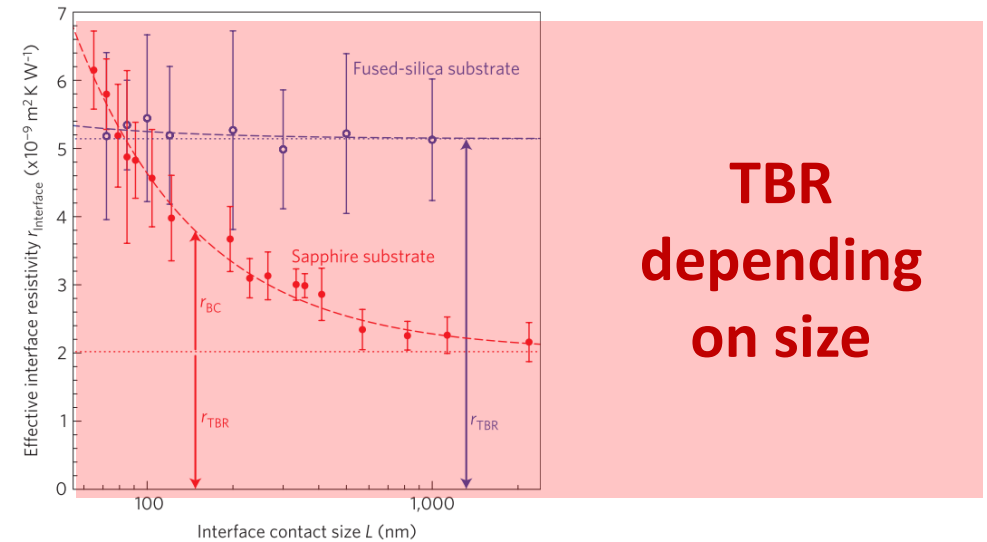
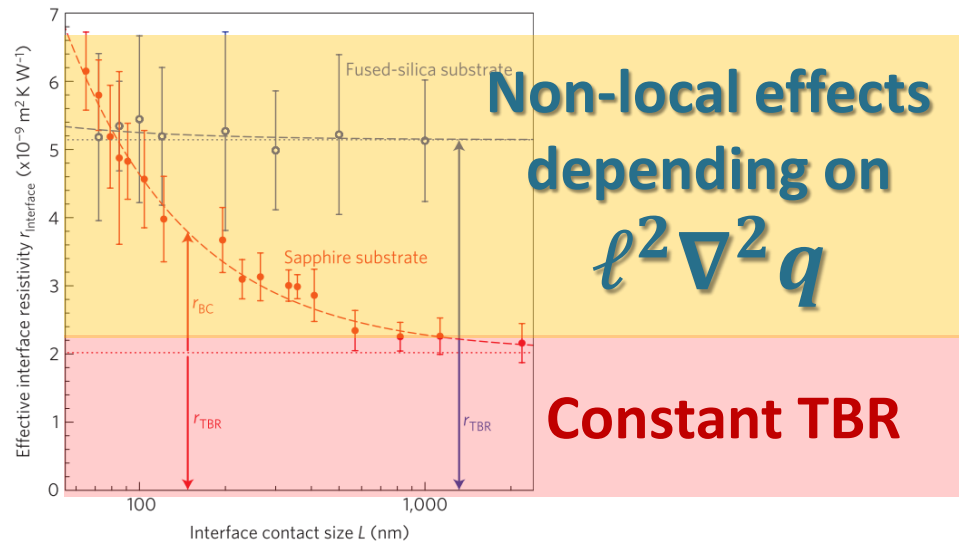
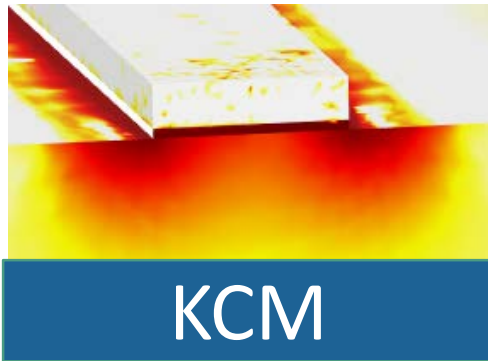
# Kinetic Collective Model + Guyer and Krumhansl

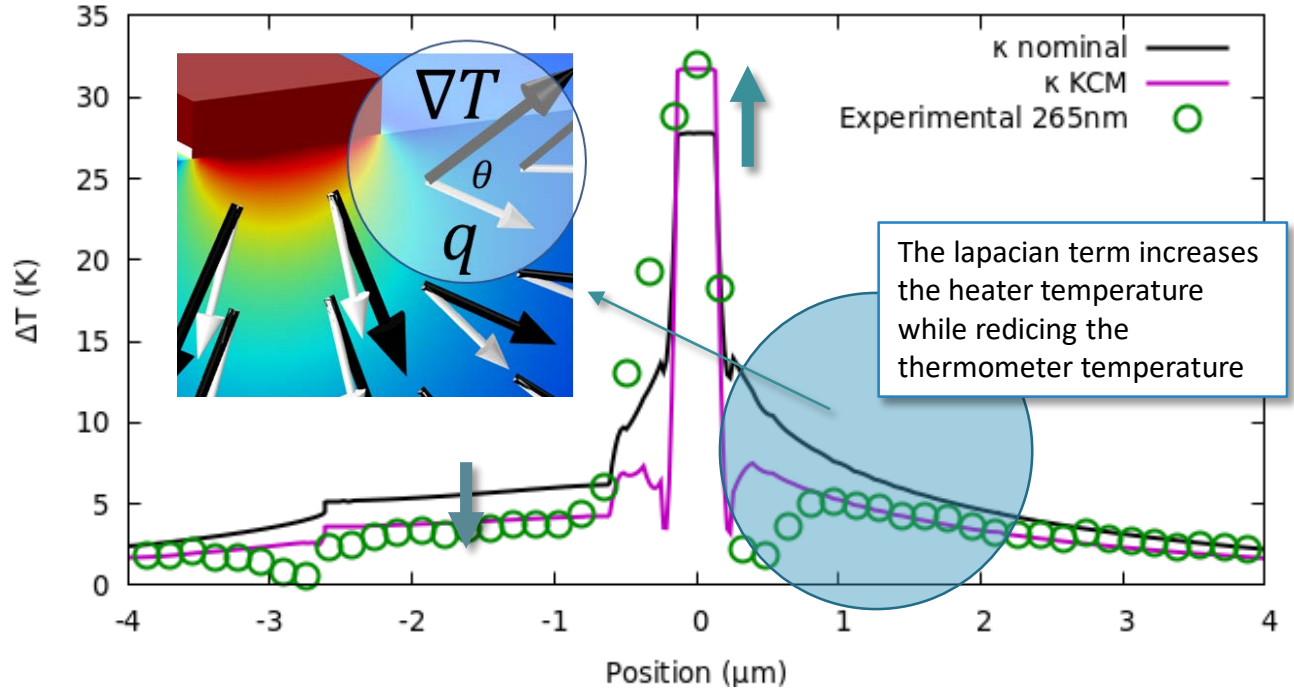
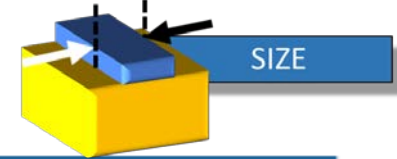
At large sizes we recover Fourier model

The smaller the size, the larger the effect

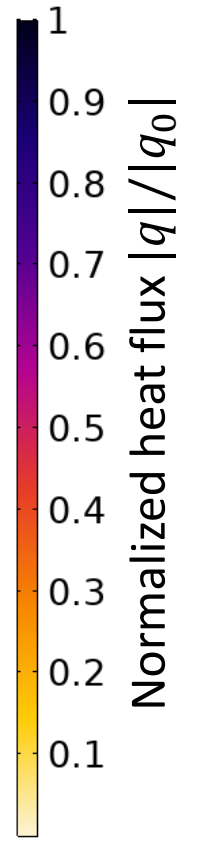
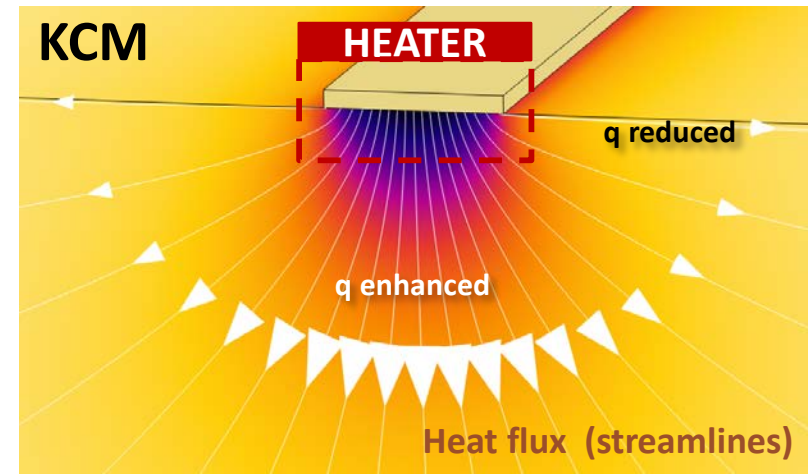
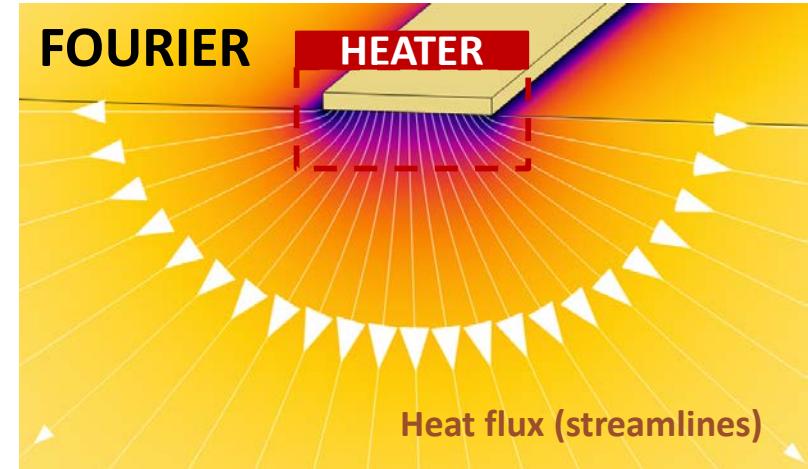


