# WKB Approximation – Part 1

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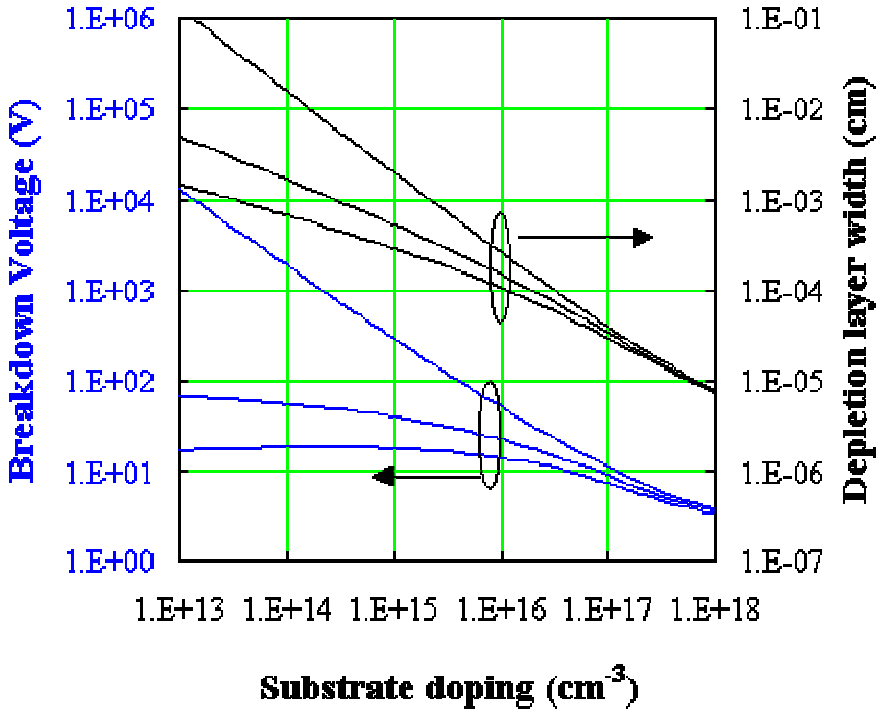
# Important Applications in which WKB Approximation is Used

- Tunneling Breakdown in normal diodes (reverse biased diode)
- Tunnel (Esaki) diode (forward + reverse bias)
- Scanning Tunneling Microscope
- Gate Leakage

# A. Tunneling Breakdown



$$|\mathcal{E}_{br}| = \frac{4x10^5}{1-\frac{1}{3}\log(N/10^{16})} \quad \text{V/cm}$$

$$|V_{br}| = -\phi_i + \frac{|\mathcal{E}_{br}|^2 \varepsilon_s}{2qN}$$

$$w_{br} = \frac{|\mathcal{E}_{br}|\varepsilon_s}{qN}$$

Breakdown voltage and depletion layer width at breakdown versus doping density of an abrupt one-sided p-n diode. Shown are the voltage and width for a planar (top curves), cylindrical (middle curves) and spherical (bottom curves) junction with 1 mm radius of curvature.

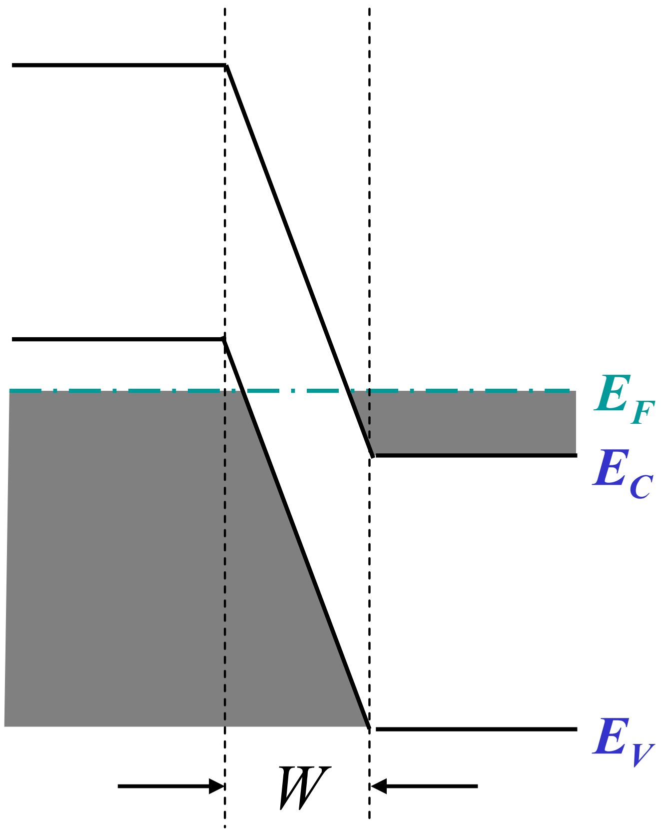
# Breakdown Mechanisms in a Diode

- Junction breakdown can be due to:
  - tunneling breakdown
  - avalanche breakdown

