Announcements

Homework #8 assigned today:
Mid-term "make-up")
Due Friday March 28th ??

Also **Homework #9** assigned today
Due Friday April 4th

This Wednesday, March 26th
Klaus Schulten: Magnetic Compasses

Talks

This Sunday, March 30th, 6-8:30pm,
½ of class gives 10-12 min + 2 min questions

Who it will be:
Janet, Ru, Tyler, Dan, Zhi, Michal
(Sukrit—be ready!)
(Pizza for everyone)

On Monday, March 31st, in class, more student presentations
Who it will be:
Sukrit, Dustin, David, Bryan, Zhuo, Mustafa

On Monday, April 7th: Amin gives talk

Diffusion

For “small” things, diffusion is a great way to get around.

For somewhat larger things, need directed motors.

How fast are small molecules moving in a cell?

How often do things come in contact?

Are chemical reactions rates limited by availability of food (ATP)?

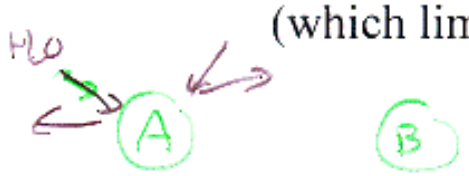
Movement by random motion: diffusion.
Limits to cell size based on oxygen diffusion/availability.

What limits how fast a cell can reproduce?
(<1 hrs for bacteria; ~day for humans)

Inertia does not matter for bacteria or anything that is small / microscopic levels.

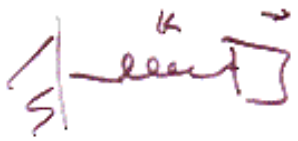
Translation & Equipartition Theorem

For two things to react, need to come in contact.
What is average speed (and distance between) molecules in cell?
Time between collisions?
How long oxygen to take to go across cell (which limits cell size)?



M_A , T

Thermal Energy $= \frac{3}{2}kT$



$P.E. = \frac{1}{2}kx^2$

free → v $\quad K.E. = \frac{1}{2}mv^2$

Equipartition Theorem

For each degree of freedom ~~that dep~~ where energy depends on (deg. of freed) $\frac{2}{=}$ $\frac{1}{2}kT$ of energy

$\frac{1}{2}kT$ PE $\frac{1}{2}kx^2$
$\frac{1}{2}kT$ KE $\frac{1}{2}mv^2$

$K_B = 1.4 \times 10^{-23}\ J/^\circ K$

$T = 300\ ^\circ K$

$K_B T = 4.2 \times 10^{-21}\ J$

$(1\ J = 1\ N \cdot m)$

$1\ N = \frac{1}{4}\ lb$

$1\ N = \frac{1}{5}\ kg$)

What is velocity of water molecule at room temperature?

$\tau_{collision} = \frac{d}{v}$ → mean free path
> dist. H_2O mol. will go before hitting another H_2O molecule.

Density of water: $1\,g/cm^3$ (55 M H_2O)

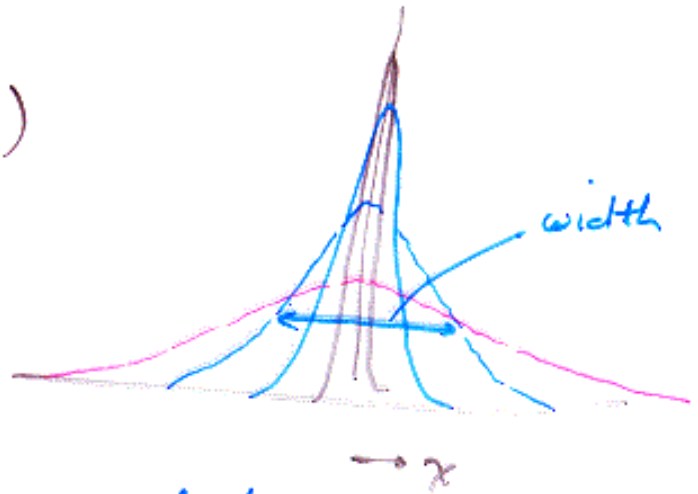
water molecules / cm^3

av. dist = ( )$^{1/3}$ → cm

$\tau_{collision}$ = ??

