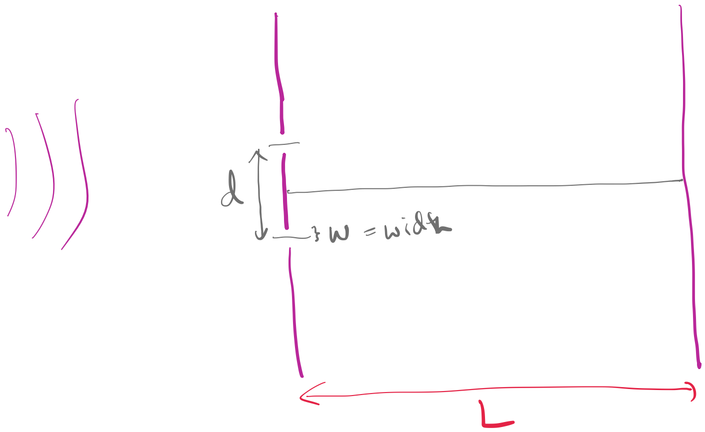
Calculating Interference Effects

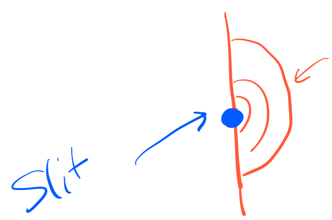


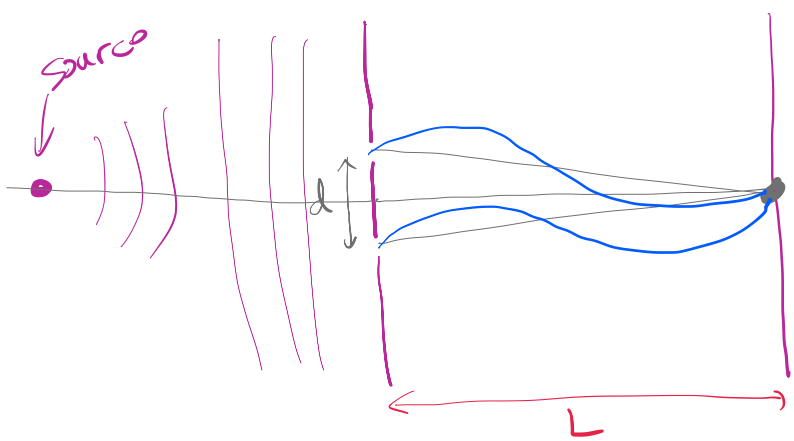
"<<" = much much less than — by a factor of 100

Assume $w << d$; $w << \lambda$

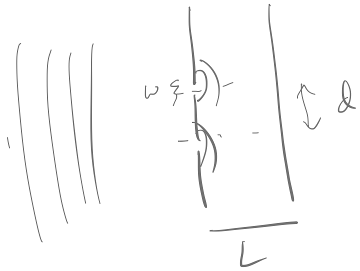
(Small parameter: $\frac{w}{d} << 1$; unitless
$\frac{w}{\lambda} << 1$; unitless)

→ Like the slits become point sources





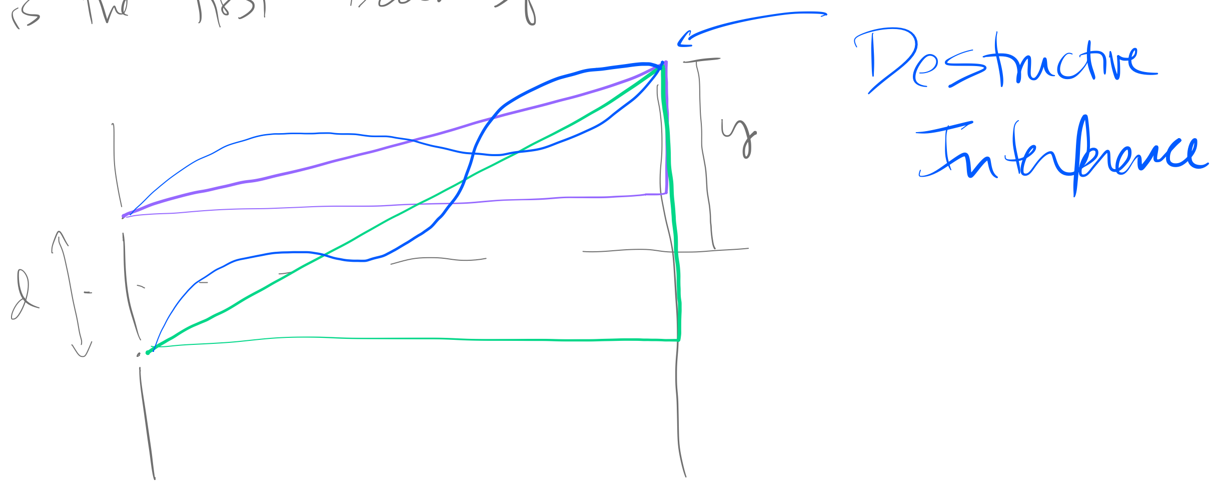
Constructive Interference
@ symmetry point
(middle)



