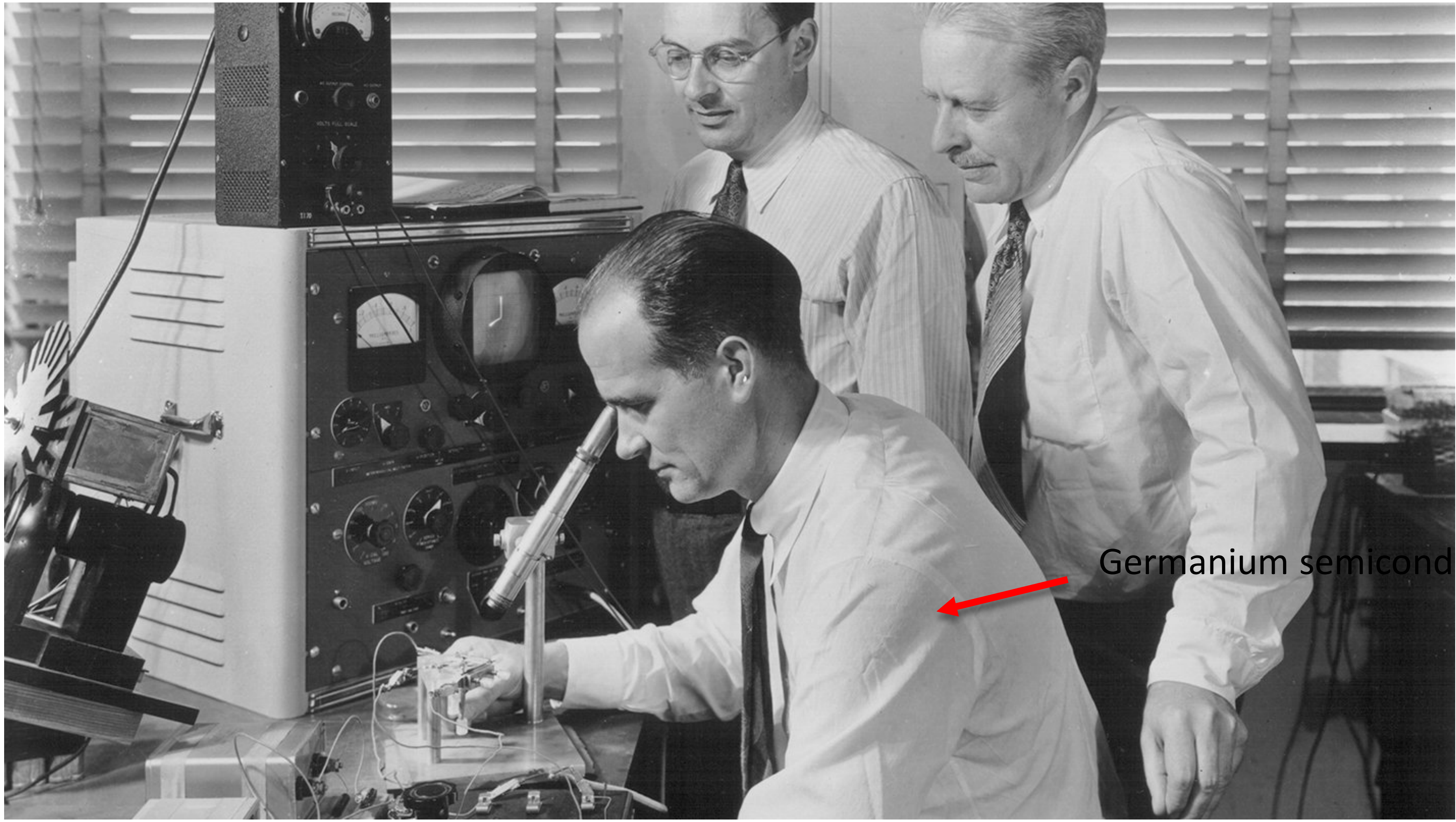
Fractionalization of charge and statistics in two dimensions

Michael J. Manfra
Hovde Distinguished Lecture
Department of Physics and Astronomy, Purdue University
September 29th, 2022

The Transistor - 1947

John Bardeen

Walter Brattain



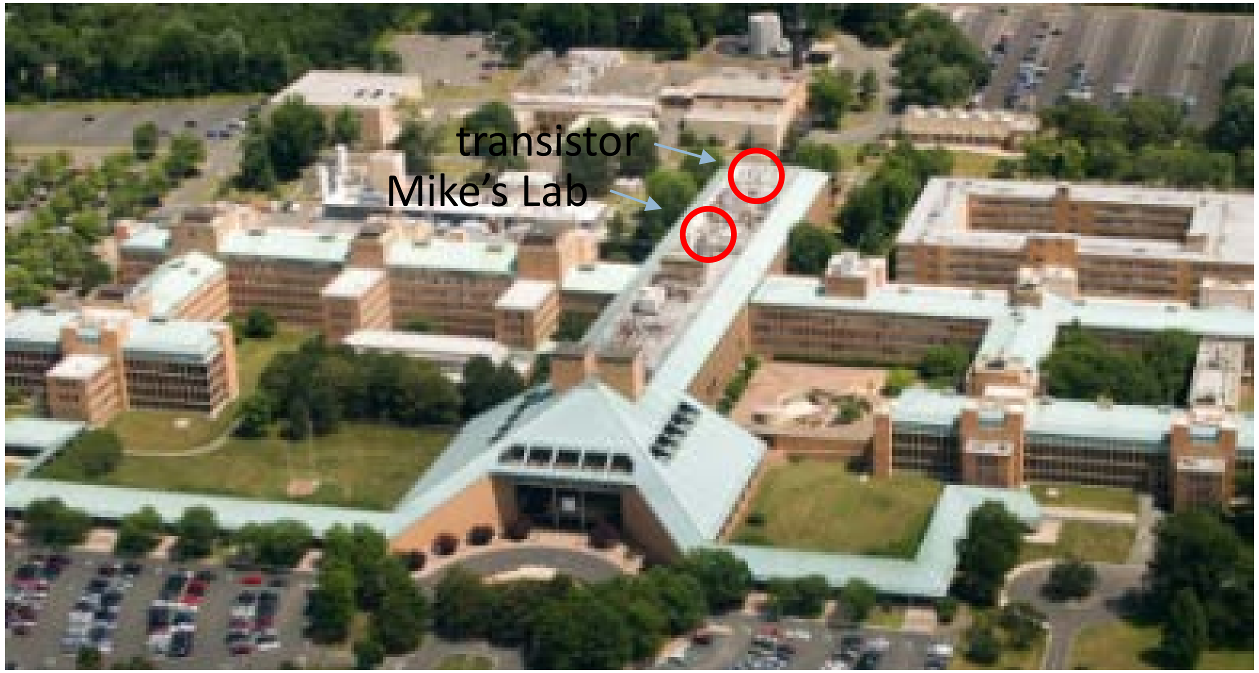
Germanium semiconductor crystal

“Solid State Physics Group”

Bill Shockley

Bell Laboratories

(circa 2008)



Bell Labs, Murray Hill New Jersey

https://www.youtube.com/watch?v=IFfdnFOiXUU

The Idea Factory: Bell Labs and the Great Age of American Innovation

Jon Gertner (2013)

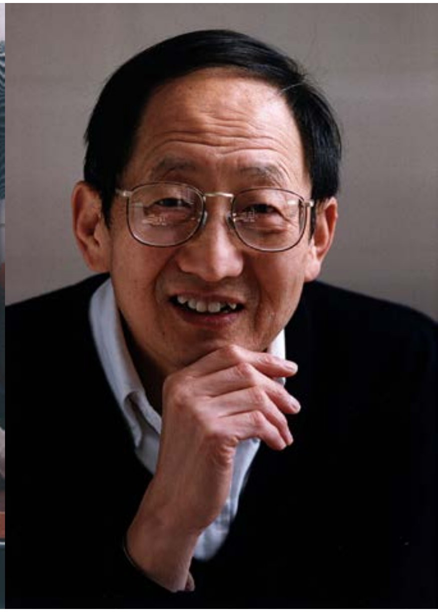
Karl Lark-Horowitz

Purdue Department of Physics, 1928-1958

- **Transformed the Physics Department into a research powerhouse.**
- **Best known for work on Germanium rectifiers during WWII (needed for radar).**
- **Collaborated/competed with Bell Labs on creating high quality germanium crystals that led to first transistors at Bell Labs in 1946.**

Crystal Fire, Michael Riordan and Lillian Hoddeson

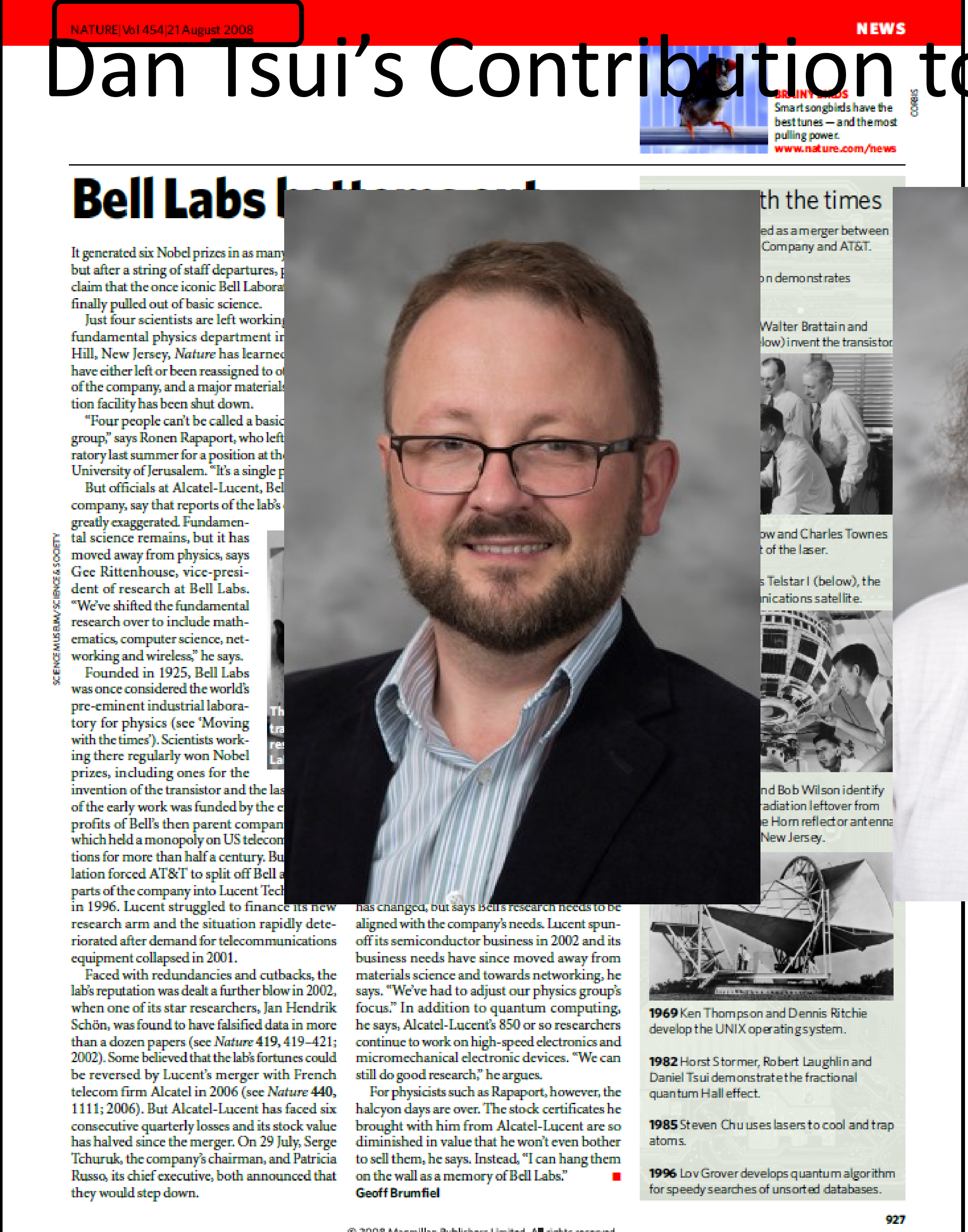
Semiconductor Physics Department early “aughts”



Horst Stormer

Dan Tsui

Dan Tsui’s Contribution to Purdue Physics and Astronomy


NATURE|Vol 454|21 August 2008

NEWS

BRAINY BIRDS
Smart songbirds have the best tunes — and the most pulling power.
www.nature.com/news

CORBIS

Bell Labs

It generated six Nobel prizes in as many
but after a string of staff departures,
claim that the once iconic Bell Labora
finally pulled out of basic science.

Just four scientists are left working
fundamental physics department in
Hill, New Jersey, *Nature* has learned
have either left or been reassigned to o
of the company, and a major materials
tion facility has been shut down.

"Four people can't be called a basic
group," says Ronen Rapaport, who left
ratory last summer for a position at th
University of Jerusalem. "It's a single p

But officials at Alcatel-Lucent, Bel
company, say that reports of the lab's
greatly exaggerated. Fundamental science remains, but it has moved away from physics, says Gee Rittenhouse, vice-president of research at Bell Labs. "We've shifted the fundamental research over to include mathematics, computer science, networking and wireless," he says.

Founded in 1925, Bell Labs was once considered the world's pre-eminent industrial laboratory for physics (see 'Moving with the times'). Scientists working there regularly won Nobel prizes, including ones for the
invention of the transistor and the las
of the early work was funded by the e
profits of Bell's then parent compan
which held a monopoly on US telecom
tions for more than half a century. Bu
lation forced AT&T to split off Bell a
parts of the company into Lucent Tech
in 1996. Lucent struggled to finance its new research arm and the situation rapidly deteriorated after demand for telecommunications equipment collapsed in 2001.

Faced with redundancies and cutbacks, the lab's reputation was dealt a further blow in 2002, when one of its star researchers, Jan Hendrik Schön, was found to have falsified data in more than a dozen papers (see *Nature* 419, 419–421; 2002). Some believed that the lab's fortunes could be reversed by Lucent's merger with French telecom firm Alcatel in 2006 (see *Nature* 440, 1111; 2006). But Alcatel-Lucent has faced six consecutive quarterly losses and its stock value has halved since the merger. On 29 July, Serge Tchuruk, the company's chairman, and Patricia Russo, its chief executive, both announced that they would step down.

has changed, but says Bell's research needs to be aligned with the company's needs. Lucent spun-off its semiconductor business in 2002 and its business needs have since moved away from materials science and towards networking, he says. "We've had to adjust our physics group's focus." In addition to quantum computing, he says, Alcatel-Lucent's 850 or so researchers continue to work on high-speed electronics and micromechanical electronic devices. "We can still do good research," he argues.

For physicists such as Rapaport, however, the halcyon days are over. The stock certificates he brought with him from Alcatel-Lucent are so diminished in value that he won't even bother to sell them, he says. Instead, "I can hang them on the wall as a memory of Bell Labs." ■

Geoff Brumfiel

SCIENCE MUSEUM/SCIENCE & SOCIETY

th the times

ed as a merger between
Company and AT&T.

on demonstrates

Walter Brattain and
low) invent the transistor

ow and Charles Townes
t of the laser.

s Telstar I (below), the
nications satellite.

nd Bob Wilson identify
radiation leftover from
e Horn reflector antenna
New Jersey.

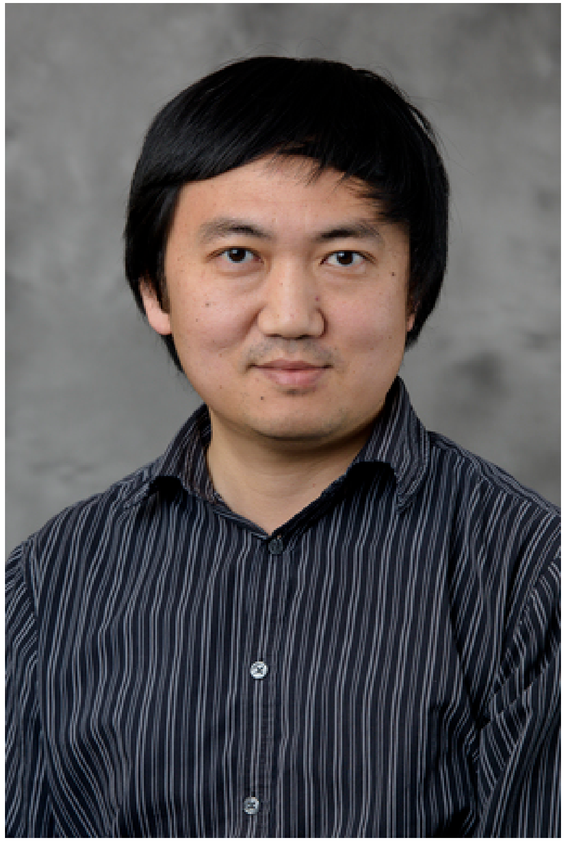
1969 Ken Thompson and Dennis Ritchie develop the UNIX operating system.

