The story so far:

- Landauer formula + scattering matrix approach: general way of treating (noninteracting, small bias) two-terminal conductance of quantum coherent system attached to classical reservoirs at absolute zero, independent of details of the quantum system.

- Subtle issues about conductance: ballistic system has finite conductance.

- Energy relaxation processes typically modeled as taking place in leads or contacts, resulting in very nonthermal / nonequilibrium electronic distributions in "active" region of device.

On the plate today:

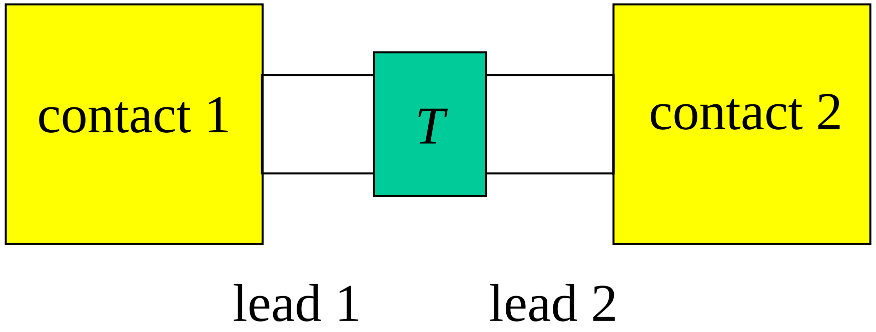
- Zeroth order effect of interactions
- Multiterminal generalization of Landauer formula: the Buttiker formula.
- Reciprocity relations
- Finite temperature and larger biases
- Combining scattering matrices

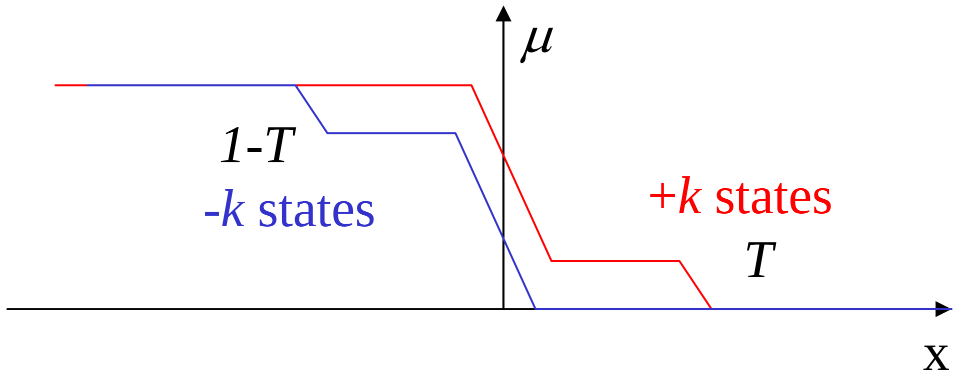
“Resistivity dipole” - Coulomb interactions

When computing chemical potential changes, we showed abrupt changes (a) at interfaces between contacts and leads; and (b) across a scatterer of transmittance *T*.

While μ may change abruptly, we know electrostatic potential cannot, because of *screening*.

Quick accounting for *averaged* electron-electron interactions: Poisson equation and screening length.





