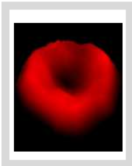

Optical Imaging
Chapter 6 – Interferometry

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6.1 Superposition of Fields

- Interference is the phenomenon by which electromagnetic fields interact with one another.
- Interferometry has various applications from the movement of charge to spectroscopy and material characterization.
- Interference is the result of the superposition principle

$$\overline{E}[\overline{r},t]=\sum_{j}\overline{E}_{j}[\overline{r},t] \tag{6.1}$$

- Intensity is the measurable quantity

$$\overline{I}[\overline{r},t]=\left\langle \left|E[r,t]\right|^{2}\right\rangle \tag{6.2}$$

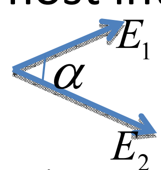
- Where $\langle\ \rangle$ stands for time (or ensemble) average

6.1 Superposition of Fields

- Consider 2 fields $\overline{E}_1$ and $\overline{E}_2$

$$\overline{I}[\overline{r},t] = \left\langle \left|E_1\right|^2 \right\rangle + \left\langle \left|E_2\right|^2 \right\rangle + \left\langle \left|E_1 \cdot E_2^*\right| \right\rangle + \left\langle \left|E_1^* \cdot E_2\right| \right\rangle \qquad (6.3)$$

- <u>Notes</u>:
  - the dot product $E_1 \cdot E_2^*$
  - Polarization is critical
  - Parallel polarization offers the most interference
  - $\left|E_1 \cdot E_2\right| = \left|E_1\right|\left|E_2\right| \cos\alpha$



  - Assume parallel polarization

$$=> I = I_1 + I_2 + 2\left|E_1\right|\left|E_2\right| \cos[\Delta\phi[\overline{r},\tau]] \qquad (6.4)$$

  - $\Delta\phi$ is generally a random variable
  - For uncorrelated (incoherent) fields $\langle \cos[\Delta\phi] \rangle \to 0$ => simplest case is a monochromatic field because it is fully coherent.

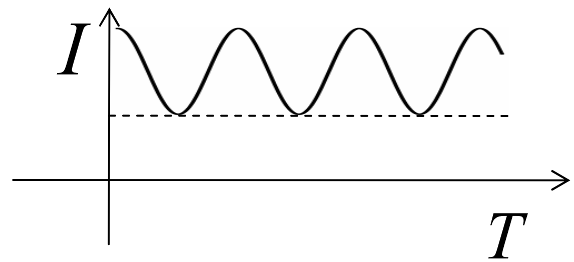
6.2 Monochromatic Fields

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cdot \cos[\omega_0 \tau + \triangle\phi] \qquad (6.5)$$

$T =$ Time difference

$\phi =$ The phase shift

$\omega_0 =$ Frequency



- Fringe contrast (visibility): $\gamma = \dfrac{I_{\max} - I_{min}}{I_{\max} + I_{min}}$ (6.6)

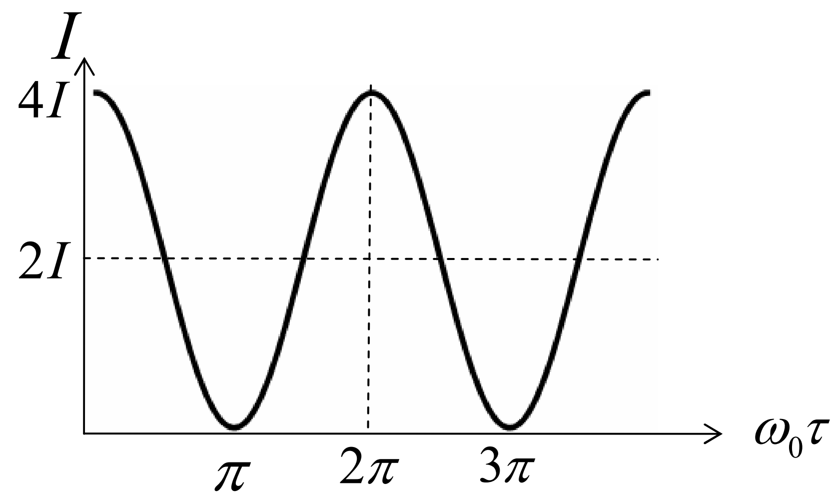
$$=> \gamma = \frac{I_1 + I_2 + 2\sqrt{I_1 I_2} \cdot 1 - I_1 - I_2 - 2\sqrt{I_1 I_2} \cdot (-1)}{2(I_1 + I_2)} = \frac{2\sqrt{I_1 I_2}}{I_1 + I_2} \in (1:0)$$

- $I_1 = I_2 => \gamma = 1$

6.2 Monochromatic Fields

- For $I_1 = I_2 = I$:

$$I = 2I(1+\cos[\omega_0\tau]) \quad (6.7)$$



- Practical Advantages of Interference:

  a) Interference term is $2\sqrt{I_1 I_2}\cos[\triangle\phi]$ so if I_1 is too small to be measured directly then I_2 can act as an amplifier. $I_1 I_2$ gives a higher sensitivity

6.2 Monochromatic Fields

- Practical Advantages of Interference:

  b) Interference $\sim \sqrt{I_1 I_2}$ => if I_1 is divided by 100, interference is divided by 10 => high dynamic range

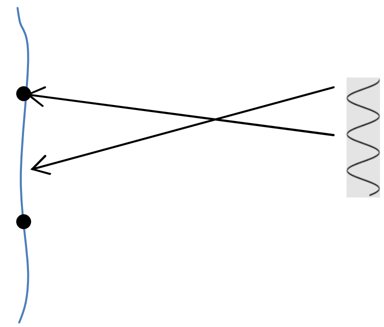
  c) Imagine we frequency shift E_1 by $\triangle\omega = \omega_1 - \omega_2$ =>

$$\sqrt{I_1 I_2} \cdot \cos[\omega_1 t - \omega_2 t] \sim \cos[\triangle\omega t]$$

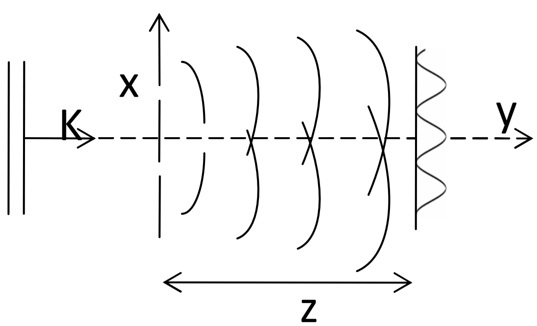
=> can tune $\triangle\omega$ to high frequency (> 1 kHz) => Low Noise

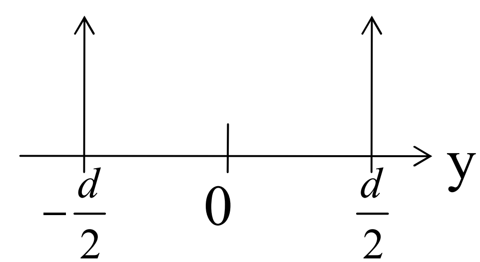
6.3 Wavefront-Division Interferometry

- Interference is obtained between different portions of the same wavefront (next: amplitude-division)



- <u>Young Interferometer</u>
  - The oldest interferometer





  - Small Slits → $1°$ δ-functions: $\delta[y-\frac{d}{2}]$; $\delta[y+\frac{d}{2}]$;

6.3 Wavefront-Division Interferometry

- The field at the plane x=0

$$E = E_1 + E_2 = E_0(\delta[y - \frac{d}{2}] + \delta[\mathrm{y} + \frac{d}{2}]) \qquad (6.8)$$

- Assume the observation plane is in the for zone => Fraunhoffer diffraction (Fourier)

