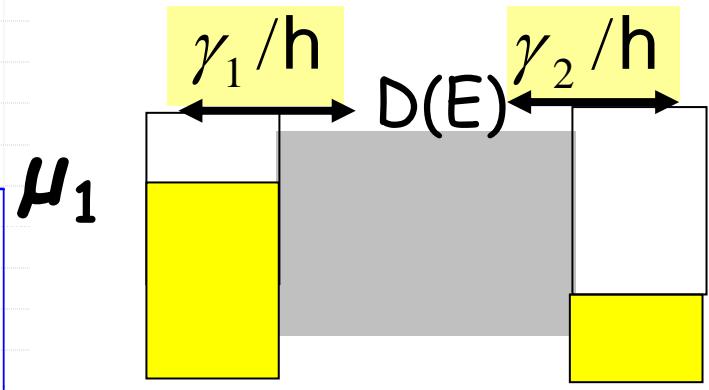
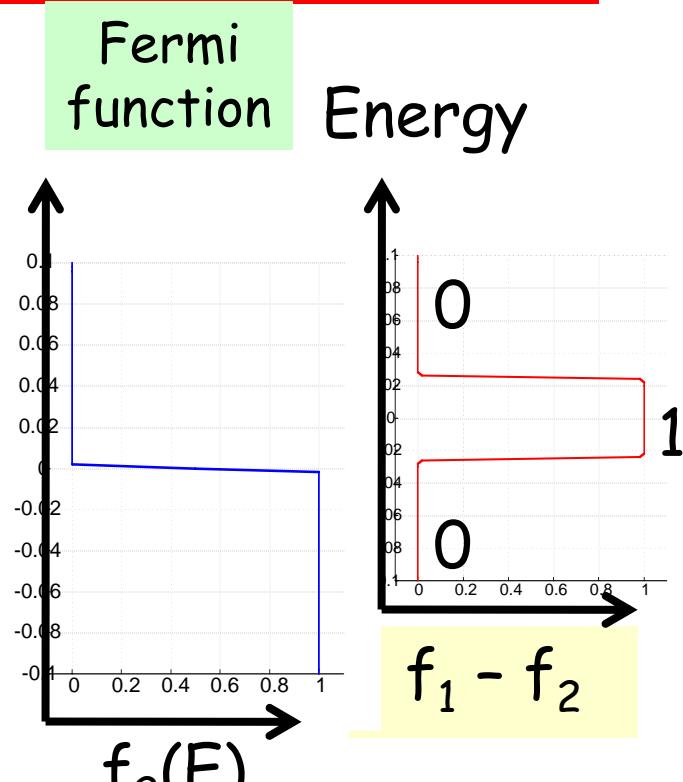
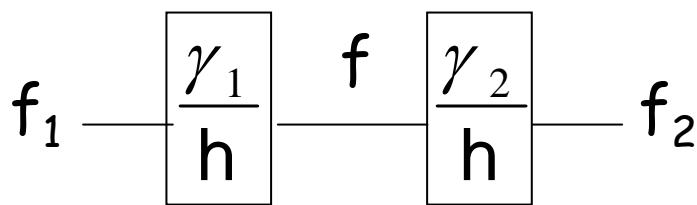
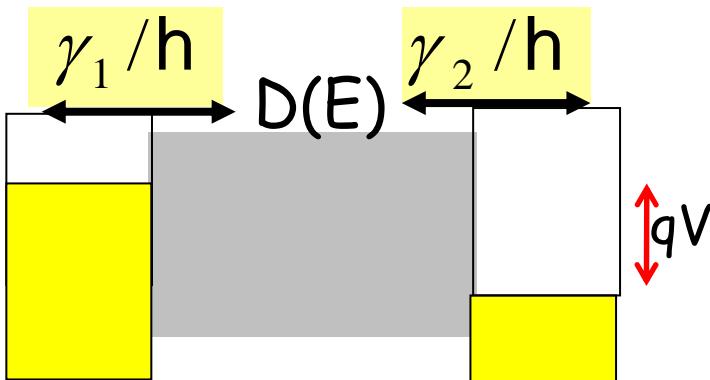


## \*\* Quantitative model


 $f_1(E)$ 

 $f_1 - f_2$ 


# Quantitative model



$$f_1 \xrightarrow{\frac{\gamma_1}{h}} f \xrightarrow{\frac{\gamma_2}{h}} f_2$$

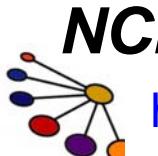
$$f(E) = \left[ \frac{\gamma_1 f_1 + \gamma_2 f_2}{\gamma_1 + \gamma_2} \right]$$

$$I(E) \sim \frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2} [f_1 - f_2]$$