# TBR vs hydrodynamics





$$q = -\kappa \nabla T + \ell^2 \nabla^2 q$$





# OTHER HYDRODYNAMIC SIGNATURES IN SILICON





**OBSERVATION OF  
HYDRODYNAMIC  
TIME SCALES**

# Temperature decay of metallic lines



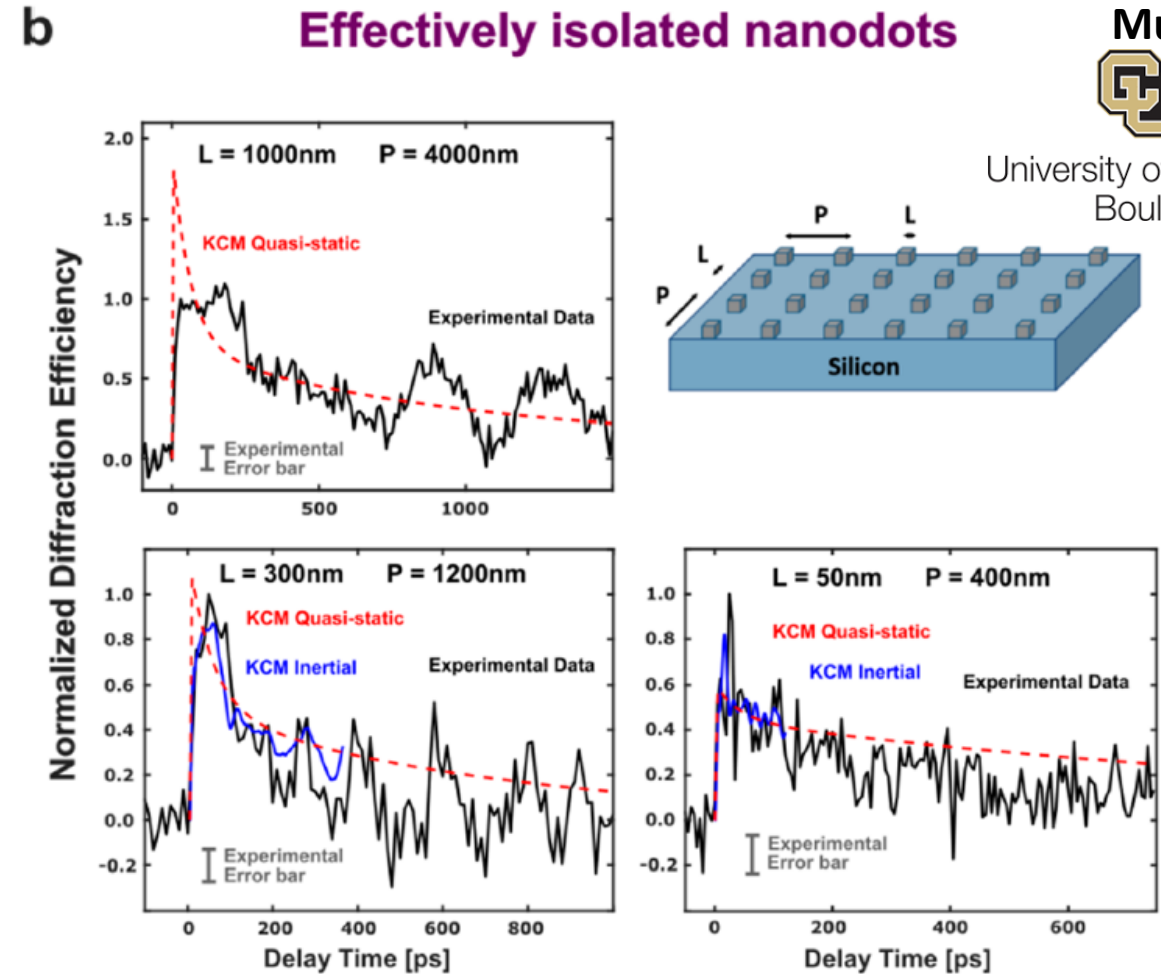
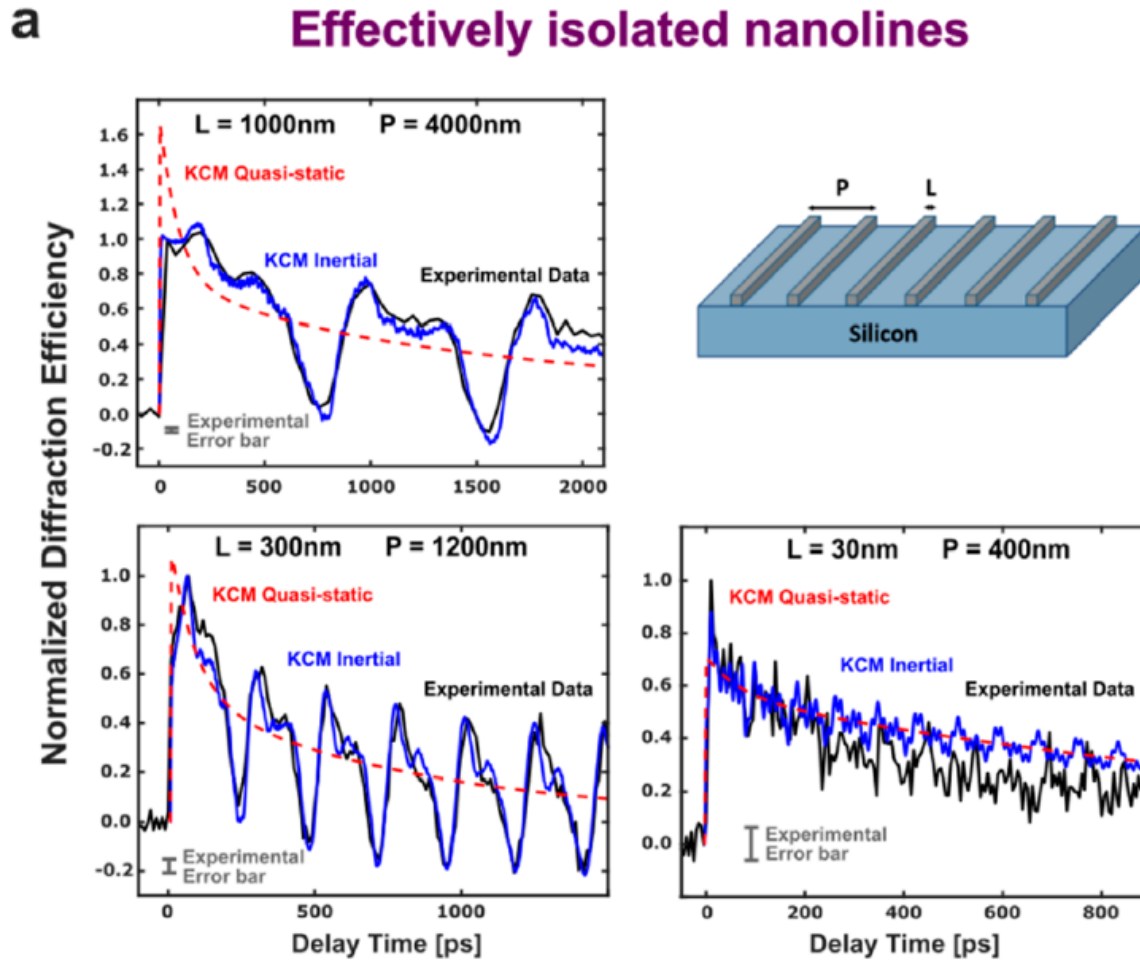
Beardo, Knobloch et al.  
*ACS Nano* 15, 13019 (2021)



Margaret Murnane



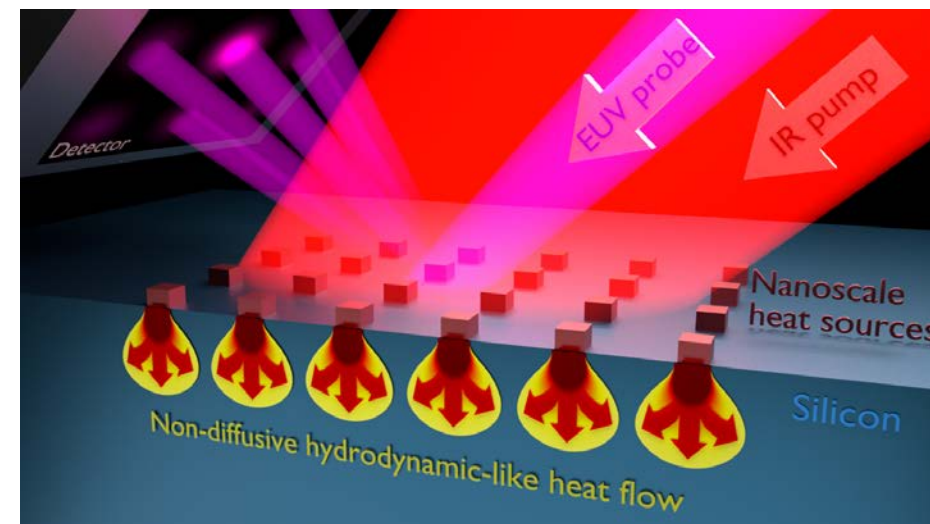
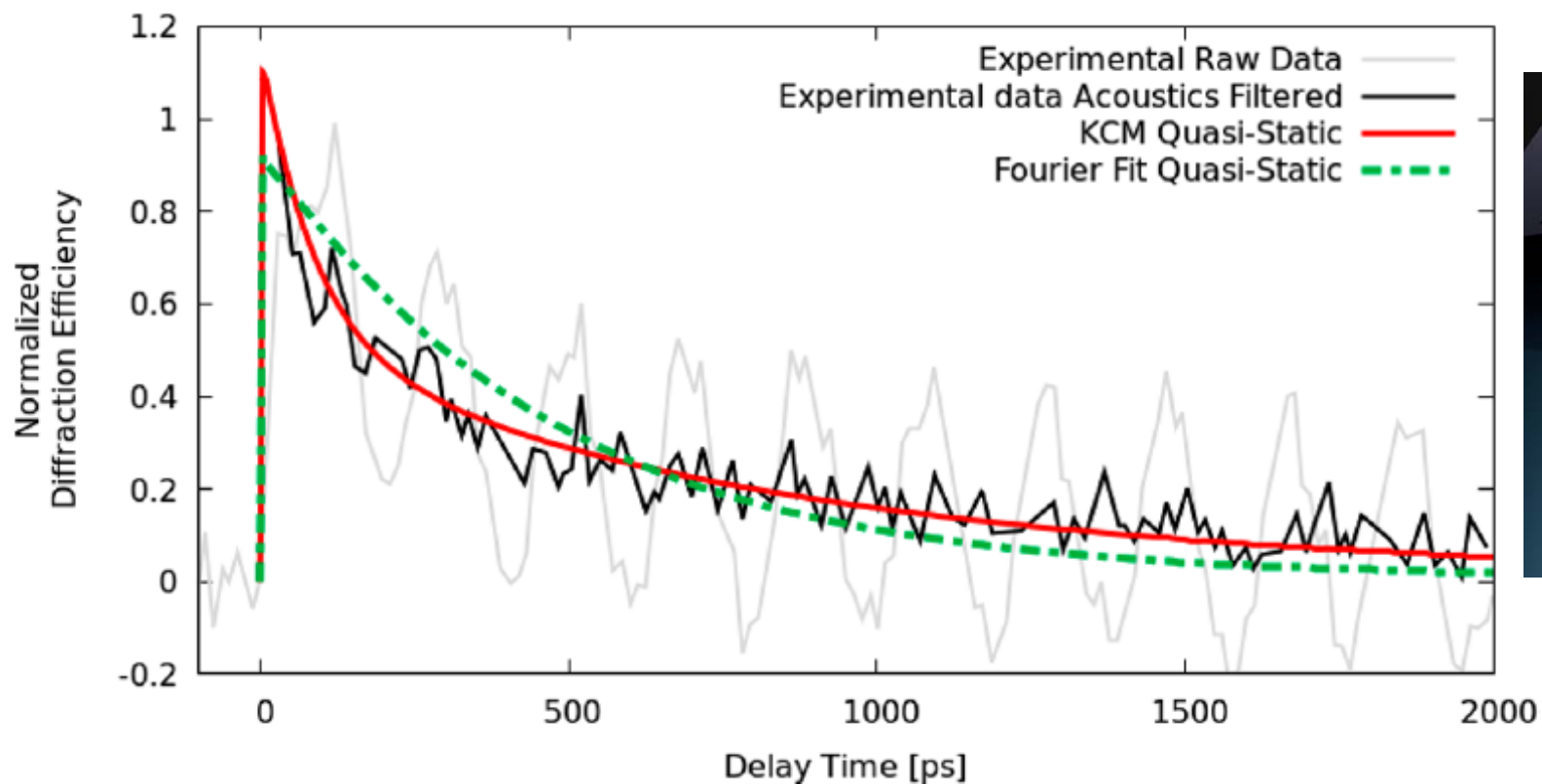
University of Colorado  
Boulder