1982 Horst Stormer, Robert Laughlin and Daniel Tsui demonstrate the fractional quantum Hall effect.

1985 Steven Chu uses lasers to cool and trap atoms.

1996 Lov Grover develops quantum algorithm for speedy searches of unsorted databases.

927



Elementary particles

Emergent particles in 2D

FERMIONS

Pauli Exclusion Principle
Fermions switch places:
(-1) * Wavefunction

electrons
quarks
muons
neutrons
protons

BOSONS

Flock Together
Bosons switch places:
(1) * Wavefunction

photons
gluons
Anderson-Higgs boson
gravitons

ANYONS

Unprecedented Quantum Effects
- Fractional charge
- Anyonic braiding statistics

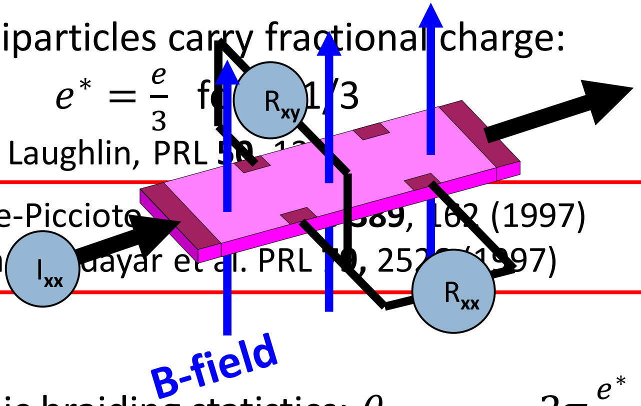
$(e^{i\theta})$ * Wavefunction

quasiparticles in the fractional quantum Hall state at $\nu=1/3$

$\theta=2\pi/3$

Slide adapted from Erica Carlson’s

Excitations of the FQHE are anyons: fractional charge and statistics



- Quasiparticles carry fractional charge:

$$e^* = \frac{e}{3} \quad \text{f} \quad 1/3$$

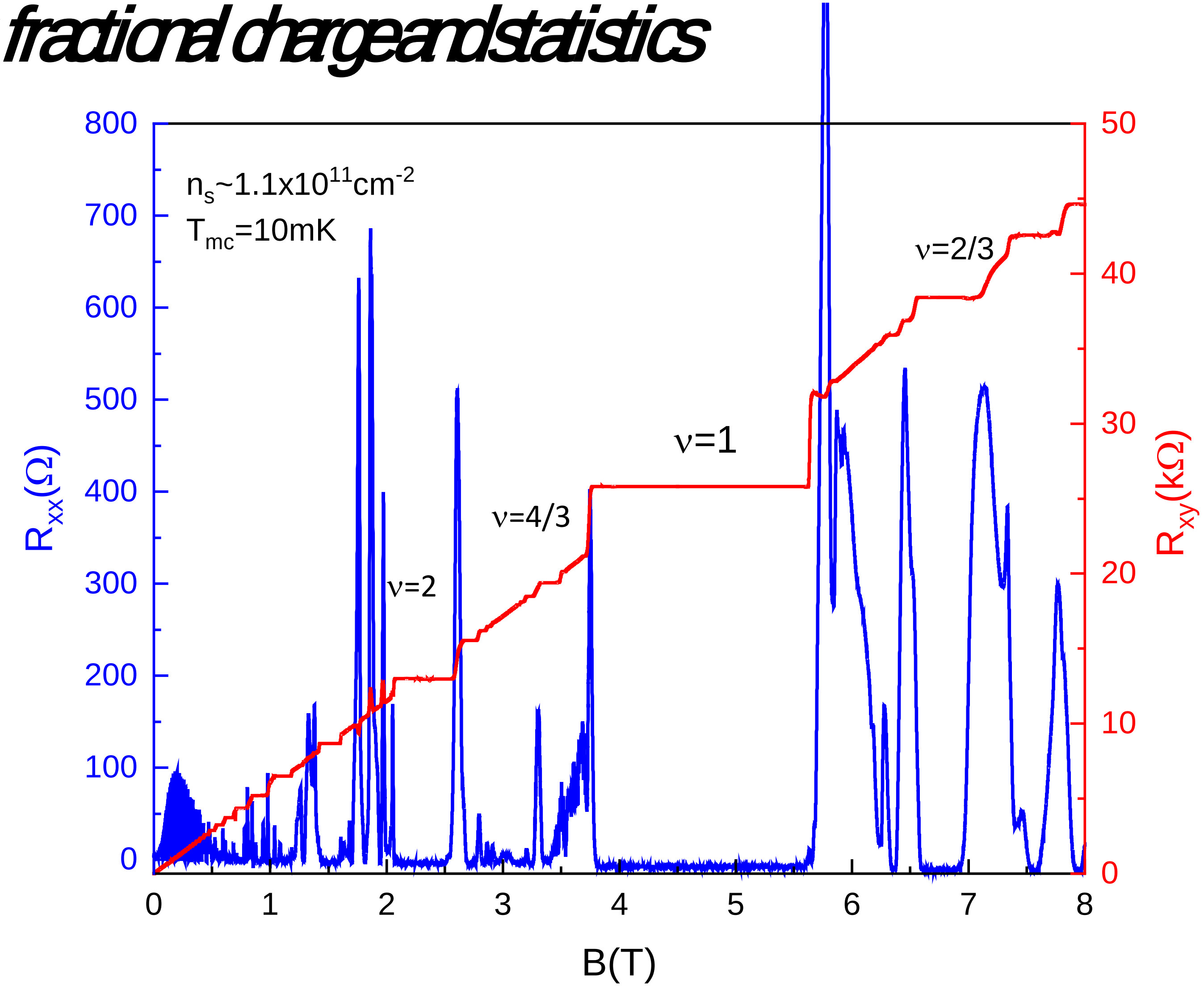
R. B. Laughlin, PRL **50**, 1

R. de-Picciote **389**, 162 (1997)
L. Sa dayar et al. PRL **79**, 252 97)

- Anyonic braiding statistics: $\theta_{anyon} = 2\pi \frac{e^*}{e}$

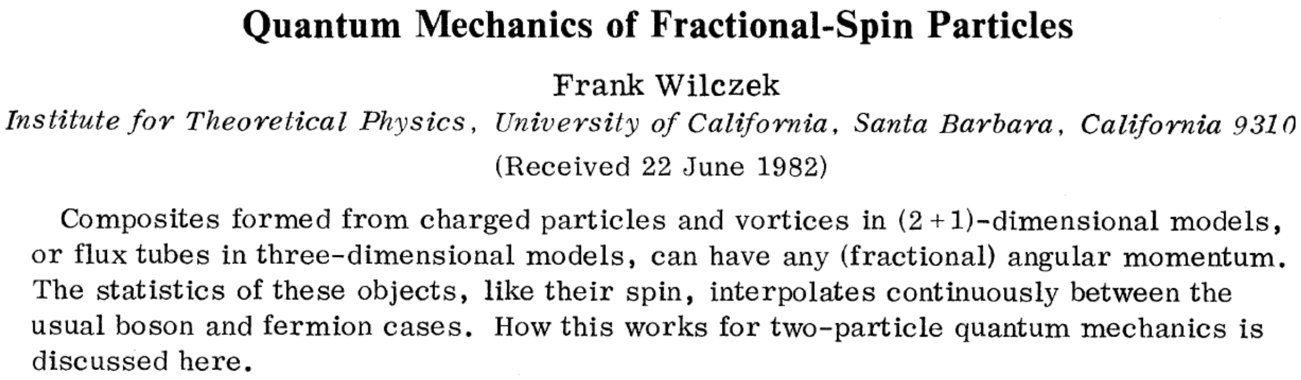
B. I. Halperin, PRL **52**, 1583 (1984)

Arovas, Schrieffer, and Wilczek, PRL **53**, 722 (1984)



$$\nu\text{=1/3: } \theta_{anyon} = \frac{2\pi}{3}$$

Early analysis of fractional statistics and the FQHE



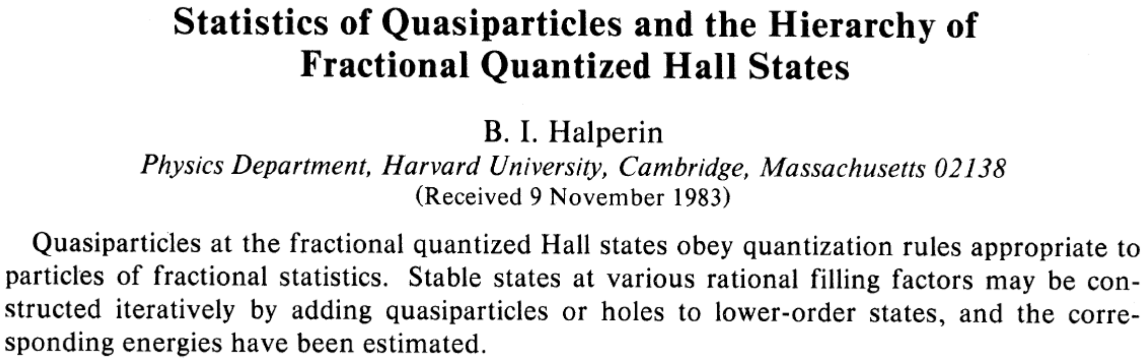
Quantum Mechanics of Fractional-Spin Particles

Frank Wilczek

Institute for Theoretical Physics, University of California, Santa Barbara, California 9310

(Received 22 June 1982)

Composites formed from charged particles and vortices in (2 + 1)-dimensional models, or flux tubes in three-dimensional models, can have any (fractional) angular momentum. The statistics of these objects, like their spin, interpolates continuously between the usual boson and fermion cases. How this works for two-particle quantum mechanics is discussed here.



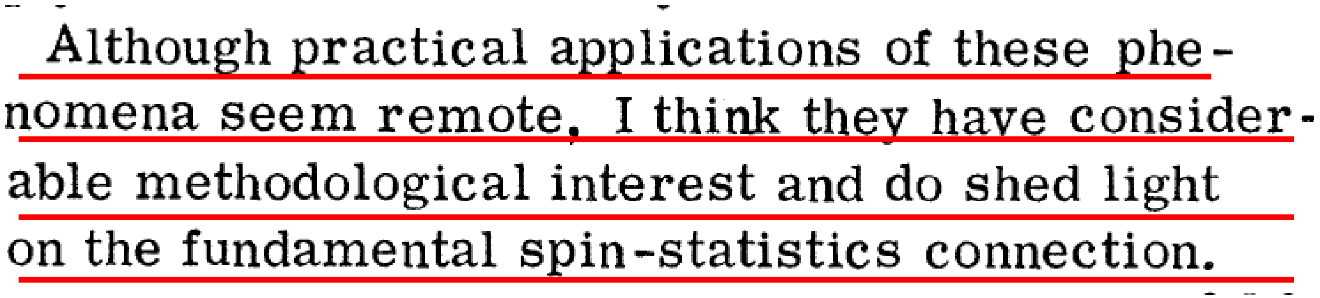
Statistics of Quasiparticles and the Hierarchy of Fractional Quantized Hall States

B. I. Halperin

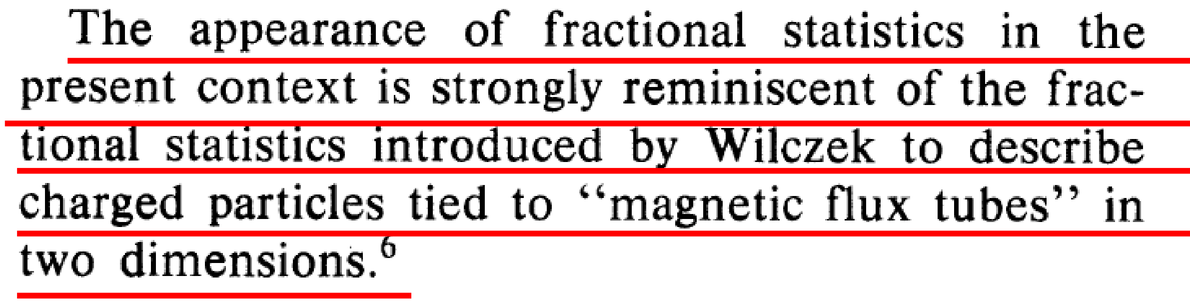
Physics Department, Harvard University, Cambridge, Massachusetts 02138

(Received 9 November 1983)

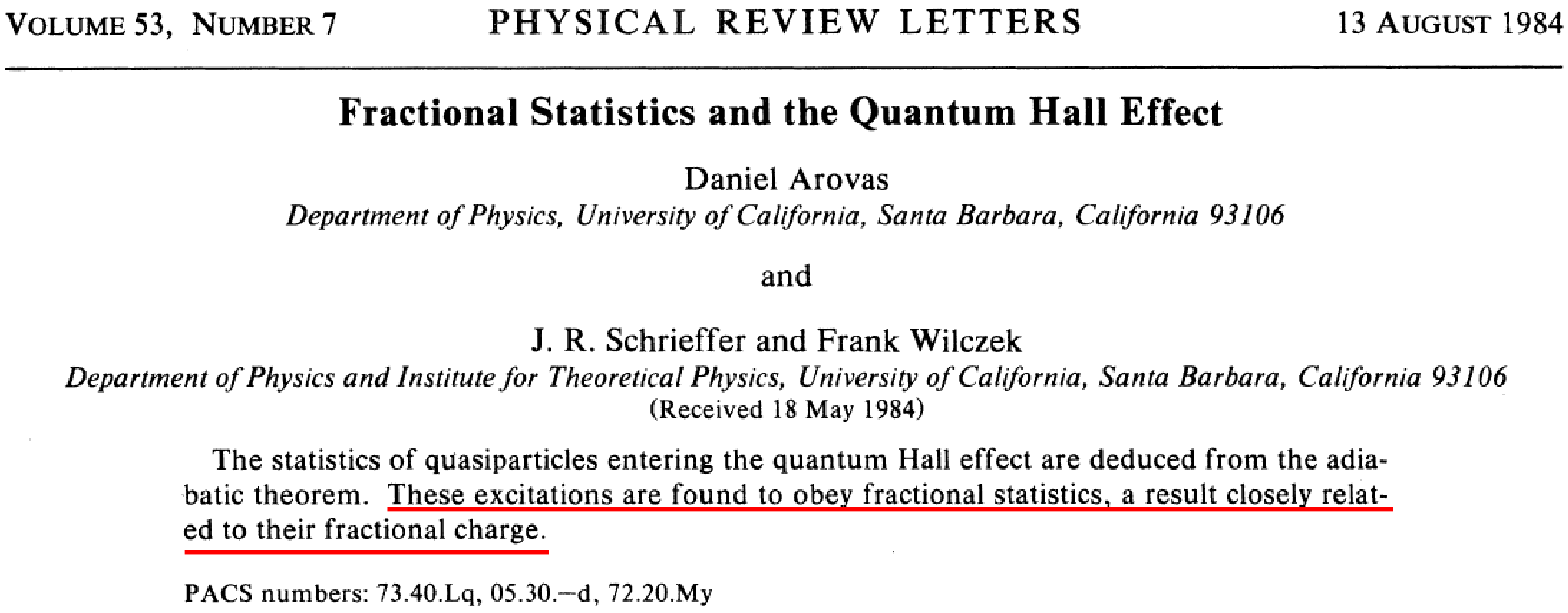
Quasiparticles at the fractional quantized Hall states obey quantization rules appropriate to particles of fractional statistics. Stable states at various rational filling factors may be constructed iteratively by adding quasiparticles or holes to lower-order states, and the corresponding energies have been estimated.



Although practical applications of these phenomena seem remote, I think they have considerable methodological interest and do shed light on the fundamental spin-statistics connection.



The appearance of fractional statistics in the present context is strongly reminiscent of the fractional statistics introduced by Wilczek to describe charged particles tied to "magnetic flux tubes" in two dimensions.[6]



VOLUME 53, NUMBER 7 PHYSICAL REVIEW LETTERS 13 AUGUST 1984

Fractional Statistics and the Quantum Hall Effect

Daniel Arovas
Department of Physics, University of California, Santa Barbara, California 93106

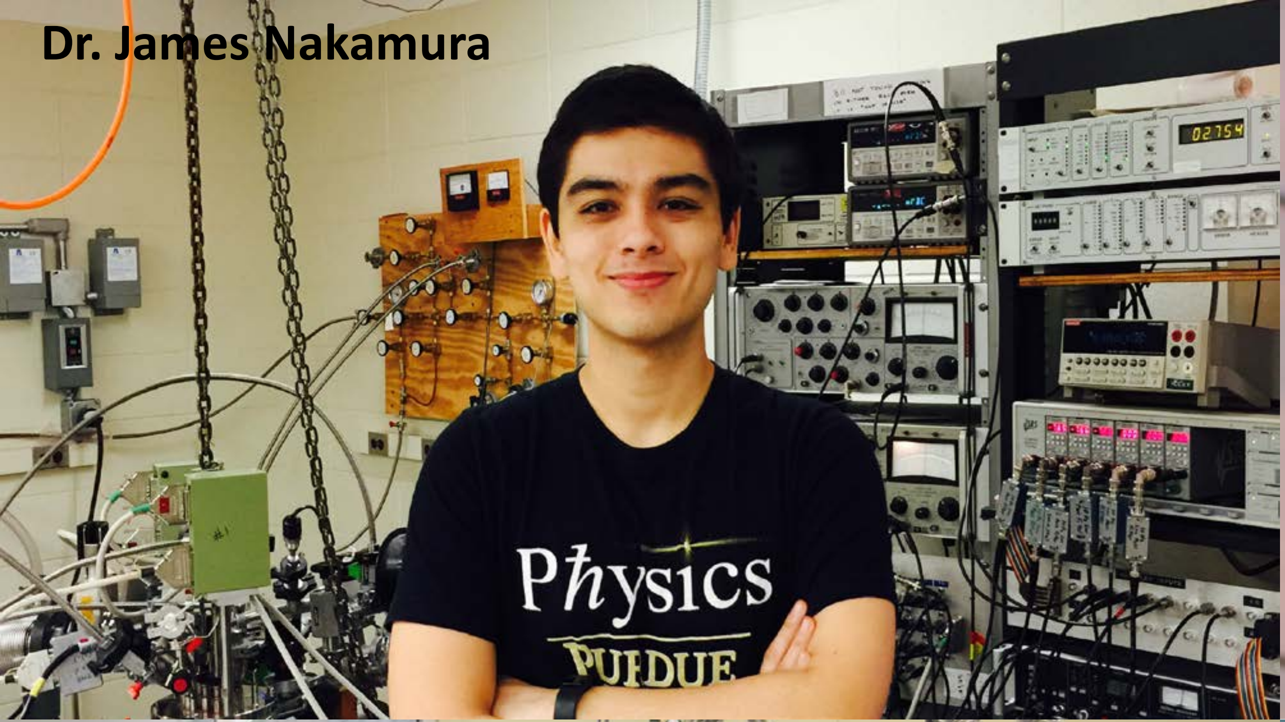
and

J. R. Schrieffer and Frank Wilczek
Department of Physics and Institute for Theoretical Physics, University of California, Santa Barbara, California 93106

(Received 18 May 1984)

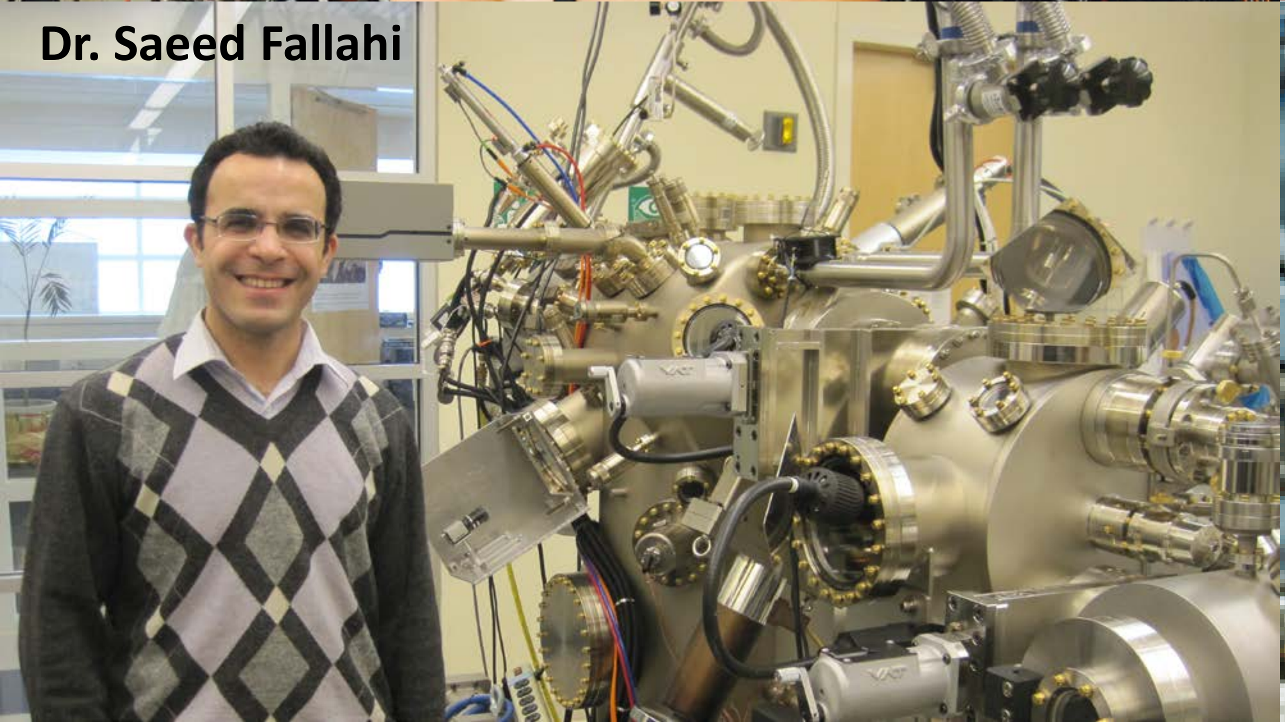
The statistics of quasiparticles entering the quantum Hall effect are deduced from the adiabatic theorem. These excitations are found to obey fractional statistics, a result closely related to their fractional charge.