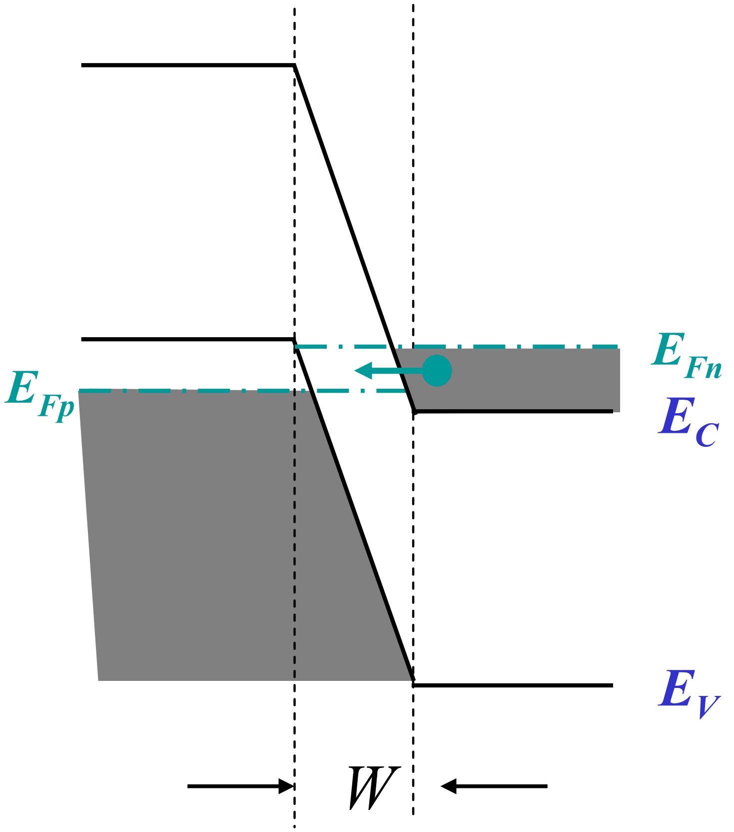
- One can determine which mechanism is responsible for the breakdown based on the value of the breakdown voltage $V_{BD}$ :
  - $V_{BD} < 4E_g/q \rightarrow$ tunneling breakdown
  - $V_{BD} > 6E_g/q \rightarrow$ avalanche breakdown
  - $4E_g/q < V_{BD} < 6E_g/q \rightarrow$ both tunneling and avalanche mechanisms are responsible

- Tunneling breakdown occurs in heavily-doped *pn*-junctions in which the depletion region width *W* is about 10 nm.

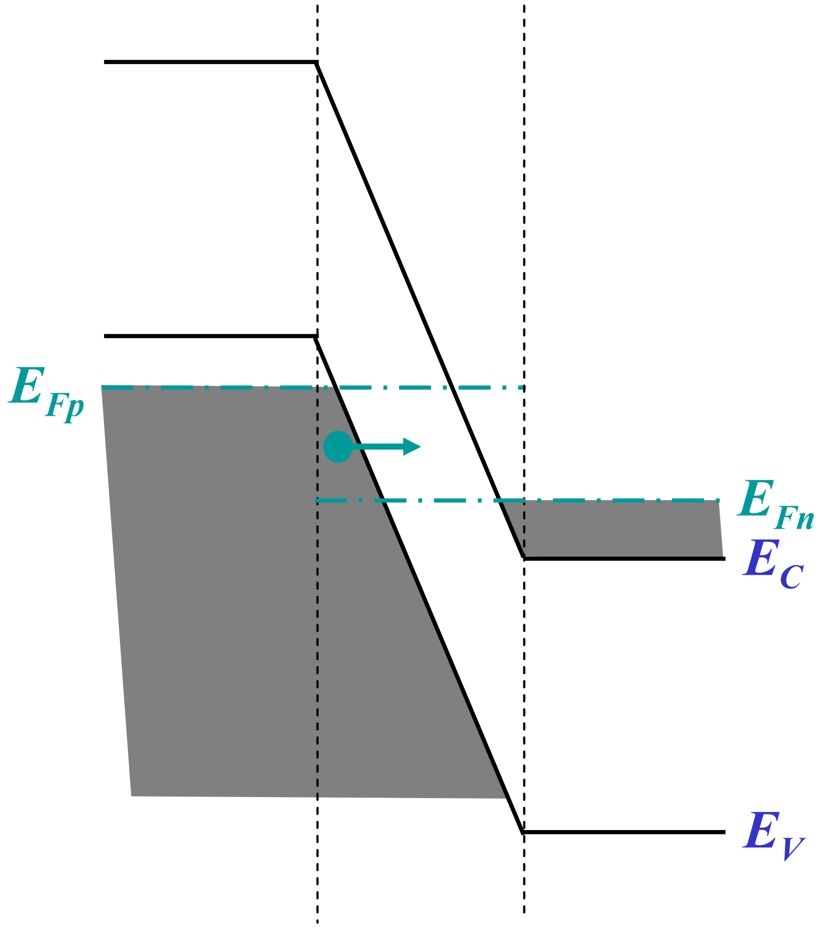
Zero-bias band diagram:



Forward-bias band diagram:



Reverse-bias band diagram:



- Tunneling current (obtained by using WKB approximation):

$$I_t = \frac{\sqrt{2m^*}q^3 F_{cr} VA}{4\pi^2\hbar^2 E_g^{1/2}} \exp\left(-\frac{4\sqrt{2m^*}E_g^{3/2}}{3\hbar q F_{cr}}\right)$$