# Non-Fourier decay / Double exponential



Beardo, Knobloch et al.  
*ACS Nano* 15, 13019 (2021)



EUV SCATTEROMETRY SETUP

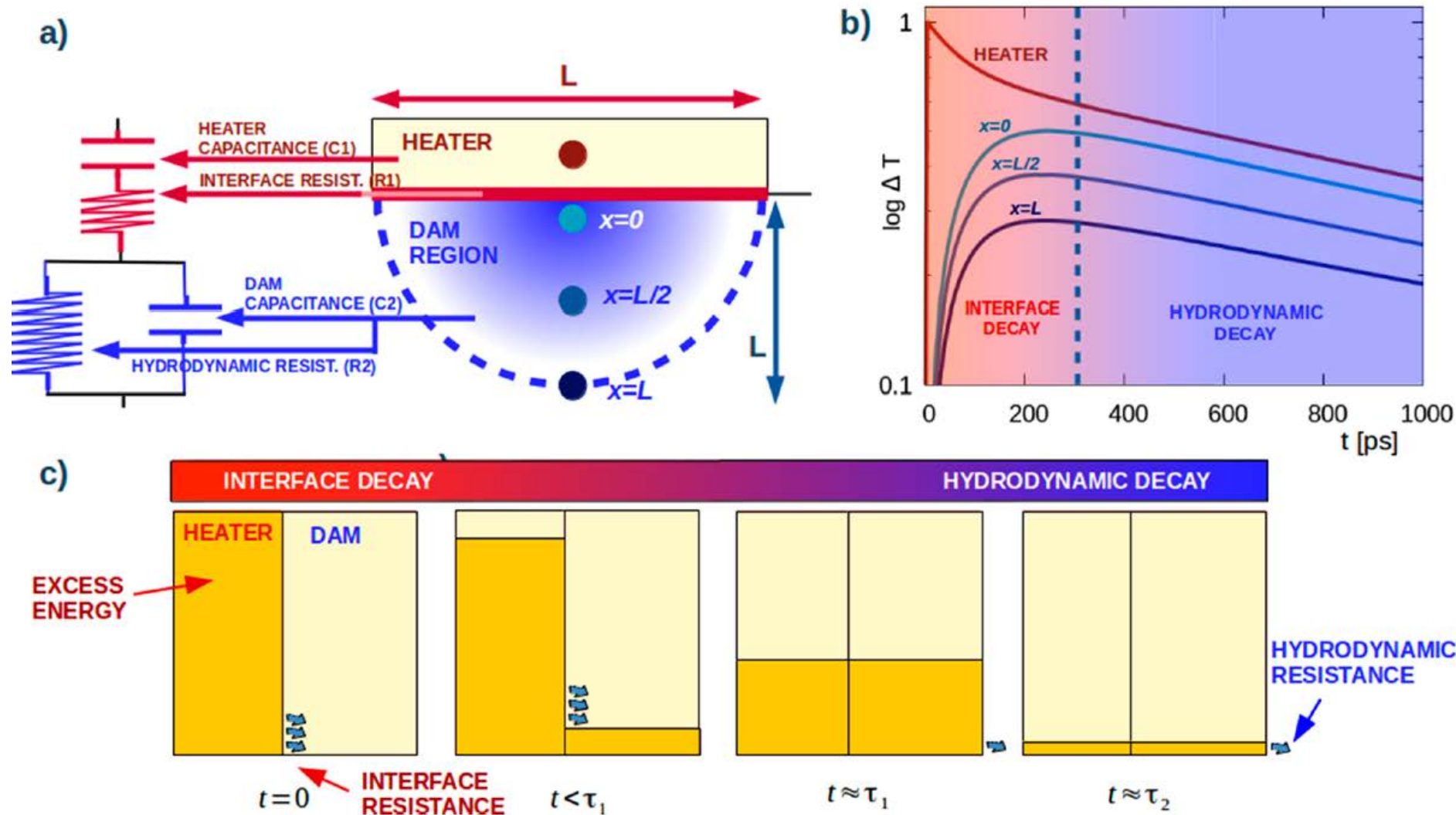


University of Colorado  
Boulder

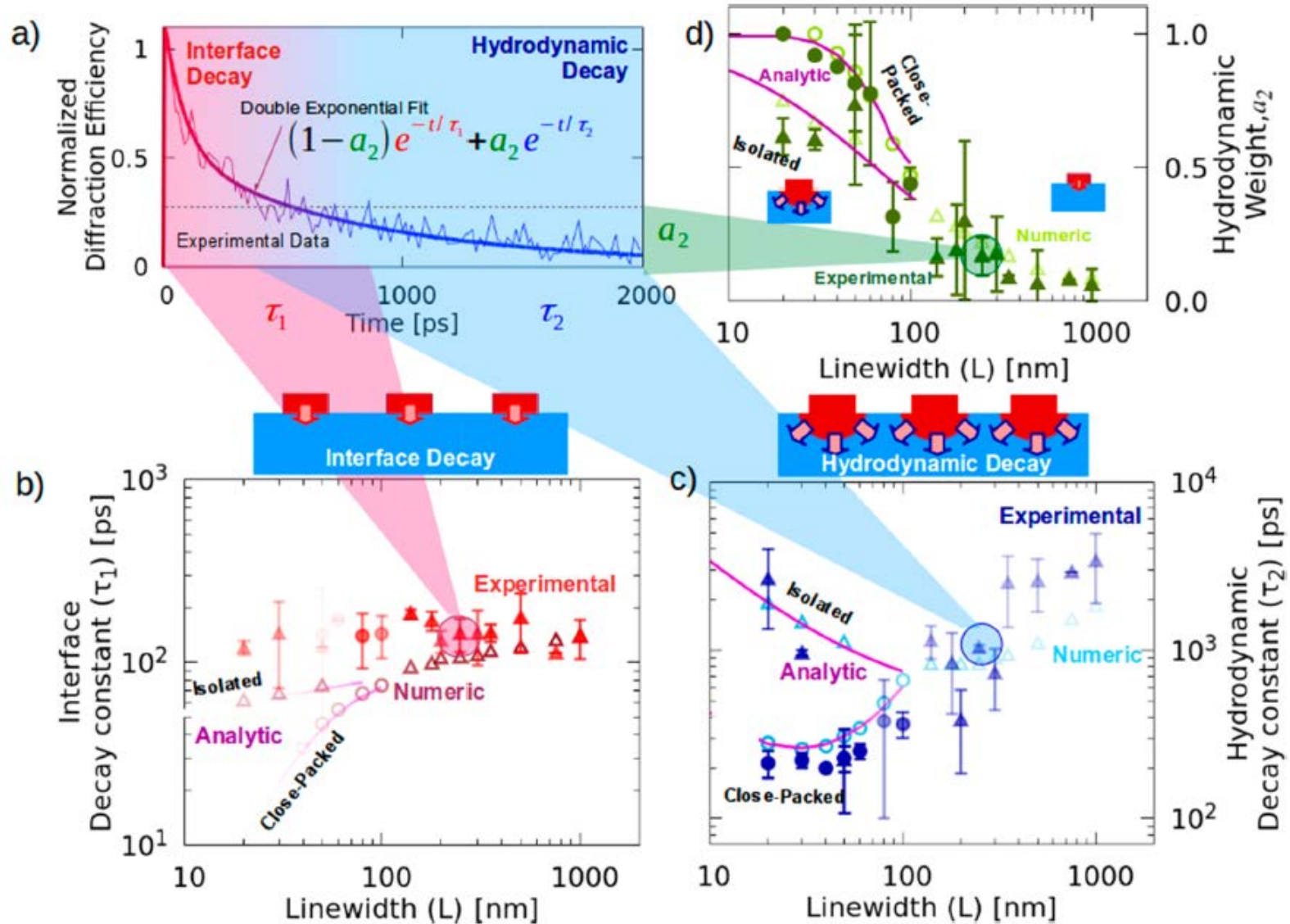
# Two Box model – The Dam region



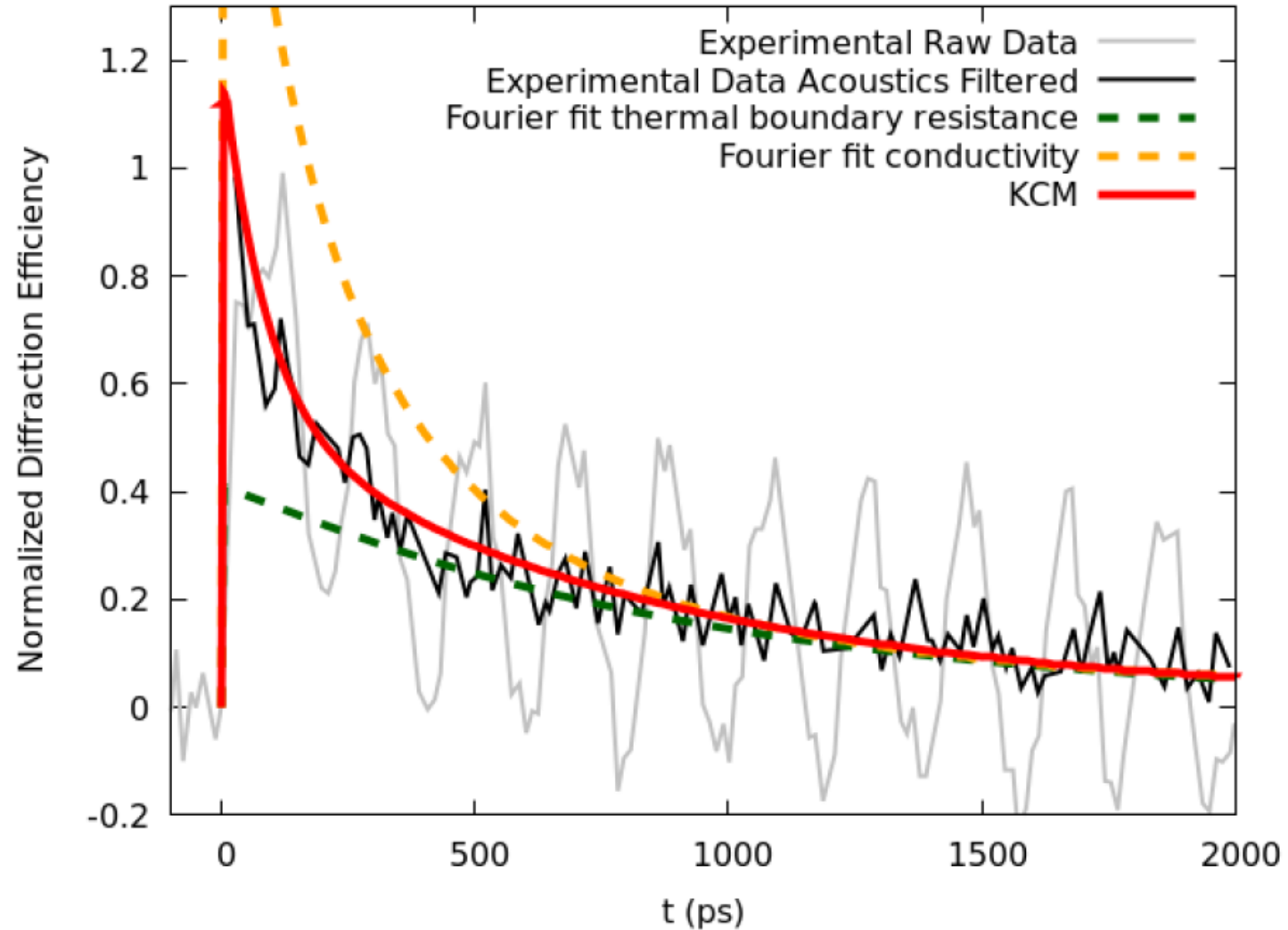
Beardo, Knobloch et al.  
*ACS Nano* 15, 13019 (2021)