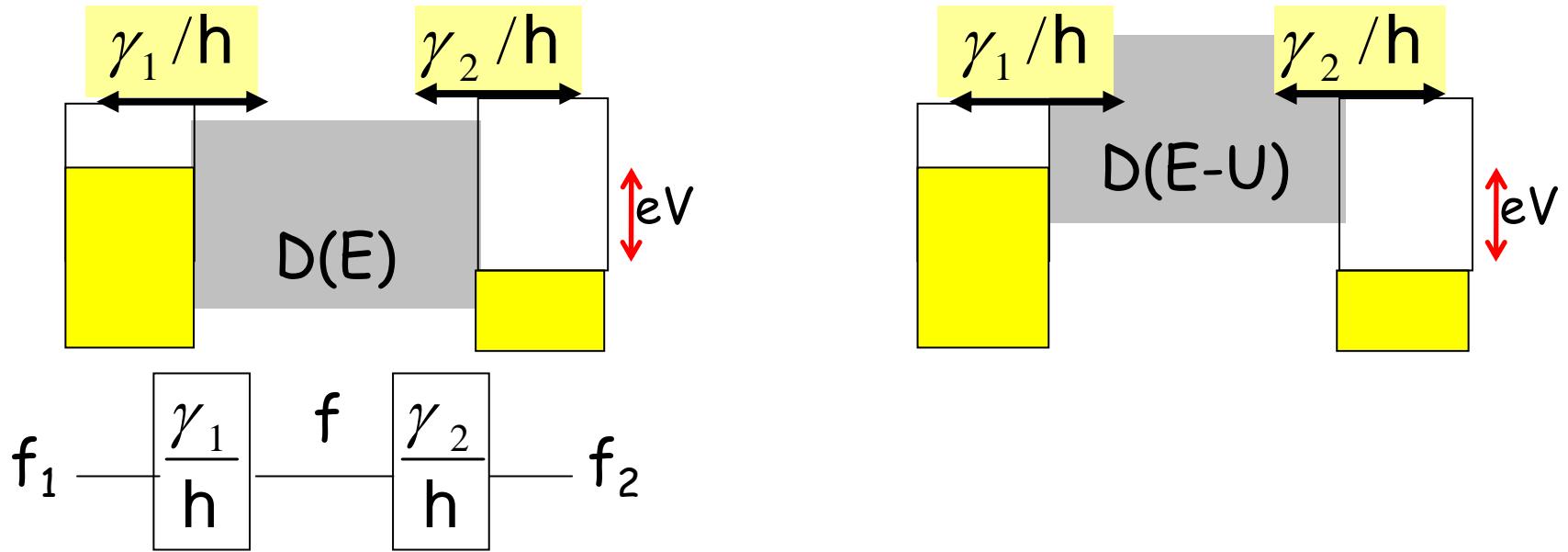
$$f(E) \equiv \frac{n(E)}{D(E)} = \left[ \frac{\gamma_1 f_1 + \gamma_2 f_2}{\gamma_1 + \gamma_2} \right]$$

$$I(E) = \frac{q}{h} D(E) \frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2} [f_1 - f_2]$$