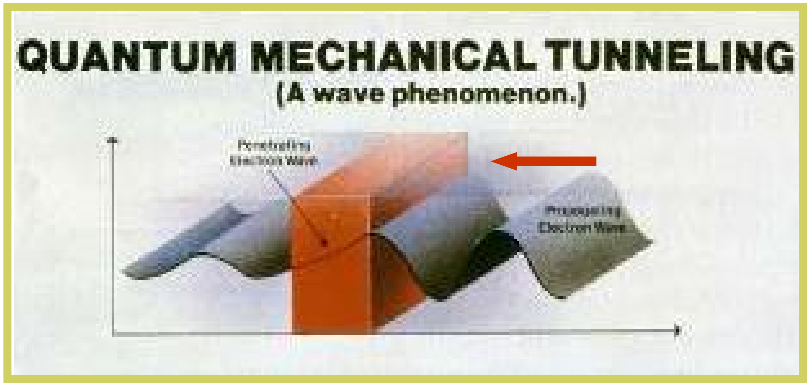
$F_{cr}$ ➔ average electric field in the junction

- The critical voltage for tunneling breakdown, $V_{BR}$, is estimated from:

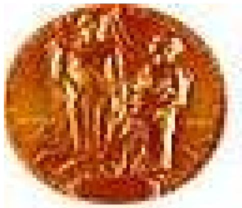
$$I_t(V_{BR}) \propto 10 I_S$$

- With $T\uparrow$, $E_g\downarrow$ and $I_t\uparrow$.