Longer Distances (>> mean free path)

Random Walk: Diffusion (more later lectures)



$t=0 \quad P(x=0)=1$
$P(x\neq 0)=0$

$\langle x \rangle = 0$

average pos. of dye = 0 ⇒ as likely to ~~drift~~ randomly drive left (−) as right (0)

$\langle |x| \rangle = \langle \sqrt{x^2} \rangle$

Can do but a little awkward…

$\langle x^2 \rangle$ at $t = 0$ $= 0$

$\langle x^2 \rangle$ at later time $t \neq 0$
$\geq$ pos. #.

$\langle x^2 \rangle$ as a measure of width of distribution

$\langle x^2 \rangle$ gets bigger in time.

if linear.

$x = \frac{\Delta x}{\Delta t} v t$

In a given period of time
X is smaller in random diffusion
than if molecules just went
at constant speed

$X \sim \sqrt{t}$

$\langle x^2 \rangle = 2Dt$ one dim

D = constant, diffusion const.
Property of molecule

2 = #
t = time

depends on dimension

1-D: # = 2

2-D: # = 4

3-D: # = 6

3-D.

$\langle r^2 \rangle = 6Dt$

$|x| \sim \sqrt{2Dt}$

if in 1 second it's gone a distance x_1
" 2 sec " " " $\sqrt{2}\, x_1$

Diffusion: $x^2 = \# Dt$

Diffusion as a Random Walk

1-D case (first)

Particle at $x=0$ $t=0$.

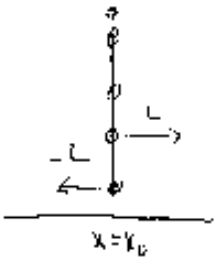
1) Assume equally likely to step to right as step to left.

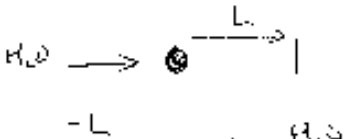
2) Take steps of length L every τ seconds.

moving with velocity $\pm V_n$ between collisions ($L = \pm V\tau$)

R steps/sec; total of N steps

(For now take V, τ as constants - they actually depend on size of particle, nature of fluid, Temp)



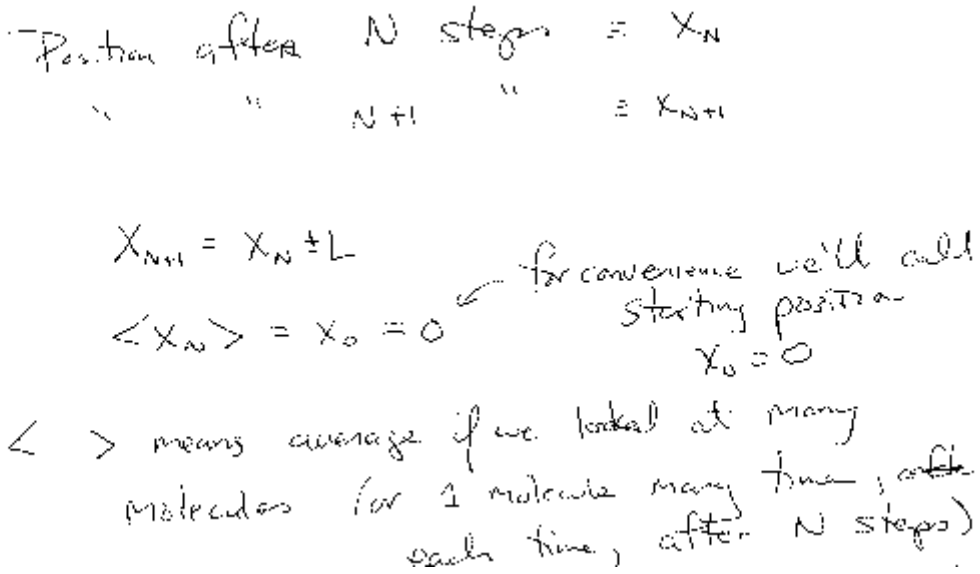


Of course in reality, distribution of step sizes but this model works amazingly well.

Thermal Motion: Move ±L

How far do particles move due to thermal motion

We cannot predict motion of individual molecules, but can make statistical (probabilistic) arguments about average/**mean** properties, as well as distribution (**standard deviation**) of these properties.



$\langle x_N \rangle$ = 0 by symmetry– equally likely to step left as right

$\langle x_N^2 \rangle = F(\langle x_{N-1} \rangle, L)$?

