Section 30
MOSFET Introduction

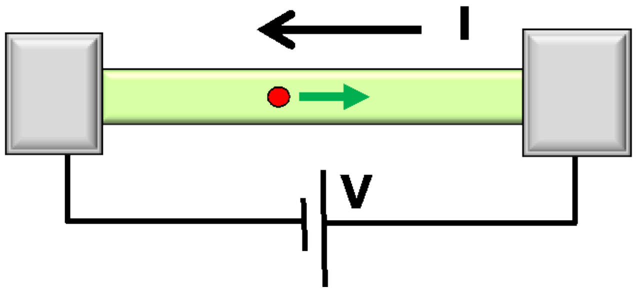
30.3 Velocity saturation in simplified theory

Gerhard Klimeck
gekco@purdue.edu

PURDUE UNIVERSITY

School of Electrical and Computer Engineering

Section 30
MOSFET Introduction



$$I = G \times V$$
$$= q \times n \times v \times A$$

charge density velocity area

1
2
3
4

- 30.1 Sub-threshold (depletion) current
- 30.2 Above-threshold, inversion current
- 30.3 Velocity saturation in simplified theory

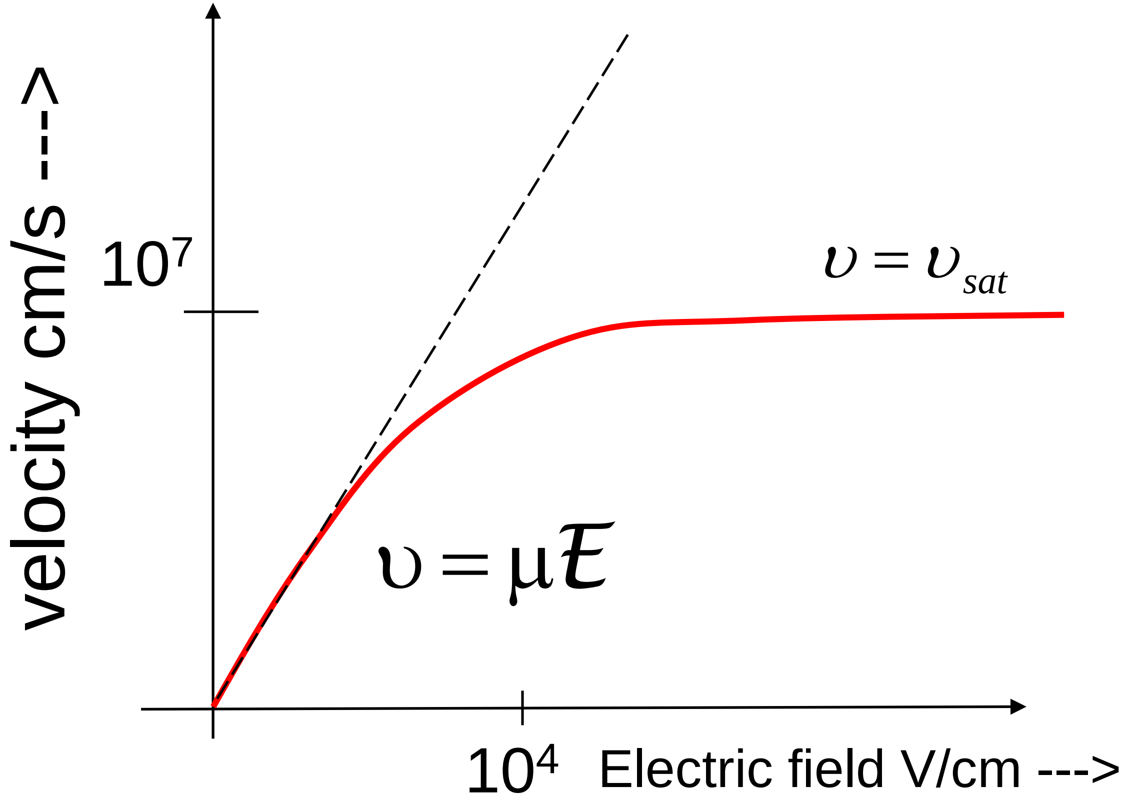
status

- 30.4 Comments on bulk charge theory & small transistors

Ref: Sec. 16.4 of SDF Chapter 18, SDF

Velocity vs. Field Characteristic (electrons)