PACS numbers: 73.40.Lq, 05.30.−d, 72.20.My

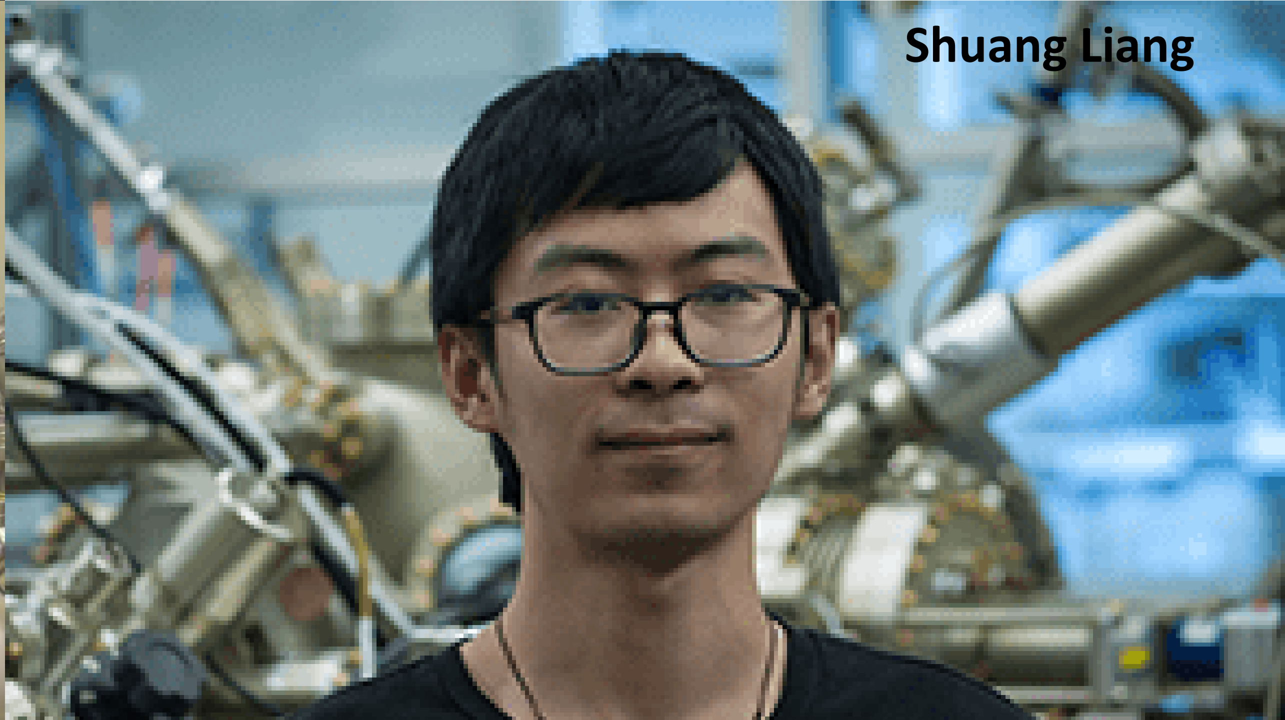


Dr. James Nakamura

Dr. Geoffrey Gardner

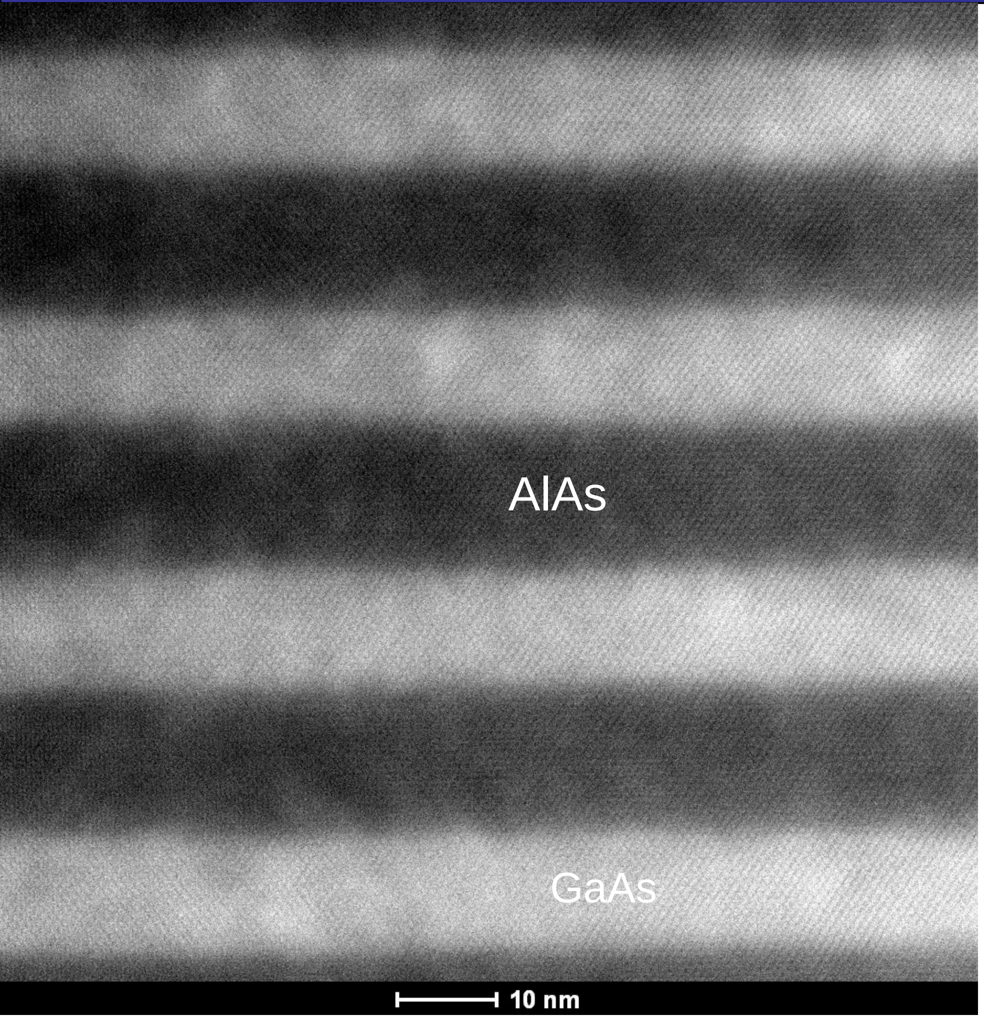


Dr. Saeed Fallahi

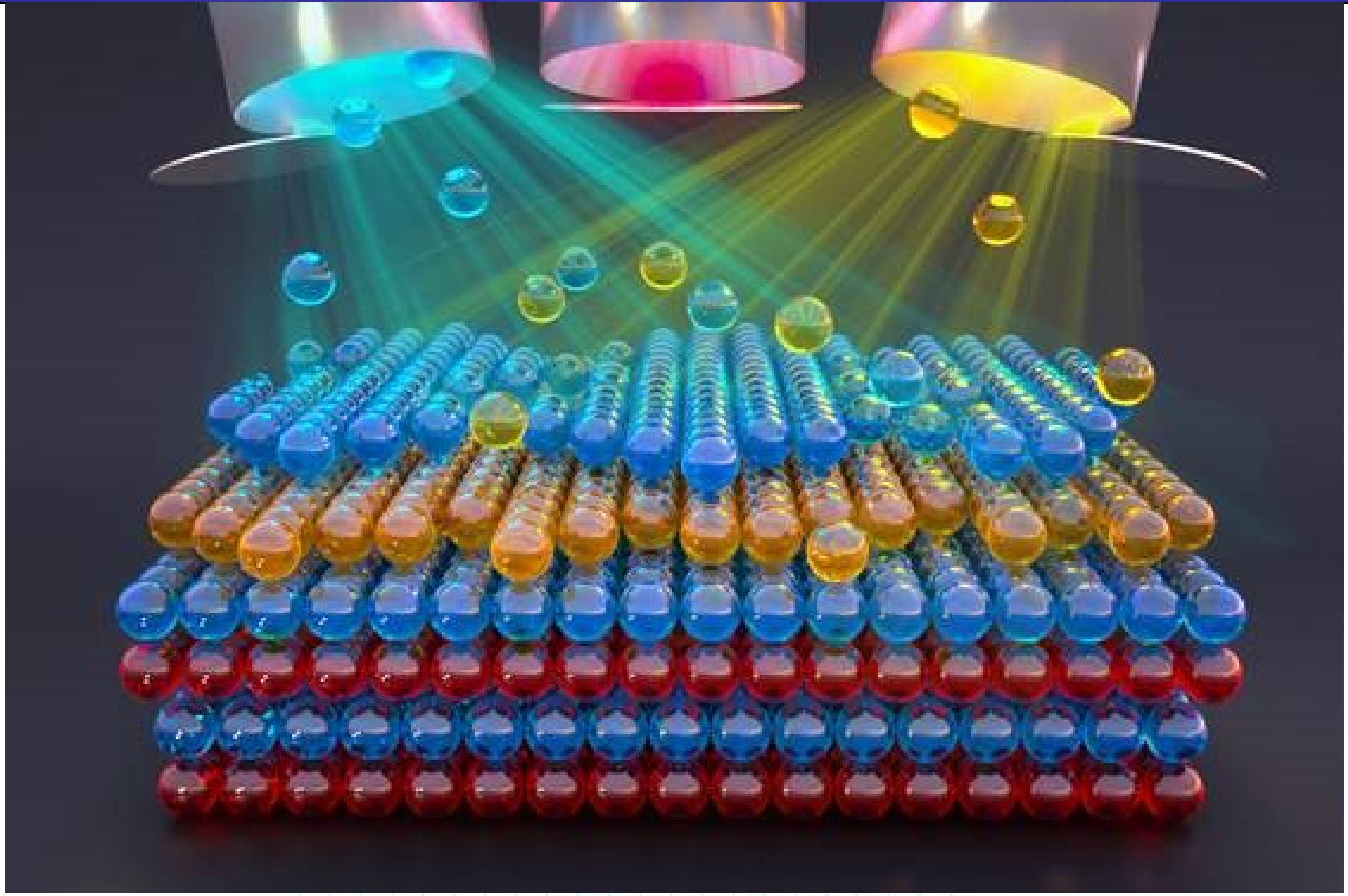


Shuang Liang

The key to low dimensional electron systems is the insulator-semiconductor heterointerface

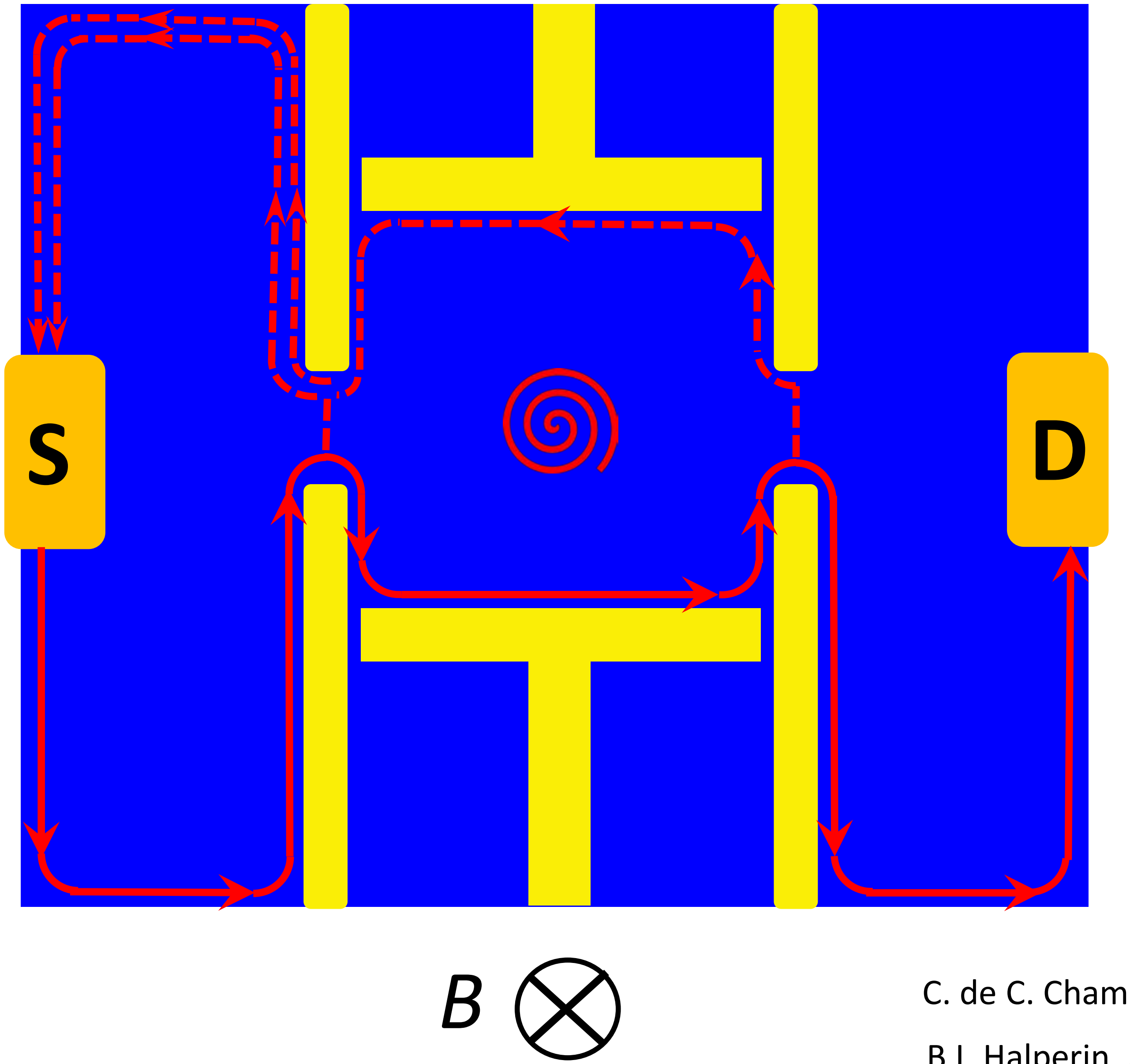


This AlAs/GaAs interface is among the most perfect in all of nature.