# B. Tunnel (Esaki) Diode

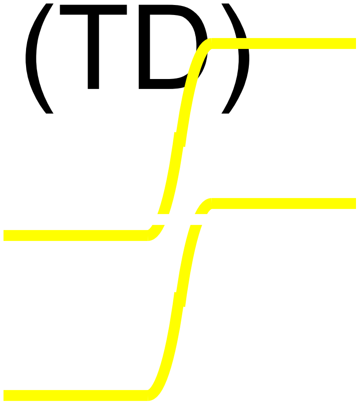


**Leo Esaki**



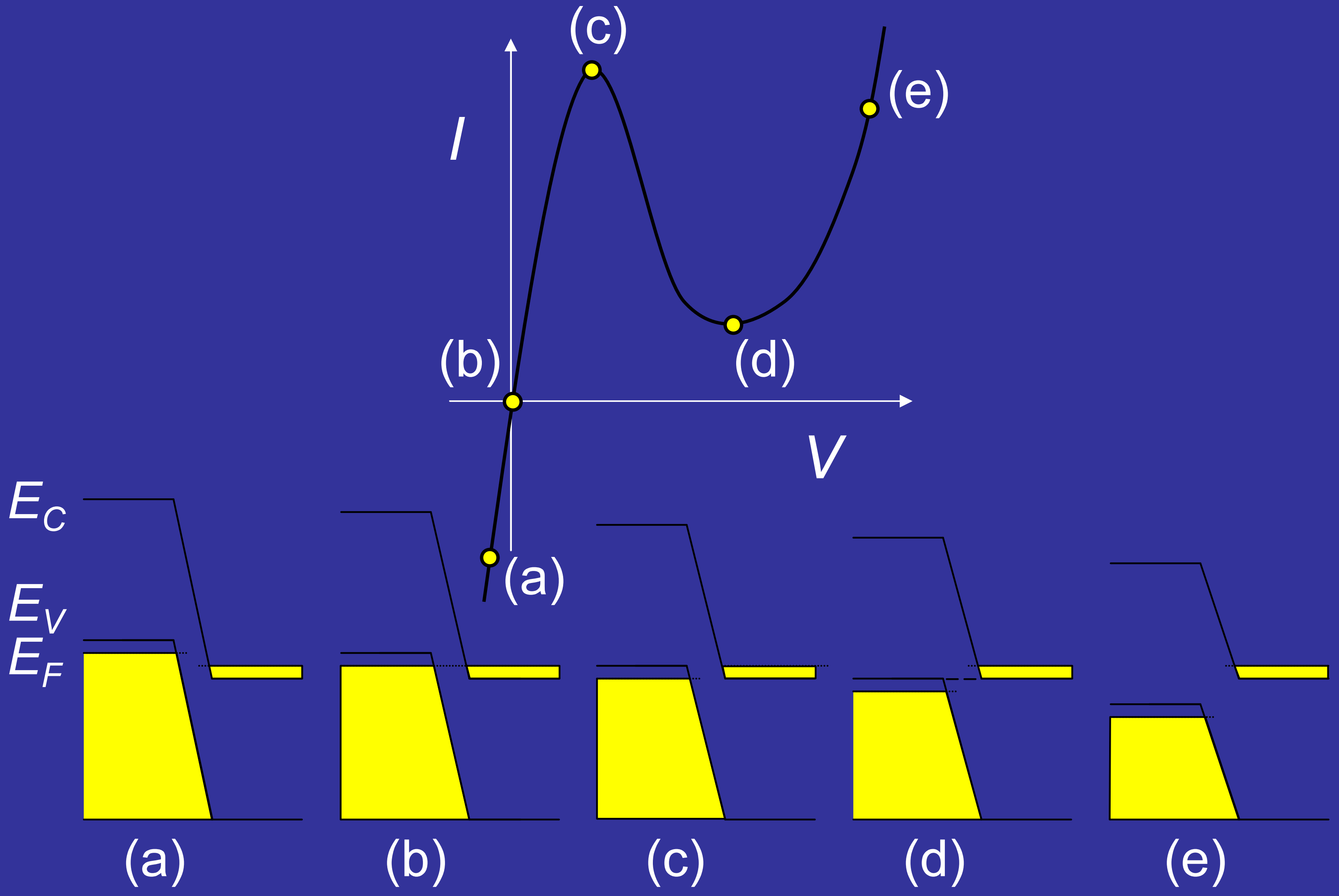
**Nobel Prize in Physics 1973**

# (Esaki) Tunnel Diode (TD)

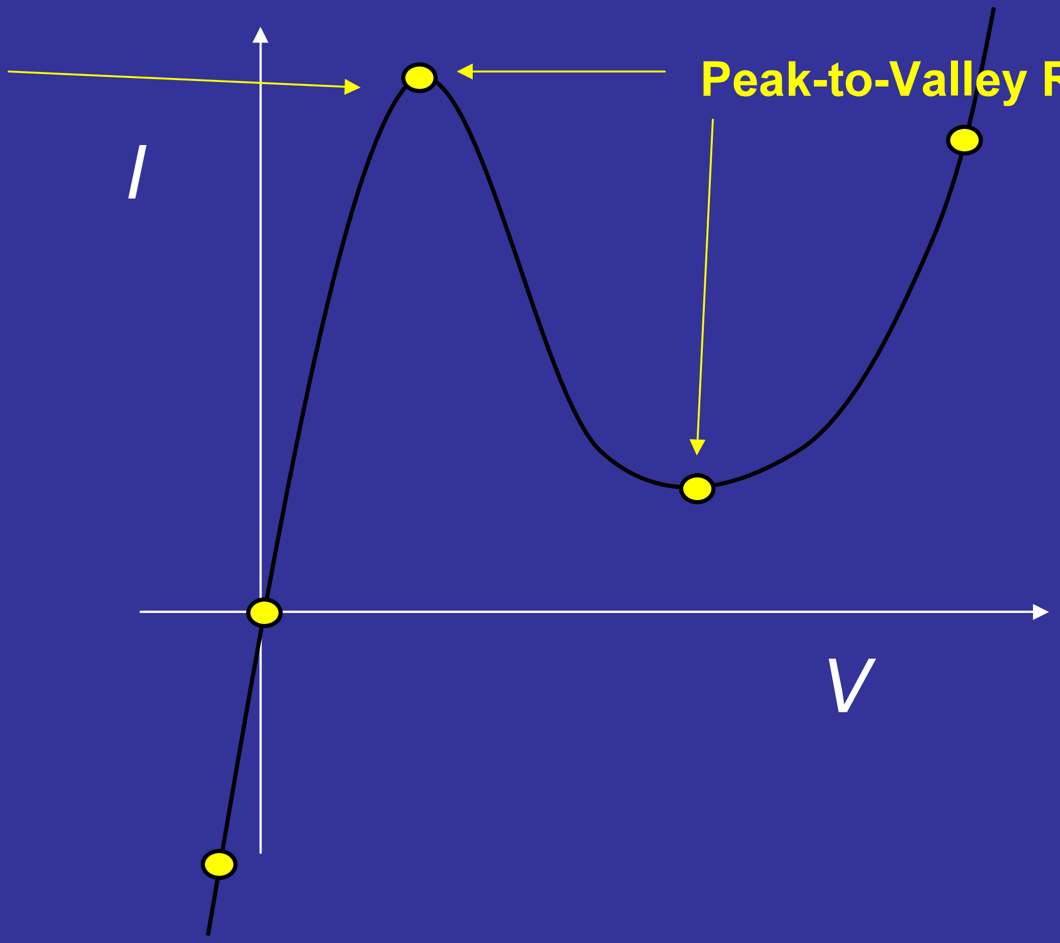


- Simplest tunneling device
- Heavily-doped pn junction
  - Leads to overlap of conduction and valence bands
- Carriers are able to tunnel inter-band
- Tunneling goes exponentially with tunneling distance
  - Requires junction to be abrupt