Velocity saturates at high fields because of scattering



$$\upsilon_d = \frac{-\mu \mathcal{E}}{\left[1 + (\mathcal{E}/\mathcal{E}_c)^2\right]^{1/2}}$$

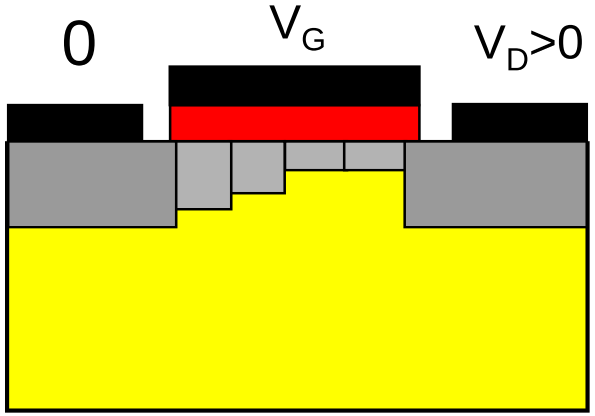
$$\upsilon_d = \frac{-\mu \mathcal{E}}{1 + (|\mathcal{E}|/\mathcal{E}_c)}$$

$$\upsilon_{d,sat} = \mu \mathcal{E}_c$$

This expression can be used to re-derive the expression for current since since mobility is now, in principle, a function of distance

$$\mu = \frac{\mu_0}{1 + \left(\frac{|\mathcal{E}|}{\mathcal{E}_c}\right)}$$

Recap - derivation for MOSFET current



$$J_1 = Q_1\,\mu_1\,\mathcal{E}_1 = Q_1\,\mu_1 \left.\frac{dV}{dy}\right|_1$$

$$J_2 = Q_2\,\mu_2\mathcal{E}_2 = Q_2\,\mu_2 \left.\frac{dV}{dy}\right|_2$$

$$J_3 = Q_3\,\mu_3\mathcal{E}_3 = Q_3\,\mu_3 \left.\frac{dV}{dy}\right|_3$$

$$J_4 = Q_4\,\mu_4\mathcal{E}_4 = Q_4\mu_4 \left.\frac{dV}{dy}\right|_4$$

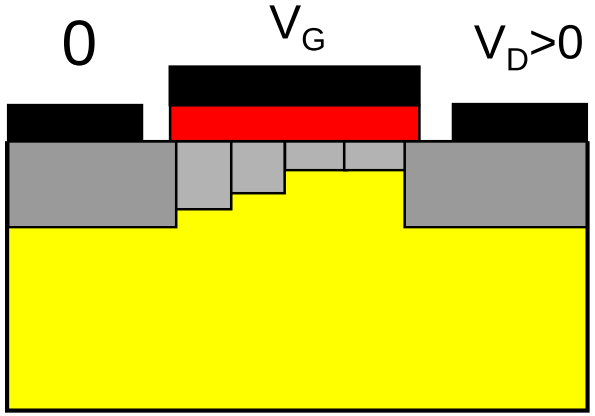
$$\Rightarrow \sum_{i=1,N} \frac{J_i dy}{\mu(y)} = \sum_{i=1,N} Q_i\,dV$$