$\langle x_N^2 \rangle = \langle (x_{N-1} \pm L)^2 \rangle$

$\langle x_N^2 \rangle = \langle x^2_{N-1} \rangle \pm 2L \cancel{\langle x_{N-1} \rangle} + L^2$

$\langle x_N^2 \rangle = \langle x^2_{N-1} \rangle + L^2$

$$\langle X_1^2 \rangle = \langle X_0^2 \rangle + L^2$$

$$\langle X_2^2 \rangle = \langle X_1^2 \rangle + L^2 = \langle X_0^2 \rangle + 2L^2$$

$$\langle X_3^2 \rangle = \langle X_2^2 \rangle + L^2 = \langle X_0^2 \rangle + 3L^2$$

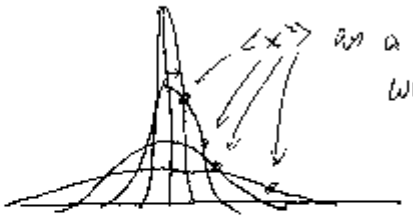
$$\vdots$$

$$\langle X_N^2 \rangle = \langle X_0^2 \rangle + NL^2$$

$$\sqrt{\Delta X_N^2} = \sqrt{\langle X_N^2 \rangle - \langle X_0^2 \rangle} = \sqrt{NL^2}$$

$$\boxed{\Delta X_N^2 = NL^2}$$

The average distance moved $\sim \sqrt{N}\, L$



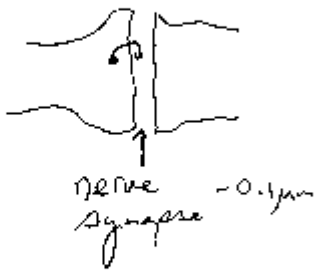
In order for a particle to wander twice as far takes 4 times longer

Plug in some #'s for $D \sim \overset{250}{300}\mu m^2/sec$ (small molecule in water)

To diffuse ~

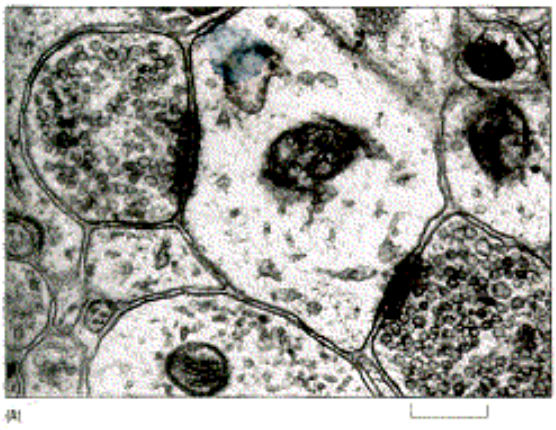
$0.1\mu m \Rightarrow \quad 0.01\mu m^2 = (2)(250\mu m^2/sec)\, t$

$t \sim 20\mu sec$ ('fast')