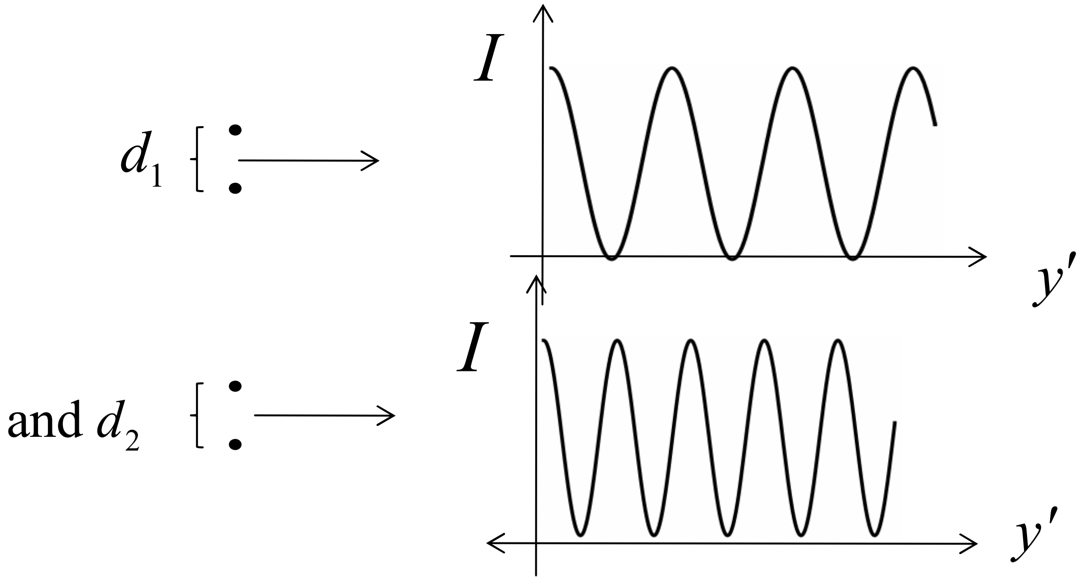
$$E[q_y] = \Im[E[y]] = E_0[e^{ig_y \frac{d}{2}} + e^{-ig_y \frac{d}{2}}] = E_0 \cdot 2\cos[q_y \frac{d}{2}] \qquad (6.9)$$

- Remember $q_y = \dfrac{y'}{\lambda z}$ (chapter 3)

- Fringes $\rightarrow \cos[\dfrac{d}{2\lambda z} \cdot y']$ (6.10)

6.3 Wavefront-Division Interferometry

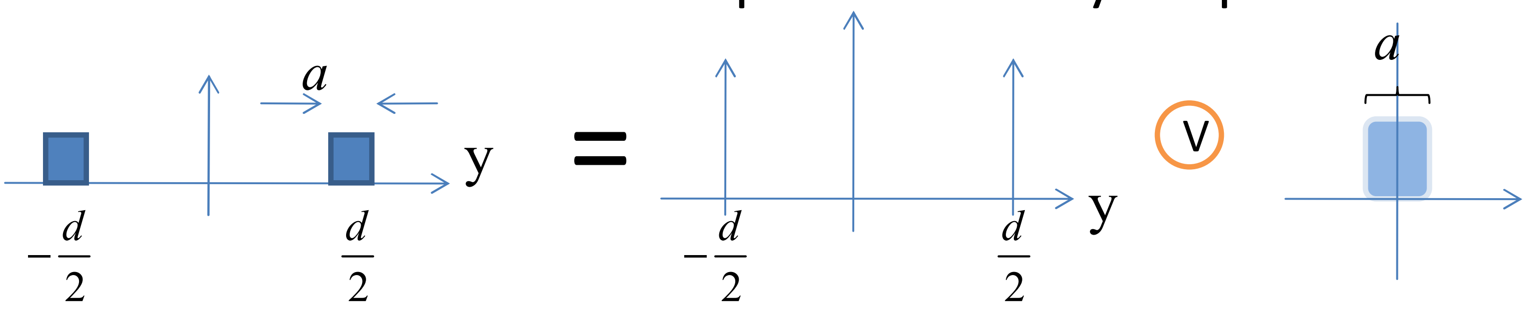
- Similarity relationship again:



- <u>Note:</u> using Fourier it is easy to generalize to arbitrary slit/particle shape; similar to scattering from ensemble of particles (chapter 5, pp 12) => use convolution to express the arbitrary shape.

6.3 Wavefront-Division Interferometry

- We can use convolution to express arbitrary shape:



- <u>Other Wavefront-Division Interferometers</u>

a) <u>Fresnel Mirrors</u>

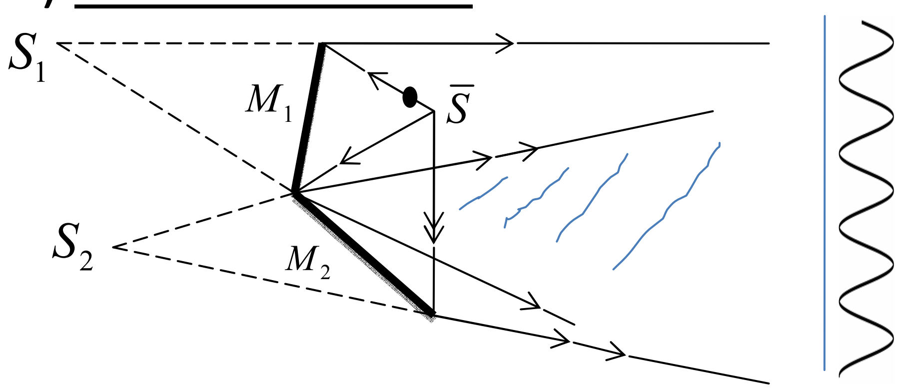
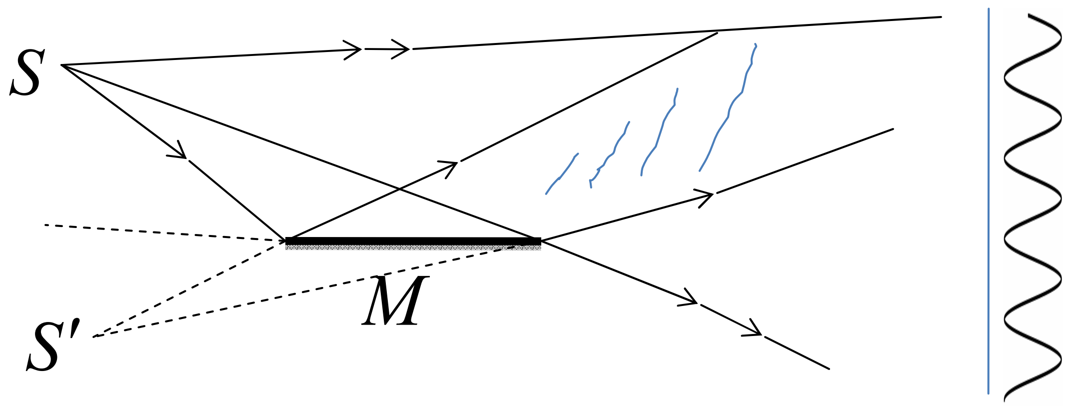


Image S thru M_1 and M_2
→virtual sources S_1 and S_2
→S_1 and S_2 act as Young's pinholes
→same equations
→ S1 and S2 are derived from the same sources and therefore coherent