# Band-to-Band Tunneling in a Tunnel Diode



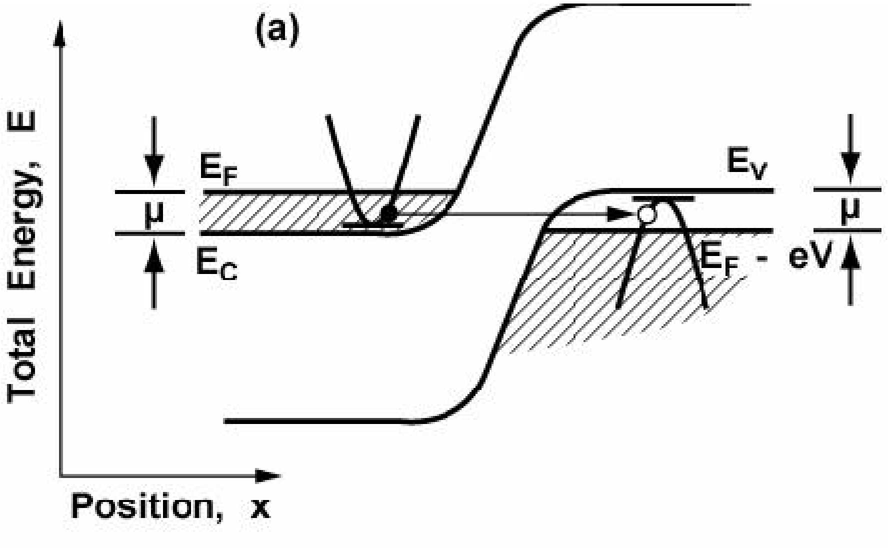
# Figures of Merit



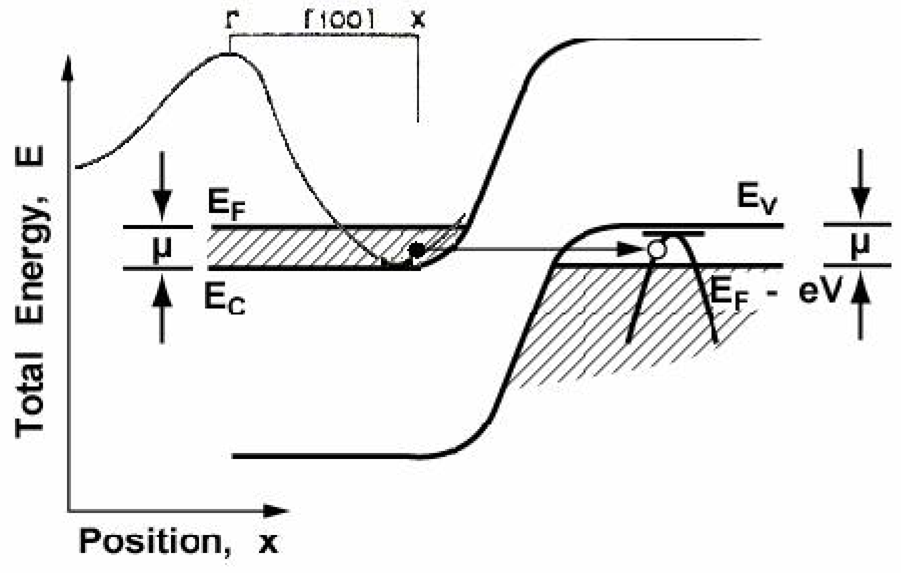
**Peak current**
**100 kA/cm$^2$**

**Peak-to-Valley Ratio (PVR)**

# Direct vs. Indirect Tunneling



**Direct**



**Indirect**

**Indirect materials require phonons to tunnel, thus reducing the probability of a tunneling event**

# Tunnel Current Expressions

$$T = \exp\left(-\frac{\pi m^{*1/2} E_G^{3/2}}{2\sqrt{2}\hbar e F}\right)\exp\left(-\frac{2E_t}{\overline{E}}\right)$$

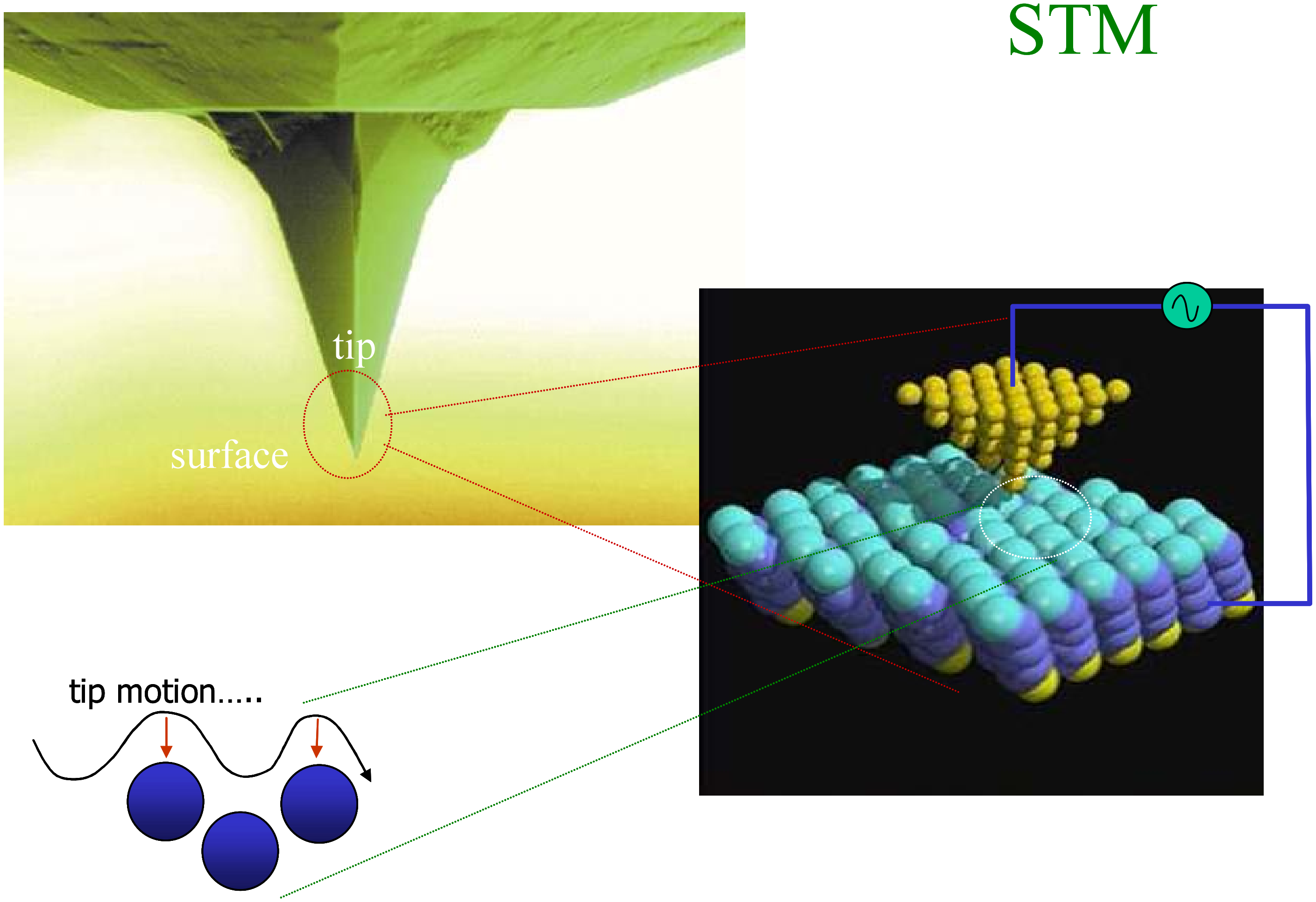
$$\overline{E} = \frac{4\sqrt{2}\hbar e F}{3\pi\sqrt{m^* E_G}}$$