# Two Box model / TBR and hydrodynamic relaxation times



## Two Box model / TBR and hydrodynamic relaxation times



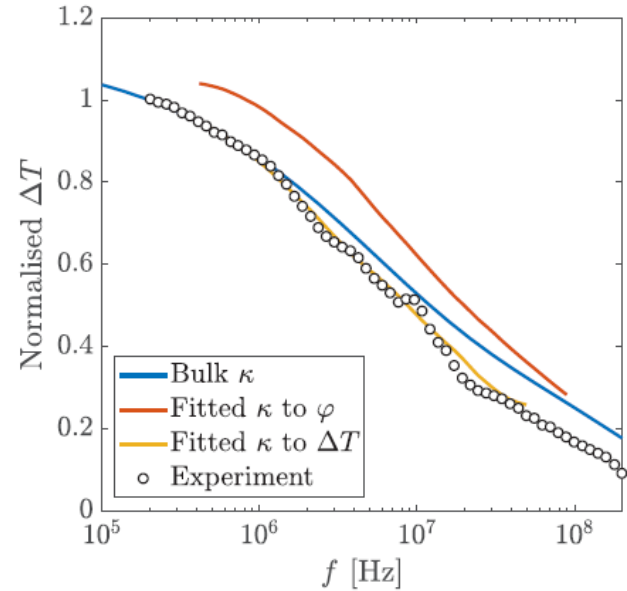
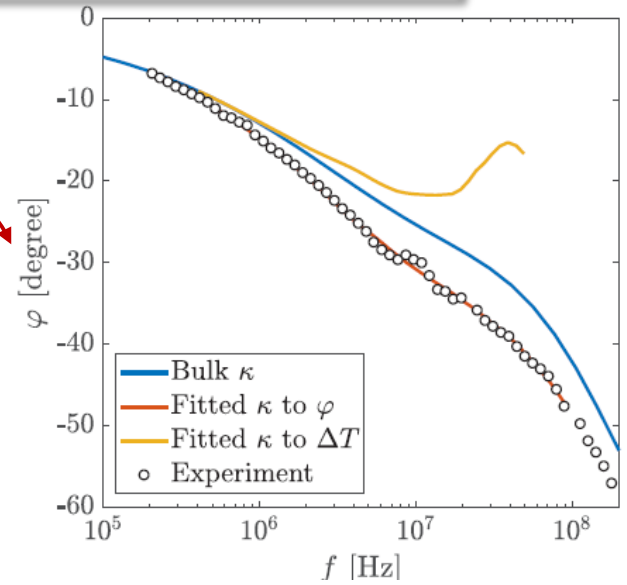
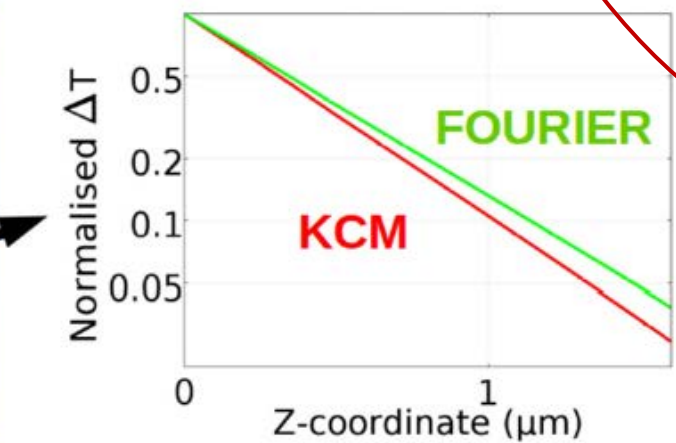
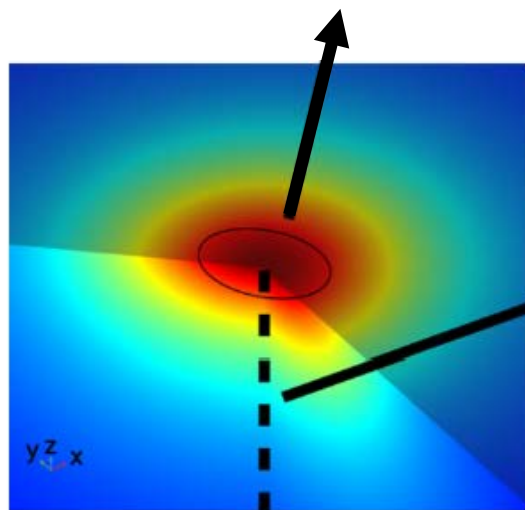
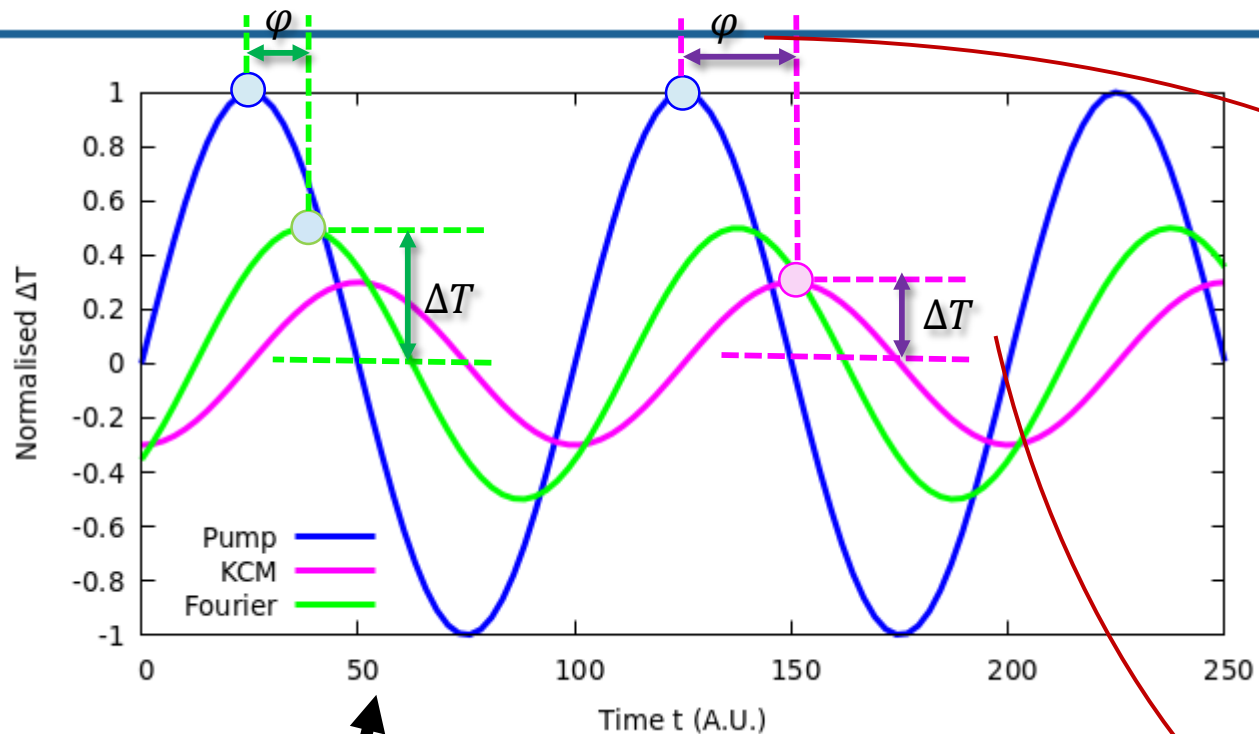
# **SECOND SOUND**



# Frequency Domain Thermoreflectance (FDTR)

Regner et al.  
*Nat. Commun.* **4**, 1640 (2013)