6.3 Wavefront-Division Interferometry

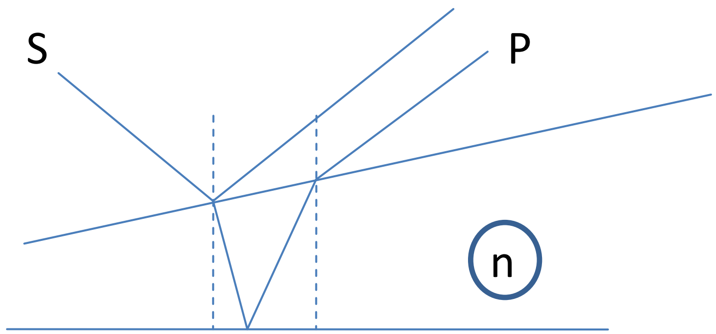
b) Lloyd's Mirror



- S and S' → Young

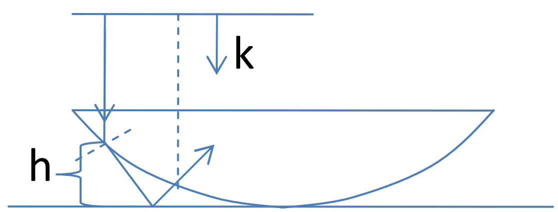
6.3 Wavefront-Division Interferometry

c) Thin films:

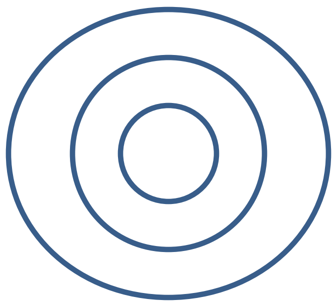


- Films of oil break the white light into colors due to interference and phase $= f(\lambda)$

d) Newton's rings:



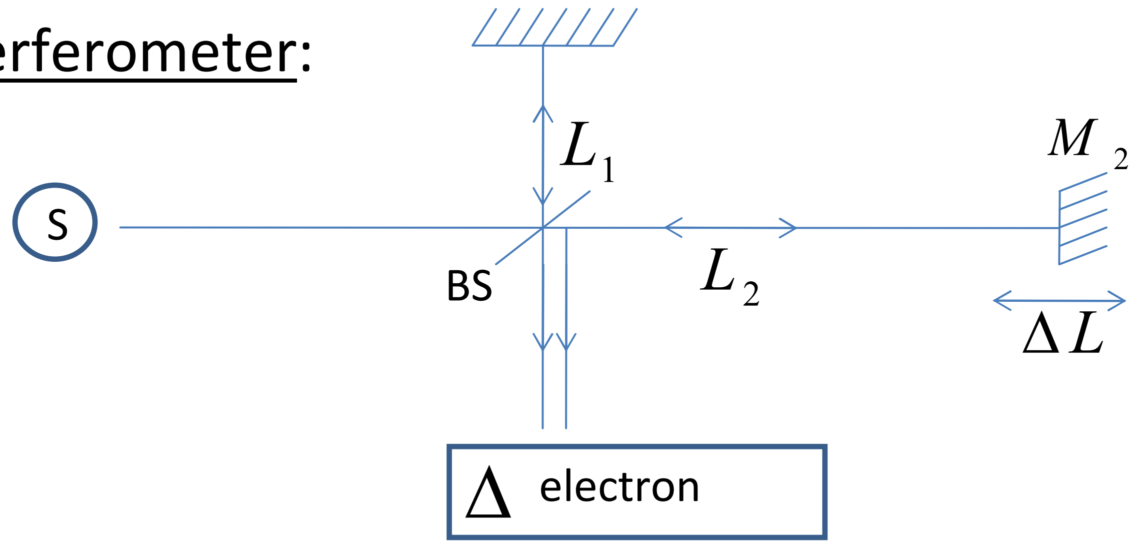
Circular fringes = rings



- Localized on the surface of lens

6.4 Amplitude-Division Interferometry

- Interference is obtained by replicating the wavefront=>less amplitude in each beam.
- The Michelson Interferometer:



- Very sensitive to path length differences between 'arms'
- Eg. It has been used to measure pressure in rarefied gases(place cell on one arm-> produce $\Delta\phi$

6.4 Amplitude-Division Interferometry

- BS- beam splitter
  - assume <u>thin</u> for now!
- Let L_1, L_2 be the lengths of the 2 areas

=> The intensity at the detector:

$$\mathrm{I}(\Delta \mathrm{L})=\mathrm{I}_1+\mathrm{I}_2+2\sqrt{I_1 I_2}\cos(2\pi\frac{\Delta L}{\lambda}) \qquad (6.11)$$

$$\text{Note: } \Delta \mathrm{L} = L_2 - L_1 = C(t_2 - t_1) = C\tau$$

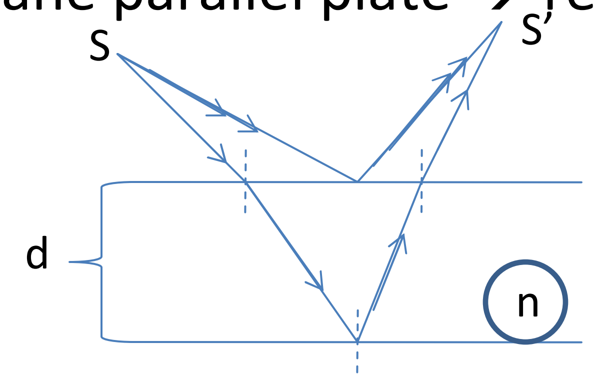
$$\tau = \text{time delay} \Rightarrow \cos(\omega\tau)$$

- We'll come back to Michelson with low-coherence light, temporal coherence, OCT, etc

6.4 Amplitude-Division Interferometry

- Other amplitude-division interferometry

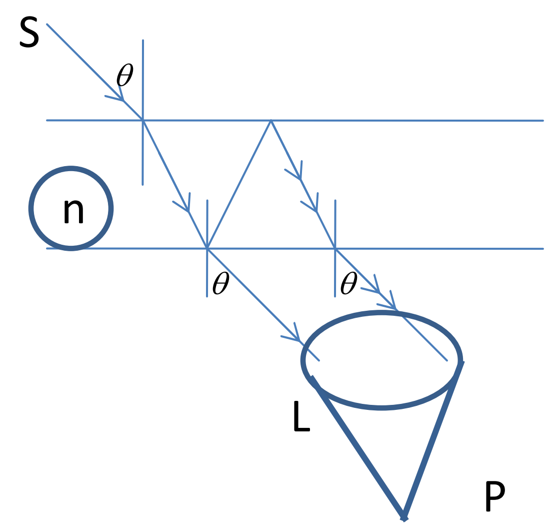
a) Plane parallel plate → reflection.



- point source-
- inter-fringe = $f(n,d) \rightarrow \text{metrology}$

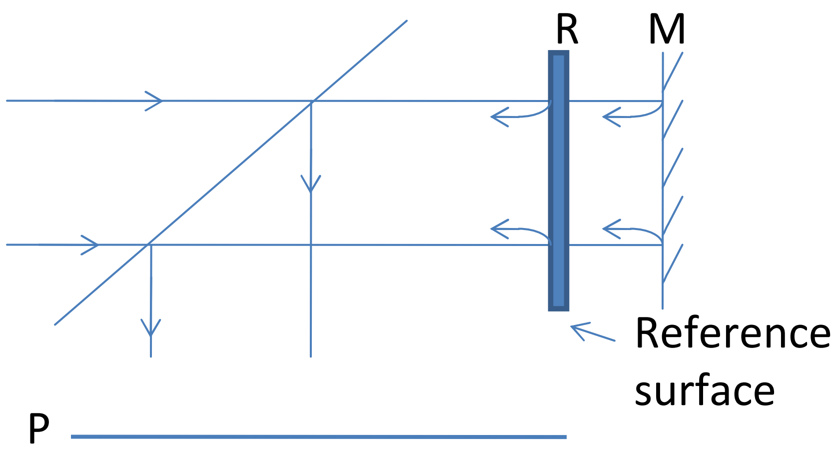
b) Plane parallel plate- transmission.

