1 Esaki Diode

• When the concentration of impurity atoms in a pn-diode is very high, the depletion layer width is reduced to about 10 nm. Classically, a carrier must have an energy at least equal to the potential-barrier height in order to cross the junction. However, quantum mechanics indicates that there is a nonzero probability that a particle might penetrate through a barrier as thin as that indicated above. This phenomenon is called tunneling, and because of this, these high-impurity density p-n devices are called tunnel diodes, or Esaki diodes.

• The condition that the barrier be less than 10 nm thick is a necessary but not a sufficient condition for tunneling. It is also required that occupied energy states exist on the side from which the electrons tunnel and that allowed empty states exist on the other side.

• The energy-band diagram for a heavily - doped diode under open circuit and reverse bias conditions is shown below.

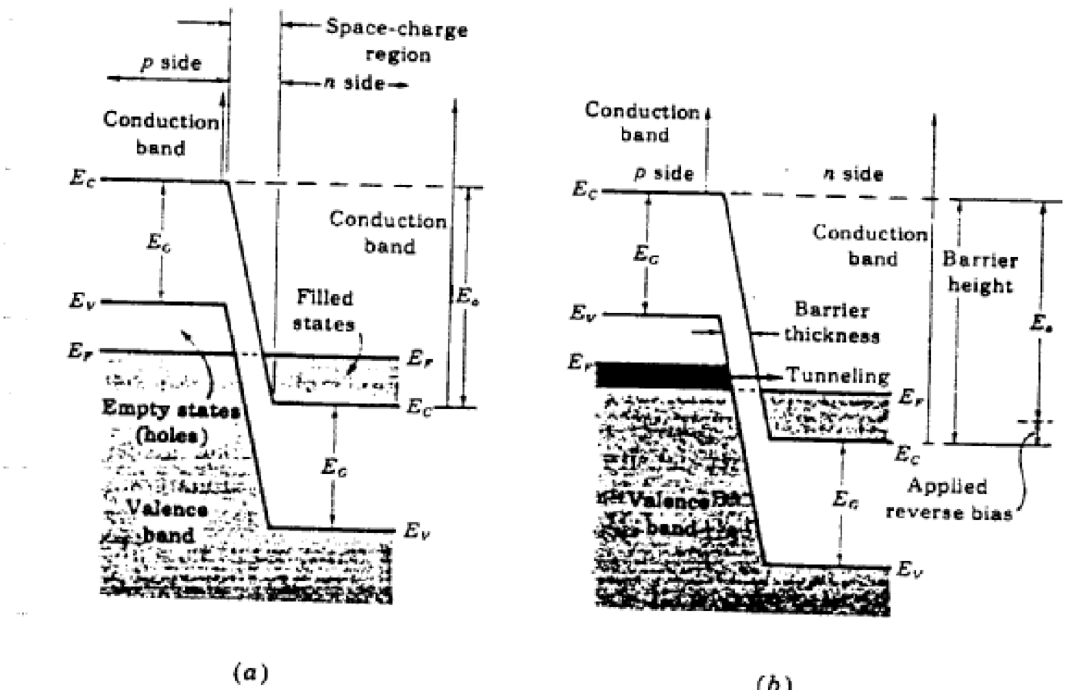

Fig. 19-11 Energy bands in a heavily doped p-n diode (a) under open-circuited conditions and (b) with an applied reverse bias. (These diagrams are strictly valid only at 0° K, but are closely approximated at room temperature, as can be seen from Fig. 19-3.)

Figure 1: Esaki diode

(a) Under zero-bias conditions, there are no filled states on one side of the junction which are at the same energy as the empty allowed states on the other side. Hence, there can be no flow of charge in either direction across the junction, and the current is zero, an obviously correct conclusion for an open - circuited diode.

(b) If a reverse bias is a applied, the height of the barrier is increased above the open-circuit value E_0. Hence, the n-side levels must shift downward with respect to the p-side levels. In this case, there are some energy states in the valence band on the p-side that lie at the same level as allowed empty states in the conduction band of the n-side. Hence, these electrons might tunnel from the p to the n - side, giving rise to the reverse diode current. As the magnitude of the reverse bias increases, the heavily - shaded are grows in size, causing the reverse current to increase, as shown by section (1) in the Figure below.