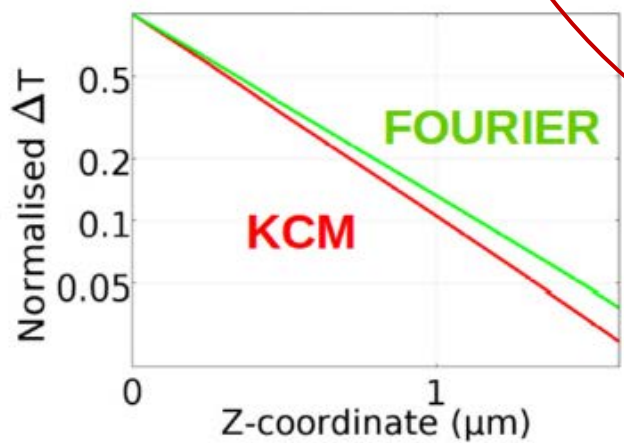
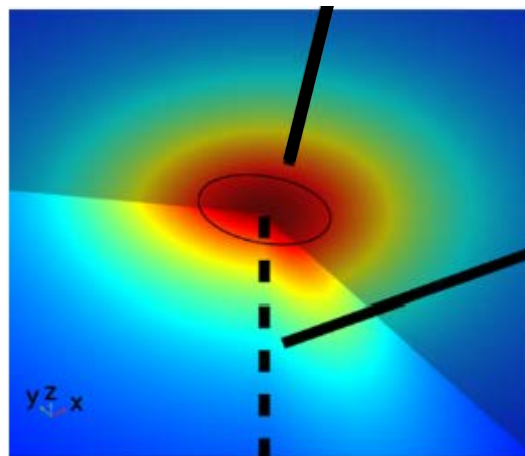
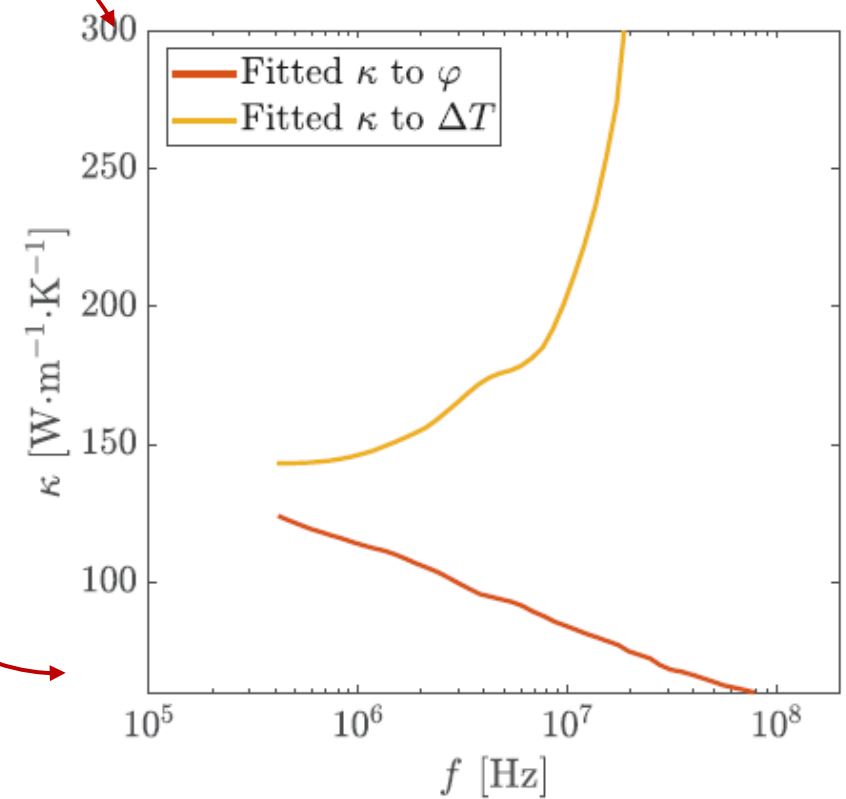
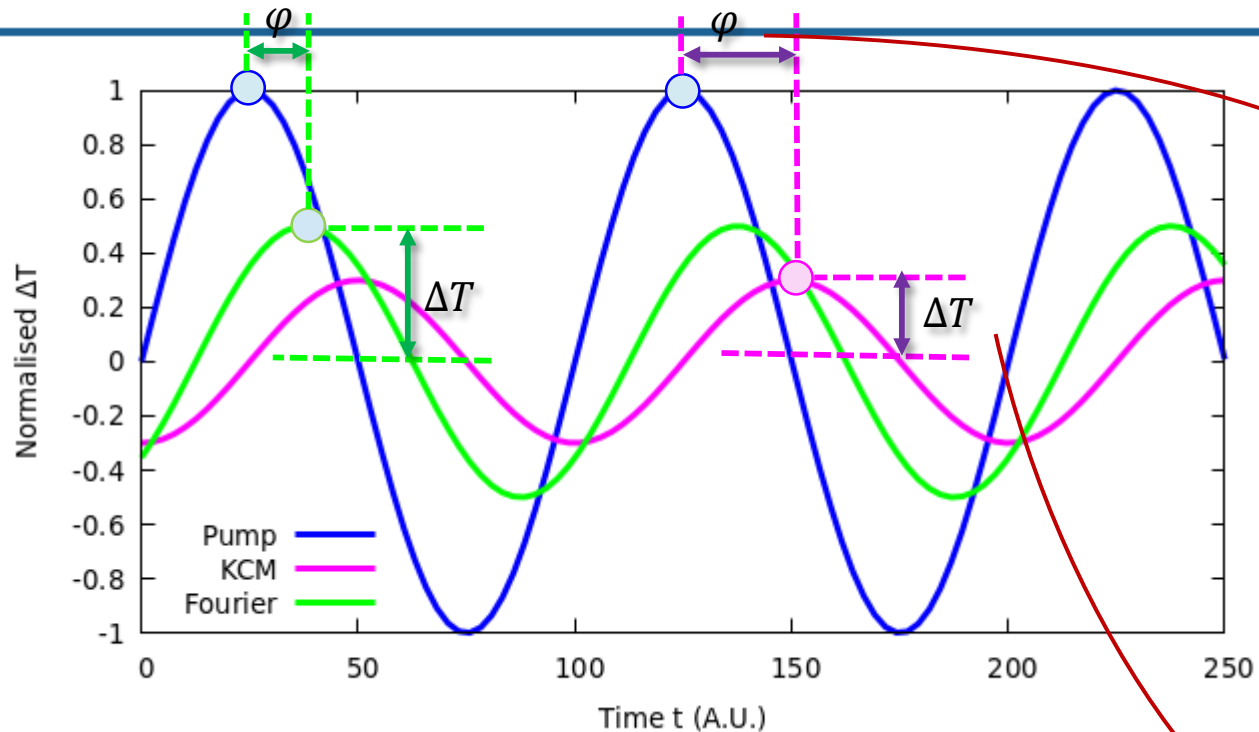
FREQUENCY



# Frequency Domain Thermoreflectance (FDTR)

Regner et al.  
*Nat. Commun.* **4**, 1640 (2013)

FREQUENCY



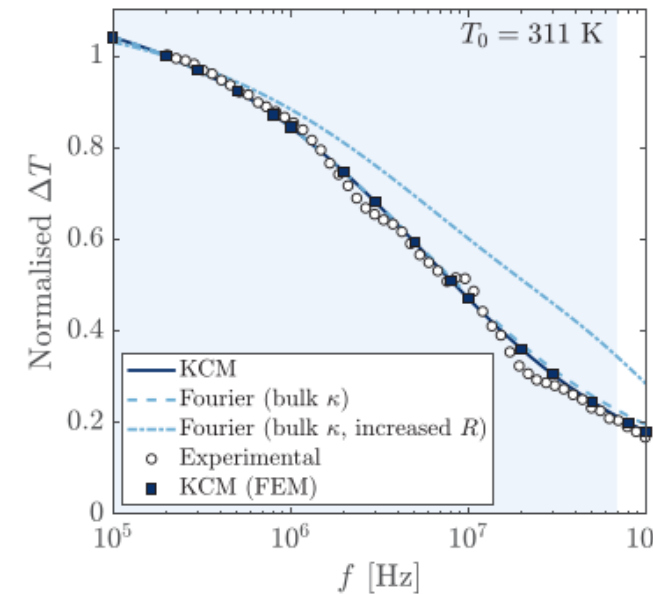
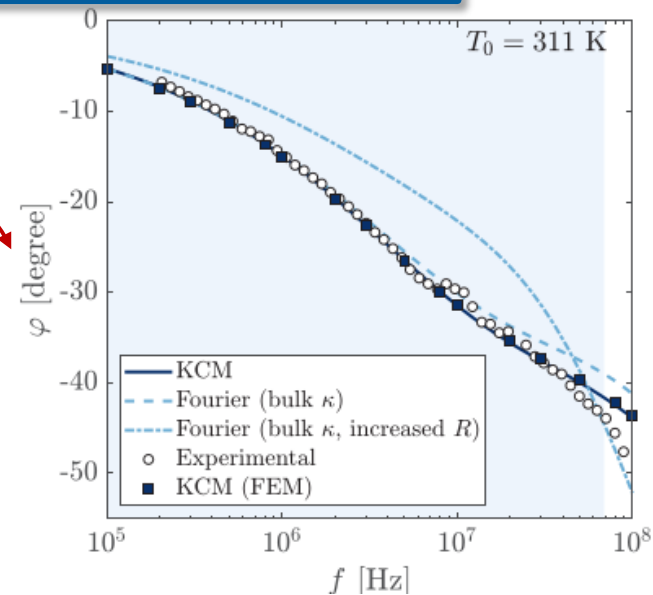
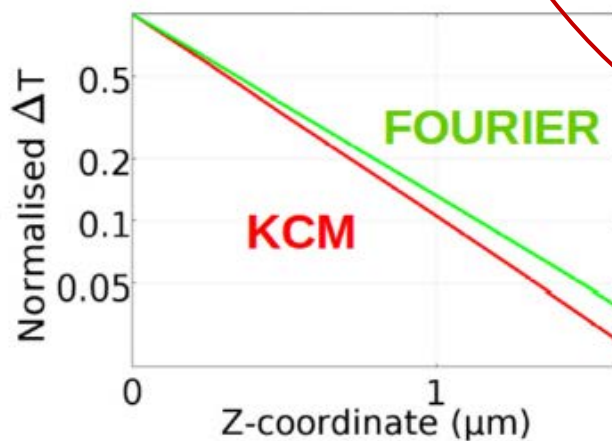
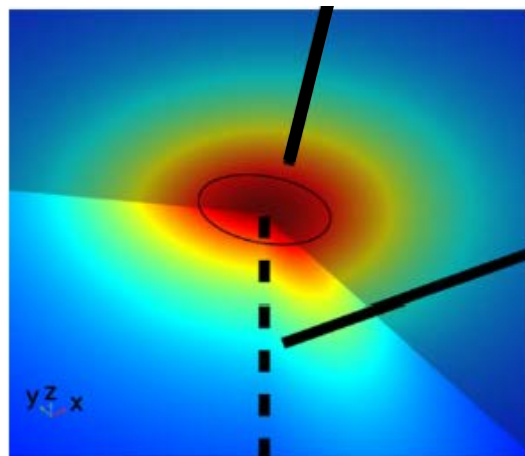
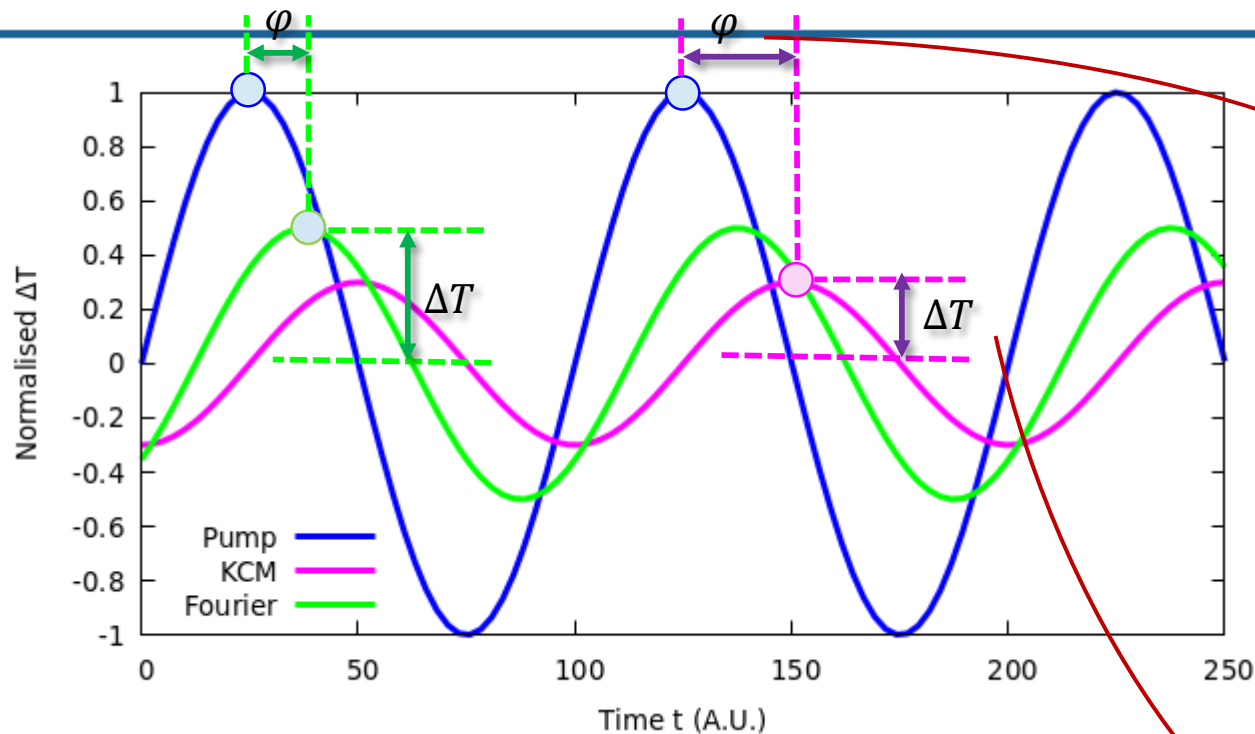
Beardo et al.  
*PRB* **101**, 075303 (2020)