MBE: painting with atoms

Electronic Fabry-Perot interferometry in the Quantum Hall regime

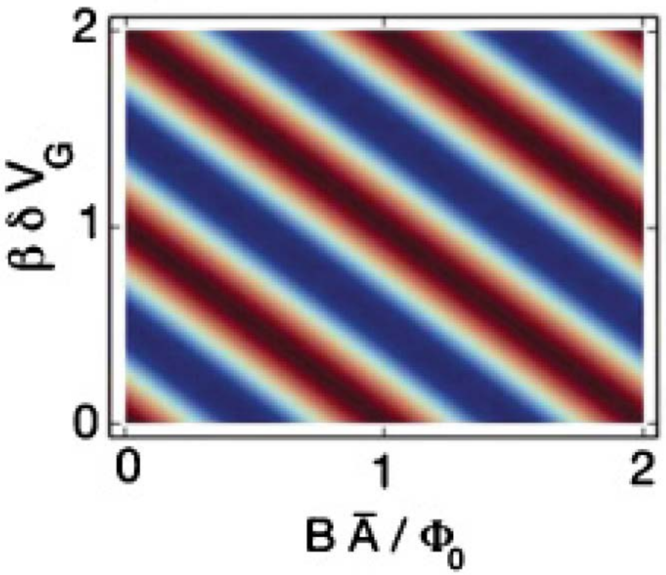


- Surface gates define electron interference path
- Quantum point contacts (QPCs) act as beam splitters

$$I\sim|t_1|^2+|t_2|^2+|t_1||t_2|\cos(\theta)$$

Aharonov-Bohm phase   Braiding phase

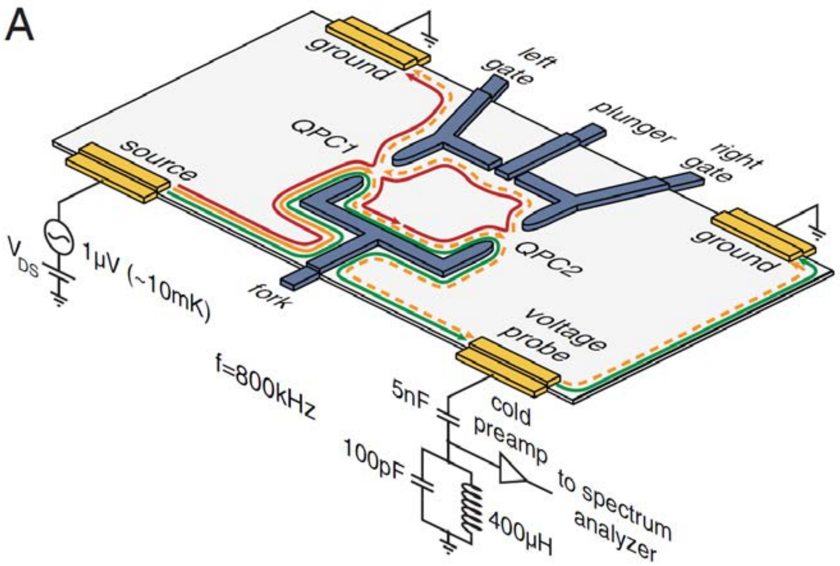
$$\theta\ =2\pi\left(\frac{AB}{\Phi_0}\right)\frac{e^*}{e}+N_L\theta_{anyon}\qquad\Phi_0\equiv\frac{h}{e}$$

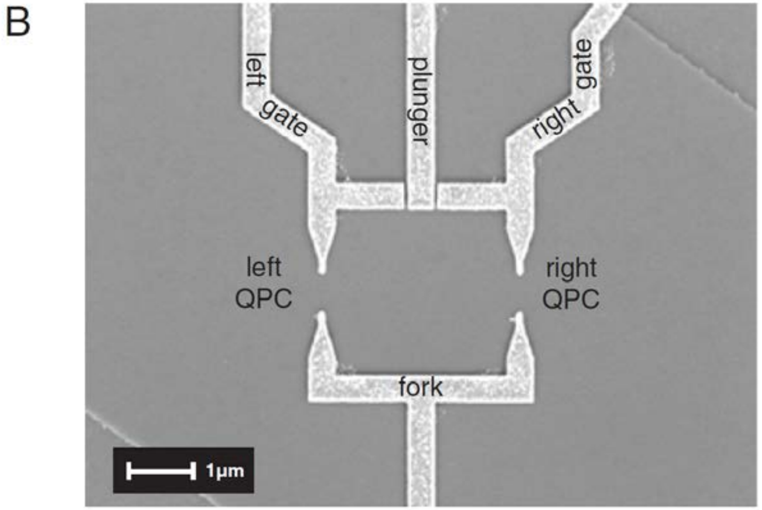


C. de C. Chamon, D. Freed, S. Kivelson, S. Sondhi, X. Wen Phys. Rev. B **55**, 2331 (1997)

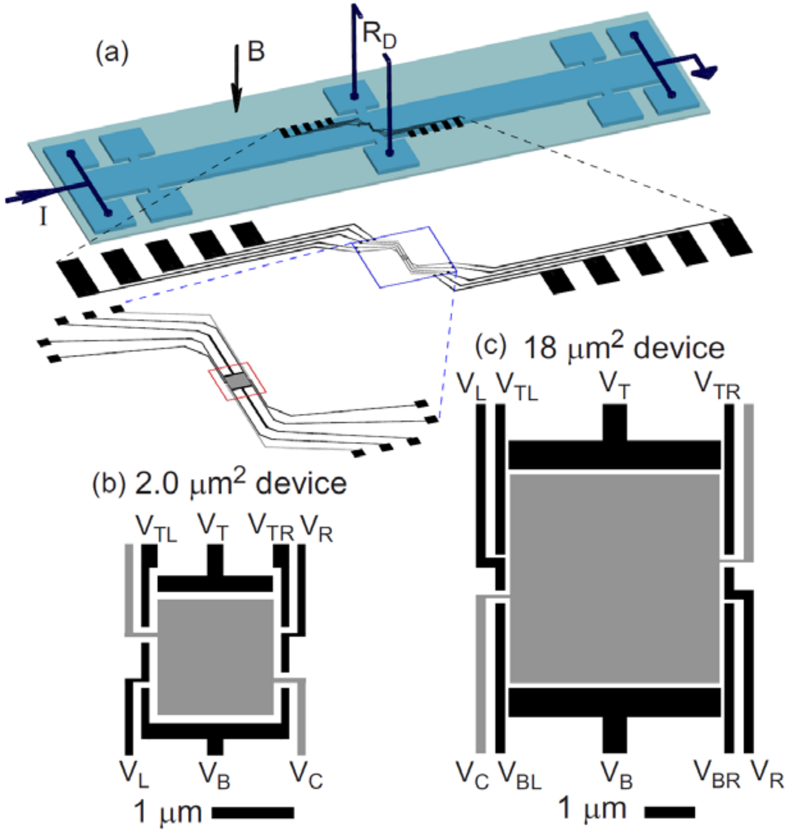
B.I. Halperin, A. Stern, I. Neder, and B. Rosenow PRB **83**, 155440 (2011)

Early Experiments: Challenges and Clues





small devices ~4μm^2
Ofek, PNAS 2010
(M. Heiblum group)

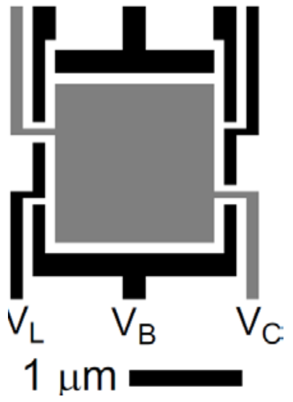


large devices ~20μm^2
Zhang, PRB **79**, 241304 (R) (2009)
(C. Marcus group)

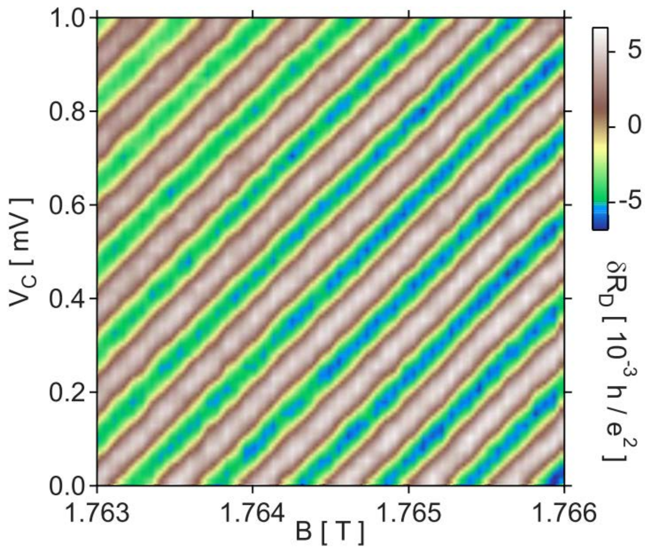
AB vs. CD in early experiments: a valuable lesson

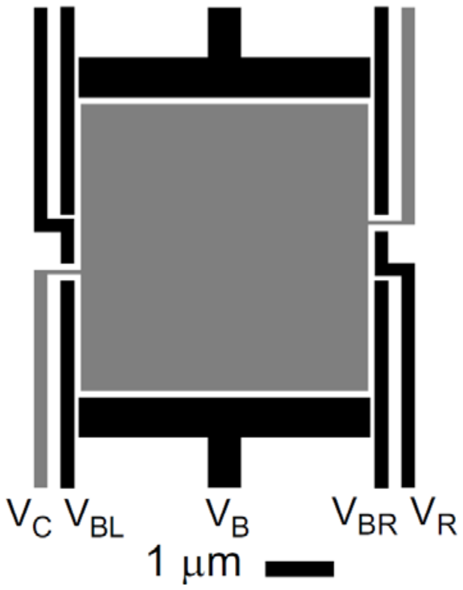
- Many early experiments observed Coulomb dominated behavior
- C. Marcus group observed AB behavior (negative slope) in devices with large area which included metal screening gates
- Coherence was poor due to large path length
- Need better way to screen to observe AB interference in smaller devices

Zhang et al. PRB **79**, 241304 (2009)

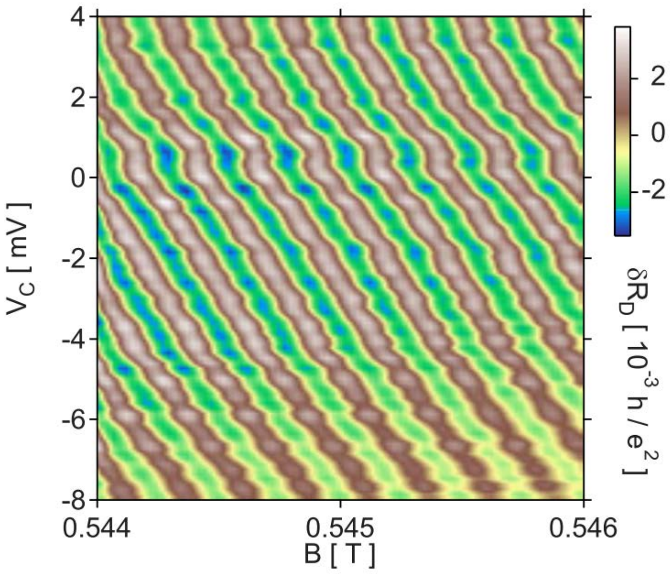


$2\mu m^2$ Device





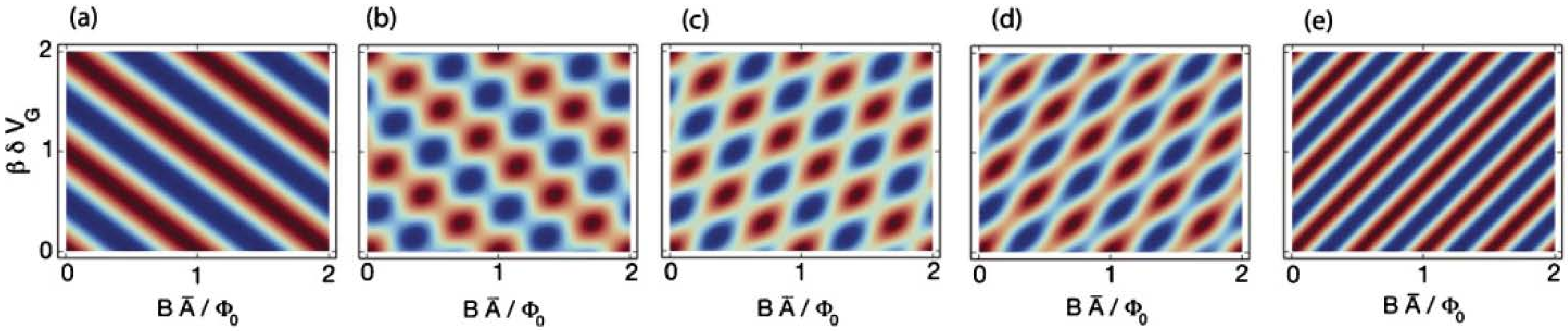
$18\mu m^2$ Device



Aharonov-Bohm vs. Coulomb-Dominated regimes

Aharonov-Bohm

Coulomb-dominated

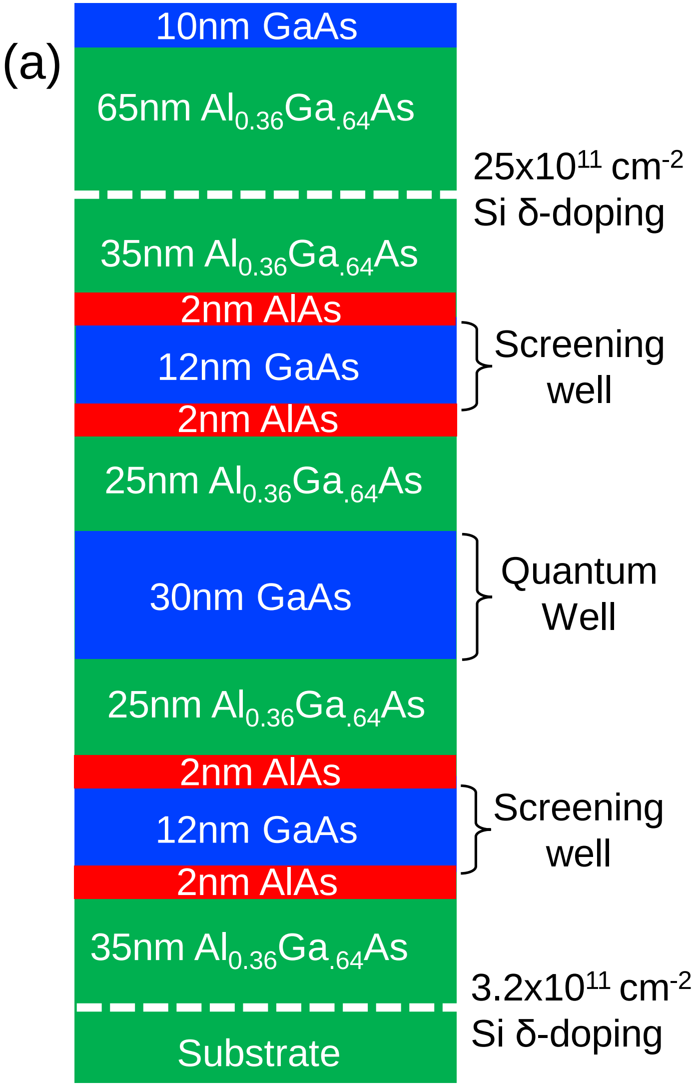


- **Regime of operation depends on the ratio of K_{IL}/K_I, where K_{IL} parameterizes bulk-edge interaction and K_I parameterizes the energy cost to add charge to the edge**

- **Critically, θ_{anyon} is unobservable in the Coulomb-dominated regime: phase change is multiple of 2π.**

B. I. Halperin, A. Stern, I. Neder, and B. Rosenow. PRB **83**, 155440 (2011)
C. W. von Keyserlingk, S. H. Simon, B. Rosenow, PRL **115**, 126807 (2015)
D. Feldman and B. Halperin, Rep. Prog. Phys. **84** (2021) 076501

Our contribution: new heterostructure and device design


(a)