$$\upsilon_d = \frac{-\mu\mathcal{E}}{1+(|\mathcal{E}|/\mathcal{E}_c)}$$

$$\mu = \frac{\mu_0}{1+\left(\frac{|\mathcal{E}|}{\mathcal{E}_c}\right)}$$

Velocity Saturation

$$\Rightarrow \sum_{i=1,N} \frac{J_i dy}{\mu(y)} = \sum_{i=1,N} Q_i \, dV$$

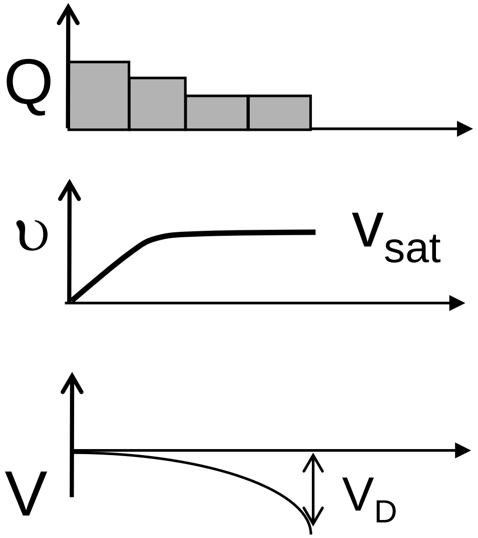


$$J_D \sum_{i=1,N} \frac{dy}{\mu_0 \Big/ \left[1+\frac{|\mathcal{E}|}{\mathcal{E}_c}\right]} = \int_0^{V_D} C_{ox}(V_G - V_{th} - mV)dV$$

$$\frac{J_D}{\mu_0}\int_0^{L_{ch}} dy\left[1+\frac{1}{\mathcal{E}_c}\frac{dV}{dy}\right] = C_{ox}\left[(V_G - V_{th})V_D - \frac{mV_D{}^2}{2}\right]$$

$$\int_0^{L_{ch}} J_D dy + \int_0^{V_{DS}} \frac{J_D}{E_c} dV = C_{ox}\left[(V_G - V_{th})V_D - \frac{mV_D{}^2}{2}\right]$$

$$J_D = \frac{\mu_0 C_{ox}}{L_{ch} + \frac{V_D}{\mathcal{E}_c}}\left[(V_G - V_{th})V_D - \frac{mV_D{}^2}{2}\right]$$



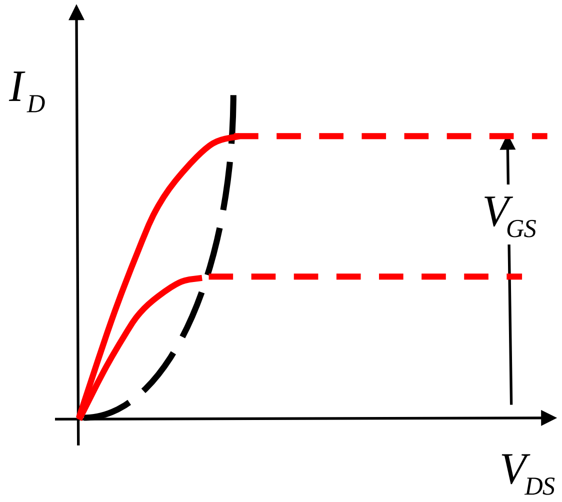
$$\upsilon_d = \frac{-\mu\mathcal{E}}{1+(|\mathcal{E}|/\mathcal{E}_c)}$$

$$\mu = \frac{\mu_0}{1+\left(\frac{|\mathcal{E}|}{\mathcal{E}_c}\right)}$$

Significance of the new expression

$$J_D = \frac{\mu_0 C_{ox}}{L_{ch} + \frac{V_D}{\mathcal{E}_c}} \left[\left(V_G - V_{th}\right) V_D - \frac{m V_D{}^2}{2} \right]$$

- At very small channel lengths and high drain biases, the current expression becomes independent of the channel length
- In the linear region in the I-V_d characteristics, you have a resistance that doesn't depend on the length of the channel

