Lenses, Apertures and Resolution

Lecture 3
Lens, apertures & resolution

Review of optics
- Ray diagrams, optical elements, lens equation, magnification, demagnification, focus

Electron lenses
- How they work, how electrons travel through them

Apertures & diaphragms
(Primary) aberrations

Resolution

Depth of focus / depth of field
Lenses

Ray diagrams

Point object

Optic axis

Convex lens

Point image

$\alpha, \beta$ - convergence semiangle
Lenses
Ray diagrams

Parallel rays brought to focus

Arrow object

Convex lens

Arrow image
Lenses
Ray diagrams

\[
\frac{1}{u} + \frac{1}{v} = \frac{1}{f}
\]
For a magnetic lens, condition (1) is the stronger excitation (i.e. more driving current)

- Stronger excitation, less magnification

\[
M = \frac{v}{u} \approx \frac{\beta}{\alpha}
\]
Lenses
Ray diagrams

Overfocused
Focused
Underfocused
Both electric and magnetic fields used to steer the electron beam

- **Scan coils are electrostatic**
- **Lenses are magnetic**

\[ p_x = \frac{eEL}{v} \]
\[ \theta = \frac{eEL}{mv^2} \]

\[ p_x = \frac{eBL}{mv} \]
\[ \theta = \frac{eBL}{mv} \]
Lenses
Magnetic fields

Lorentz force:
\[
F = -e(E + \mathbf{v} \times \mathbf{B}) = e(\mathbf{v} \times \mathbf{B})
\]
\[
F = evB \sin \theta \approx evB = \frac{mv^2}{r}
\]
\[
r = \frac{mv}{eB}
\]
\[
r = \sqrt{\frac{2m_0E\left(1 + \frac{E}{2E_0}\right)}{B}}
\]

Relativistic correction

w/ B=1 Tesla, E = 100 kV \( \Rightarrow r \approx 1\text{mm} \)

Cyclotron frequency:
\[
\omega = \frac{2\pi}{T} = \frac{eB}{m}
\]
Lenses
Magnetic fields

Rotation of electron results in image rotation

Old microscopes must have this calibrated

New microscopes add an extra projection lens

- Lens action coordinated to remove this rotation
Lenses

Electron lenses
Lenses
Electron lenses

Objective lens must be strong
- Want specimen close to plane of objective lens (small u, large M)

Side entry
- Greater flexibility for sample rotation / probing

Top entry
- Maximum resolution
  - Less aberration
  - Smaller u
Lenses
Electron lenses

**Quadropole**
- Point object focused to a line image
- Used as stigmators

**Hexapole & Octupole**
- Combinations for aberration correction
Apertures
Apertures

\[ \beta_2 < \beta_1 \]
Aberrations

Magnetic lenses are far from perfect
Suffer from a host of aberrations

“Third order isotropic” aberrations:
- Spherical
- Astigmatism
- Field curvature
- Distortion
- Coma

Chromatic aberration

Astigmatism (first order)

Can be extended even further:
- Third order anisotropic
- Fifth order aberrations
Aberrations
Spherical aberration

Off-axis rays focused more strongly than on-axis rays

Disk of least confusion:

\[ d_{s,\text{min}}' = C_s \beta^3 \]

Larger in image:

\[ d_{s,\text{min}} = 2C_s \beta^3 \]

\( C_s \) usually 0.5 to 2 mm

- About equal to focal length
Aberrations
Chromatic aberration

Not from differences in $\Delta E$ from the HV Tank & source per se.

- HV ripple is 1 part in $10^6$ $\cong 0.1 \text{eV}$
- $\Delta E$ is source dependent

$\Delta E$ arises from inelastic scattering

- Up to 2keV difference
- Most between 15-25 eV

Disk of least confusion

$$ r_{chr} = C_c \frac{\Delta E}{E_o} \beta $$
Theoretical resolution given by Rayleigh criterion:

\[ r_{th} = 0.61 \frac{\lambda}{\beta} \]
Aberrations

Astigmatism

Caused by inhomogeneities in the lens, aperture defects and aperture centering problems

Fortunately, can be corrected

– Stigmator octupoles

Learning how is a big part of initial labs
Aberrations
Coma

Oblique, off-axis rays focused at different magnifications

This can be corrected through ‘coma-free’ alignment

Necessary for high resolution imaging
– Not as important in other work
Aberrations

Distortions

Pincushion  
Barrel  
Spiral

Only a worry in low magnification modes (Lorentz imaging)
Presence of any aperture causes diffraction

At minimum, the main tube that runs down the column provides to provide vacuum acts as an aperture

Diffraction from a circular aperture yield an intensity known as an “Airy disc”
Recall: \( r_{\text{sph}} = C_s \beta^3 \)

Add in quadrature (arbitrary):

\[
   r = \left[ r_{\text{th}}^2 + r_{\text{sph}}^2 \right]^{1/2}
\]

Variation w/ \( \beta \):

\[
   r(\beta) = \left[ \left( \frac{0.61 \lambda}{\beta} \right)^2 + \left( C_s \beta^3 \right)^2 \right]^{1/2}
\]

Find minimum:

\[
   \frac{dr(\beta)}{d\beta} = 0 = -2 \left( \frac{0.61 \lambda}{\beta^3} \right)^2 + 6C_s^2 \beta^5 \quad \Rightarrow \quad \beta_{\text{opt}} = 0.77 \frac{\lambda^{1/4}}{C_s^{1/4}}
\]

\[
   r_{\text{min}} = 0.91 \left( C_s \lambda^3 \right)^{1/4}
\]
**Depth of field & depth of focus**

**Depth of field:**
- Depth of ‘sharpness’ in object space
  \[ D_{ob} = \frac{d_{ob}}{\beta_{ob}} \]
- 2Å detail → 20 nm thick
- 2 nm detail → 200 nm thick

**Depth of focus:**
- Depth of ‘sharpness’ in image space
  \[ D_{im} = \frac{d_{ob} M^2}{\beta_{ob}} \]
- 2Å detail → 500 kX → 5 km
- 2 nm detail → 50 kX → 5 m