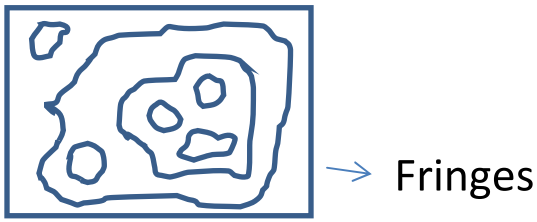
- Note: With plane wave incident, fringes are localized at infinity => Need lens



6.4 Amplitude-Division Interferometry

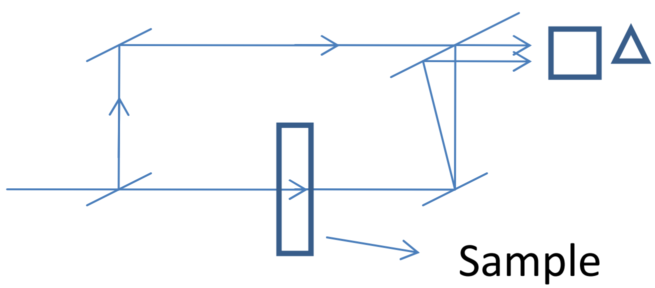
c) Fizeau Interferometer:





- Used to test mirrors and other surfaces.

d) Mach-Zender Interferometer

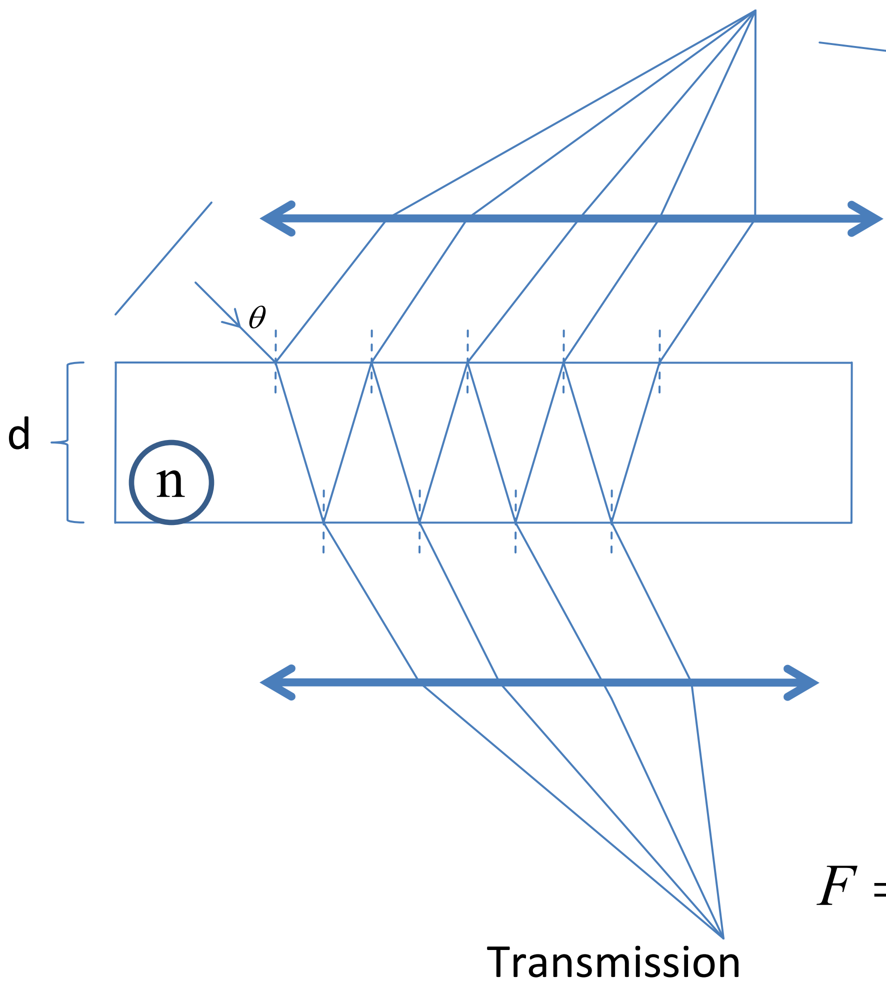


- Very Common
- Shear interferometry

→ Tilt one mirror

→ Fringes → analysis of surfaces

6.5 Multiple Beam Interference



Fabny-Penot interferometry

$$\frac{I^{(r)}}{I^{(i)}} = \frac{F\sin^2\frac{\delta}{2}}{1+F\sin^2\frac{\delta}{2}};$$

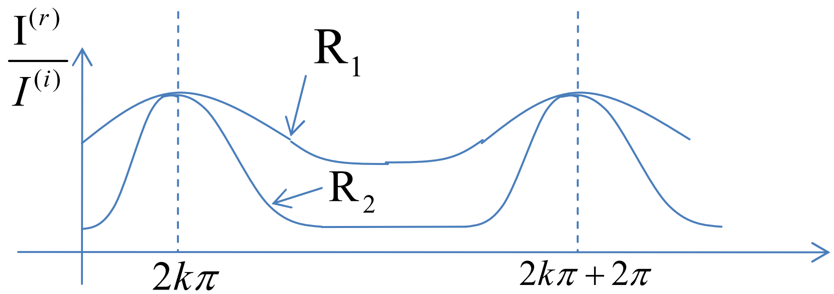
$$I^{(t)} = I^{(i)} - I^{(r)}$$

$$F = \frac{4R}{(1-R)^2} \Rightarrow \text{Finess coeff.} \qquad (6.12)$$

$$\delta = 2k_0 d\cdot n\cdot\cos\theta \Rightarrow \text{ one pass phase shift}$$

6.5 Multiple Beam Interference

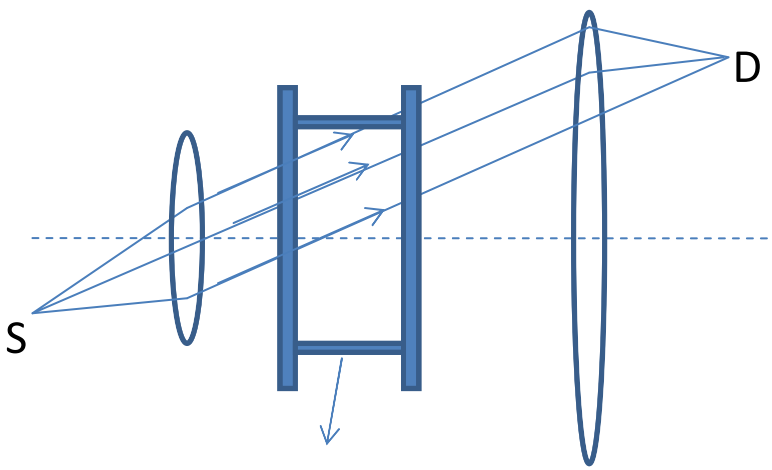
- Transmitted Intensity



- R_1 , R_2 → reflectivities
- R increases → narrower lines
- i.e. More reflection orders

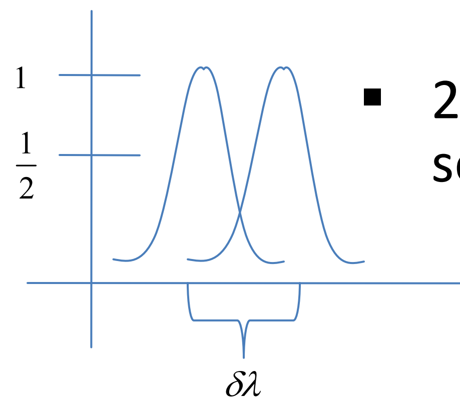
participate in interference

- In practice, Fabny-Perot gave accurate information about spectral lines(also called etalon)



Δ is fixed $\Rightarrow$ etalon

- Rayleigh Criterion:



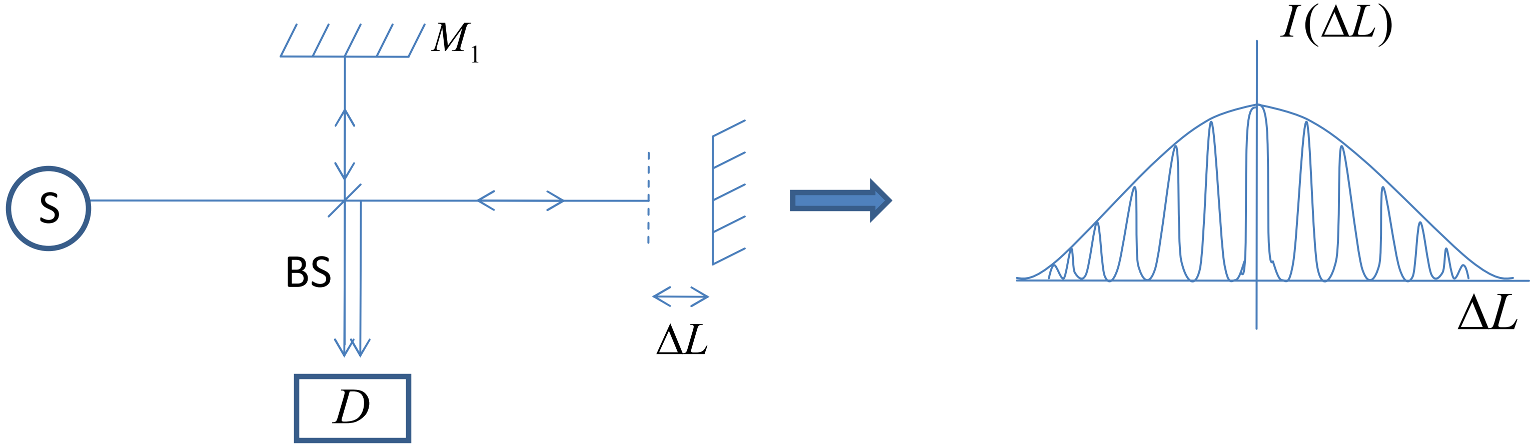
- 2 lines are separated if:

$$\delta\lambda \geq FWHM$$

6.6 Interference with Partially Coherent Light

- So far, we assumed monochromatic light = fully coherent.
- What happens when an arbitrary, broad-band, extended source is used for interference?

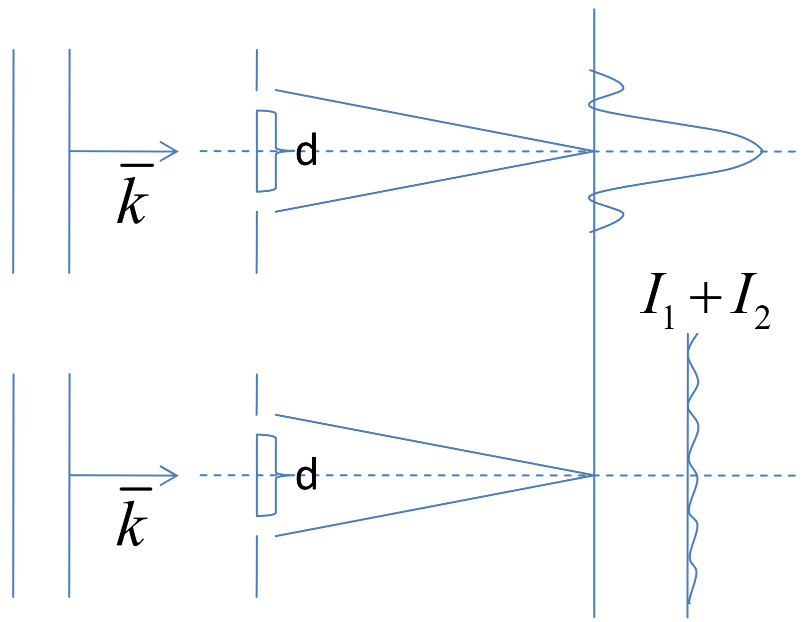
a) <u>michelson interferometry</u>

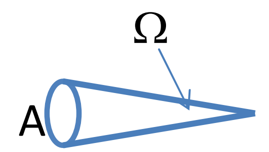


- The contrast of the fringe is decreasing
- i.e. limited temporal coherence.

6.6 Interference with Partially Coherent Light

b) Young Interferometer :





$$A_c \cong \frac{\bar{\lambda}^2}{\Omega}$$

- i.e. Limited Spatial Coherence

6.7 Temporal Coherence

- Coherence defines the degree of correlation between fields:
  - Typical correlation at one point in space (typical coherence)
  - Temporal correlation between fields at two points (spatial coherence)
- Given the field at one point $E(\bar{r},t)$, the mutual coherence function is:

$$\Gamma(\tau) = < E(t)\cdot E^*(t+\tau) >_t \qquad (6.13)$$

$$= \int_{-\infty}^{\infty} E(t)\cdot E^*(t+\tau)dt = \text{autocorrelation function}$$

6.7 Temporal Coherence

- Note: $\Gamma(0) = \left\langle \left| E(t) \right|^2 \right\rangle$ (6.14)

  $= \mathrm{I} \Rightarrow \text{ irradiance}$

- So $\Gamma = E \otimes E$
- Appl y again the correlation theorem (Eq 2.30)

$$\Im[\Gamma] = \tilde{E}(\omega)\tilde{E}^*(\omega) = \left|\tilde{E}(\omega)\right|^2 \quad (6.15)$$

$$= S(\omega) = \text{ Spectrum } \Rightarrow \text{ FTIR}$$

- So, the autocorrelation function $\Gamma(\tau)$ relates to the optical spectrum of the field:

$$\Gamma(\tau) = \int_0^\infty S(\omega) \cdot e^{-iw\tau} d\omega$$ Wiener-Kintchin theorem. (6.16)

6.7 Temporal Coherence

- ! Compare 6.16 with 5.28
- ! It applies even when E(t) does not have a Fourier Transform
- Typically, spectrum is centered on ω_0
- Assume spectrum is $S(\omega-\omega_0)$; apply shift theorem (Eq 2.31)

$$\Rightarrow \Gamma(\tau)=\left|\Gamma(\tau)\right|\cdot e^{i\omega_0\tau}, \qquad (6.17a)$$

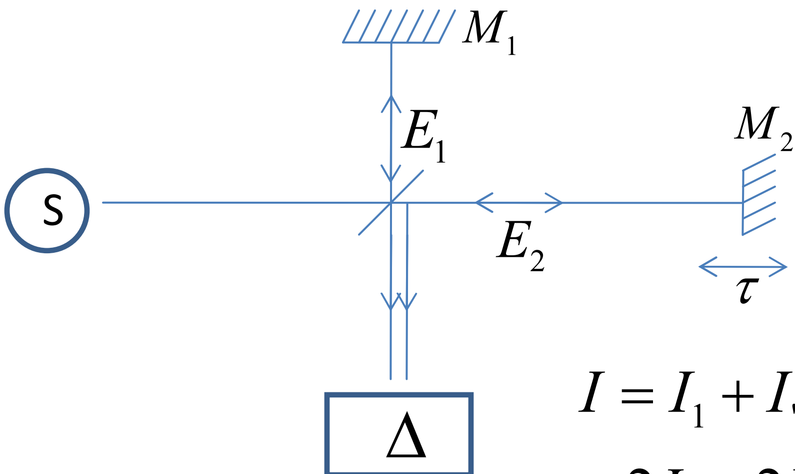
$$\text{where } \left|\Gamma(\tau)\right| = \int_{-\omega_0}^{\infty} S(u)\cdot e^{iu\tau}du;\ u=\omega-\omega_0 \qquad (6.17b)$$