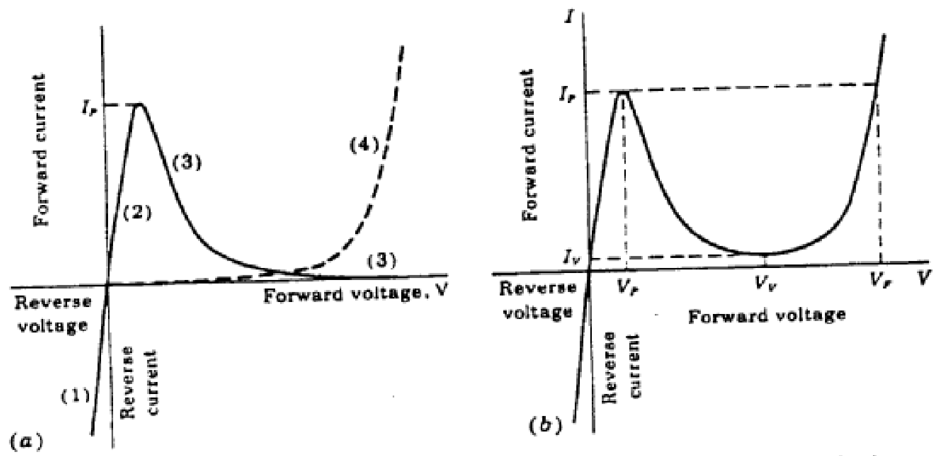

Fig. 19-13 (a) The tunneling current is shown solid. The injection current is the dashed curve. The sum of these two gives the tunnel-diode volt-ampere characteristic, which is shown in (b).

Figure 2: IV characteristics.

• Consider now the forward bias case when the potential barrier is decreased below E_0. The n-side level shifts upward with respect to those on the p-side, and the energy-band picture for this situation is indicated in Figure 3.

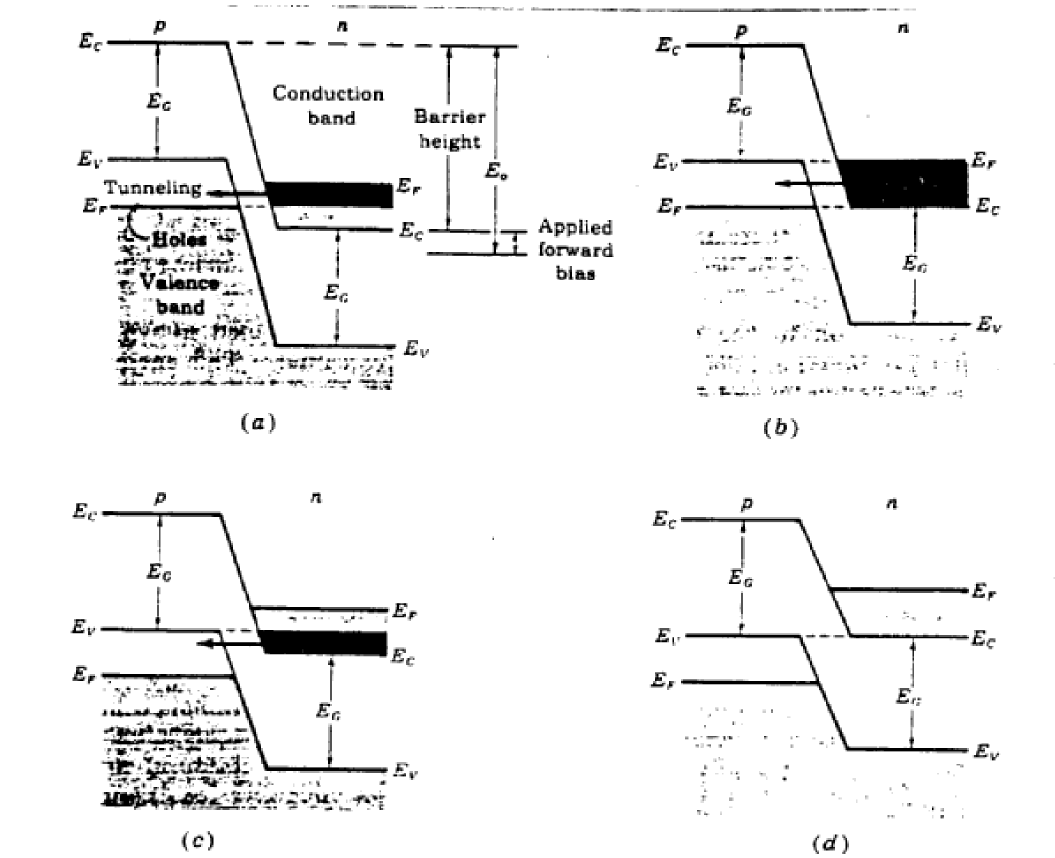

Fig. 19-12 The energy-band diagrams in a heavily doped p-n diode for a forward bias. As the bias is increased, the band structure changes progressively from (a) to (d).