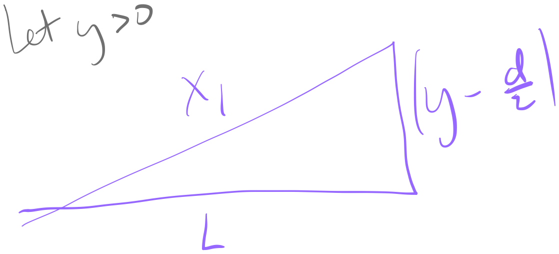
Assume $w \ll d$

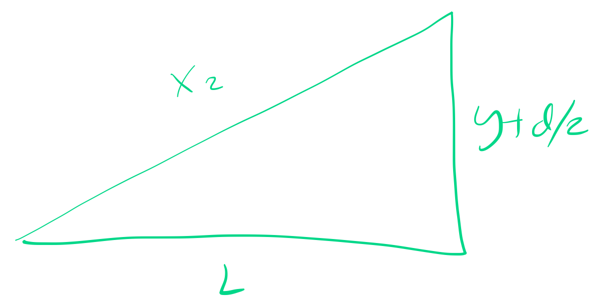
→ i.e. slits act like point source.

Where is the first Dark Spot:



Let $y > 0$





$$x_1 = \sqrt{L^2 + \left(y - \frac{d}{2}\right)^2}$$

$$x_2 = \sqrt{L^2 + \left(y + d/2\right)^2}$$

Dark spots:

$$x_2 - x_1 = \left(n + \tfrac{1}{2}\right)\lambda \qquad n = 0, 1, 2 \ldots$$

1st Dark Spot: $n = 0$

Solve for y . . .

1st Dark spot: Understand the plot ... ← (Mathematica)

$$y = \frac{\lambda}{4}\underbrace{\sqrt{\frac{16L^2 + 4d^2 - \lambda^2}{4d^2-\lambda^2}}}\times\frac{1/4}{1/4}$$

Need $d > \lambda/2$ ☆

Basic form: $\propto \sqrt{\frac{L^2+c}{c}}$

$$y = \frac{\lambda}{4}\sqrt{\frac{(16L^2+4d^2-\lambda^2)/16}{(4d^2-\lambda^2)/16}}$$

$$= \frac{\lambda}{4}\sqrt{\frac{L^2 + \frac{4d^2-\lambda^2}{16}}{\frac{4d^2-\lambda^2}{16}}} = \frac{\lambda}{4}\sqrt{\frac{L^2+c}{c}}$$

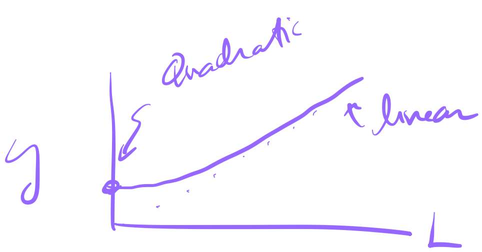
where $c = \frac{4d^2-\lambda^2}{16}$

$$\propto \sqrt{\frac{L^2+c}{c}}$$

↑ Proportional to

$y \sim a + L^2$

My plot looks like:



$y \sim L$

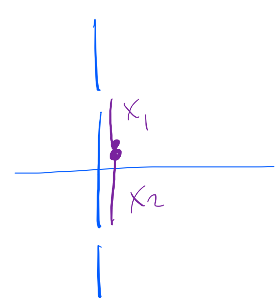
For large L: $L^2 >> c$

$$y \rightarrow \frac{\lambda}{4}\sqrt{\frac{L^2+\cancel{c}}{c}} \rightarrow \frac{\lambda}{4}\frac{L}{\sqrt{c}} \propto L$$

Linear in L @ large L

For small L: $L^2 << c$

$$y \rightarrow \frac{\lambda}{4}\sqrt{\frac{\cancel{L^2}+c}{c}} \rightarrow \frac{\lambda}{4}$$



} $\lambda/4$ ⇒ $\lambda/2$ phase difference

But how does small L limit depend on L?

→ Taylor series...

$$y = \frac{\lambda}{4}\sqrt{1+\frac{L^2}{c}}$$

Taylor is the SMALL parameter:

$$\frac{L^2}{c}$$

General:
$$\boxed{\begin{aligned} f(x) &= (1+x)^n \\ &\simeq 1 + nx + \mathcal{O}(x^2) \end{aligned}}$$

$$y = \frac{\lambda}{4}(1+x)^{1/2}$$

$x = L^2/c$

$n = 1/2$

$$\simeq \frac{\lambda}{4}\left[1 + \frac{1}{2}\frac{L^2}{c}\right] + \mathcal{O}\left(\frac{L^4}{c^2}\right)$$

Small L behavior is Quadratic in L

$$= \frac{\lambda}{4} + \frac{\lambda}{8}\frac{L^2}{c}$$

y
$\sim L^2$
$\sim L$
$\frac{\lambda}{4}$
L