- Complex degree of coherence

$$\gamma(\tau)=\frac{\Gamma(\tau)}{\Gamma(0)}\in\mathbb{C} \qquad (6.18) \qquad \Rightarrow \left|\gamma(\tau)\right|\in(0;1) \qquad (6.19)$$

6.7 Temporal Coherence

- Measuring the temporal coherence: Michelson interferometer



- $E_{total} = E_1 + E_2$
- Irradiance:

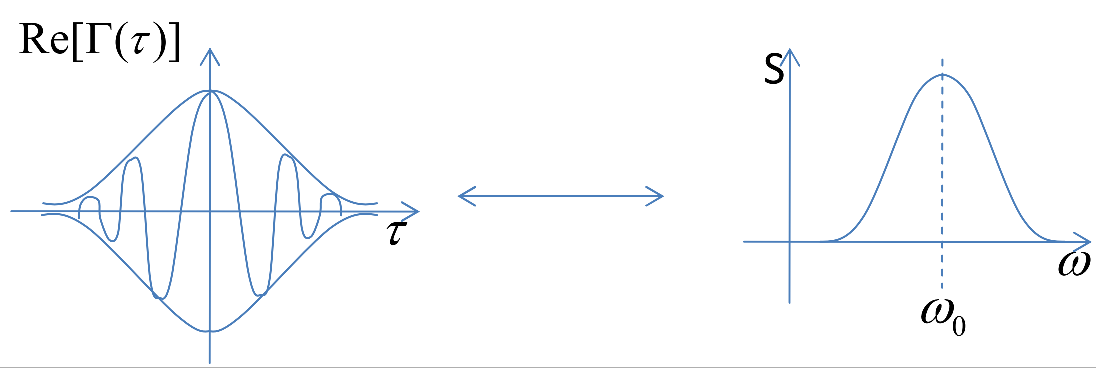
$$I = \langle |E_1(t) + E_2(t+\tau)|^2 \rangle \qquad (6.20)$$

- Assume: $|E_1| = |E_2|$

$$I = I_1 + I_{'2'} + \langle E_1(t)E_2^*(t+\tau)\rangle + \langle E_1^*(t)E_2(t+\tau)\rangle$$

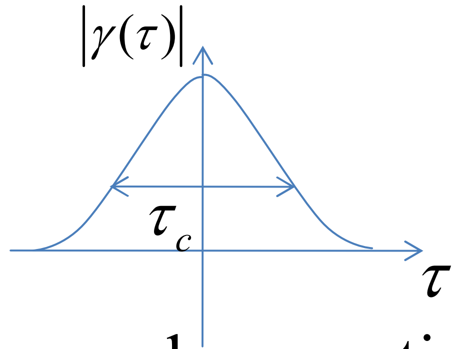
$$= 2I_1 + 2\,\mathrm{Re}[\Gamma(\tau)] \qquad (6.21)$$

- $I_{1,2}$ can be measured separately → access to $\mathrm{Re}(\Gamma)$ directly, by moving M_2



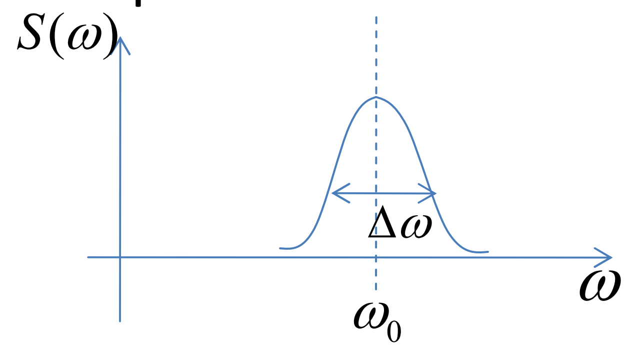
6.7 Temporal Coherence

- The degree of coherence



$$\tau_c = \text{coherence time}$$
$$\equiv \text{width of } |\gamma(\tau)|$$
$$\Gamma(\tau) = \Im[S(\omega)]$$
$$\Rightarrow \Delta\omega \cdot \tau_c = \text{constant} \qquad (6.22)$$

- Spectral width



- Coherence Length:

$$l_c = C \cdot \tau_c \qquad (6.23)$$

- Rule of thumb:

$$l_c = \frac{\lambda_0{}^2}{\Delta\lambda} \qquad (6.24)$$

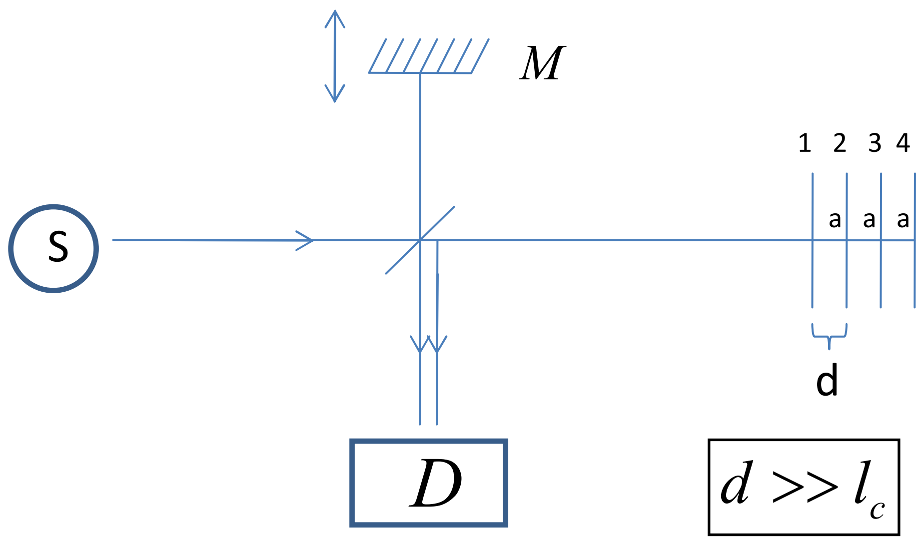
$$\lambda_0 = c \cdot T = c \cdot \frac{2\pi}{\omega_0}; \Delta\lambda = \Delta\left(\frac{2\pi c}{\omega}\right) = 2\pi c \frac{\Delta\omega}{\omega^2} = \lambda \frac{\Delta\omega}{\omega} \Rightarrow \frac{\Delta\lambda}{\lambda} = \frac{\Delta\omega}{\omega}$$

- Interference occurs only if $\Delta L = l_c$
- Broad spectrum$(\Delta\lambda = 100nm)$ => short $l_c (\cong 2-3\mu m)$

6.8 Optical Domain Refrectometry

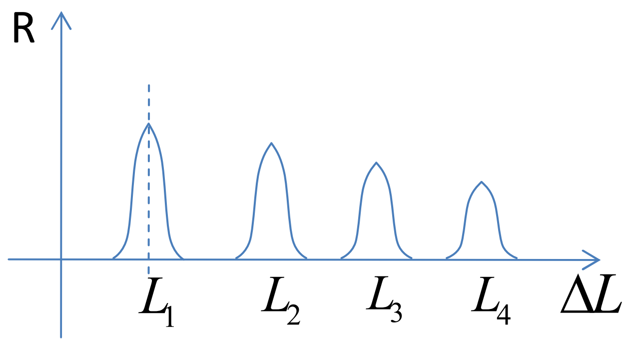
- Low-Coherence interferometry (inter. With broad band light).
- Sources: SLD, LED, white light, femtosecond laser, etc.

→ consider a transparent, layered structure under investigation.