$$J_t = \frac{q^3 m^{*1/2}\,\xi V_a}{4\sqrt{2}\pi^2\hbar^2 E_g^{\;1/2}}\exp\left(\frac{-4\sqrt{2m^*}E_g^{\;3/2}}{3q\hbar\xi}\right)$$

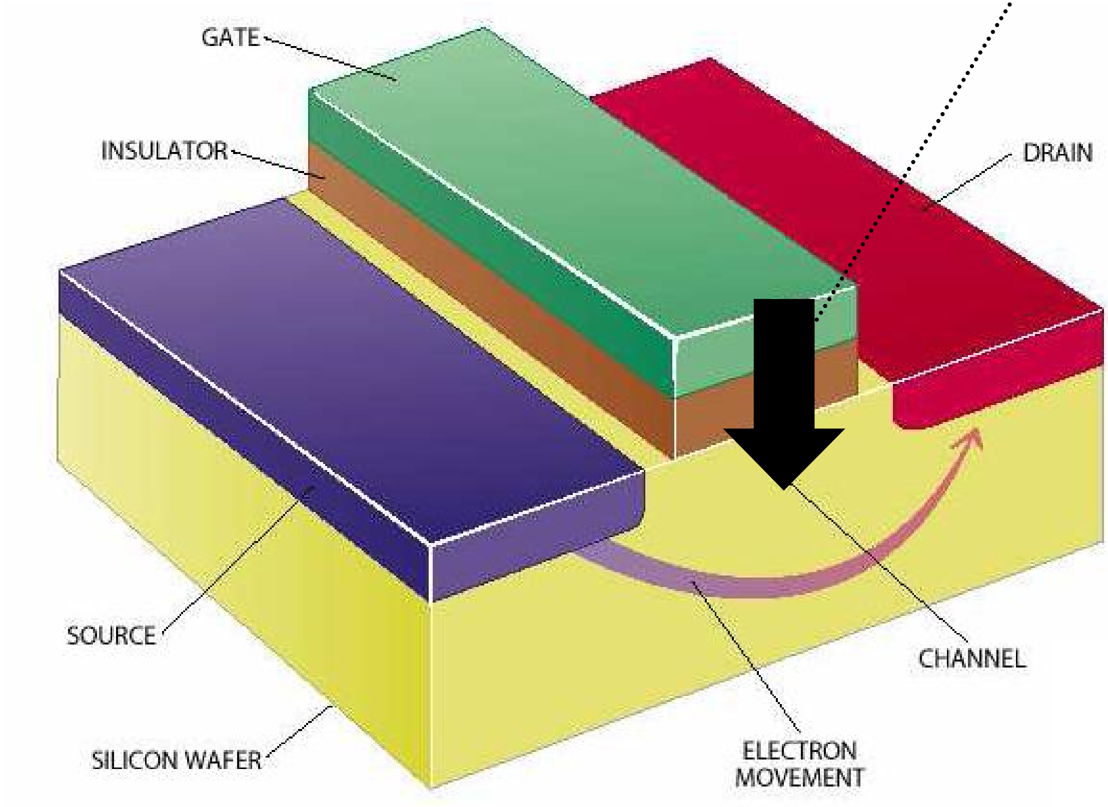
# C. Scanning Tunneling Microscope

**revolution of tunnelling: Scanning Tunnelling Microscope**

STM



# Gate Leakage



gate leakage
tunnelling current

# Gate Leakage

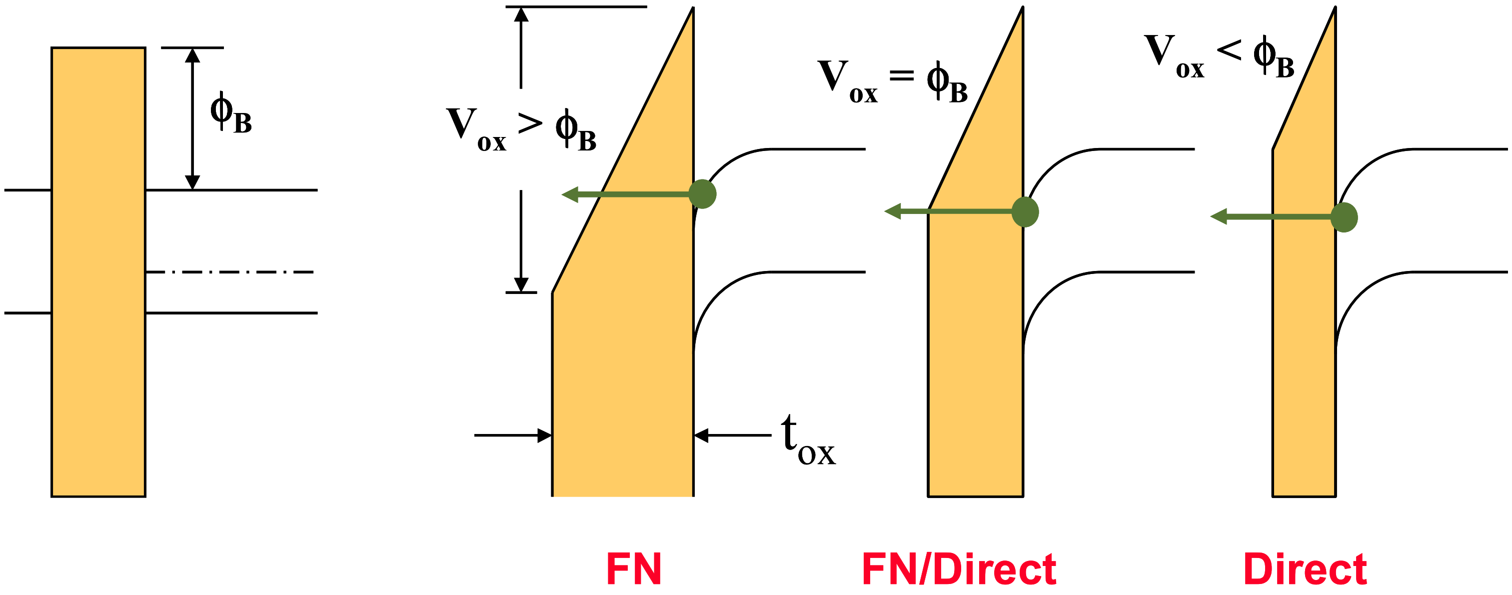
- For sub-micrometer devices, due to smaller oxide thickness, there is significant conductance being measured on the gate contact. The finite gate current gives rise to the following effects:
  - Negative => degradation in the device operating characteristics with time due to oxide charging; larger off-state power dissipation