Figure 3: Forward Bias

It is clear that there will be occupied states in the conduction band on the n-side which are at the same energy as the allowed empty sates (holes) in the valence band on the p-side. Hence electrons will tunnel from the n to the p material giving rise to the forward current of section (2). As the forward bias increases further, the condition in (b) is reached, giving rise to maximum current value. If more forward bias is applied, the situation in (c) is obtained and the tunneling current decreases, giving to section (3). At even larger bias, there will not be empty allowed states on one side of the junction at the same energy as the occupied states on the other side and the tunneling current drops to zero.

• In addition to the quantum-mechanical current described above, the regular pn-junction current is being collected at larger voltages, giving rise to section (4).

• Mathematical analysis of the tunneling current is based on the results described in the previous sections. In other words, one can use the WKB approximation to calculate the tunneling coefficient (or probability) and then use the result into the Landauer expression for the current.

• To calculate the tunneling coefficient, we consider the case when $E_\perp = 0$ and $E_z \neq 0$, and we approximate the barrier height by E_G:

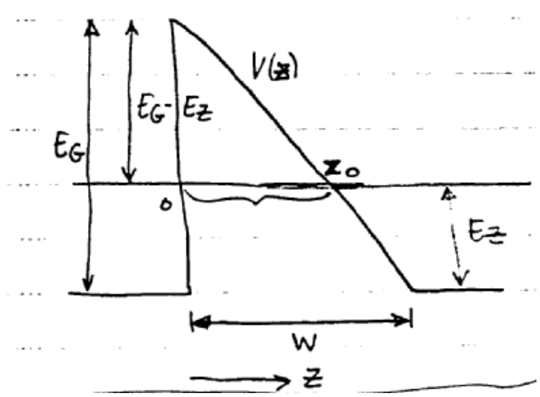

Figure 4: Tunneling barrier

$$V(Z) = E_G\left(1 - \frac{Z}{W}\right) \tag{1}$$

The attenuation of the barrier is thus

$$2\alpha = 2\int_0^{Z_0} \sqrt{\frac{2m}{\hbar^2}\left[V(Z) - E_Z\right]}dZ$$

$$\begin{aligned}
&= 2\int_0^{Z_0} \sqrt{\frac{2m}{\hbar^2}\left[E_G\left(1-\frac{Z}{W}\right)-E_Z\right]}dZ \\
&= 2\int_0^{Z_0} \sqrt{\frac{2mE_G}{\hbar^2}\left[1-\frac{Z}{W}-\frac{E_Z}{E_g}\right]}dZ \\
&= 2\int_0^{Z_0} \sqrt{\frac{2mE_G}{\hbar^2}}\sqrt{1-\frac{Z}{W}-\frac{E_Z}{E_g}}d\left(-\frac{Z}{W}\right)(-W) \\
&= (-2W)\int_0^{Z_0} \sqrt{\frac{2mE_G}{\hbar^2}}\sqrt{1-\frac{Z}{W}-\frac{E_Z}{E_g}}d\left(-\frac{Z}{W}\right) \\
&= (-2W)\sqrt{\frac{2mE_G}{\hbar^2}}\frac{\left(1-\frac{Z}{W}-\frac{E_Z}{E_g}\right)^{3/2}}{3/2}\Bigg|_0^{Z_0} \\
&= -\frac{4W}{3}\sqrt{\frac{2mE_G}{\hbar^2}}\left[\left(1-\frac{Z_0}{W}-\frac{E_Z}{E_g}\right)^{3/2}-\left(1-\frac{E_Z}{E_g}\right)^{3/2}\right] \qquad (2)
\end{aligned}$$

where

$$\begin{aligned}
1-\frac{Z_0}{W}-\frac{E_Z}{E_g} &= 1-\frac{E_g-E_Z}{E_g}-\frac{E_Z}{E_g} \\
&= \frac{E_g-E_Z}{E_g}-\frac{E_g-E_Z}{E_g} \qquad (3)
\end{aligned}$$

Hence

$$\begin{aligned}
\alpha &= \frac{4W}{3}\sqrt{\frac{2mE_G}{\hbar^2}}\left(1-\frac{E_Z}{E_g}\right)^{3/2} \\
&\approx \frac{4W}{3}\sqrt{\frac{2mE_G}{\hbar^2}}\left(1-\frac{3}{2}\frac{E_Z}{E_g}\right) \qquad (4)
\end{aligned}$$

provided that $E_Z/E_G \ll 1$. Substituting the last expression into the WKB result for the transmission coefficient gives

$$\begin{aligned}
T(E_Z) &= e^{-2\alpha} \\
&= \underbrace{e^{-\frac{4W}{3}\sqrt{\frac{2mE_G}{\hbar^2}}}}_{T_0} e^{\frac{2W}{E_g}\sqrt{\frac{2mE_G}{\hbar^2}}E_Z} \\
&= T_0 e^{E_Z/E_0} \qquad (5)
\end{aligned}$$

where we have defined

$$
\begin{aligned}
E_0 &= \frac{E_g}{2W}\sqrt{\frac{\hbar^2}{2mE_g}} \\
&= \sqrt{\frac{\hbar^2 E_g^2}{2mE_g}}\frac{1}{2W} \\
&= \frac{1}{2W}\sqrt{\frac{\hbar^2 E_g}{2m}} \qquad (6)
\end{aligned}
$$

The expression for $T(E_Z)$ suggests that the larger the energy E_Z, the larger the probability for transmission, which should be expected.