6.8 Optical Domain Refrectometry

- Scanning M, we retrieve



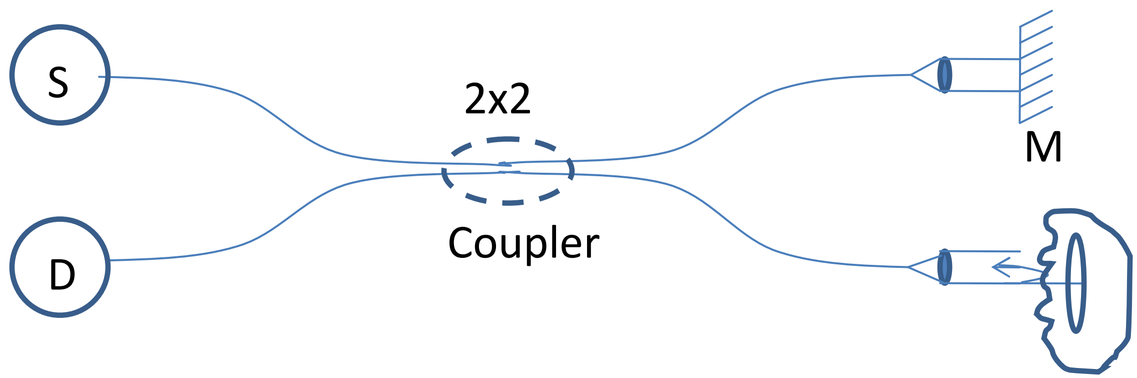
$\Rightarrow$ The interface are resolved
$\Rightarrow$ Reflectivity give info about refractive index
$\Rightarrow$ $L_{1,2,3,4}$ determine position of interfaces.

- ODR:
  - Successful for quantifying lasers in waveguides, fiber optics, etc.
  - 1987-HP- fiber optic reflectometer.

6.9 Optical Coherence Tomography(OCT)

- Optical technique capable of rendering 3D images from think biological samples.
- Penetrates 1-2 mm deep in tissue.

=> Typically implemented in optical fiber configuration.



1991, Fujimoto's group, MIT

Scan beam x-y
Scan M -> z

- Tissue = continuous superposition of interfaces.
- Scanning M, a depth-resolved reflectivity signal is retrieved

=> can resolve regions inside tissue(e.g. Tumors).

6.9 Optical Coherence Tomography(OCT)

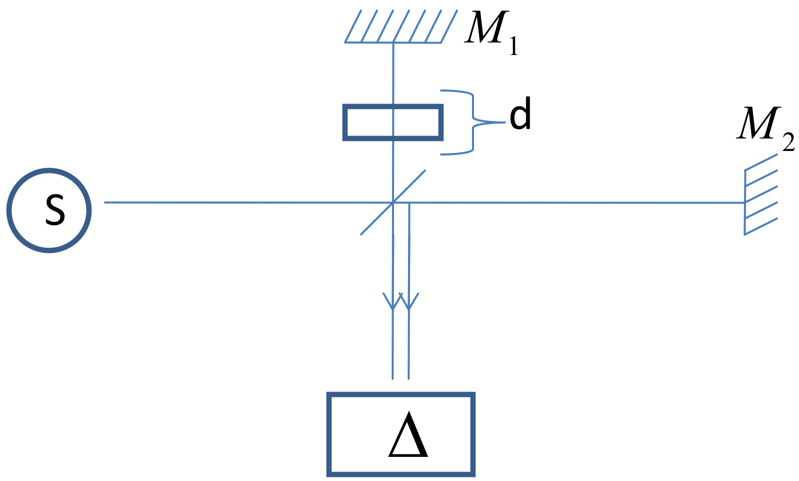
- If mirror is swept at constant speed v:
  $\Rightarrow$ z=vt
  $\Rightarrow$ Phase delay: $\phi = 2kz = 2kvt$ (2 means back and forth)
  $\Rightarrow$ Frequency shift: $\Delta\omega = \phi = 2kvt$ → Dopler Shift (6.25)
- The detector is recording a high-frequency signal→ Low-noise
- Dynamic range can easily reach 10 orders of magnitude! i.e. can record reflectivities from 1 to 1/10 billion!(100 dB)
- <u>Various Technological Improvements</u>:
  - Spectral domain OCT: instead of scanning M, measure $S(\omega) \xrightarrow{\Im} \Gamma(\tau)$
  - Galvo-scanning group delay- fast

6.9 Optical Coherence Tomography(OCT)

- Various Technological Improvements:
  - Spectral encoding- instead of scanning on x, illuminate with $\lambda(x)$
  - Spectroscopic OCT- trade z-resolution for $S(\omega)$ information
- Since 1991, ~1,000 OCT papers published.
- Currently applied in: Oftalmology, Dermathology, cardio, etc
- Recently combined with SHG, molecular imaging

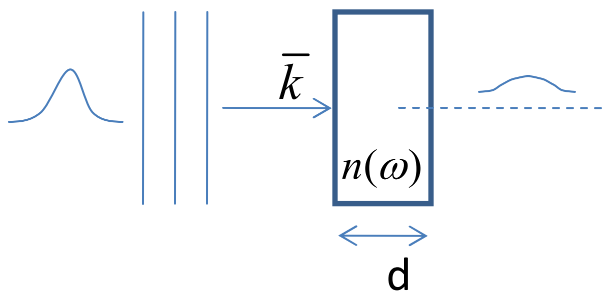
6.10 Dispersion Effects on Temporal Coherence

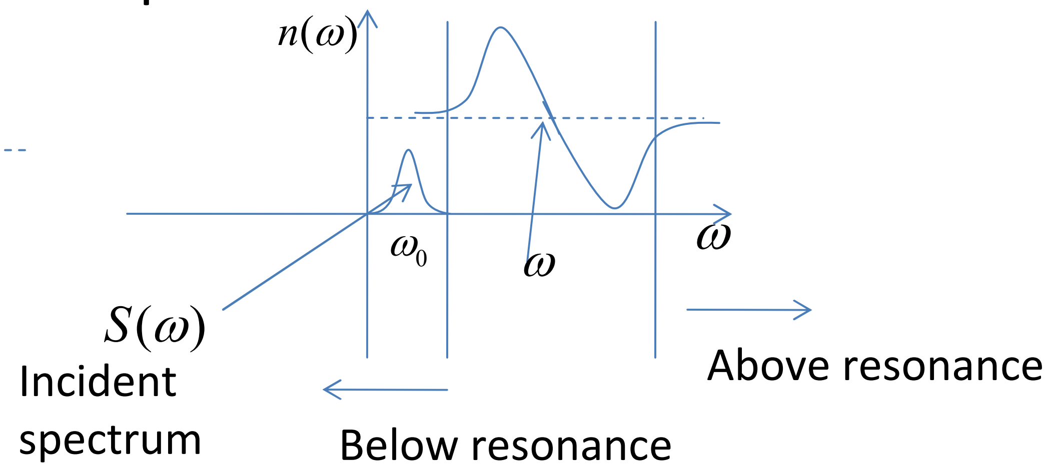
- What happens if on one area of the Michelson interferometer, there is extra material (eg. Glass)?



6.10 Dispersion Effects on Temporal Coherence

- How does broad band fields propagate through dispersive materials (such as glass)? Think pulses!