10nm GaAs

65nm $Al_{0.36}Ga_{.64}As$

25×10^{11} cm^{-2}
Si δ-doping

35nm $Al_{0.36}Ga_{.64}As$

2nm AlAs

12nm GaAs

2nm AlAs

Screening well

25nm $Al_{0.36}Ga_{.64}As$

30nm GaAs

Quantum Well

25nm $Al_{0.36}Ga_{.64}As$

2nm AlAs

12nm GaAs

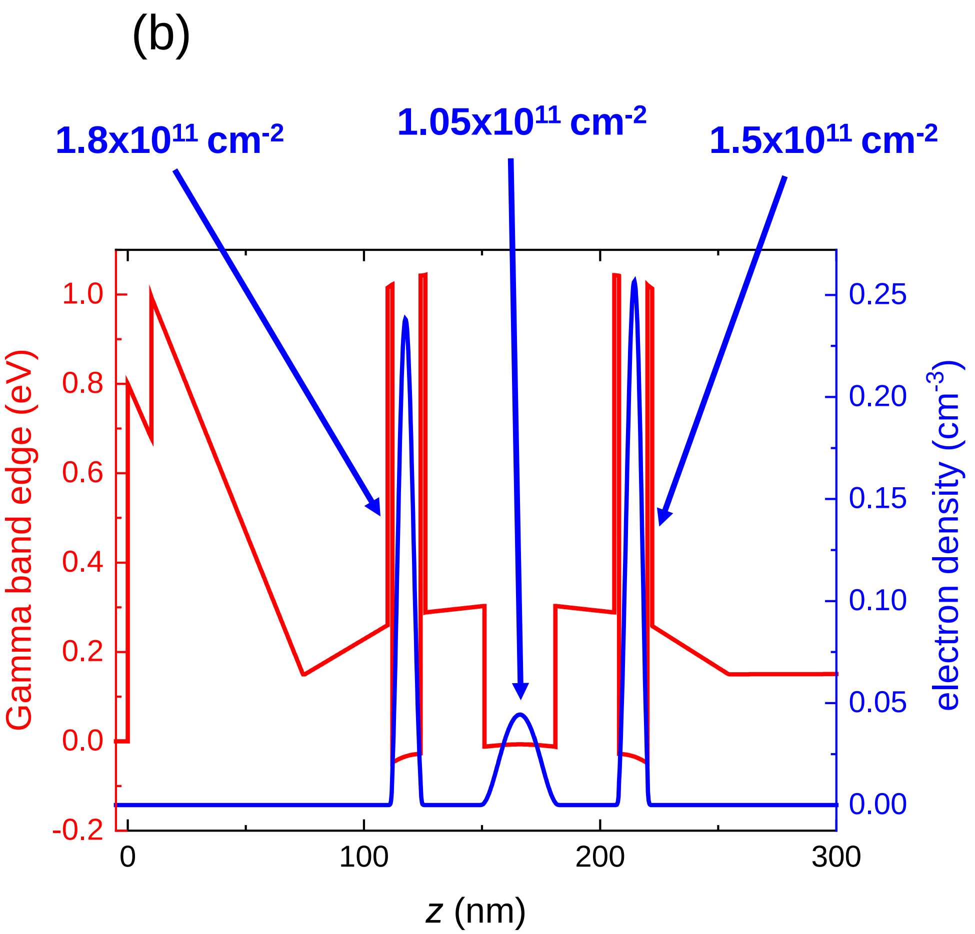
2nm AlAs

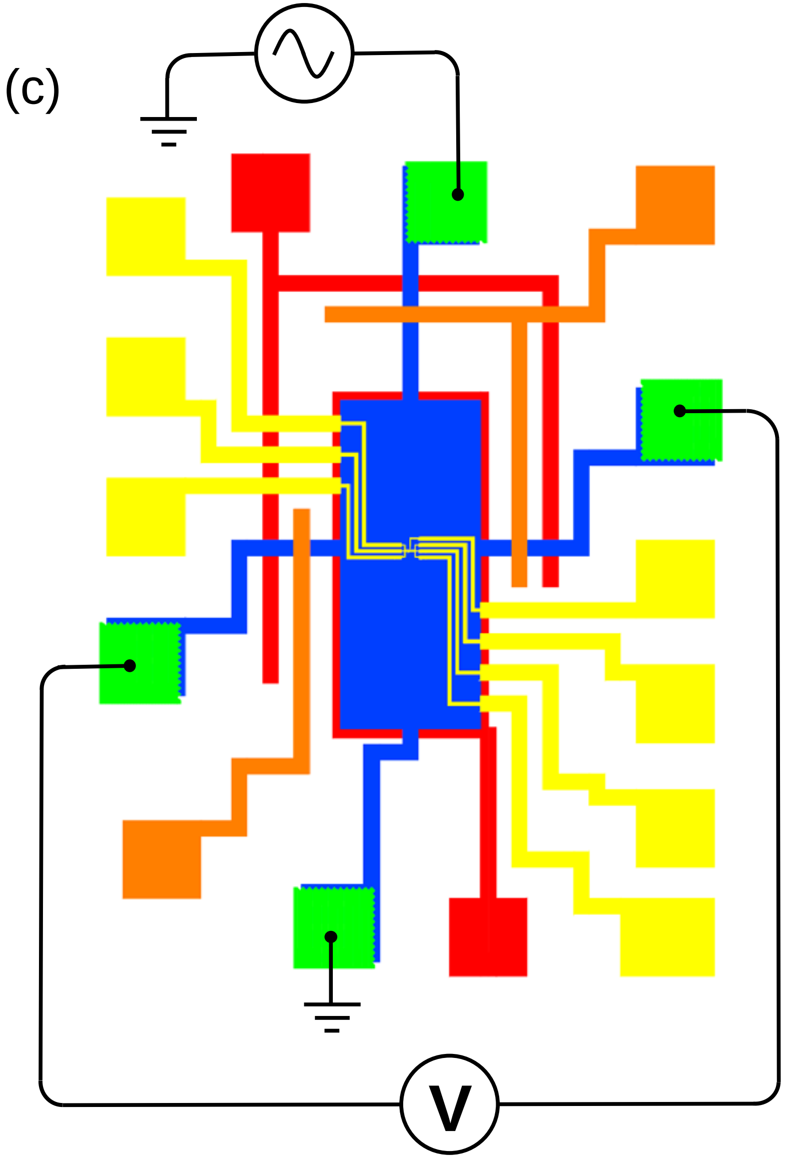
Screening well

35nm $Al_{0.36}Ga_{.64}As$

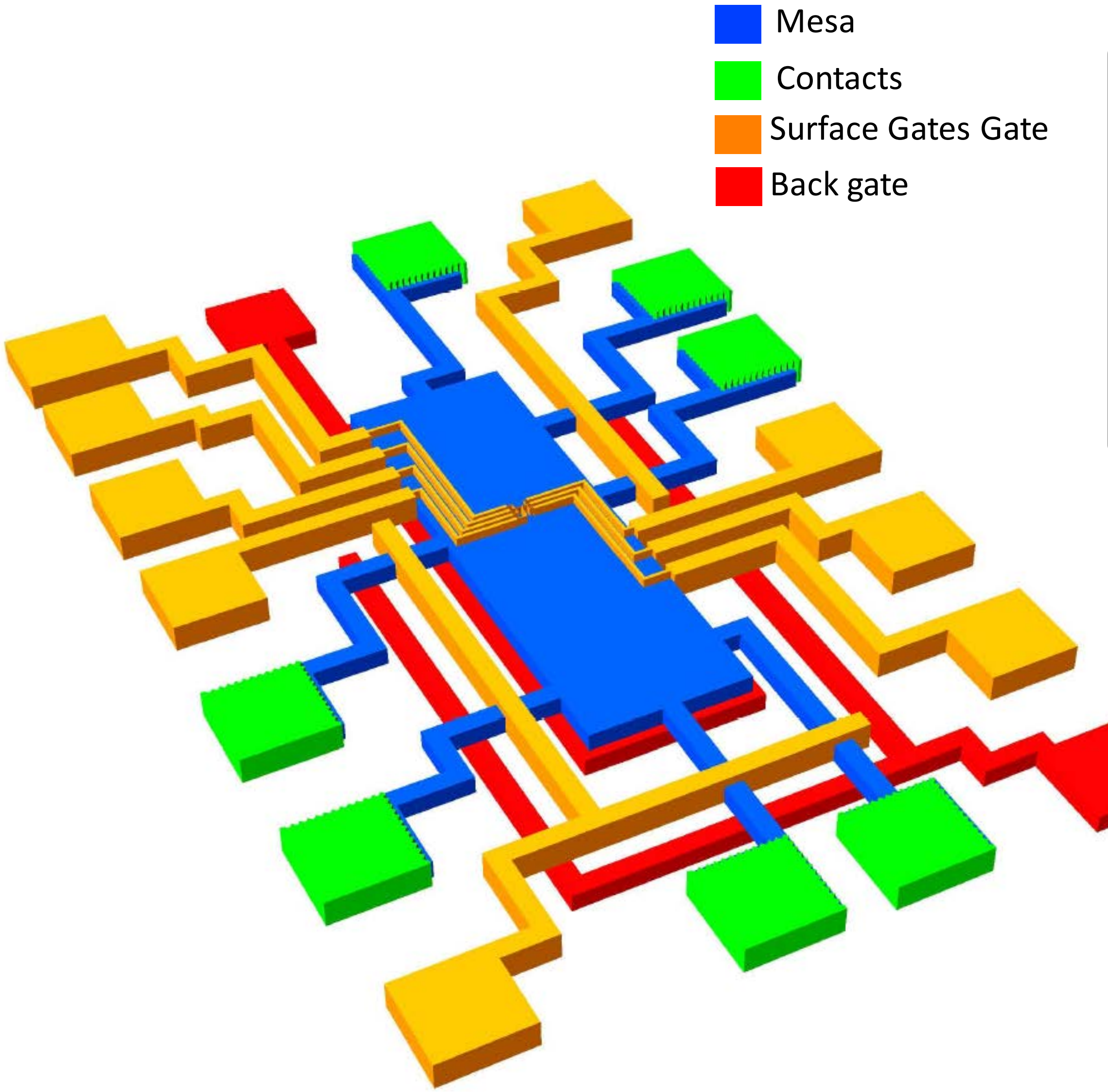
3.2×10^{11} cm^{-2}
Si δ-doping

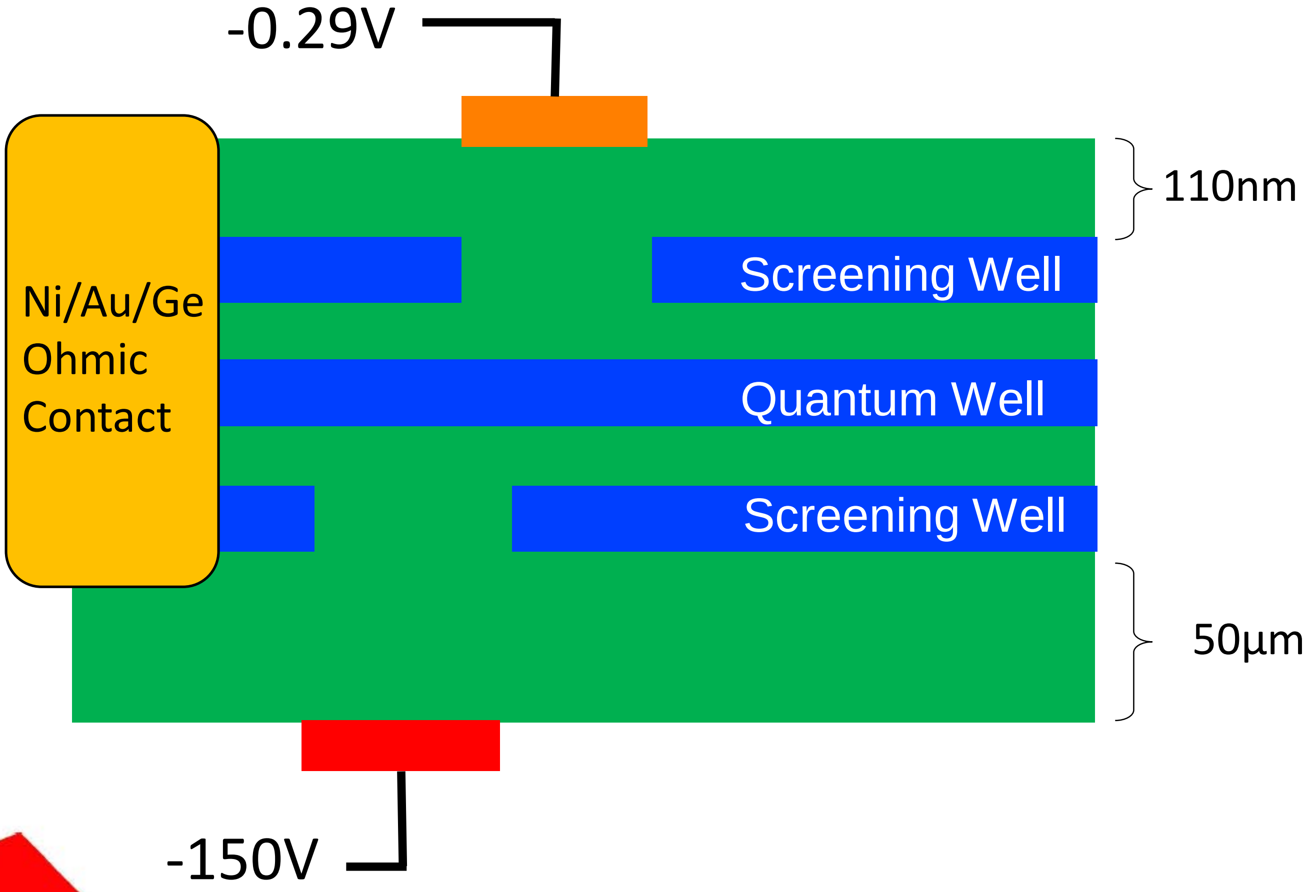
Substrate





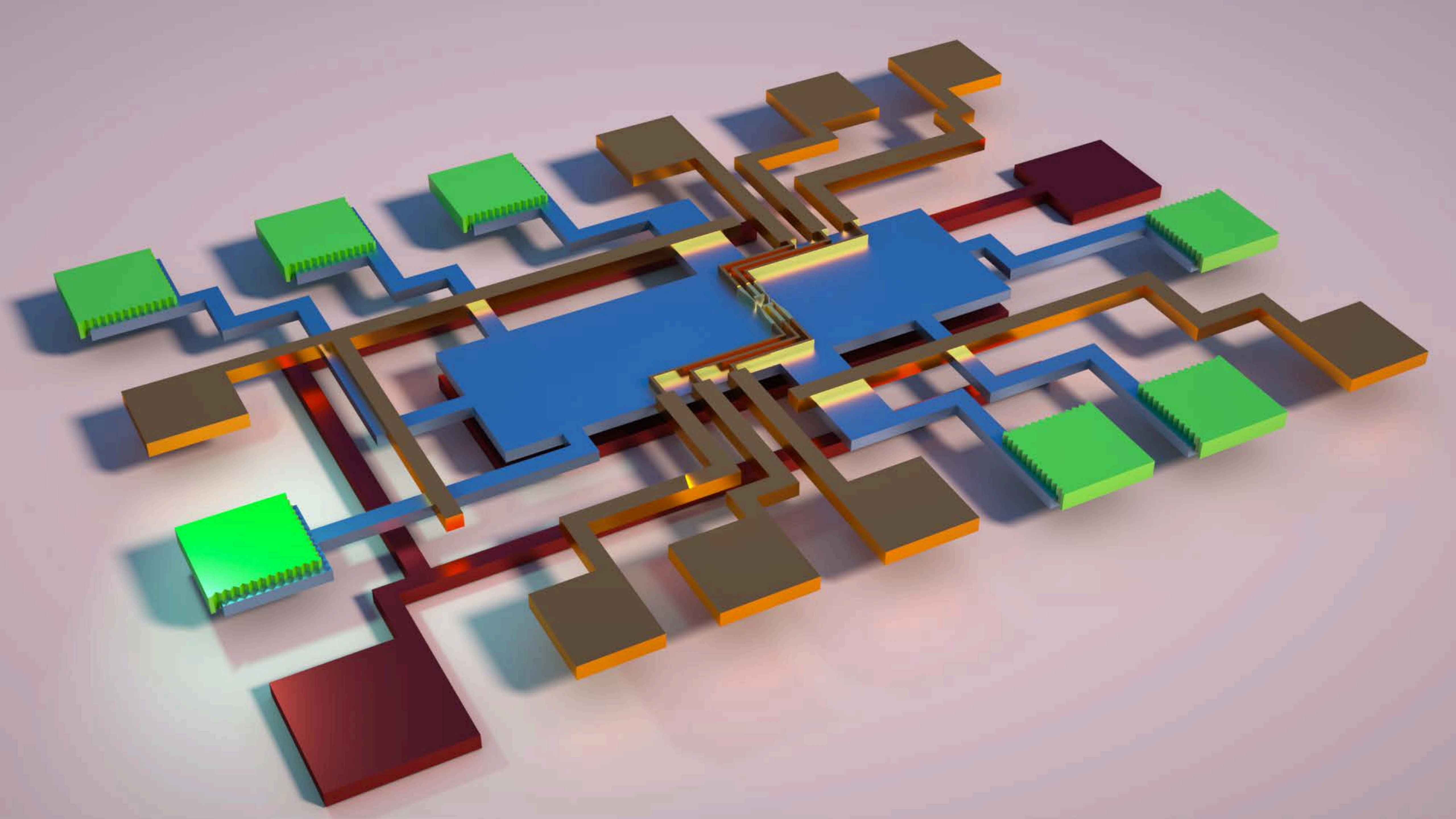
Top and back-gated interferometer operation

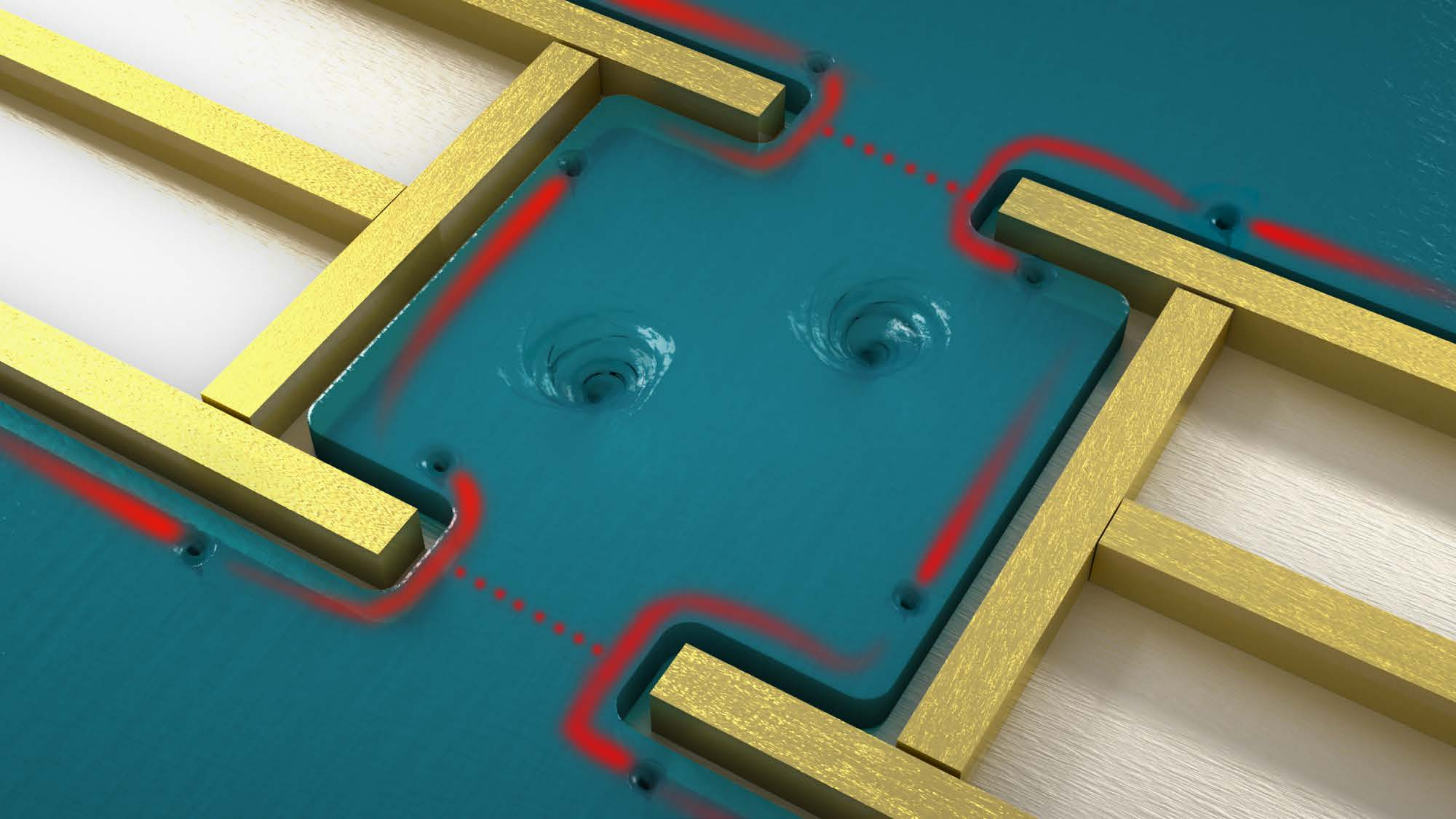


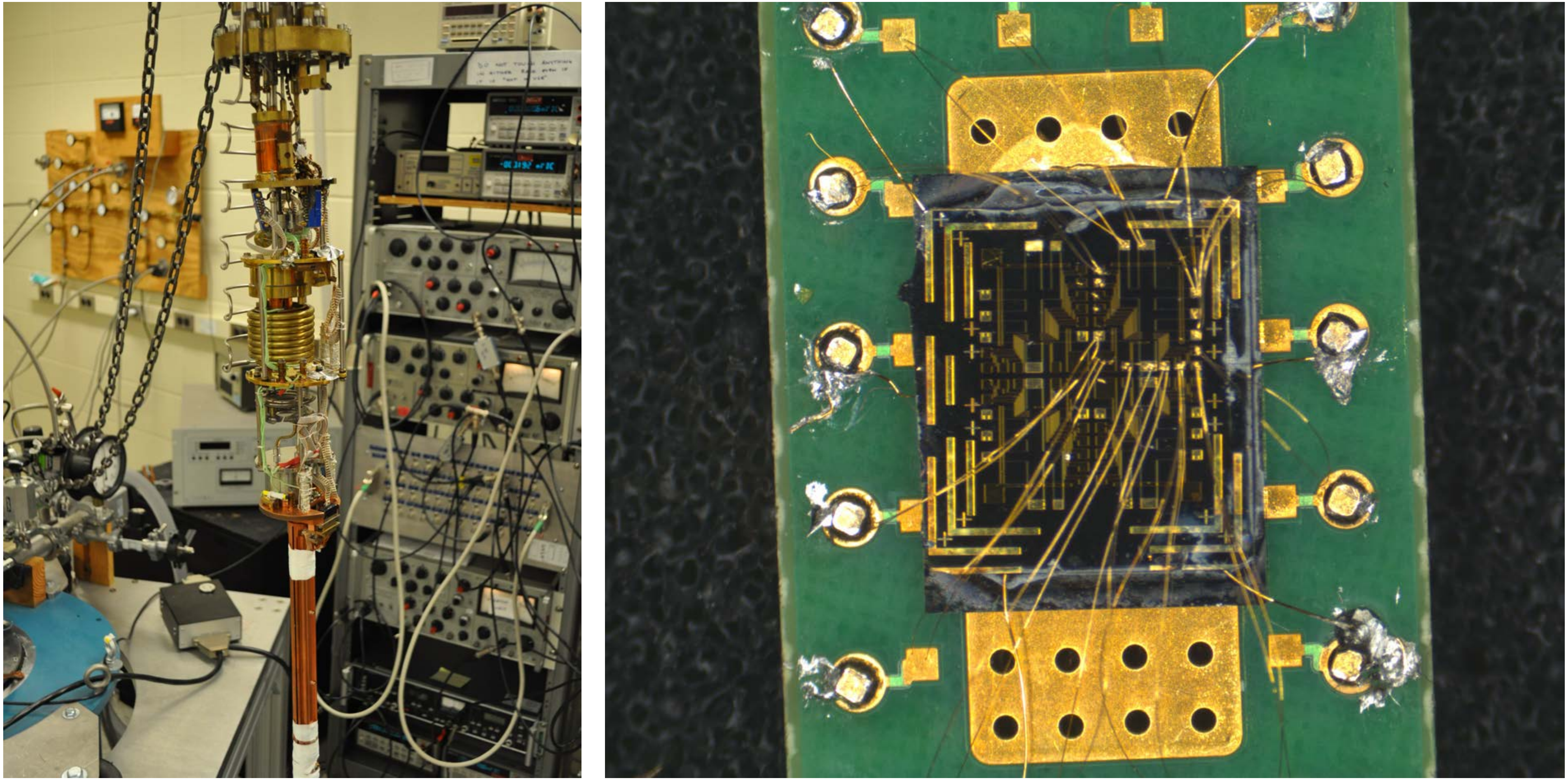


- Need to eliminate parallel conduction through screening wells
- Adapt technique used in bilayer systems – use gates around Ohmics to disconnect SWs from contacts