  - Positive => non-volatile memories utilize the gate current to program and erase charge on the "floating contact" – FLASH, FLOTOX, EEPROM
- There are two different types of conduction mechanisms to the insulator layer:
  - Tunneling: *Fowler-Nordheim* or *direct* tunneling process
  - Hot-carrier injection: *lucky electron model* or *Concannon model*

Electron is emitted into the oxide when it gains sufficient energy to overcome the insulator/semiconductor barrier.

- Similar to the lucky electron model, but assumes non-Maxwellian high energy tail on the distribution function.
- Requires solution of the energy balance equation for carrier temperature.

# Tunneling Currents

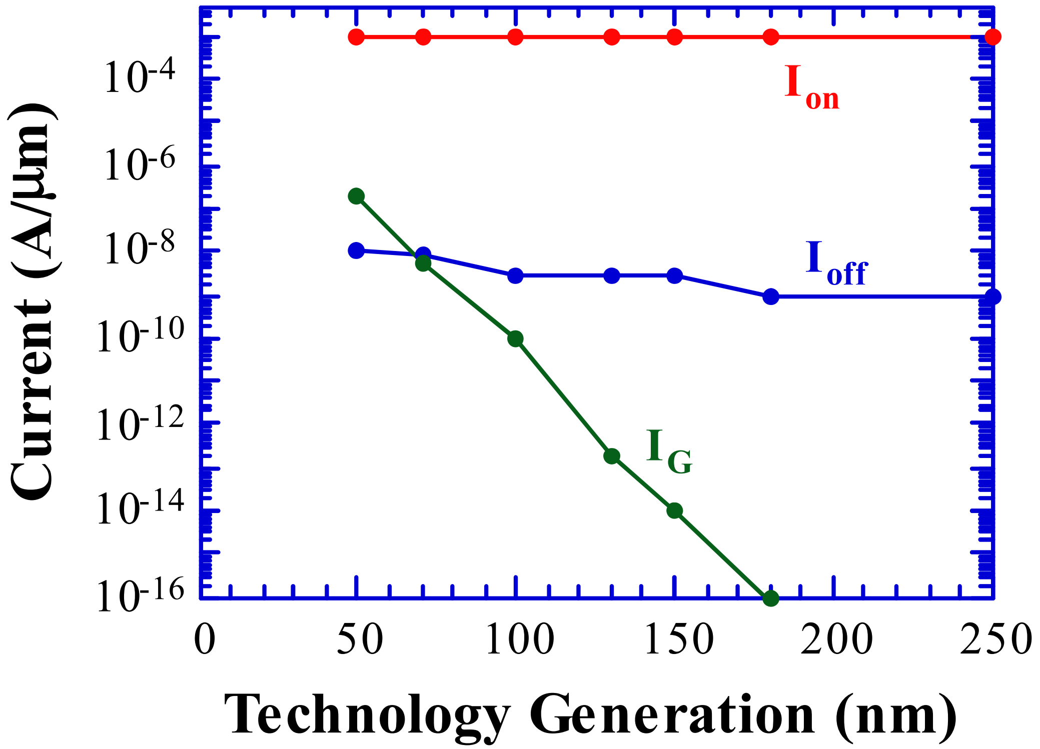
- Three types of tunneling processes are schematically shown below (courtesy of D. K. Schroder)