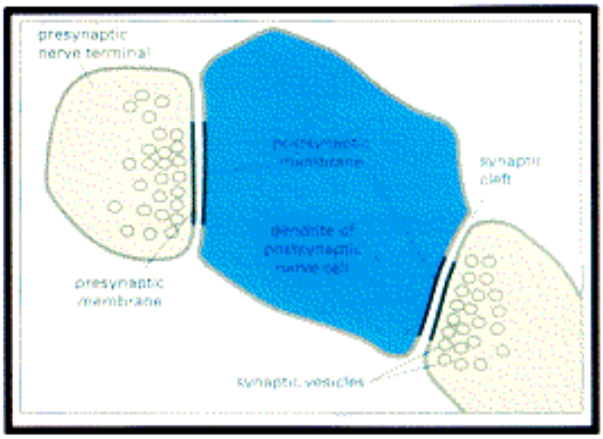


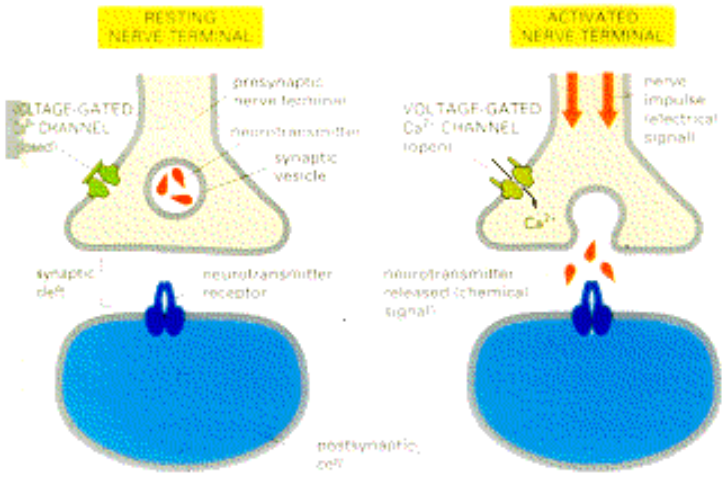
Diffusion across nerve synapse

Cross-sectional slice of nerve synapse



2 μm





How long for neurotransmitter to cross synapse via difffusion?

D = 250 μm^2/sec
Nerve synapse: 0.1 μm $X^2 = 2Dt$

0.01 μm^2 = (2)(250 μm^2/sec)t
t = 20 μsec (fast!)

$\langle x^2 \rangle = 6Dt$

D - diffusion const.

if molecule gets bigger

$D\uparrow \sim D\downarrow$

Object is large $D\downarrow$

because $x\downarrow$ for a given time

How long does it take for O_2 to

from edge of cell to middle?

20 μm cell

$x \sim 10\mu m$

$\langle x^2 \rangle = 6Dt$

$t = \frac{\langle x^2 \rangle}{6D} = \frac{(10\mu m)^2}{(6)(1000\mu m^2/sec)}$

$\frac{10^2}{6 \cdot 1000} \sim 0.016\,sec = 16\,msec$

Bacterial cell ~1 μm

10x less distance than eukaryotic cell

100x less time

Metabolism of bacterial can (and is) much higher than eukaryotic cell

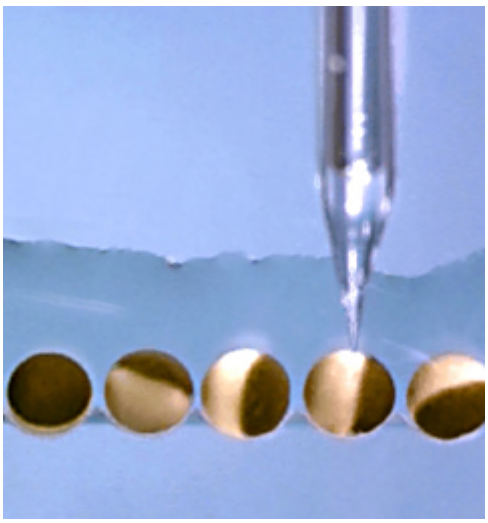
↳ 10-100 μm.

bacterial cell ~1 x 3 μm

eukaryotic cell → 10-100 μm

Size of eukaryotes limited by size (diffusion time of O_2). As size gets bigger, everything happens more slowly.

Large cell: frog oocytes– basically everything happens slowly.

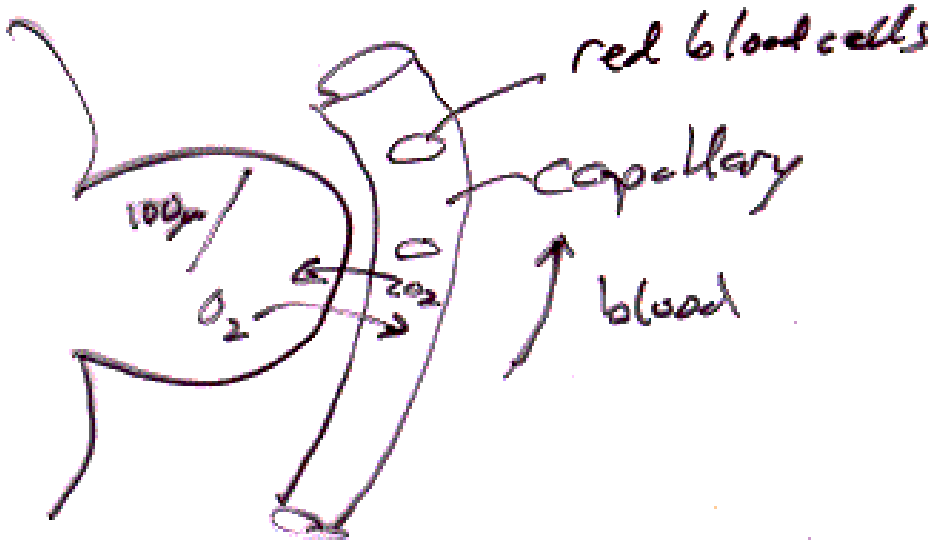


Oocyte:1-2 mm!