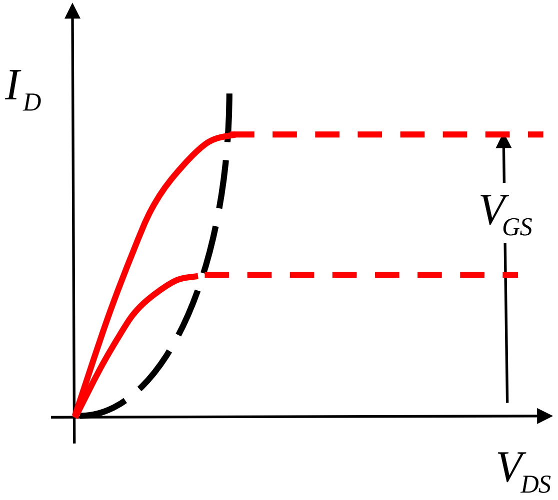


Calculating V_{DSAT} & I_{DSAT}

$$J_D = \frac{\mu_0 C_{ox}}{L_{ch} + \frac{V_D}{\mathcal{E}_c}}\left[(V_G - V_{th})V_D - \frac{mV_D{}^2}{2}\right]$$

$$\frac{dJ_D}{dV_{DS}} = 0$$

Take log on both sides and then set the derivative to zero ….



$$V_{DSAT} = \frac{2(V_G - V_{th})/m}{1+\sqrt{1+2\mu_o(V_G - V_{th})/m\upsilon_{sat}L_{ch}}} < \frac{(V_{GS} - V_T)}{m}$$

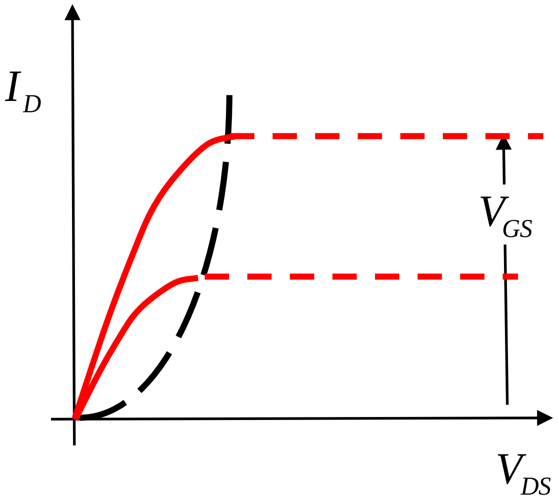
$$J_{D,sat} = \frac{\mu_0 C_{ox}}{L_{ch} + \frac{V_{D,sat}}{\mathcal{E}_C}}\left[(V_G - V_{th})V_{D,sat} - \frac{mV_{D,sat}{}^2}{2}\right]$$

‘Linear Law’ Expression at the limit of L→ 0

$$V_{DSAT} = \frac{2\left(V_G - V_{th}\right)/m}{1+\sqrt{1+2\mu_o\left(V_G - V_{th}\right)/m\upsilon_{sat}L_{ch}}}$$

$$\boxed{V_{DSAT} \rightarrow \sqrt{2\upsilon_{sat}L_{ch}\left(V_G - V_{th}\right)/m\mu_0}}$$

$$I_{DSAT} = W\,C_{ox}\upsilon_{sat}\left(V_G - V_{th}\right)\frac{\sqrt{1+2\mu_0\left(V_G - V_{th}\right)/m\upsilon_{sat}L_{ch}}-1}{\sqrt{1+2\mu_0\left(V_G - V_{th}\right)/m\upsilon_{sat}L_{ch}}+1}$$