# Frequency Domain Thermoreflectance (FDTR)



Regner et al.  
*Nat. Commun.* **4**, 1640 (2013)

FREQUENCY



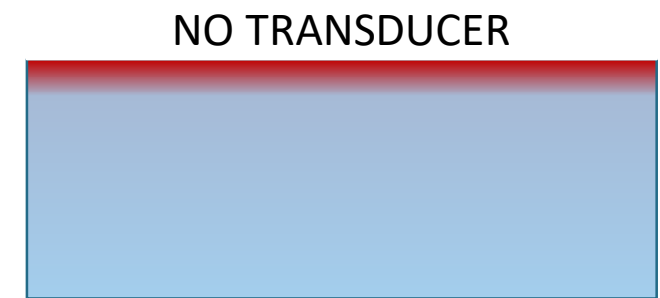
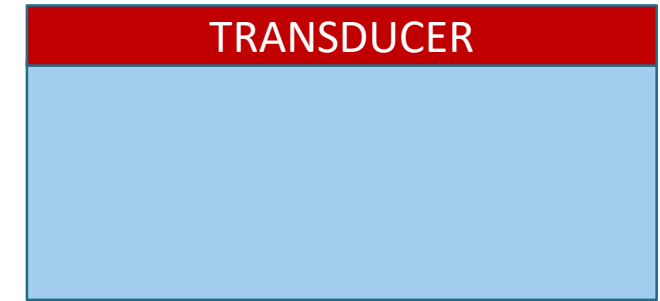
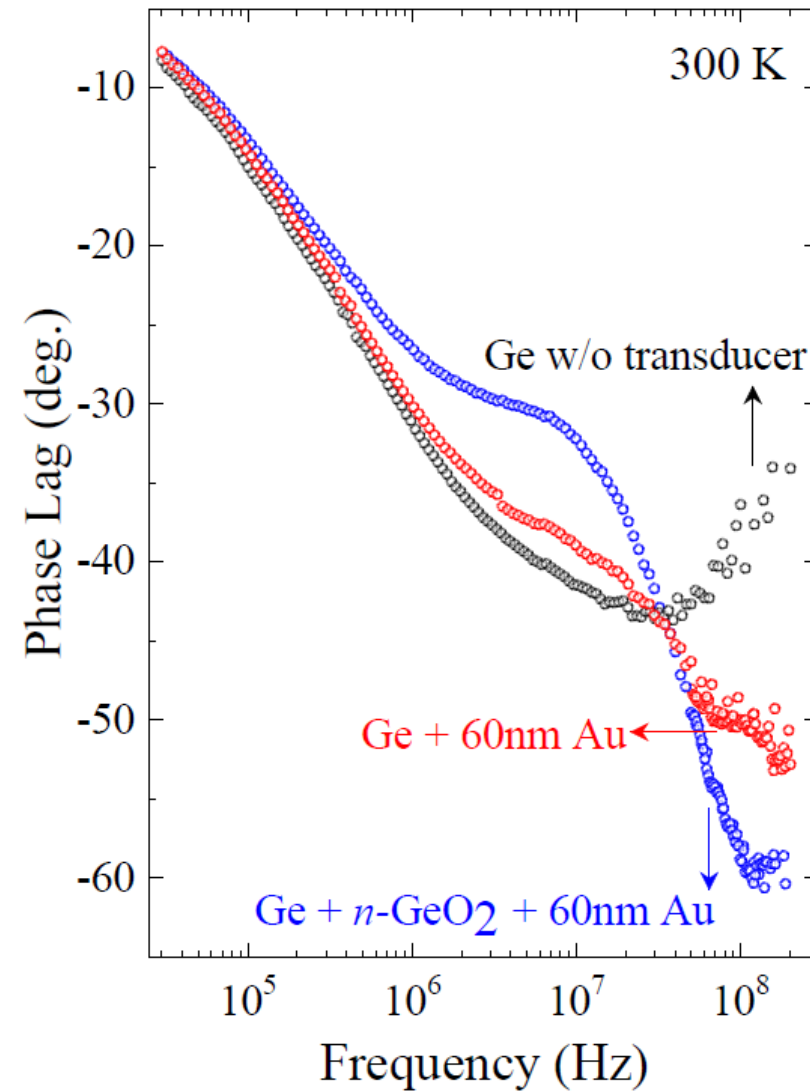
# Memory effects / Second sound



Beardo et al.  
*Sci. Adv.* 7, eabg4677 (2021)



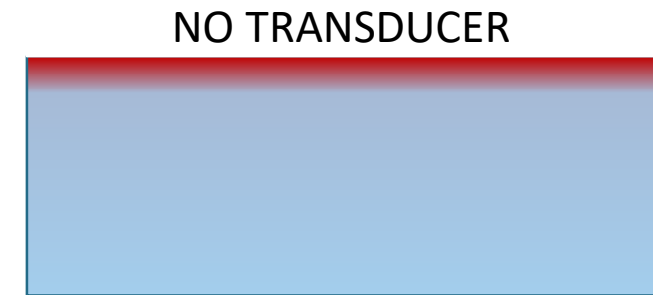
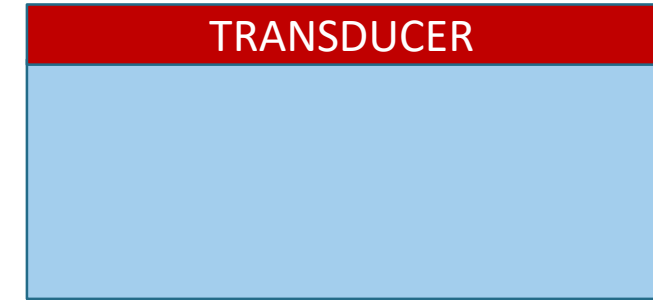
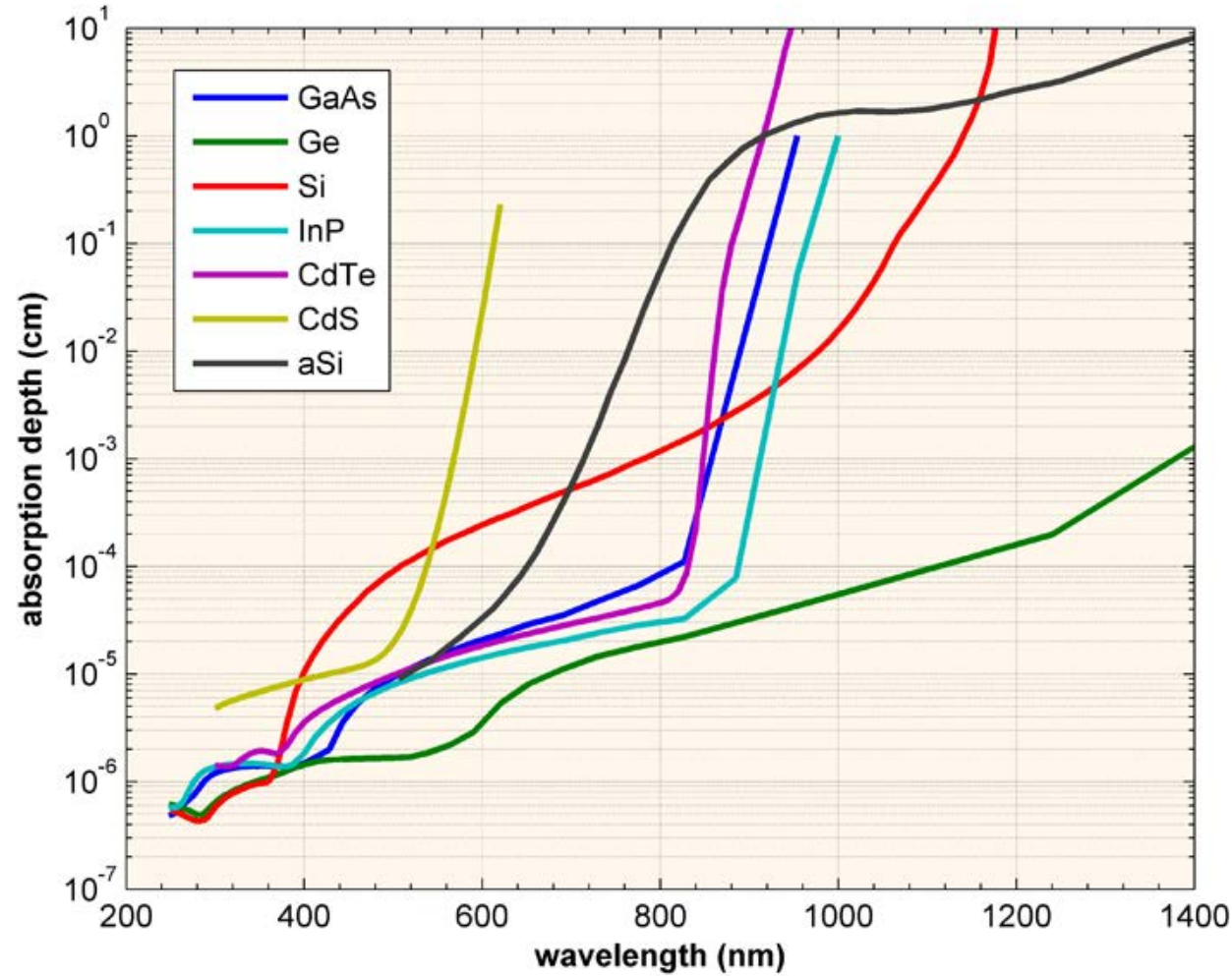
Sebastian  
Reparaz