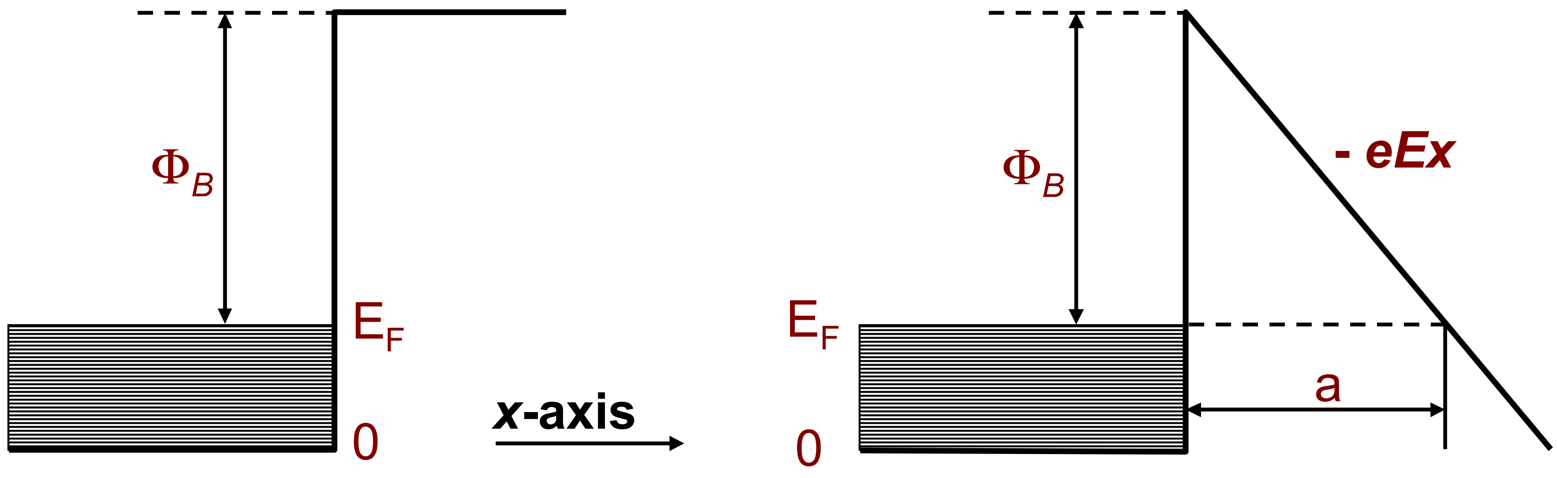
- For $t_{ox} \geq 40$ Å, Fowler-Nordheim (FN) tunneling dominates
- For $t_{ox} < 40$ Å, direct tunneling becomes important
- $I_{dir} > I_{FN}$ at a given $V_{ox}$ when direct tunneling active
- For given electric field:
  - $I_{FN}$ independent of oxide thickness
  - $I_{dir}$ depends on oxide thickness

# Significance of Gate Leakage

- As oxide thickness decreases, the gate current becomes more important. It eventually dominates the off-state leakage current ($I_D$ at $V_G$ = 0 V)
- The drain current $I_D$ as a function of technology generation is shown below (courtesy of D. K. Schroder)



# Fowler-Nordheim Tunneling



*No applied bias*

*With applied bias*

- The difference between the Fermi level and the top of the barrier is denoted by $\Phi_B$
- According to WKB approximation, the tunneling coefficient through this triangular barrier equals to:

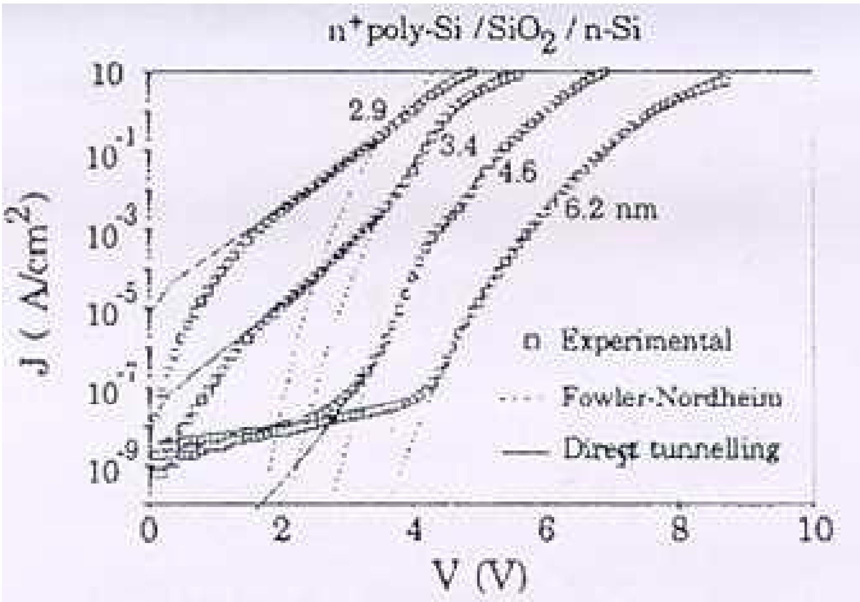
$$T \propto \exp\left[-2\int_0^a \gamma(x)\,dx\right] \quad \text{where:} \quad \gamma(x) = \sqrt{\frac{2m^*}{\hbar^2}(\Phi_B - eEx)}$$

# Fowler-Nordheim Tunneling

- The final expression for the Fowler-Nordheim tunneling coefficient is:

$$T \propto \exp\left[-\frac{4\sqrt{2m^*}\Phi_B^{3/2}}{3eE\hbar}\right]$$

- Important notes:
  - The above expression explains tunneling process only qualitatively because the additional attraction of the electron back to the plate is not included
  - Due to surface imperfections, the surface field changes and can make large difference in the results



**Calculated and experimental tunnel current characteristics for ultra-thin oxide layers.**

(M. Depas et al., *Solid State Electronics*, Vol. 38, No. 8, pp. 1465-1471, 1995)