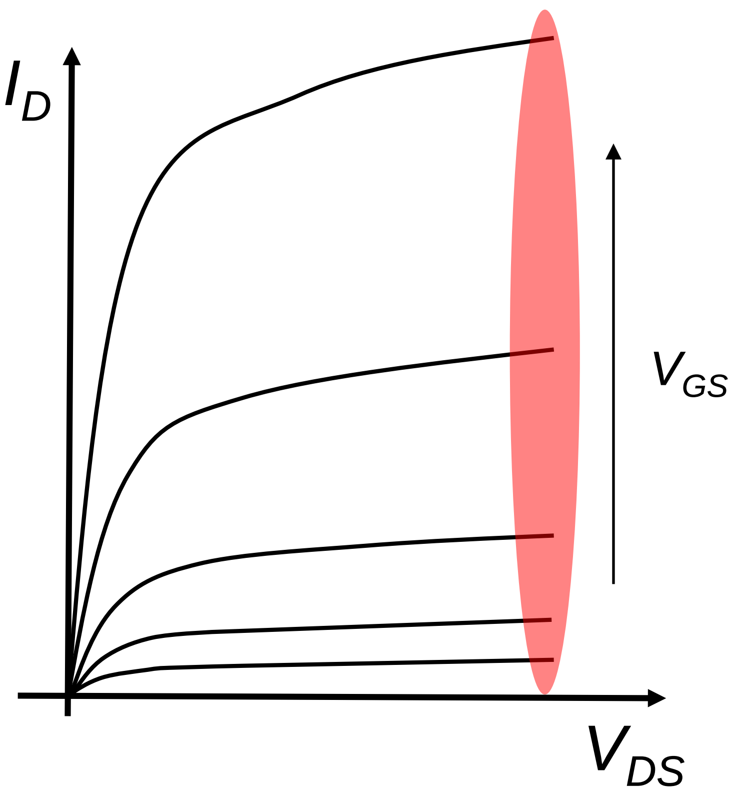


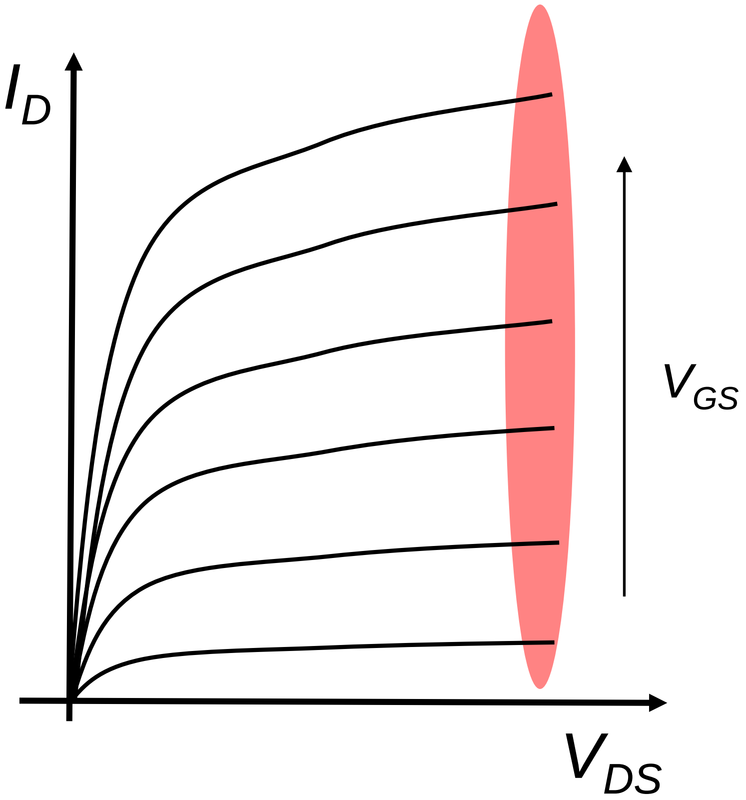
$$\boxed{I_{DSAT} = W\,C_{ox}\upsilon_{sat}\left(V_G - V_{th}\right)}$$