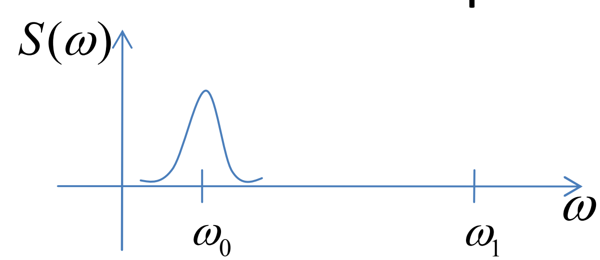


- The phase delay through a transparent material:

$$\phi(\omega) = k(\omega)\cdot d$$

$$= k_0 \cdot d \cdot n(\omega) \qquad (6.26)$$

- Taylor expansion of $n(\omega)$ around the central frequency:



6.10 Dispersion Effects on Temporal Coherence

$$n(\omega) = n(\omega_0) + \left.\frac{dn}{d\omega}\right|_{\omega_0} (\omega - \omega_0) + \frac{1}{2}\frac{\partial^2 n}{\partial \omega^2}(\omega - \omega_0)^2 + ...$$

- Note:

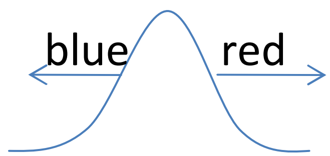
$n(\omega_0) \Rightarrow \phi_0 = n(\omega_0) \cdot k_0 \cdot d =$ constant → not important

$$\phi(\omega) = \phi_0 + \frac{dk}{d\omega} \cdot d \cdot (\omega - \omega_0) + \frac{1}{2} \cdot \frac{d^2 k}{d\omega^2} \cdot d \cdot (\omega - \omega_0)^2 ....$$

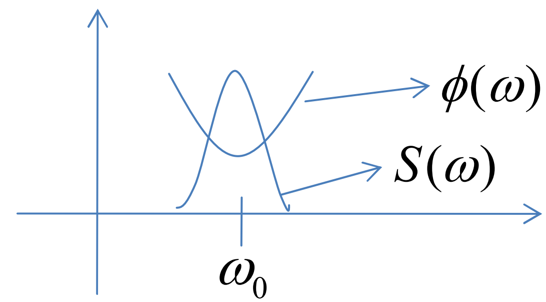
- Definitions:
  - $\frac{dk}{d\omega}$ = v = group velocity (6.27)
  - $\frac{d^2 k}{d\omega^2}$ = β_2= group velocity dispersion (GVD)
- Different colors have different group velocities

6.10 Dispersion Effects on Temporal Coherence

- E.g. Pulse:



$$v_{blue} < v_{red}$$



- Riding with pulse (v=0) → parabolic phase:
- So $$\phi(\omega) \simeq \frac{1}{2}\beta_2 \cdot \omega^2 \qquad (6.28)$$
- Cross-Spectral density:

$$W(\omega) = \left\langle E_1(\omega) \cdot E_2{}^*(\omega) \right\rangle \qquad (6.29)$$

- Then, the cross-correlation function is

$$\Gamma_{12}(\tau) = \int_0^\infty W(\omega) e^{-i\omega\tau} d\omega \qquad (6.30)$$

6.10 Dispersion Effects on Temporal Coherence

- $\Gamma_{12}(\tau) = \int_0^\infty W(\omega) e^{-i\omega\tau} d\omega$ is the generalization of (6.31) i.e generalized Wiener-Kinntilin Theorem.

- For the "unbalanced" Michealson, the cross-spectral density is

$$W(\omega) = E_1(\omega) \cdot E_2^*(\omega) = |E_1||E_2^*| e^{i\frac{1}{2}\beta_2\omega^2} = S(\omega) e^{i\frac{1}{2}\beta_2\omega^2} \qquad (6.32)$$

- The cross-correlation function:

$$\Gamma_{12}(\tau) = \Im[W(\omega)] = \Im[S(\omega) e^{-i\frac{1}{2}\beta_2\omega^2}] \qquad (6.33)$$

- Remember convolution theorem:

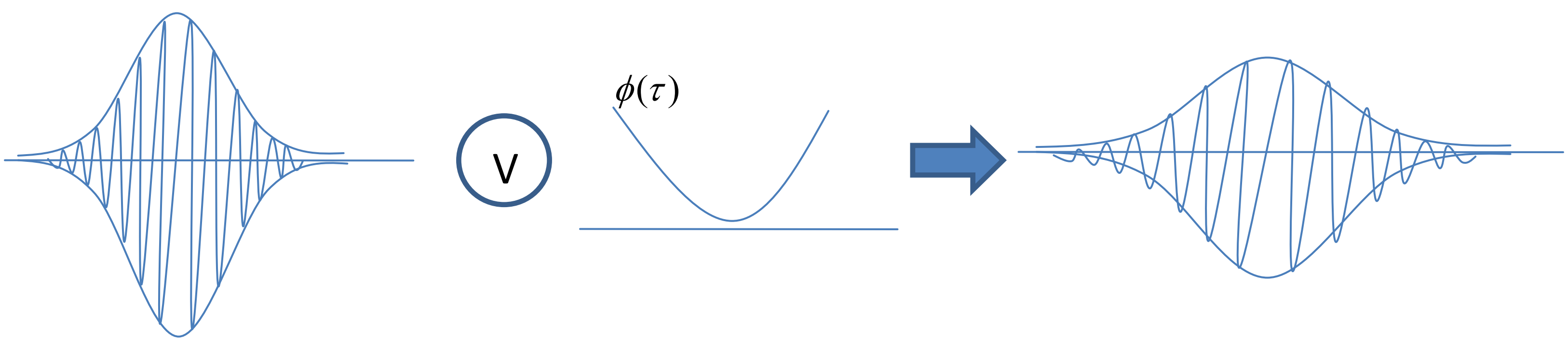
$$\Gamma(\tau) = \Im[S(\omega)] \circledvee \Im[e^{-i\frac{1}{2}\beta\omega^2}] = \Gamma_0(\tau) \circledvee h(\tau) \qquad (6.34)$$

6.10 Dispersion Effects on Temporal Coherence

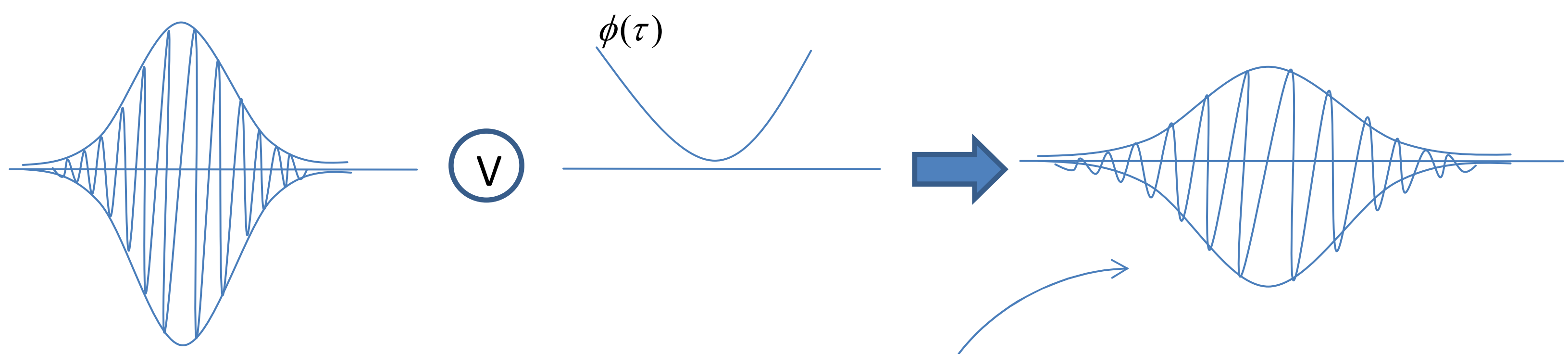
- $$h(\tau) = \Im[e^{i\frac{1}{2}\beta\omega^2}]$$
- Useful Fourier Transform relationship for Gauss functions:

$$e^{-b\omega^2} \xrightarrow{\mathcal{F}} \frac{1}{\sqrt{2b}} e^{-\frac{t^2}{4b}} \qquad (6.35)$$

$$h(\tau) = \frac{e^{i\frac{\tau^2}{2\beta}}}{\sqrt{i\beta}} \quad \phi(\tau) \sim \tau^2 \text{ also parabolic}$$



6.10 Dispersion Effects on Temporal Coherence



- <u>The coherence time is increased</u>
- Frequency is "chirped"

- So, in OCT, is important to balance the interferometer=> minimum coherence length
- l_c gives the depth resolution