# Memory effects / Second sound



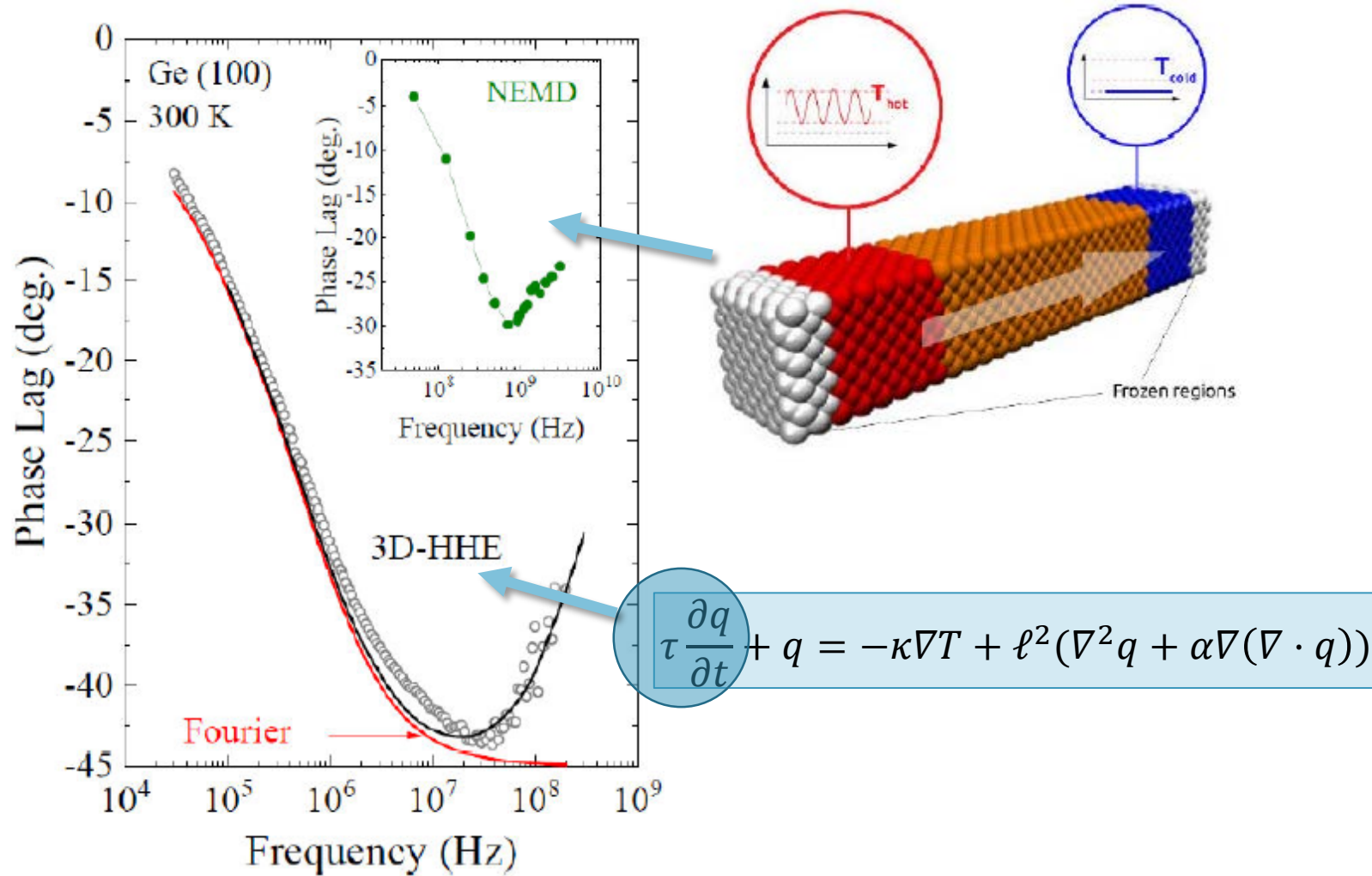
Beardo et al.  
*Sci. Adv.* 7, eabg4677 (2021)



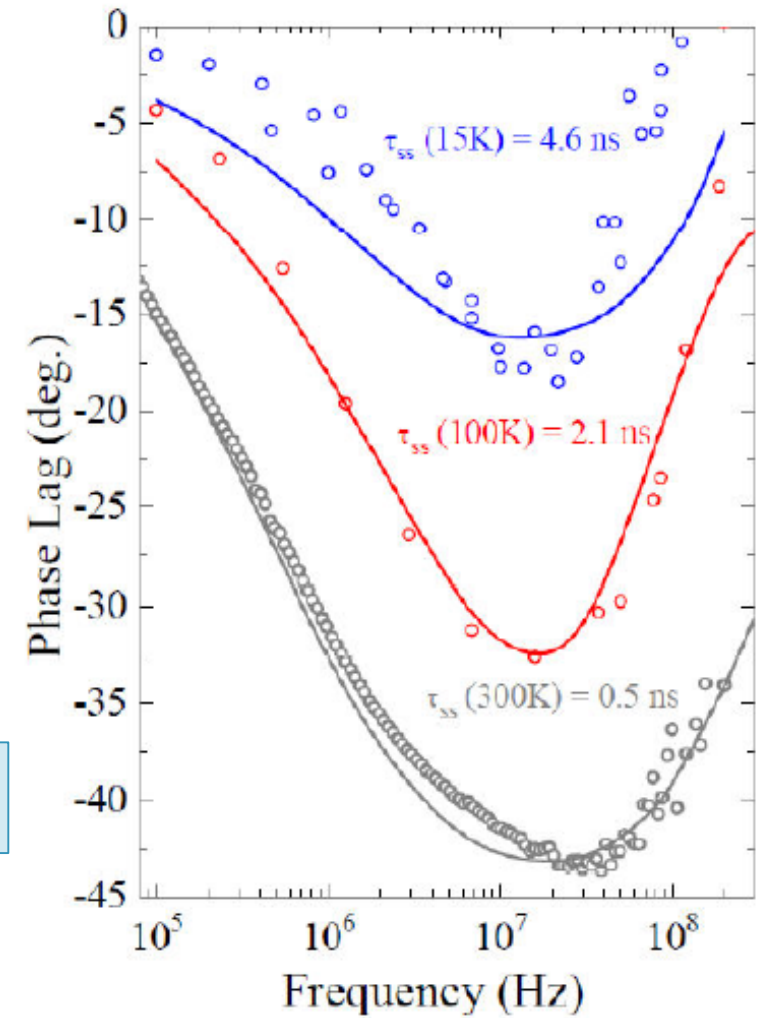
# Memory effects / Second sound



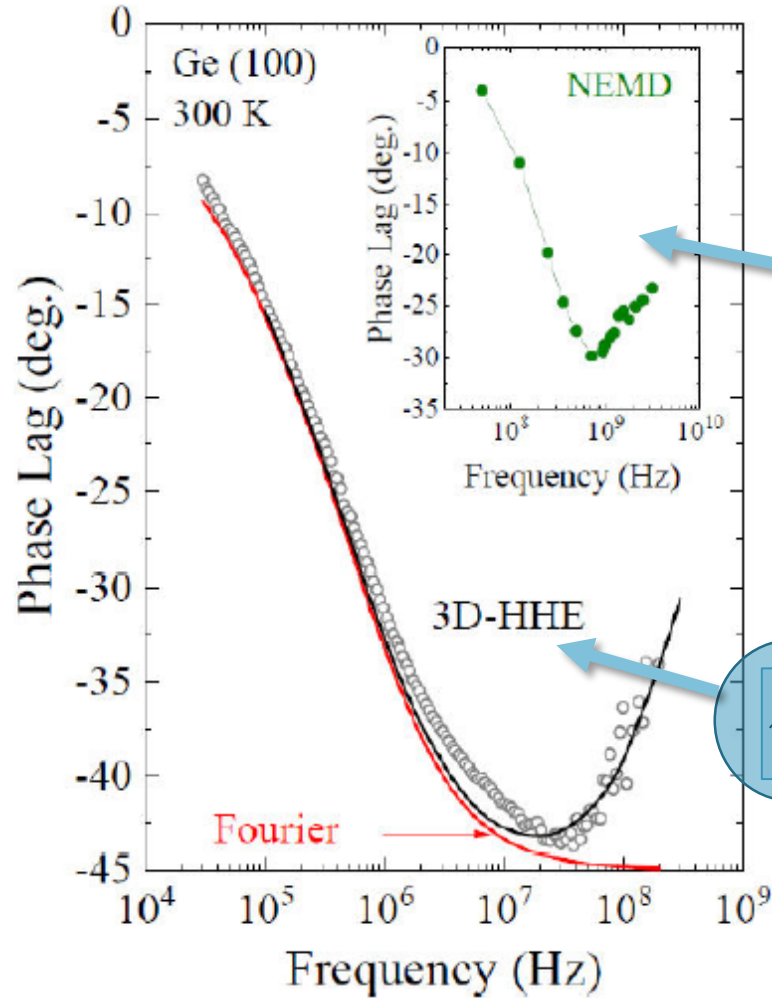
Beardo et al.  
*Sci. Adv.* 7, eabg4677 (2021)



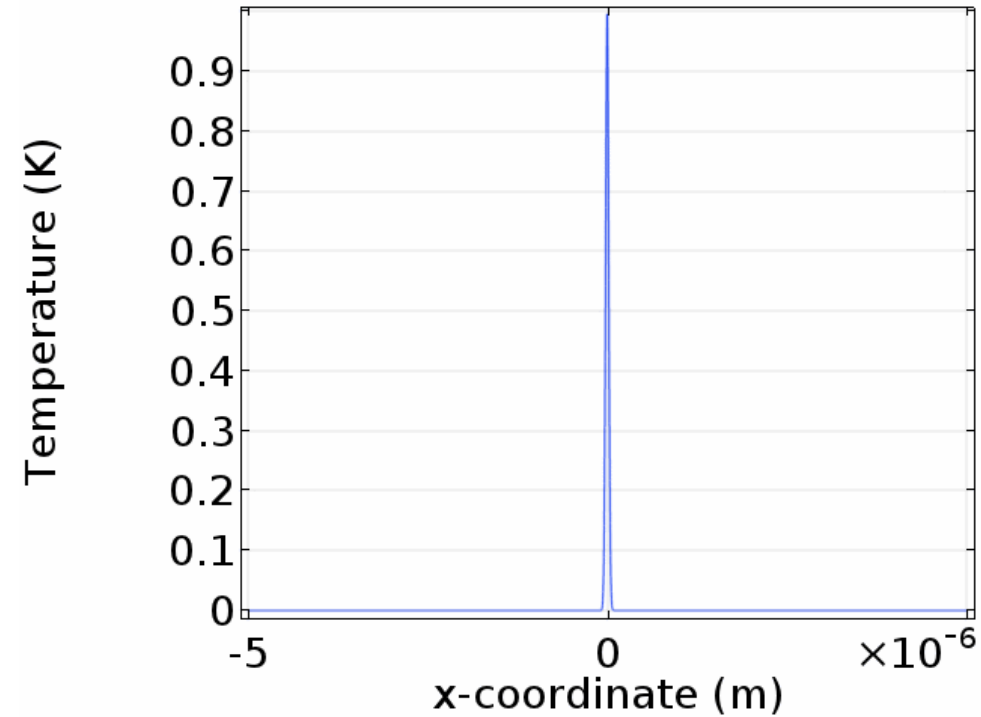
$$\tau \frac{\partial q}{\partial t} + q = -\kappa \nabla T + \ell^2 (\nabla^2 q + \alpha \nabla (\nabla \cdot q))$$



