Optical Imaging
Chapter 1 - Introduction

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1.1 Properties of EM Fields

- Amplitude $A$ and phase $\phi$ are random functions of both time and space:

$$\vec{E}(\vec{r}, t) = \vec{A}(\vec{r}, t).e^{i\phi(\vec{r}, t)} \quad (1.1)$$
1.1 Properties of EM Fields

a) Polarization:
   - Gives the direction of field oscillation
   - Generally, light is a transverse wave (unlike sound = longitudinal)

\[ | \vec{k} | = \frac{2\pi}{\lambda} \]

- Anisotropic materials: different optical properties along different axis → useful
1.1 Properties of EM Fields

a) Polarization:
   - There is always a basis \((\hat{x}, \hat{y})\) for decomposing the field into 2 polarizations (eigen modes); equivalently (right, left) circular polarization is also a basis.
   - Dichroism: preferential absorption of one component \(\rightarrow\) one way to create polarizers:

     \[
     \begin{align*}
     |E_1| &= |E_2| \cdot \cos \theta \\
     \end{align*}
     \]  

     Malus Law: \( |E_1| = |E_2| \cdot \cos \theta \) (1.2)
1.1 Properties of EM Fields

a) Polarization:
   - Natural Light $\rightarrow$ unpolarized $\rightarrow$ superposition $E_x = E_y$ with no phase relationship between the two
   - Circularly polarized $\rightarrow$ $E_x = E_y$, $\phi_x - \phi_y = \pi/2$
   - Matrix formalism of polarization transformation
     (Jones – 2x2, complex & Muller – 4x4, real)
1.1 Properties of EM Fields

b) Amplitude: \[ A(r,t) = \frac{V}{m} \]
1.1 Properties of EM Fields

c) Phase: \([\Phi] = \text{rad}\)

- Thermal source
- Random field
- Laser at freq \(\omega_o\)
- Plane Wave \(\Phi = kz\)
1.1 Properties of EM Fields

c) Phase: $[\Phi] = \text{rad}$
   - For quasi-monochromatic fields, plane wave
     \[ \phi = \omega t - \vec{k} \cdot \vec{r} \]
     \[ k = \frac{\omega}{c} = \frac{2\pi \nu}{c} = \frac{2\pi}{Tc} = \frac{2\pi}{\lambda} = \text{wave number} \quad (1.3) \]
1.2 The frequency domain representation

- Random variable $E(t)$ has a frequency-domain counterpart:

$$E(\omega) = A(\omega)e^{i\phi(\omega)} \quad (1.4)$$

- Similarly $E(x)$ has a frequency-domain pair:

$$E(\xi) = A(\xi)e^{i\phi(\xi)} \quad (1.5)$$
1.2 The frequency domain representation

a) Spectral amplitude:

- Optical Spectrum: \( s(\omega) = |A(\omega)|^2 \)
- Angular Spectrum: \( s(\xi) = |A(\xi)|^2 \)

\[
\begin{bmatrix} \xi \end{bmatrix} = m^{-1} = \text{Spatial Frequency} \quad \text{(connects to angular spectrum)}
\]

- \( t \rightarrow \omega \)
- \( x \rightarrow \xi \)

- Will follow similar equations

- The information contained is the same \((t, \omega)\) and \((x, \xi)\)
1.2 The frequency domain representation

b) Spectral phase:

- Phase delay of each spectral component

  Optical Frequency
  \[ \Phi(\omega) \sim \omega^2 \]
  
  • Dispersive material (linear chirp)

  Spatial Frequency
  \[ \Phi(\xi) \sim \xi^2 \]
  
  • Defocused point source (1st order aberration)

- Full similarity between \((t, \omega)\) and \((x, \xi)\)
1.3 Measurable Quantities

- The information about the system under investigation may be contained in polarization and:
  - $A(t), \phi(t)$
  - $A(\omega), \phi(\omega)$
  - $A(x), \phi(x)$
  - $A(\xi), \phi(\xi)$

- Experimentally, we have access only to:
  
  $I = \left\langle |A(t)|^2 \right\rangle = \text{time average}$
1.3 Measurable Quantities

- Experimentally, we have access only to:
  \[ I = \left\langle |A(t)|^2 \right\rangle = \text{time average} \quad (1.6) \]

- i.e the photodetectors (photodiode, CCD, retina, etc) produce photoelectrons:
  \[ h\nu = E_{e^-} + W \quad \text{(Einstein)} \quad (1.7) \]
1.3 Measurable Quantities

- All detectors sensitive to power/energy
- However, all 8 quantities can be accessed via various tricks
  - **Eg1:** Want $I(\lambda) \rightarrow$ measure $I(\theta)$ and use a device with $\theta(\lambda)$
  - **Eg2:** Want $\phi \rightarrow$ use interferometry $\rightarrow I(\phi) \propto |E_1||E_2|\cos(\phi_1 - \phi_2)$
1.4 Uncertainty Principle

- Space - momentum or energy-time cannot be measured simultaneously with infinite accuracy

\[
\begin{align*}
\Delta x \cdot \Delta p &= \text{constant} \approx \hbar \\
\Delta E \Delta t &= \text{constant}
\end{align*}
\]

- For photons:

\[
\begin{align*}
E &= \hbar \omega \\
p &= \hbar k
\end{align*}
\]
1.4 Uncertainty Principle

a) \( t - \omega \)

\[ \hbar \Delta \omega \Delta t = \text{constant} \]

\[ \Rightarrow \Delta \omega \Delta t \approx 2\pi \]

- Implications:
  1- short pulses require broad spectrum
  2- high spectral resolution requires long time of measurement
1.4 Uncertainty Principle

b) \( x - \xi \)

\[ \Delta x |q| \approx \lambda \pi \]

\[ \Delta p = h(\kappa_s - \kappa_i) = h|q| \]

\[ \Delta x \frac{2 \sin(\theta / 2)}{\lambda} \approx 1 \]

\[ \Delta x_{\text{min}} = \frac{\lambda}{2} \]

- meaning of resolution
### 1.4 Uncertainty Principle

- Smaller aperture $\rightarrow$ Higher angle

- If aperture $< \frac{\lambda}{2}$, light doesn’t go through
- **Eg:** Microwave door
1.4 Uncertainty Principle

- We will encounter these relationships many times later
- Fourier seems to have understood this uncertainty principle way before Heisenberg!