•The next task is to calculate the current under forward bias conditions, for which we use the previously derived expression

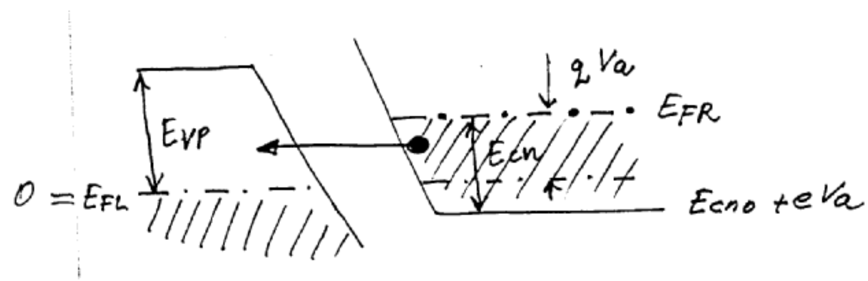

Figure 5: Forward bias conditions.

$$
\begin{aligned}
J &= J_{L\to R} - J_{R\to L} \\
&= -\frac{em^*}{2\pi^2\hbar^3}\int_0^\infty dE_Z T(E_Z)\int_0^\infty dE_t\left[f_L(E_Z+E_t) - f_R(E_Z+E_t+eV_a)\right] \qquad (7)
\end{aligned}
$$

where

$$
\begin{aligned}
E &= E_Z + E_t \\
&= E_{Z_1} + E_t - E_{cn} \qquad (8)
\end{aligned}
$$

Using the approximation (valid at $T \to 0$);

$$
f_L(E_Z+E_t) - f_R(E_Z+E_t+eV_a) =
$$

$$
\begin{aligned}
&= f(E_Z+E_t) - f(E_Z+E_t) - eV_a \left.\frac{\partial f}{\partial E}\right|_{E_Z+E_t} = \\
&= -eV_a \delta(E_Z+E_t-E_F) \qquad (9)
\end{aligned}
$$

we get for the forward current

$$
\begin{aligned}
J &= -\frac{em^*}{2\pi^2\hbar^3}(-eV_a)T_0 \int_0^\infty dE_t \int_0^\infty dE_Z e^{E_Z/E_0}\delta(E_Z+E_t-E_F) \\
&= \frac{e^2m^*V_aT_0}{2\pi^2\hbar^3}\int_0^{E_t^{\max}} dE_t e^{(\cancelto{0}{E_F} - E_t)/E_0} \\
&= \frac{e^2m^*V_aT_0}{2\pi^2\hbar^3}(-E_0)e^{-E_t/E_0}\Big|_0^{E_t^{\max}} \\
&= -\frac{e^2m^*V_aT_0E_0}{2\pi^2\hbar^3}\left[e^{-E_t^{\max}/E_0} - 1\right] \qquad (10)
\end{aligned}
$$

where

$$
E_t^{\max} = E_{VP} + E_{cn} - eV_a \qquad (11)
$$

Therefore

$$
\begin{aligned}
e^{-E_t^{\max}/E_0} - 1 &= e^{-(E_{VP}+E_{cn}-eV_a)/E_0} - 1 \\
&\approx \cancel{1} - \frac{E_{VP}+E_{cn}-eV_a}{E_0} - \cancel{1} \qquad (12)
\end{aligned}
$$

i.e.,

$$
\begin{aligned}
J &= -\frac{e^2m^*V_aT_0E_0}{2\pi^2\hbar^3}\left(-\frac{E_{VP}+E_{cn}-eV_a}{E_0}\right) \\
&= \frac{e^2m^*V_aT_0}{2\pi^2\hbar^3}\left(\underbrace{E_{VP}+E_{cn}}_{E_t} - eV_a\right) \\
&= \frac{e^2m^*T_0E_t}{2\pi^2\hbar^3}V_a\left(1 - \frac{eV_a}{E_t}\right) \qquad (13)
\end{aligned}
$$

The maximum transverse energy is obtained using

$$
-E_{cno} + eV_a + E_\perp^{\max} = E_{VP} \qquad (14)
$$

$$
E_{\perp^{\max}} = E_{VP} + E_{cn} - eV_a \qquad (15)
$$