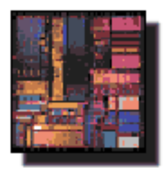
Digital Systems Design Automation

Unit 2: Advanced Boolean Algebra

Lecture 2.1: Boolean Algebra - Quick Review



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Outline

Reading for Unit 2

- Review basic Boolean algebra (your favorite book)
  - Digital Design: Principles & Practices, 4th Ed., John F. Wakerly, Prentice Hall, 2005
  - Purdue ECE270 course material
    - *Module 2: Boolean Algebra and Combinational Logic Circuits*
- Advanced Boolean algebra
  - De Micheli, Chapter 2.5
  - Hachtel & Somenzi, Chapter 3 (3.1 – 3.3)

Boolean Algebra (a.k.a. Boolean Logic)

- A set of two symbols or values ({0,1} or {TRUE,FALSE}, or …) and a family of operations on them that obey certain laws
- Basic Operations
  - Conjunction / AND : $x \wedge y$, $x.y$, xy
  - Disjunction / OR : $x \vee y$, $x+y$
  - Negation / Complement / NOT : $\neg x$, x', $\bar{x}$
- Complex operations: Any formula that can be composed of basic operations
  - Exclusive OR / XOR : $x \oplus y$
  - Implication : $x \rightarrow y$
- Operations obey certain laws or axioms
- Analogy to the algebra of real numbers
  - $(B, \vee, \wedge, \neg, 0, 1) \leftrightarrow (\mathbb{R}, +, *, -, 0, 1)$

Boolean Operators as Set Operations

- The operators of Boolean algebra can also be interpreted in terms of sets

Boolean operators

$\wedge$	0	1
0	0	0
1	0	1

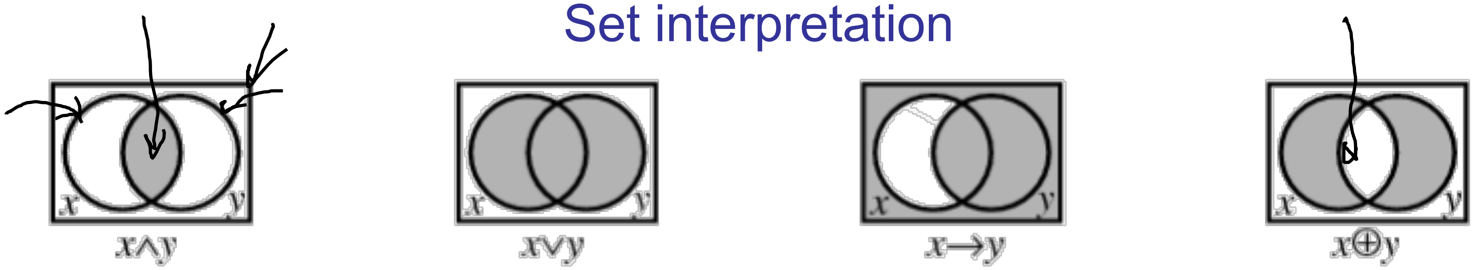
$\vee$	0	1
0	0	1
1	1	1

$\rightarrow$	0	1
0	1	1
1	0	1

$\oplus$	0	1
0	0	1
1	1	0

In each table, the rows are the values of x and the columns are the values of y.

Set interpretation



Basic Laws of Boolean Algebra

Law	Description
Commutativity	$x \vee y = y \vee x$ $x \wedge y = y \wedge x$
Associativity	$x \vee (y \vee z) = (x \vee y) \vee z$ $x \wedge (y \wedge z) = (x \wedge y) \wedge z$
Distributivity	$x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z)$ $x \vee (y \wedge z) = (x \vee y) \wedge (x \vee z)$
Identity	$x \vee 0 = x$ $x \wedge 1 = x$
Annihilation	$x \wedge 0 = 0$ $x \vee 1 = 1$
Idempotence	$x \vee x = x$ $x \wedge x = x$
Absorption	$x \wedge (x \vee y) = x$ $x \vee (x \wedge y) = x$

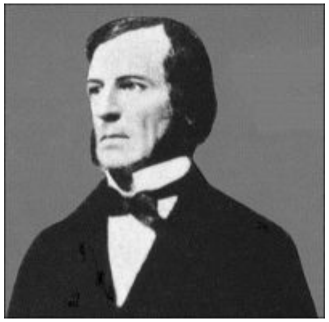
Basic Laws of Boolean Algebra

Law	Description
Complementation	$x \wedge \neg x = 0$ $x \vee \neg x = 1$
Double Negation	$\neg\neg x = x$
De Morgan	$(\neg x) \wedge (\neg y) = \neg(x \vee y)$ $(\neg x) \vee (\neg y) = \neg(x \wedge y)$

- Duality principle
  - Boolean algebra is unchanged when 0,1 and $\wedge$, $\vee$ are interchanged

Did You Know?

- Boolean logic was the invention of George Boole (1815-1864), an English mathematician and philosopher
- Published his first paper at the age of 24
- Landmark papers
  - "The mathematical analysis of logic," 1847
  - "An Investigation of the Laws of Thought, on Which Are Founded the Mathematical Theories of Logic and Probabilities," 1854
- Argued that there was a strong analogy between logic (then considered a sub-discipline of philosophy) and mathematics
- Initially, his theory was ignored or criticized by the academic community
- Followed up later by a student at MIT for his M.S thesis in 1937
  - Showed how to use Boolean logic to optimize electromechanical relay networks



- Boole's life was tragically cut short at the age of 49, when he was at the peak of his intellectual abilities
- After walking 2 miles through a drenching rain to get to class and then lecturing in wet clothes, Boole caught a 'feverish cold'
- It is believed that Mary Everest Boole, also a mathematician and Boole's wife, dumped buckets of water on him based on the theory that the remedy for an illness ought to bear resemblance to its cause

That was easy … what else?

- Quite a bit!
- Basic Boolean algebra helps you design simple digital circuits by hand
- Need more advanced concepts to create design automation algorithms and tools