Every cell needs to be within 50-100 μm of blood supply!

Lung + Diffusion of O_2 / CO_2

- Billions of air sacks (alveoli)



Can diffusion move O_2, CO_2 ? enough?

Efficiency of Diffusion

Diffusion moves things short distances very fast!

$$\langle x^2(t) \rangle = 2Dt$$

$$\sqrt{\langle x^2(t) \rangle} = \sqrt{2Dt}$$

The mean squared displacement increases linearly with time

The mean (average) absolute distance increases as square-root of time.

Distance moved in diffusion $\sim \sqrt{t}$

$$V_{ave} = \frac{d\langle x_{ave}^2 \rangle}{dt}$$

$$\frac{d}{dt}\left(x^2 = 2Dt\right)$$

$$x\frac{dx}{dt} = D$$

$$\frac{dx}{dt} = \frac{D}{x} \sim \sqrt{\frac{D}{t}}$$

$t \to 0 \quad v \to \infty$!

Very fast spreading for short times

What's wrong? Special Relativity doesn't allow this!

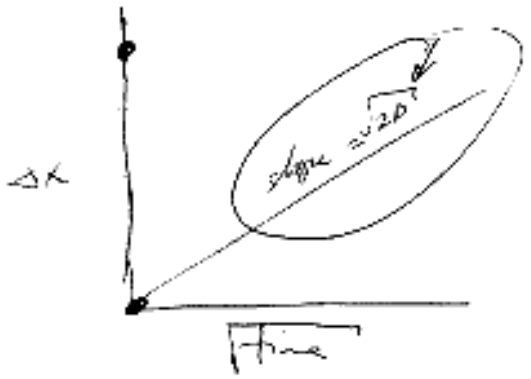
Experimentally: How do you measure D?

Brownian Motion

Robert (?) Brown - botanist saw diffusion
i.e. thermal jiggling of pollen
with my. light microscope
- res. ~ λ ~ 500nm
+ Big eye t~ 0.1 sec.
How could he have seen this?

Ans: you see only very rare events that
lead to big fluctuations
slow
Most of jiggling you don't see by eye.

Label/look at pollen grain - note $x(t=0)$
Wait time t, measure Δx, repeat many times or measure at diff times

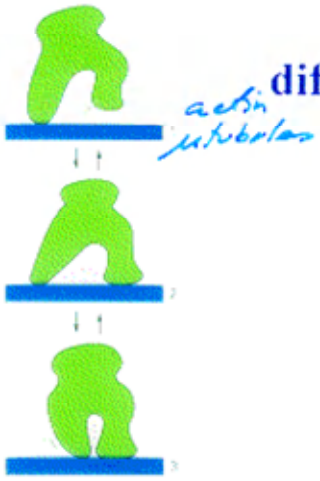


Diffusion good for small distance:

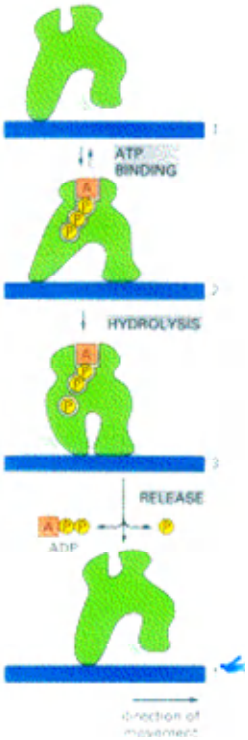
But not good for bringing you from A to B.

➔ Molecular Motors

Directed vs. Random Motion



No energy input, diffusion leads to up and back motion, spread in position.



Energy input, leads to unidirectional motion.

