Quantum Mechanics:
Homework on Harmonic Oscillator

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1. Determine the expectation values for the kinetic and potential energies in a harmonic oscillator. What can you say about $\langle T \rangle$ and $\langle V \rangle$?

2. By methods similar to those we used for the evaluation of $\langle x^2 \rangle$, evaluate $\langle x^4 \rangle$ and $\langle p^4 \rangle$ for the case that $\psi = \psi_n(x)$, where *n* represents arbitrary eigenvalue of the microscopic simple harmonic oscillator.

3. A one-dimensional microscopic simple harmonic oscillator is in a quantum state represented by the state function:

$$\psi(x,t) = \frac{1}{\sqrt{2}}\left[\psi_0(x)e^{-iE_0 t/\hbar} + \psi_1(x)e^{-iE_1 t/\hbar}\right]$$

   (a) Evaluate the position and the momentum uncertainties $\Delta x(t)$ and $\Delta p(t)$, and show that these functions oscillate at the natural frequency of the simple harmonic oscillator ω_0.

   (b) Graph the position probability density for this state at the following times (where T_0 is the classical period): $t = T_0/4$, $T_0/2$, $3T_0/4$, T_0, $3T_0/2$ and $2T_0$.

4. Small-amplitude vibrations of a diatomic molecule can be studied using as a model of the true molecular vibrational potential the simple harmonic oscillator potential

$$V(x) = \frac{1}{2}k_f x^2,$$

   where k_f is the force constant. For a typical diatomic molecule, $k_f = 10^3\ Jm^{-2}$.

   (a) Using this value, estimate the value of the zero-point vibrational energy of a diatomic molecule (in eV).

   (b) Estimate the energy spacing (in eV) between the ground vibrational state and the first excited vibrational state of the molecule.

(c) Suppose the molecule undergoes a transition from the first excited vibrational state to the ground state, emitting a phonon in the process. Estimate the energy, frequency, and the wavelength of the emitted phonon. In what range of the electromagnetic spectrum is this phonon found?

5. Using the Bound State Calculation Lab calculate the bound states in a harmonic potential with energy E=0.02 eV.