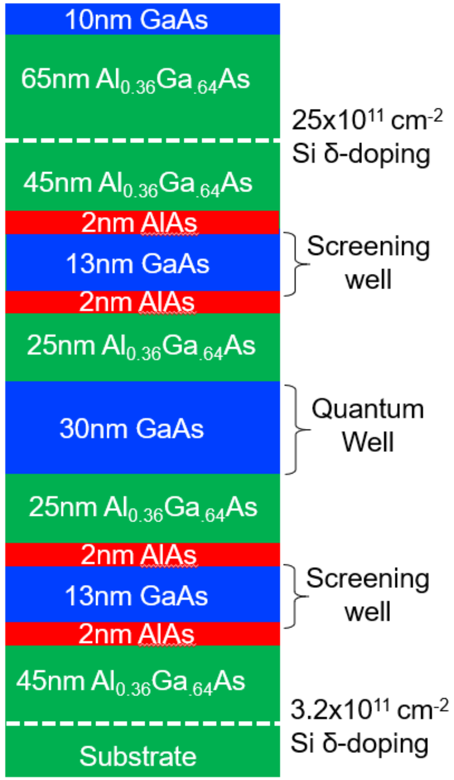
J. P. Eisenstein, L. N. Pfeiffer, & K. W. West. APL 57, 2324 (1990)






DO NOT TOUCH ANYTHING IN EITHER RACK EVEN IF IT IS "NOT IN USE"

In-Situ screening wells enable Aharonov-Bohm oscillations


10nm GaAs

65nm $Al_{0.36}Ga_{.64}As$

25×10^{11} cm^{-2} Si δ-doping

45nm $Al_{0.36}Ga_{.64}As$

2nm AlAs

13nm GaAs

2nm AlAs

Screening well

25nm $Al_{0.36}Ga_{.64}As$

30nm GaAs

Quantum Well

25nm $Al_{0.36}Ga_{.64}As$

2nm AlAs

13nm GaAs

2nm AlAs

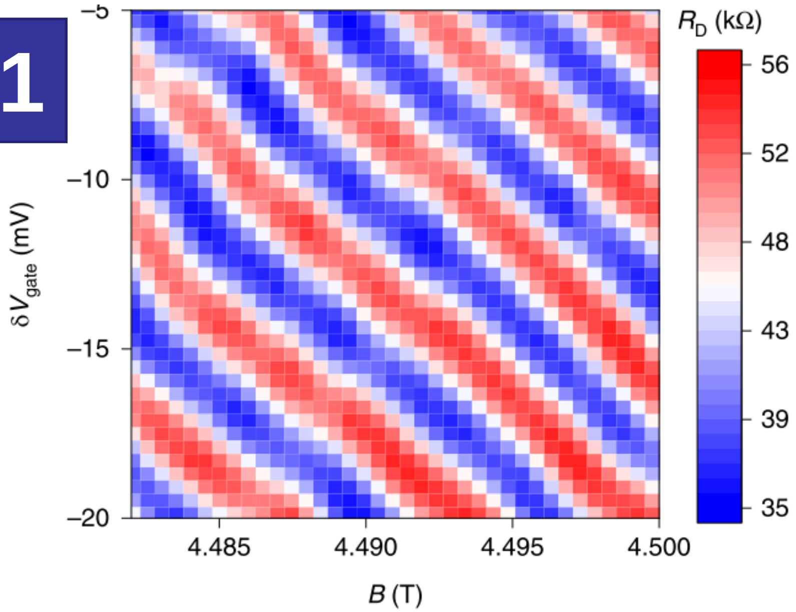
Screening well

45nm $Al_{0.36}Ga_{.64}As$

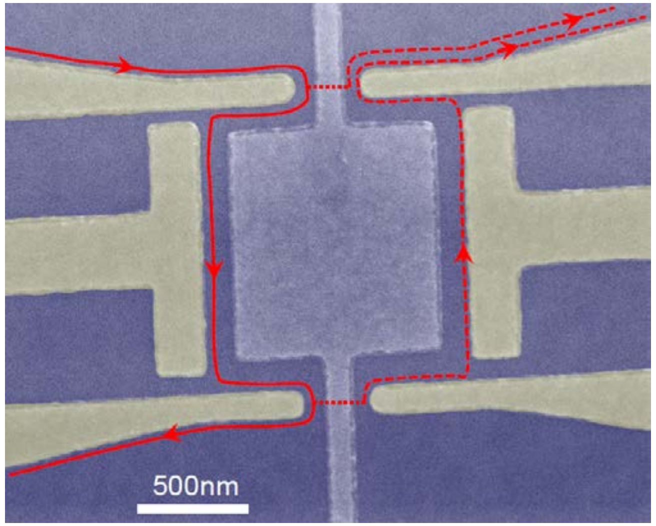
3.2×10^{11} cm^{-2} Si δ-doping

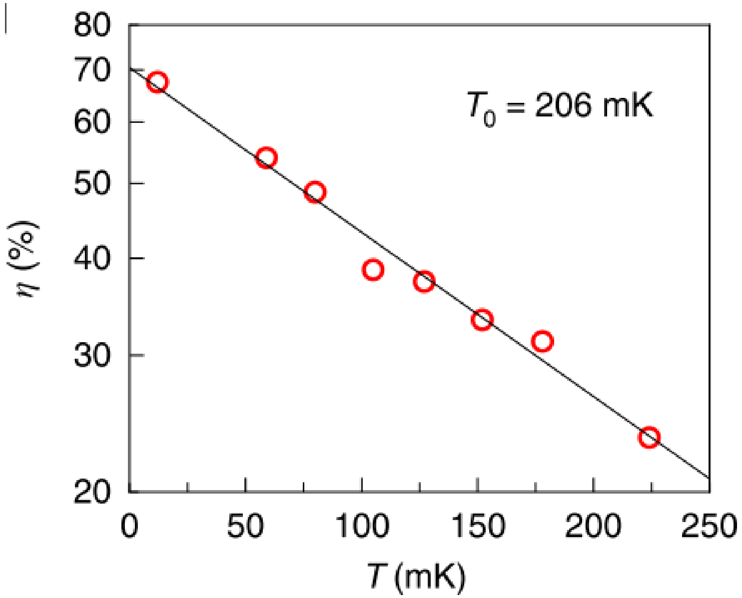
Substrate

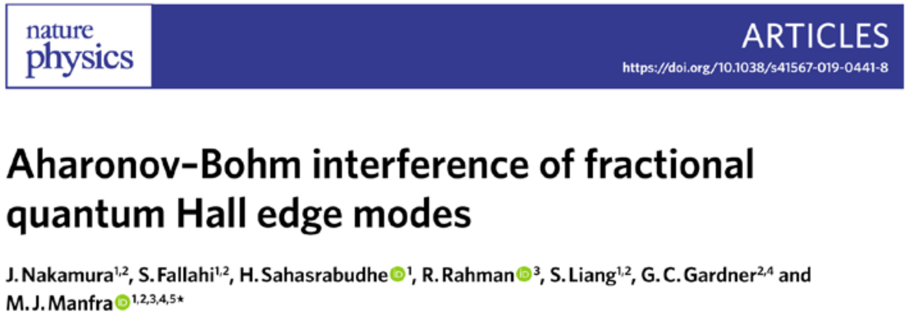
ν=1



- reduces bulk-edge coupling and ***increases*** sharpness of confining potential
- Enables high edge mode velocity ***and*** small interferometers
- High amplitude, robust interference in the FQHE






nature physics

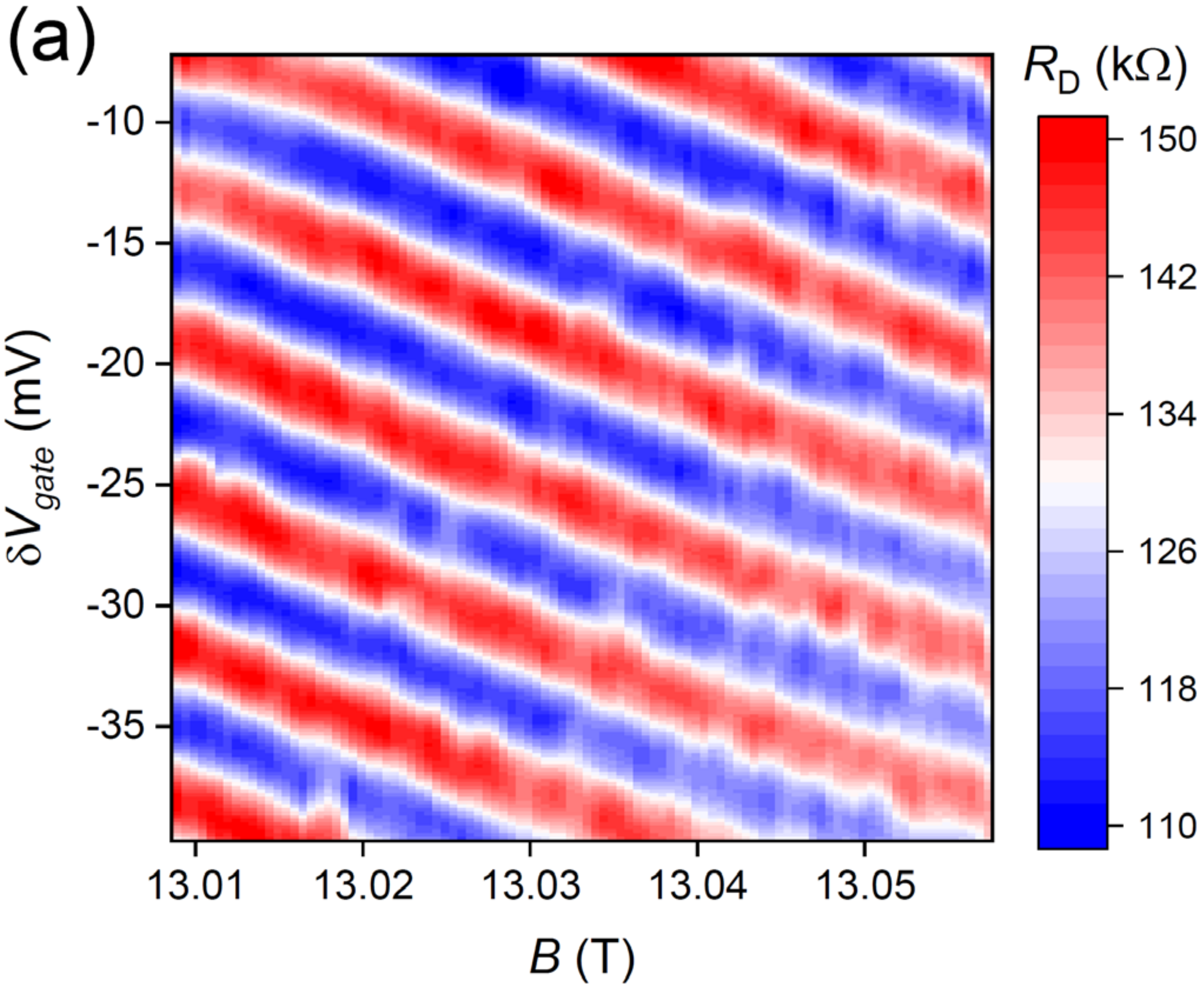
ARTICLES

https://doi.org/10.1038/s41567-019-0441-8

Aharonov–Bohm interference of fractional quantum Hall edge modes

J. Nakamura[1,2], S. Fallahi[1,2], H. Sahasrabudhe[1], R. Rahman[3], S. Liang[1,2], G. C. Gardner[2,4] and M. J. Manfra[1,2,3,4,5*]

Aharonov-Bohm interference at $\nu = 1/3$ FQHE state



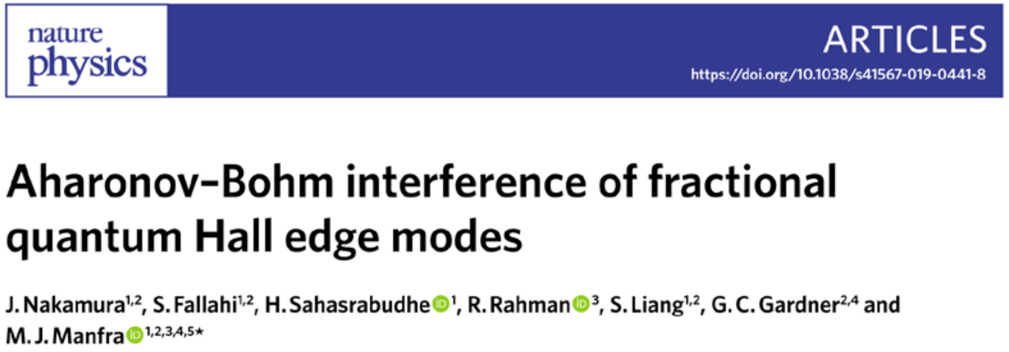
$$\frac{e^*}{e} = \frac{\Phi_0}{B\Delta V_g \frac{\partial A}{\partial V_g}}$$

$$\nu_{bulk} = \frac{1}{3}$$

$$\Delta B = 22.2\ \mathrm{mT}$$

$$\Delta V_{gate} = 6.1\ \mathrm{mV}$$

$$\frac{e^*}{e} = 0.29$$



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Aharonov–Bohm interference of fractional quantum Hall edge modes

J. Nakamura[1,2], S. Fallahi[1,2], H. Sahasrabudhe[1], R. Rahman[3], S. Liang[1,2], G. C. Gardner[2,4] and M. J. Manfra[1,2,3,4,5*]

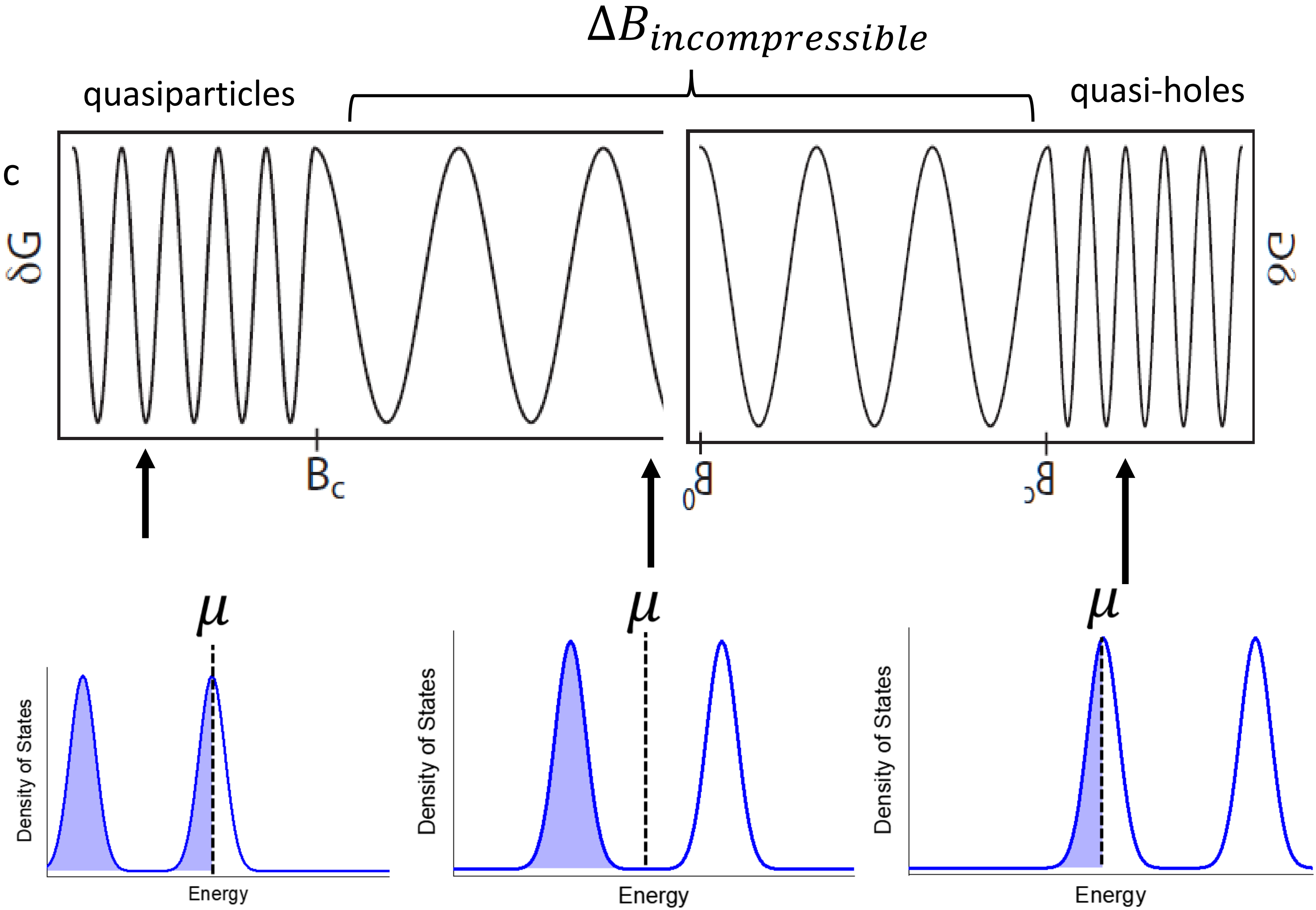
Theoretical analysis: transition from incompressible to compressible droplet