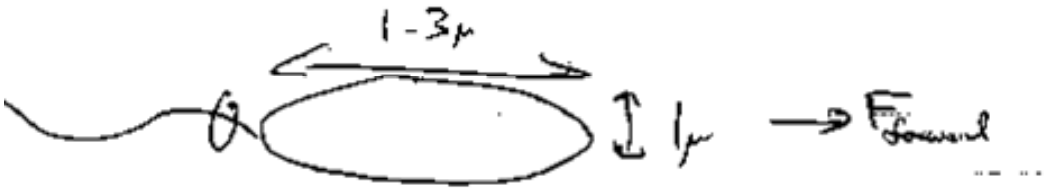
How Bacteria move

Inertia doesn't matter for microscopic world
Life at low Reynold's number

Why study?

1) Simple Ex of F=ma
2) Don't need much bio
3) Results are broadly applicable to bio @ μ level

Bacteria



Go ~ 25 μm/sec ~ 10 body lengths/sec

Compare to you: running Walk: 4 miles/hr ~ 6ft/sec
~ 1 body length/sec

Swimming: pool length lap ~ (50 yards) ~ min.
~ yard/sec ~ ½ body length/sec
Olympic swimmer ~ body length/sec

Bacteria: good swimmers

If turn off “propeller,” how far Bacteria coast?

$F = ma$

Solve eq'n of motion

(8) $\frac{m dv}{dt} = -\gamma v$

$\int \frac{m dv}{v} = \int -\gamma dt$

$m \ln v = -\frac{\gamma}{m} t$

$v = v_0 e^{-\frac{\gamma}{m} t}$

$= v_0 e^{-t/\tau} \qquad \tau = m/\gamma$

What is mass of bacteria?

$m \sim \frac{4}{3}\pi r^3 \rho \qquad r \sim 1\mu m \quad \rho = 1 g/cm^3$

$\boxed{\sim 4 \times 10^{-15} kg = m}$

$\gamma = 6\pi\eta r \qquad \eta = 0.001 \quad r = 10^{-6}$ meters

$\gamma \sim 20 \times 10^{-9} \frac{N-s}{m} \sim \boxed{\frac{20 nN-s}{m} = \gamma}$

Plugging in #'s.

$m = 4 \times 10^{-15}$ kg

$\gamma = \frac{20 nN\text{-}s}{m}$

$\tau = \frac{m}{\gamma} = 0.2 \mu sec$

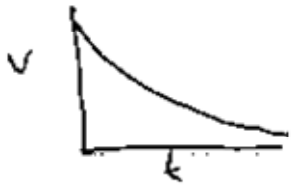
So bacteria stops in 200 nsec! Very fast.

So once forces are turned off, bacteria forgets about history very quickly.

History does not matter to bacteria

How far does bacteria coast in 0.2sec?

$$x = \int v dt = \int_0^\infty V_0 e^{-t/\tau_0} dt$$



$V_0 \sim 25 \mu m/sec$

$x = 0.05 Å$! < diameter of H-atom!

Inertia is irrelevant to bacteria

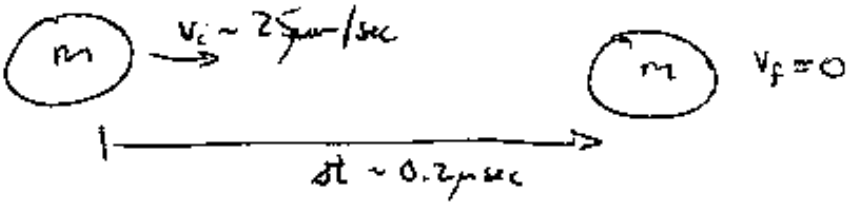
Once force is over, no forward motion!

Person swimming

- a good swimmer coasts ~ 1 body length.

Inertia is much more important to bigger organisms

Size of drag force on Bacteria



$$F = \frac{\Delta p}{\Delta t} = \frac{(4\times10^{-15}kg)(25\mu m/sec)}{0.2\mu sec}$$

$$= 5\times10^{-13}\ N$$

$$= 0.5\,pN \qquad (p\times10^{-12})$$

How compare to its weight?

$$w = mg = (4\times10^{-15}kg)(10m/s/s) = 0.04\,pN$$

So:

> Bacteria swim as if dragging 10× their own weight!

Class evaluation

1. What was the most interesting thing you learned in class today?

2. What are you confused about?

3. Related to today's subject, what would you like to know more about?

4. Any helpful comments.

Answer, and turn in at the end of class.