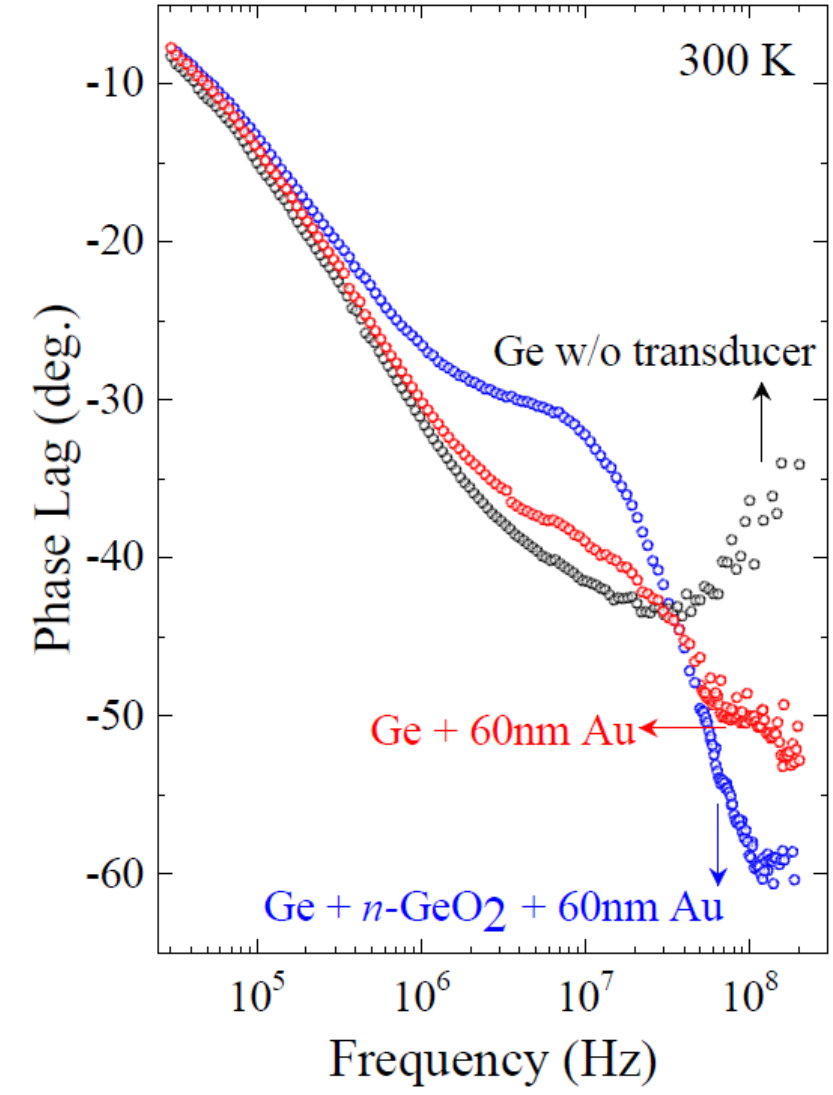
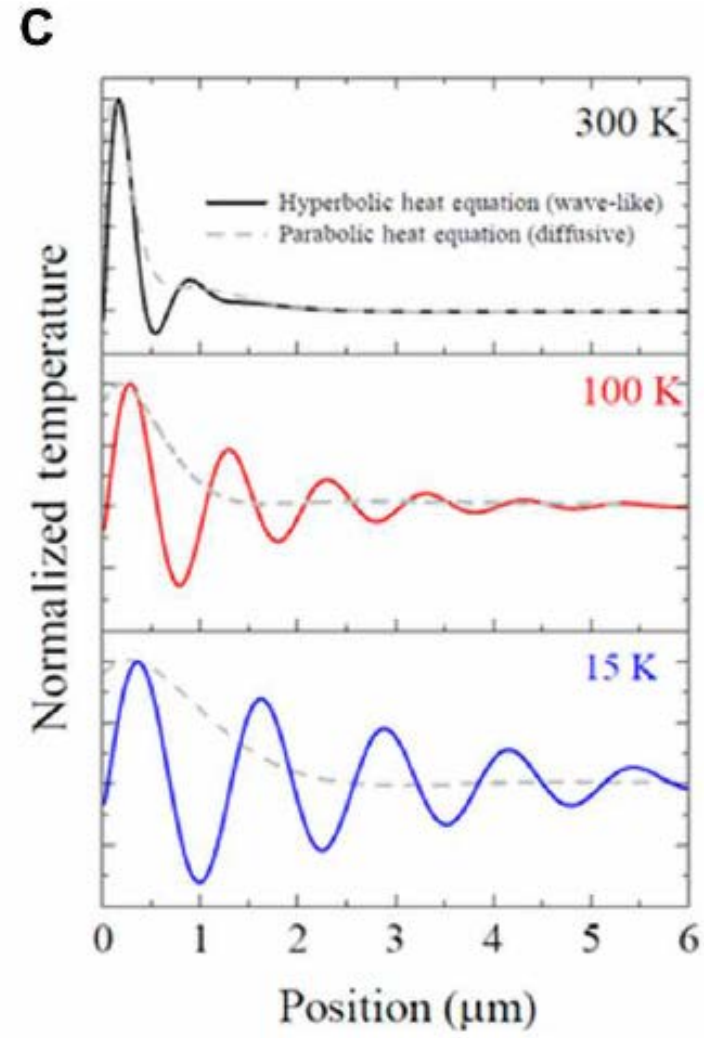
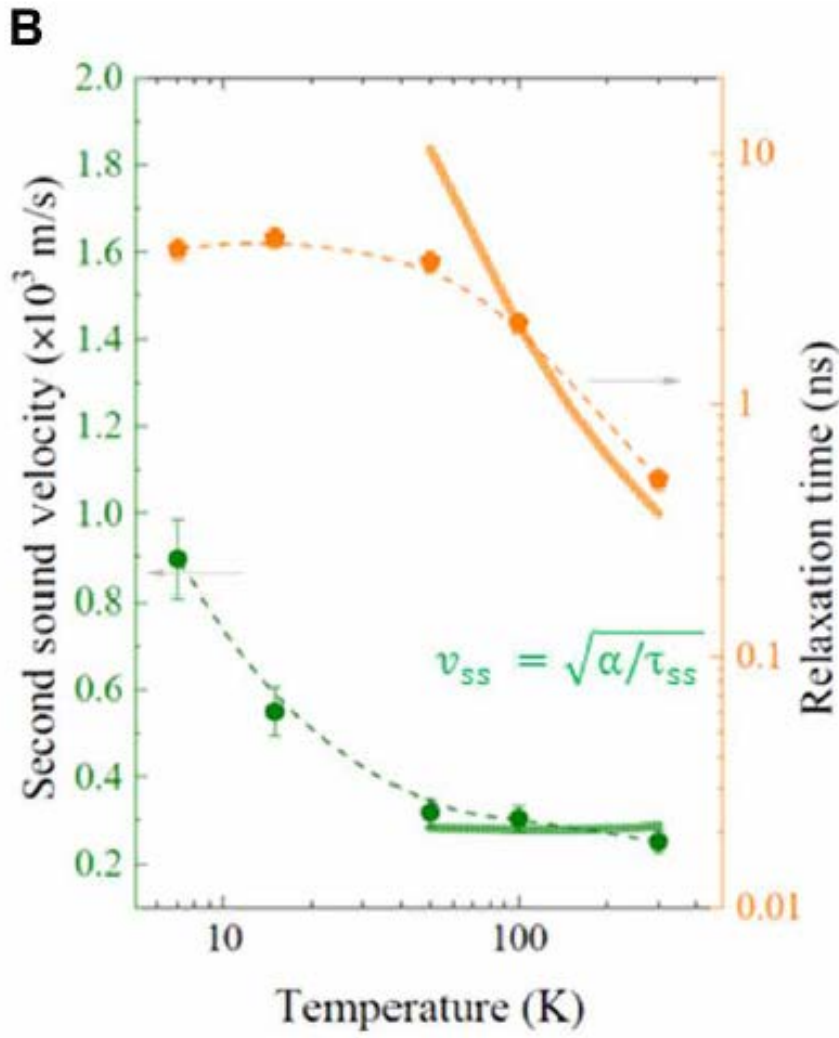
# Memory effects / Second sound



$$\tau \frac{\partial q}{\partial t} + q = -\kappa \nabla T + \ell^2 (\nabla^2 q + \alpha \nabla (\nabla \cdot q))$$



# Memory effects / Second sound





## Conclusions



My approach is fundamental

I'm pure



He's a phenomenological approach

He's a wrong approximation to the field

## Conclusions

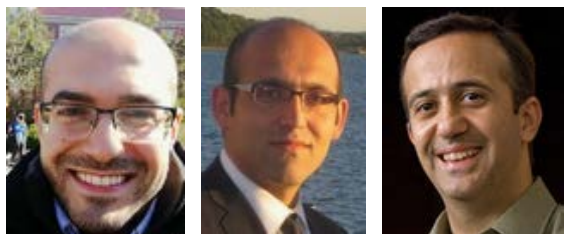


H and S approaches are connected!

- Combination of **Guyer and Krumhansl** equation with **ab-initio Kinetic Collective Model** for the transport properties allows the prediction of a large set of experiments
- The possibility to solve this model in a **Finite Element (COMSOL)** allows the direct comparison with any experimental setup despite its geometrical complexity
- The large set of experimental data on silicon explained by GK with a single abinitio set of parameters is an evidence in favor of a **hydrodynamic regime** in silicon

# Thank you

## THERMOREFLECTANCE IMAGING

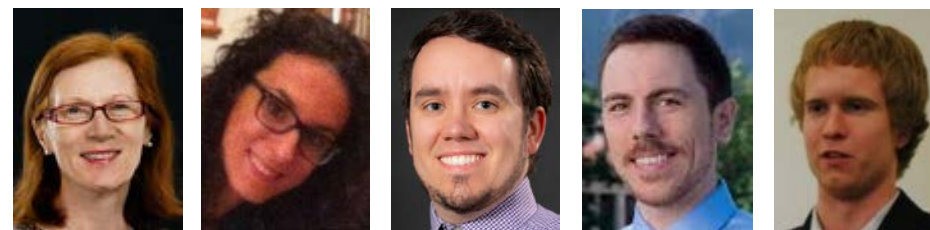


**Sami Alajlouni** **Amir Koushyar Ziabari** **Ali Shakouri**

## EUV SCATTEROMETRY SETUP

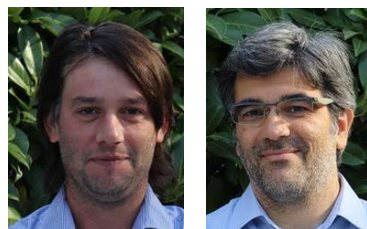


University of Colorado  
Boulder



**Margaret Murnane** **Begoña Abad** **Joshua L. Knobloch** **Travis Frazer** **Brendan McBennett**

## FDTR SETUP



**Sebastian Reparaz** **Riccardo Rurali**

## GK-KCM MODEL



**Lluc Sendra** **Albert Beardo** **Javier Bafaluy** **Juan Camacho**