B. Rosenow and A. Stern. *PRL* **124**, 106805(2020)

- Competition between energy cost to create quasiparticles Δ and electrostatic energy cost to keep ν fixed
- Predicted transition from AB (incompressible) with $3\Phi_0$ period to AB+qp creation with Φ_0 period (compressible bulk)

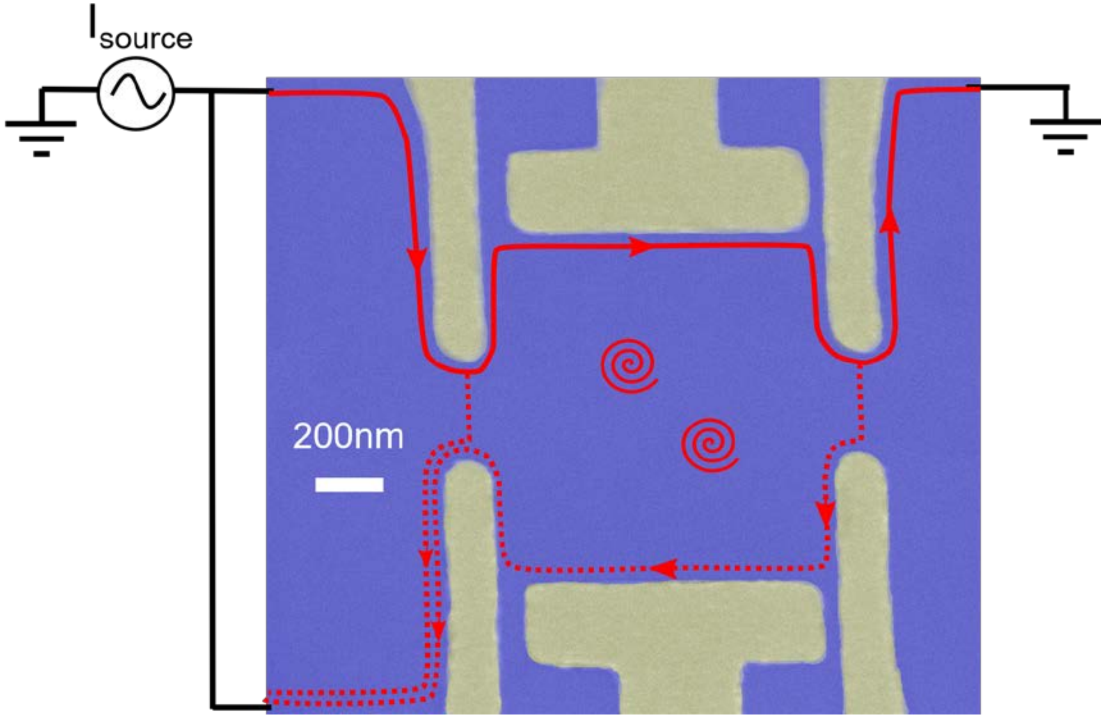
width in B with fixed ν where bulk is incompressible and $3\Phi_0$ oscillations:

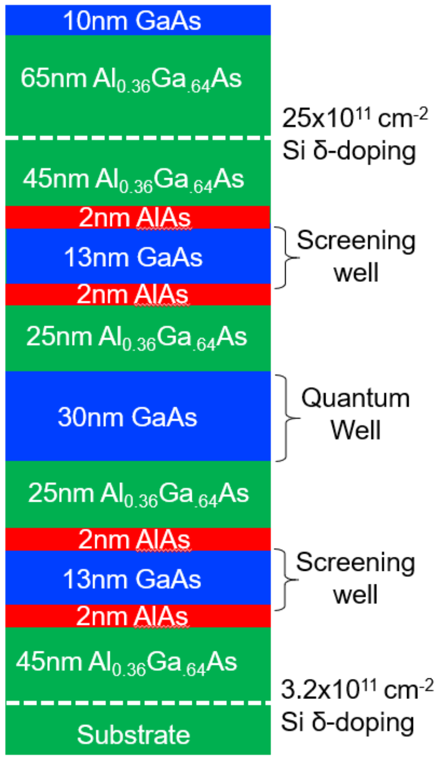
$$\Delta B_{incompressible} = \frac{\Delta \times \Phi_0}{\nu e^* \times \frac{e^2}{C}}$$



Δ = Energy gap of quantum Hall state
C = capacitance to screening layers (per unit area)

New round of experiments with modified devices: reduce size and reduce 2DEG density





- *Smaller interferometer: 1 μm x 1 μm lithographic area*
- *Lower density 2DEG: $n \sim 7x10^{10} cm^{-2}$: enhanced gate stability*
- *Examine interference over broad range of magnetic field around $\nu=1/3$ with $B \sim 9$ Tesla*

Observation of discrete phase slips at $\nu = 1/3$

- Primarily negative sloped constant-phase lines, but few discrete jumps in interference pattern

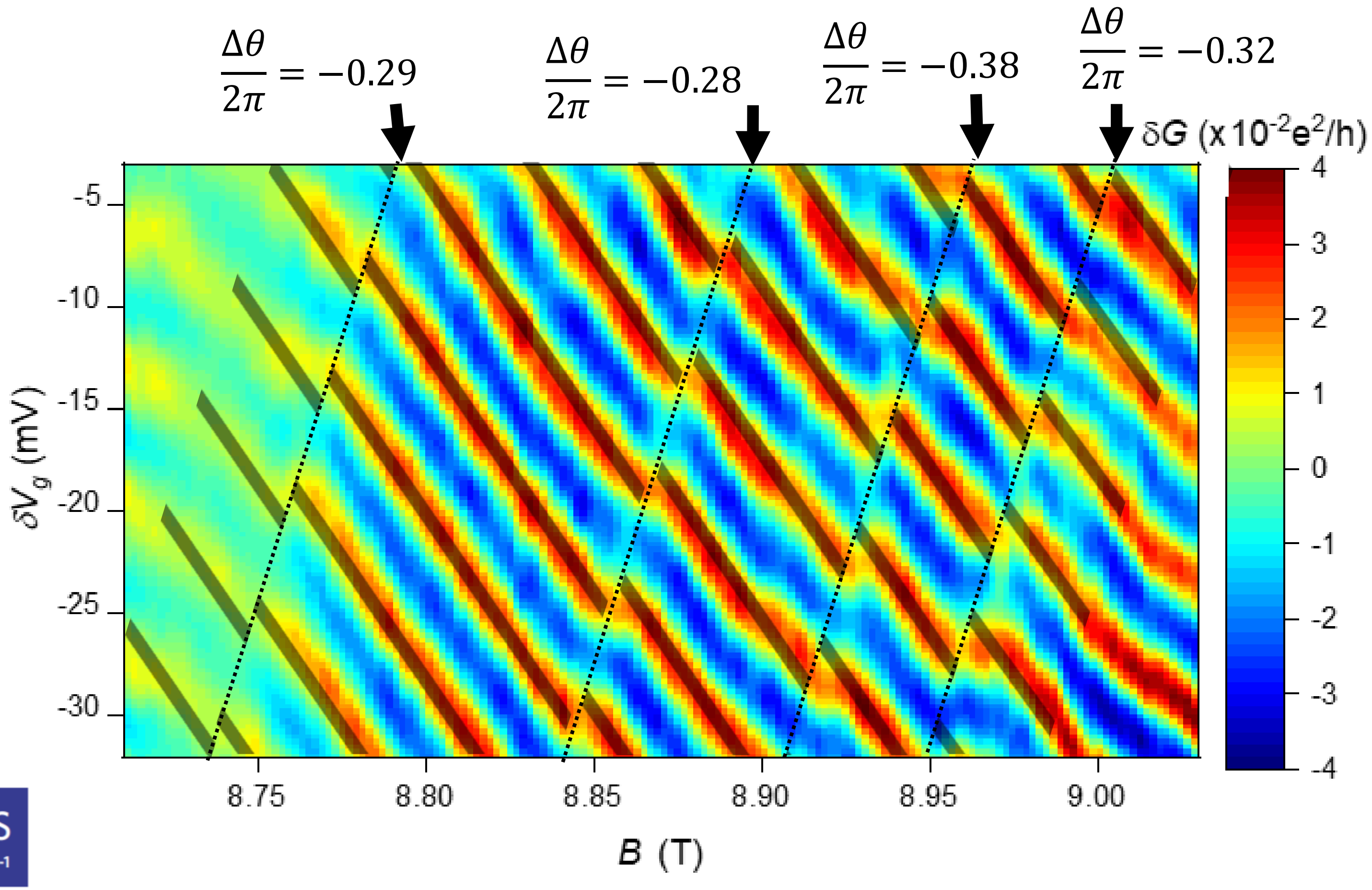
$$\theta = 2\pi \left(\frac{AB}{\Phi_0}\right)\frac{e^*}{e} + N_L\theta_{anyon}$$

- Both ΔB and ΔV_g indicate $e^* = \frac{1}{3}$

- Discrete jumps in phase: $\Delta\theta = -2\pi \times (0.31 \pm 0.04)$

Theory: $\theta_{anyon} = \frac{2\pi}{3}$

- Negative sign consistent with removing QPs (or creating quasi-holes) with increasing *B*

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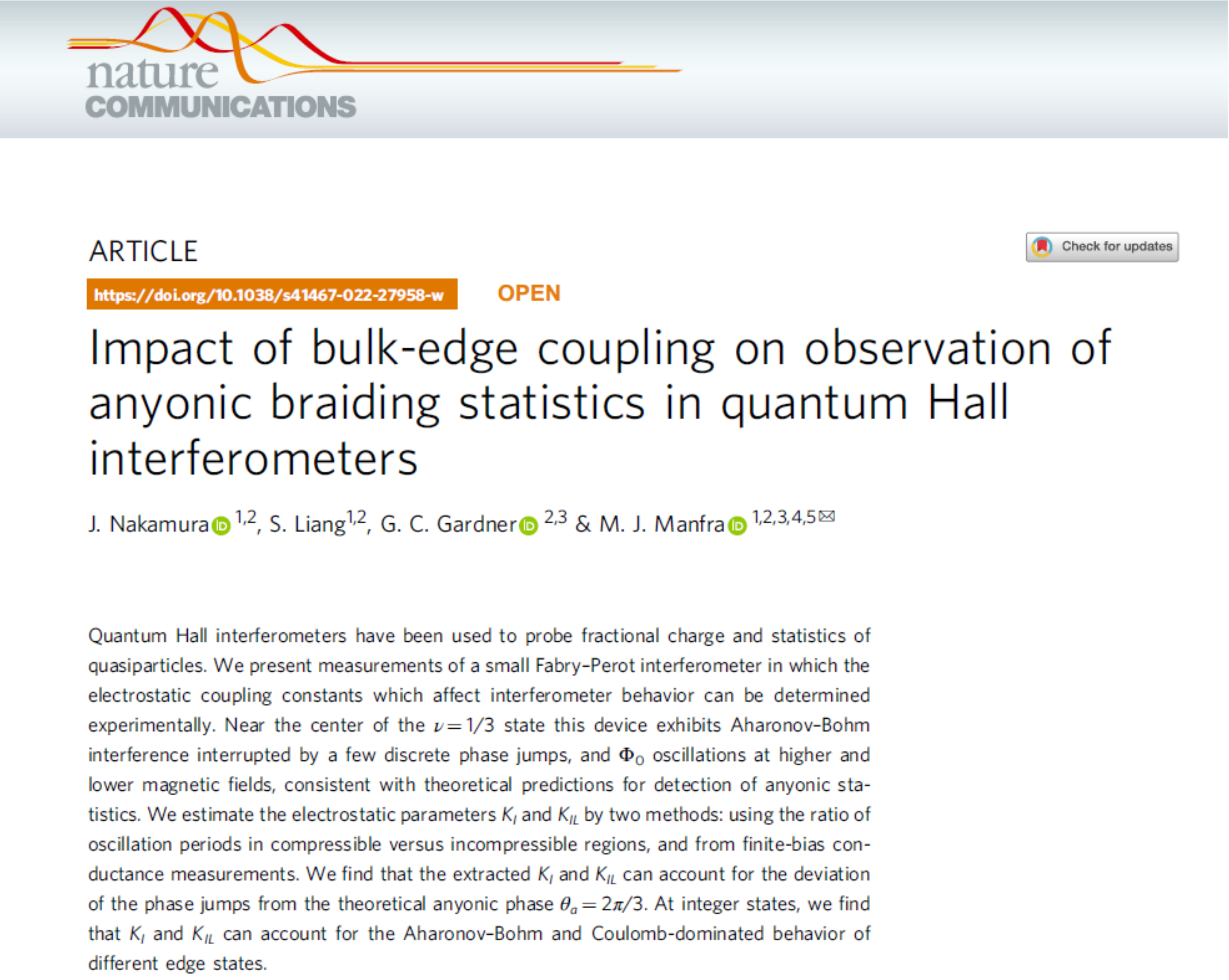
https://doi.org/10.1038/s41567-020-1019-1

Check for updates

Direct observation of anyonic braiding statistics

J. Nakamura[1,2], S. Liang[1,2], G. C. Gardner[2,3] and M. J. Manfra[1,2,3,4,5]

Enhance bulk-edge interaction to understand couplings and probe anyon response



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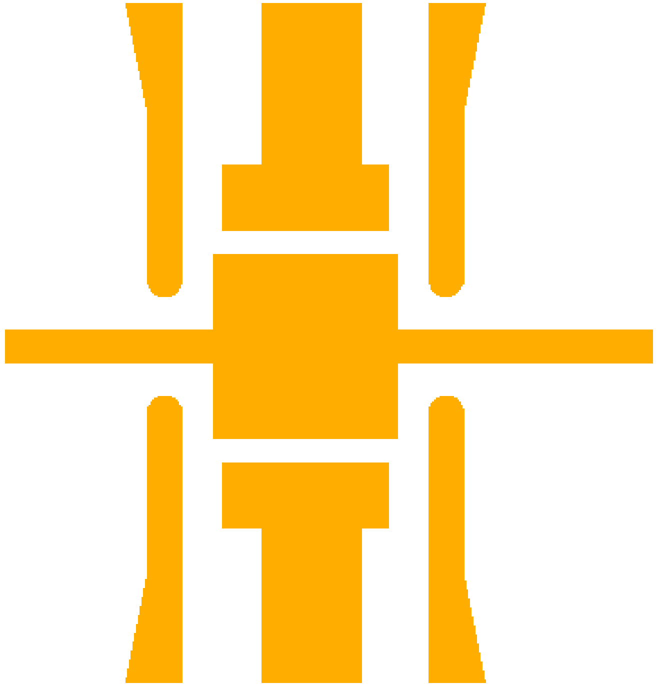
https://doi.org/10.1038/s41467-022-27958-w

OPEN

Impact of bulk-edge coupling on observation of anyonic braiding statistics in quantum Hall interferometers

J. Nakamura [1,2], S. Liang [1,2], G. C. Gardner [2,3] & M. J. Manfra [1,2,3,4,5] ✉

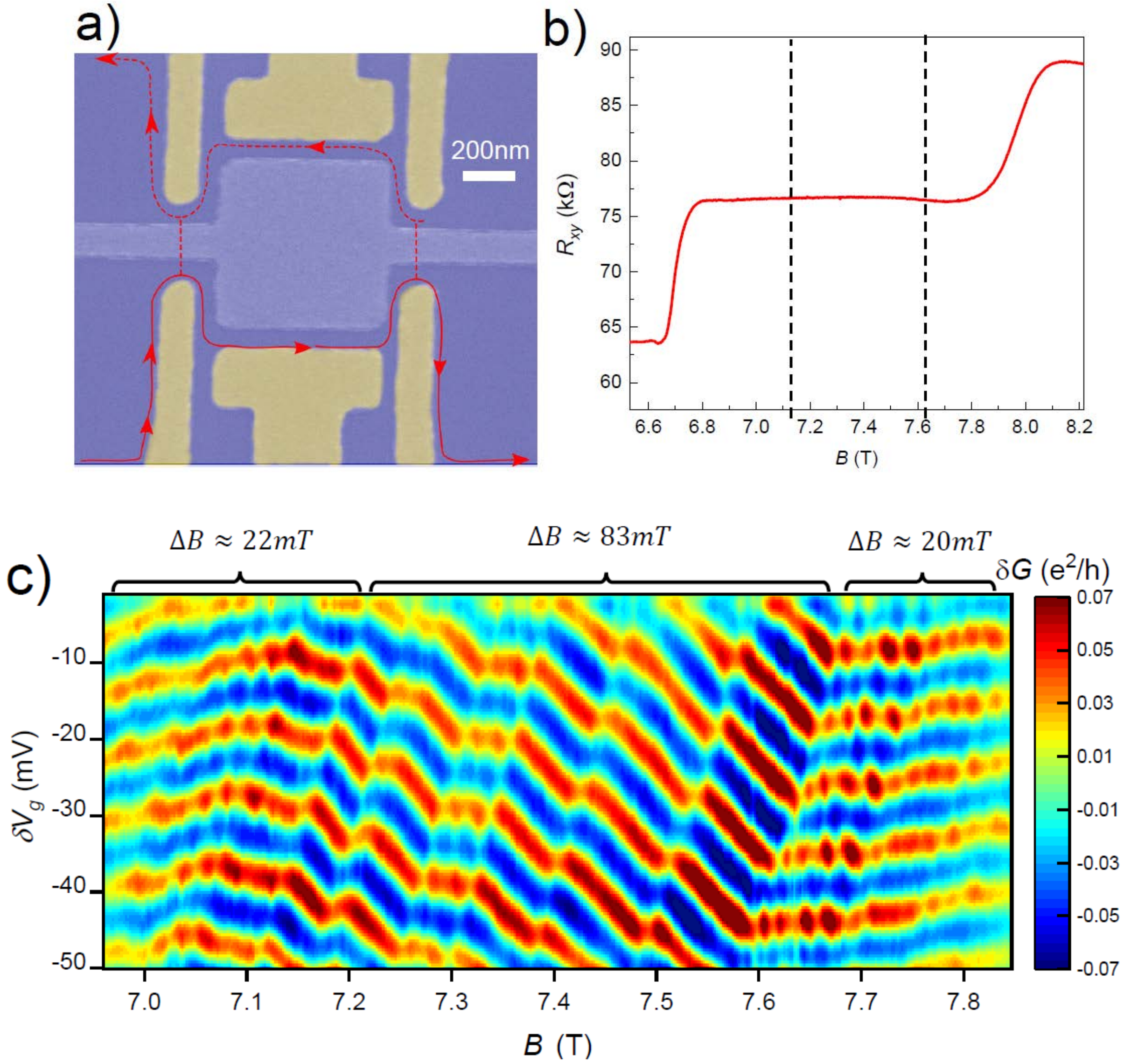
Quantum Hall interferometers have been used to probe fractional charge and statistics of quasiparticles. We present measurements of a small Fabry–Perot interferometer in which the electrostatic coupling constants which affect interferometer behavior can be determined experimentally. Near the center of the $\nu = 1/3$ state this device exhibits Aharonov-Bohm interference interrupted by a few discrete phase jumps, and Φ_0 oscillations at higher and lower magnetic fields, consistent with theoretical predictions for detection of anyonic statistics. We estimate the electrostatic parameters K_I and K_{IL} by two methods: using the ratio of oscillation periods in compressible versus incompressible regions, and from finite-bias conductance measurements. We find that the extracted K_I and K_{IL} can account for the deviation of the phase jumps from the theoretical anyonic phase $\theta_a = 2\pi/3$. At integer states, we find that K_I and K_{IL} can account for the Aharonov-Bohm and Coulomb-dominated behavior of different edge states.