Complete velocity saturation

Current independent of *L*

‘Signature’ of Velocity Saturation



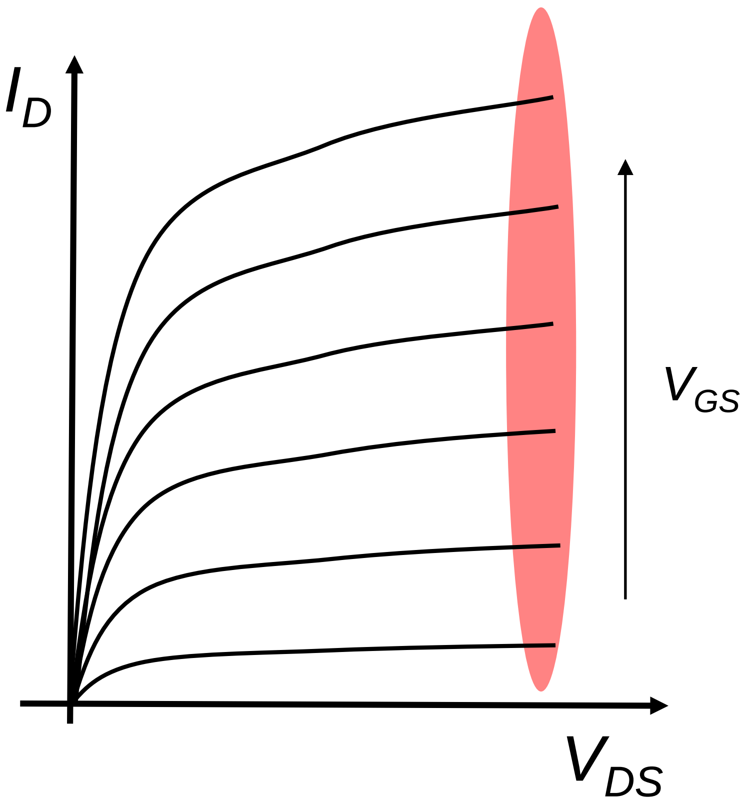


$$I_D = \frac{W}{2L_{ch}} \mu_0 C_{ox} \frac{(V_G - V_{th})^2}{m}$$

$$I_D = W \upsilon_{sat} C_{ox} (V_G - V_{th})$$

Can pull out oxide thickness from experimental curves… How?

I_D and $(V_{GS}$ - $V_T)$: In practice …..



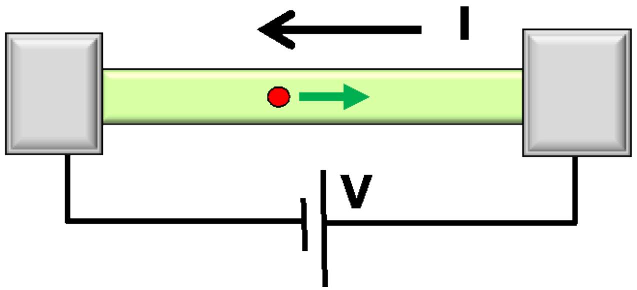
$$I_D(V_D = V_{DD}) \sim \left(V_G - V_{th}\right)^{\alpha}$$

$$1 < \alpha < 2$$

Complete velocity saturation

Long channel

Section 30
MOSFET Introduction



$$I = G \times V$$
$$= q \times n \times v \times A$$

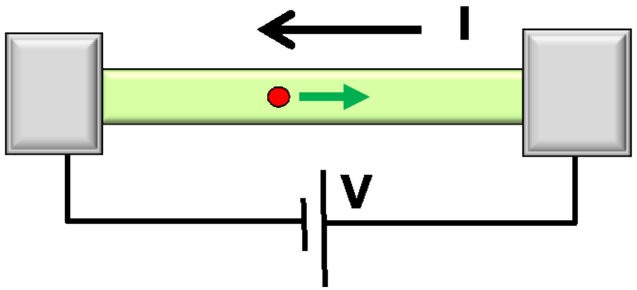
charge density velocity area

1
2
3
4

- 30.1 Sub-threshold (depletion) current
- 30.2 Above-threshold, inversion current
- 30.3 Velocity saturation in simplified theory
- 30.4 Comments on bulk charge theory & small transistors

Ref: Sec. 16.4 of SDF Chapter 18, SDF

Section 30
MOSFET Introduction



$$I = G \times V$$
$$= q \times n \times v \times A$$

charge density velocity area

- 1 • 30.1 Sub-threshold (depletion) current
- 2 • 30.2 Above-threshold, inversion current
- 3 • 30.3 Velocity saturation in simplified theory
- 4 • 30.4 Comments on bulk charge theory & small transistors

status

Ref: Sec. 16.4 of SDF Chapter 18, SDF