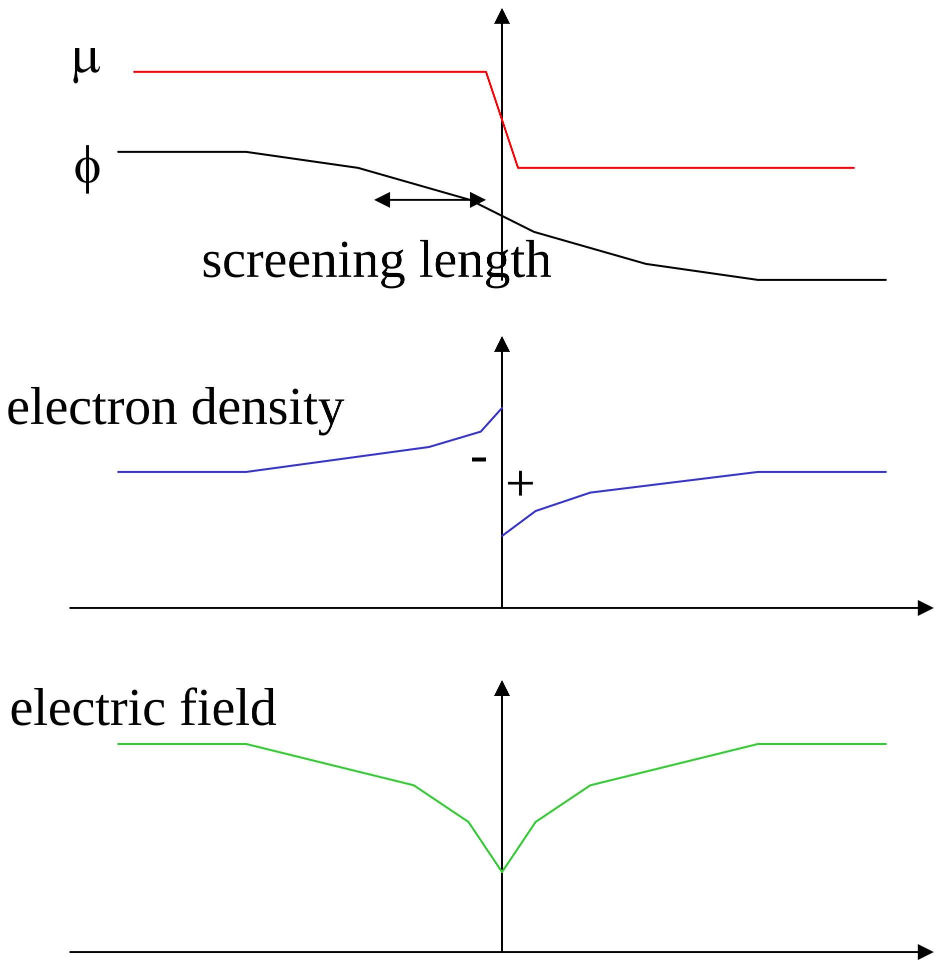
“Resistivity dipole” - Coulomb interactions

Discontinuity in chemical potential leads to smeared discontinuity in electrostatic potential.

Charge builds up microscopically like a dipole around the scatterer.

Whole system is solved self-consistently.

In systems with poor screening, effects of interfaces can be very big!



Buttiker formula (1988)

Treats multiple probe measurements such that all probes are on equal footing:

$$I_p = \frac{2e}{h}\sum_q (\bar{T}_{q\leftarrow p}\mu_p - \bar{T}_{p\leftarrow q}\mu_q)$$

Net current *out* of terminal *p*

Contributions from scattering with to/from terminals *q*.

Rewriting

$$G_{pq} \equiv \frac{2e^2}{h}\bar{T}_{p\leftarrow q}$$

$$I_p = \sum_q (G_{qp}V_p - G_{pq}V_q)$$

Sum rule (guarantees $I = 0$ when all V are same): $\sum_q G_{qp} = \sum_q G_{pq}$

$\longrightarrow$ $$I_p = \sum_q G_{pq}(V_p - V_q)$$ equivalent to Kirchhoff's law

Buttiker formula

Using this formula, potential of terminal *n* is determined by potentials of other terminals weighted by transmission functions:

$$V_n = \frac{\sum_{q \neq n} G_{nq} V_q}{\sum_{q \neq n} G_{nq}}$$

Note that, in general, $G_{qp} \neq G_{pq}$

though $G_{qp}(+B) = G_{pq}(-B)$

“reciprocity” -- not easy to show in general.

Buttiker formula: 4-terminal example

$$\begin{pmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{pmatrix} = \begin{bmatrix} G_{12}+G_{13}+G_{14} & -G_{12} & -G_{13} & -G_{14} \\ -G_{21} & G_{21}+G_{23}+G_{24} & -G_{23} & -G_{24} \\ -G_{31} & -G_{32} & G_{31}+G_{32}+G_{34} & -G_{34} \\ -G_{41} & -G_{42} & -G_{43} & G_{41}+G_{42}+G_{43} \end{bmatrix} \begin{pmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{pmatrix}$$

Can set $V_4 = 0$ without loss of generality….

$$\begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix} = \begin{bmatrix} G_{12}+G_{13}+G_{14} & -G_{12} & -G_{13} \\ -G_{21} & G_{21}+G_{23}+G_{24} & -G_{23} \\ -G_{31} & -G_{32} & G_{31}+G_{32}+G_{34} \end{bmatrix} \begin{pmatrix} V_1 \\ V_2 \\ V_3 \end{pmatrix}$$