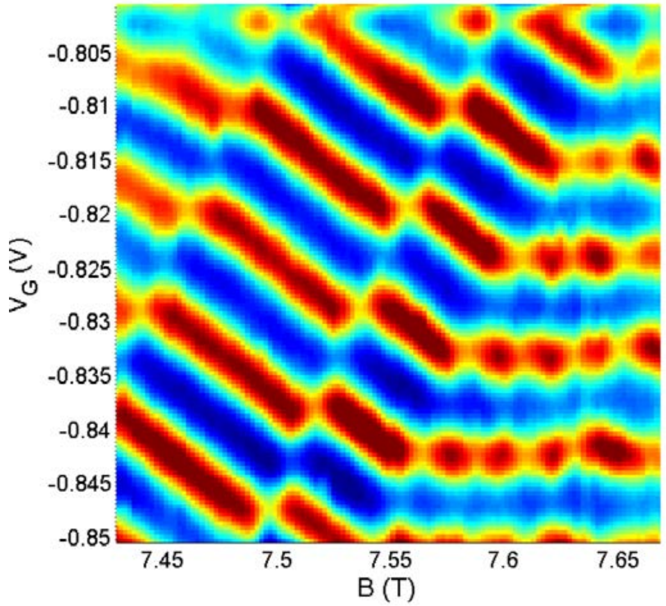
800nm x 800nm interferometer

2DEG density ~6×10^{10}cm^{-2}

Interference at $\nu = \frac{1}{3}$ in 800nm x 800nm Fabry-Perot interferometer



Extremely sharp phase slips &
Clear transition to compressible regime



REACTION OF RESEARCH COMMUNITY

- “To me as a condensed matter physicist, they are at least as interesting as the Higgs particle,” says Bernd Rosenow theoretical physicist at University of Leipzig

- “It is absolutely convincing. Theoretical physicists have long thought that anyons exist but to see it in reality takes it to a whole another level,” says Nobel Laurate Frank Wilzcek of MIT who coined the term “anyon” in the 1980s.

PHYSICISTS FIND BEST EVIDENCE YET FOR ELUSIVE 2D STRUCTURES

Strange quasiparticles called anyons could herald a way to build quantum computers.

By Davide Castelvecchi

Physicists have reported what could be the first incontrovertible evidence for the existence of unusual particle-like objects called anyons, which were first proposed more than 40 years ago. Anyons are the latest addition to a growing family of phenomena called quasiparticles, which are not elementary particles, but are instead collective excitations of many electrons in solid devices. Their discovery – made using a 2D electronic device – could represent the first steps towards making anyons the basis of future quantum computers.

“This does look like a very big deal,” says Steven Simon, a theoretical physicist at the University of Oxford, UK. The results, which have not yet been peer-reviewed, were posted on the arXiv preprint server last week[1].

Known quasiparticles display a range of exotic behaviours. For example, magnetic monopole quasiparticles have only one magnetic pole – unlike all ordinary magnets, which always have a north and a south. Another example is Majorana quasiparticles, which are their own antiparticles.

Anyons are even more strange. All elementary particles fall into one of two possible categories – fermions and bosons. Anyons are neither. The defining property of fermions (which include electrons) is Fermi statistics: when two identical fermions switch spatial positions, their quantum-mechanical wave – the wavefunction – is rotated by 180°. When bosons exchange places, their wavefunction doesn’t change. Switching two anyons should produce a rotation by some intermediate angle. This effect, which is called fractional statistics, cannot occur in 3D space, but only as collective states of electrons confined to move in two dimensions.

Fractional statistics

Fractional statistics is the defining property of anyons, and the latest work – led by Michael Manfra, an experimental physicist at Purdue University in West Lafayette, Indiana – is the

ScienceNews
INDEPENDENT JOURNALISM SINCE 1921

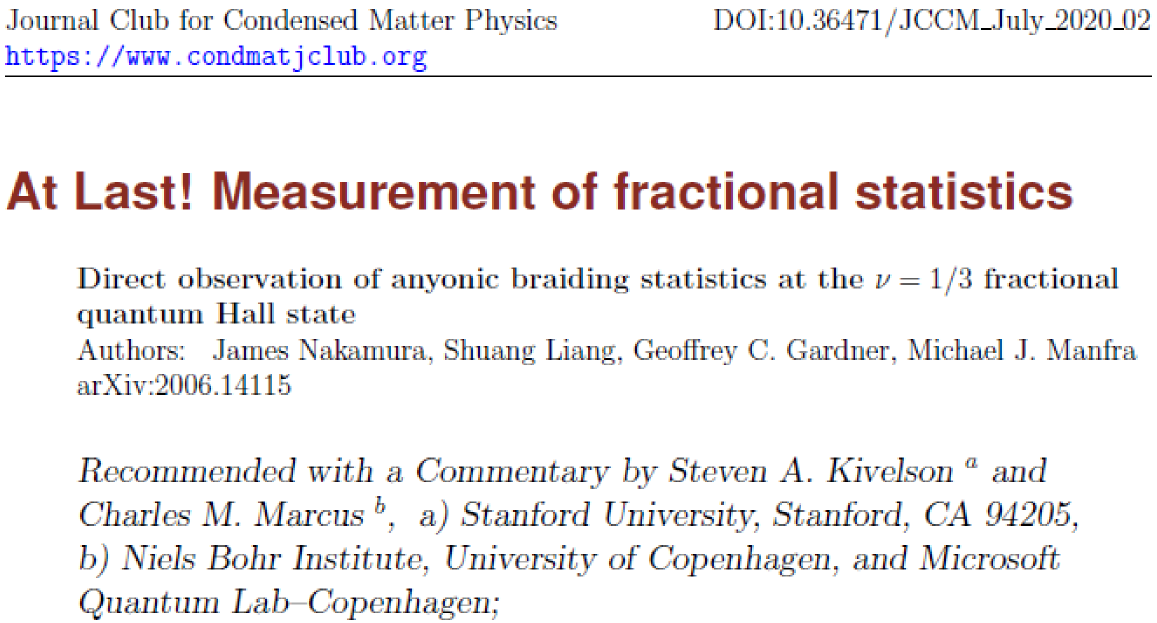
Physicists have ‘braided’ strange quasiparticles called anyons

Looping the structures around one another strengthens the case that anyons really exist

Anyons, which show up within 2-D materials, can be looped around one another like rope. Now physicists have observed this "braiding" effect.

BELCHONOCK/ISTOCK/GETTY IMAGES PLUS

By Emily Conover
JULY 9, 2020 AT 6:00 AM



Journal Club for Condensed Matter Physics DOI:10.36471/JCCM_July_2020_02
https://www.condmatjclub.org

At Last! Measurement of fractional statistics

Direct observation of anyonic braiding statistics at the $\nu = 1/3$ fractional quantum Hall state
Authors: James Nakamura, Shuang Liang, Geoffrey C. Gardner, Michael J. Manfra
arXiv:2006.14115

Recommended with a Commentary by Steven A. Kivelson [a] and Charles M. Marcus [b], a) Stanford University, Stanford, CA 94205, b) Niels Bohr Institute, University of Copenhagen, and Microsoft Quantum Lab–Copenhagen;

Dr. James Nakamura:
2019 Karl Lark-Horovitz Award, Purdue Physics and Astronomy
2022 Lee-Osheroff-Richardson Science Prize
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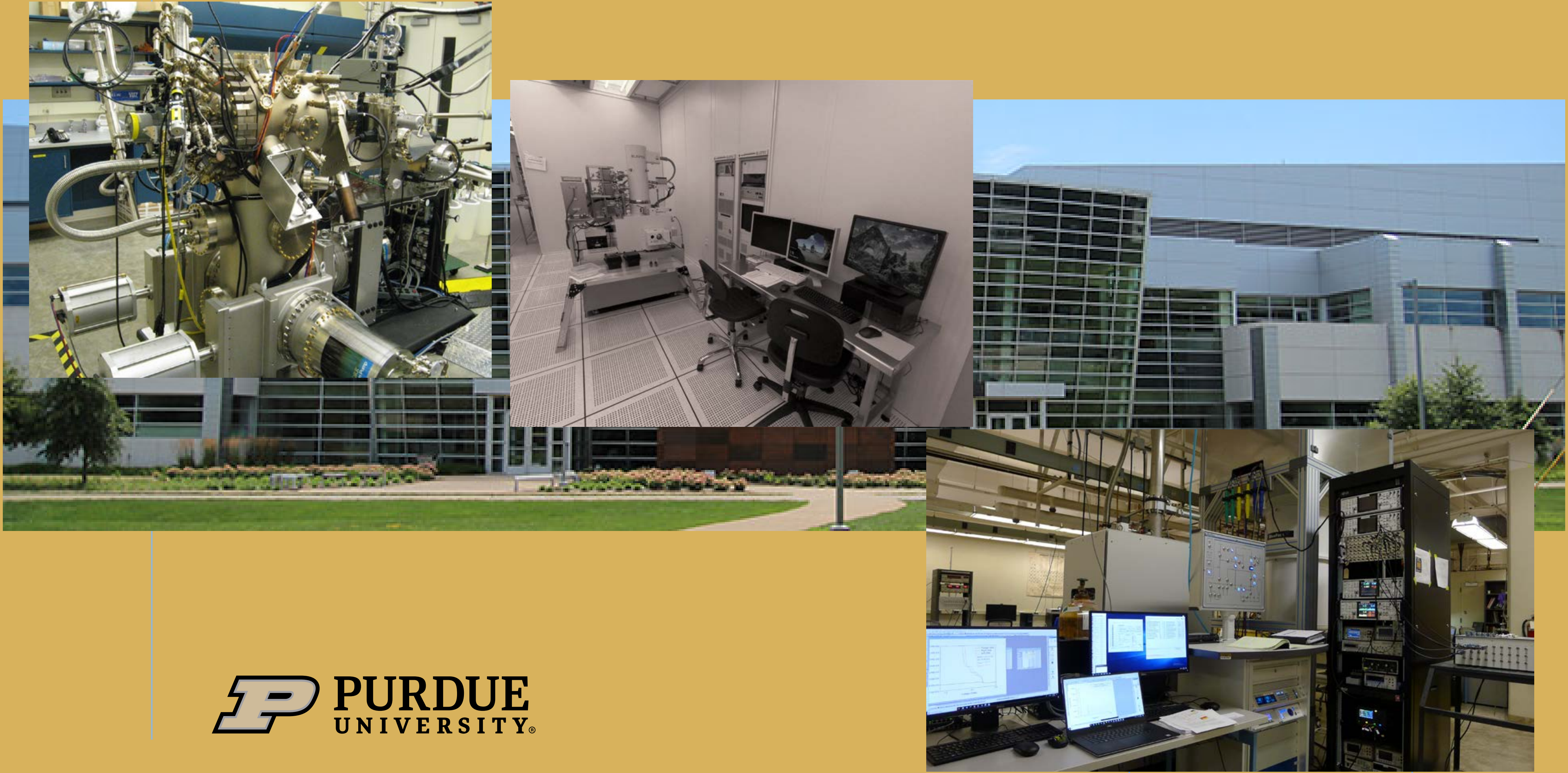
Opportunities at Purdue – Birck Nanotechnology Center



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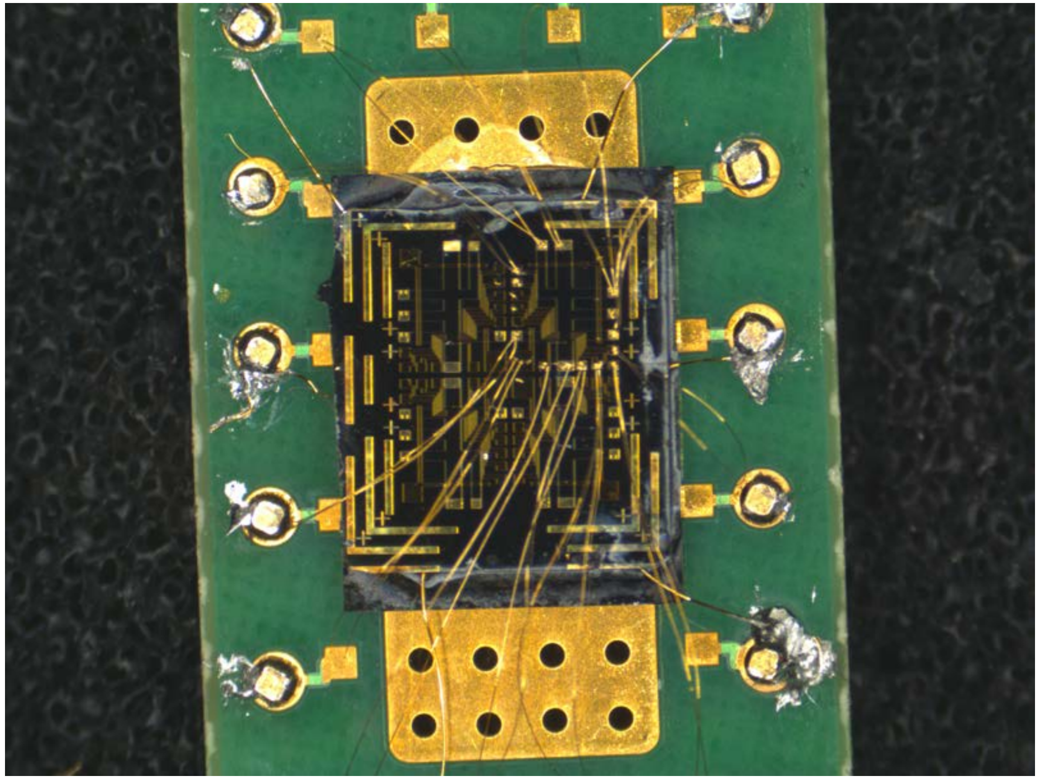
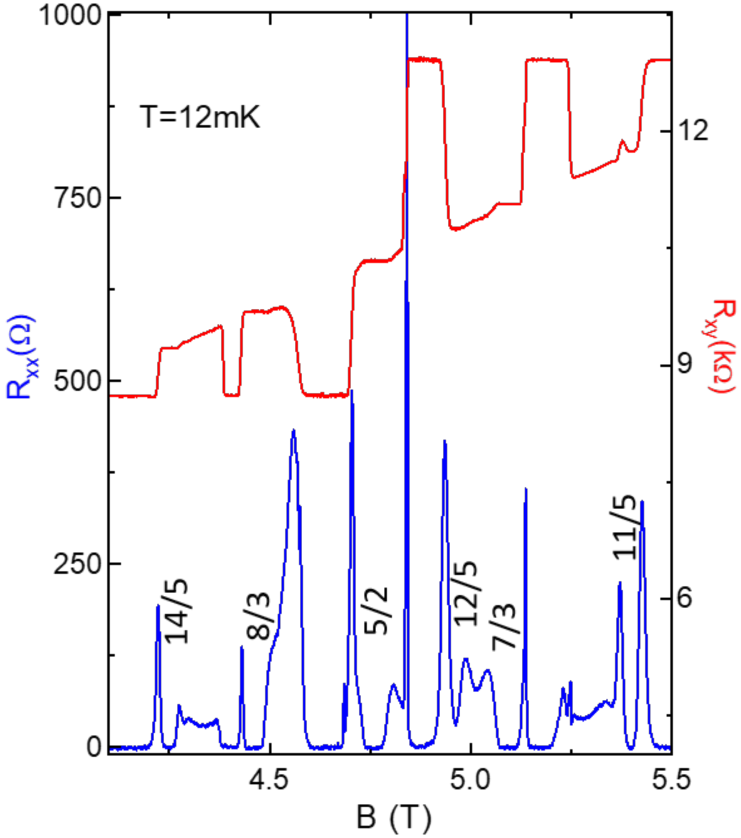
Summary and Outlook

- At ν=1/3, anyonic braiding phase of $\theta_{anyon} = \frac{2\pi}{3}$ has been measured
- Heterostructure + device design was critical to observation of new physics
- Physics discovery and technology development are not linear, nor sequential

What's Next? Non-Abelian braiding…





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