$$\rightarrow \frac{q\gamma}{2h} D [f_1 - f_2]$$



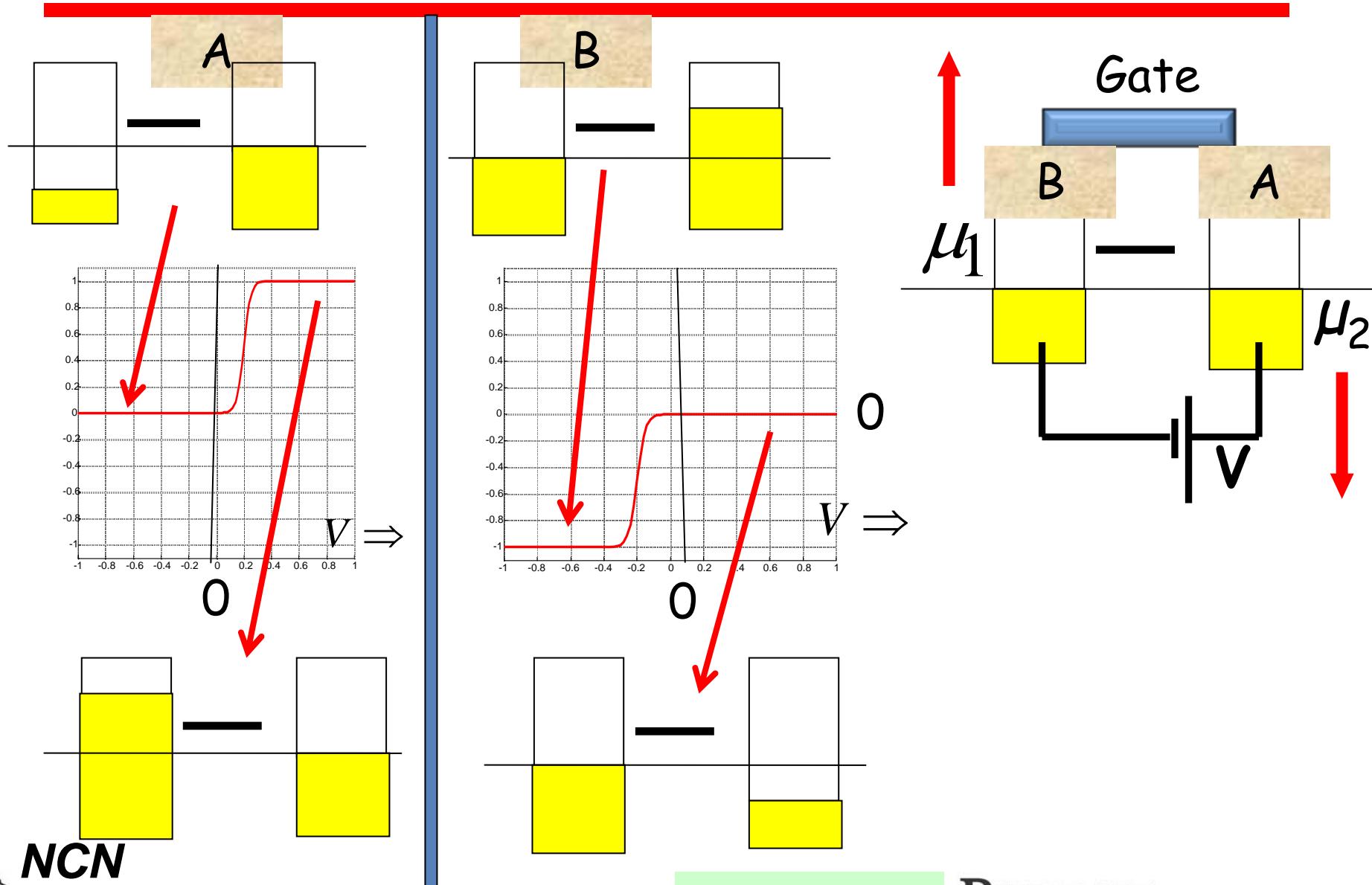
# Quantitative model: Introducing U



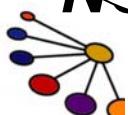
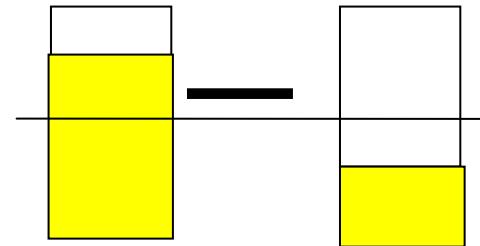
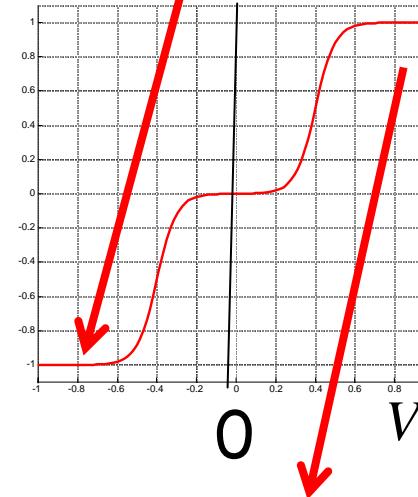
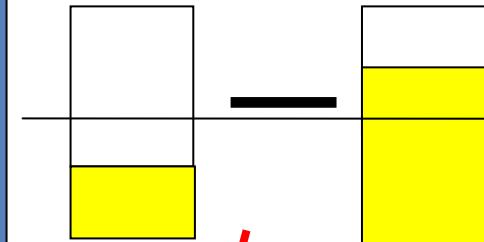
$$n(E) = D(E) \left[ \frac{\gamma_1 f_1 + \gamma_2 f_2}{\gamma_1 + \gamma_2} \right] \rightarrow D(E - U) \left[ \frac{\gamma_1 f_1 + \gamma_2 f_2}{\gamma_1 + \gamma_2} \right]$$

$$I(E) = \frac{q}{h} D(E) \frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2} [f_1 - f_2] \rightarrow \frac{q}{h} D(E - U) \frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2} [f_1 - f_2]$$

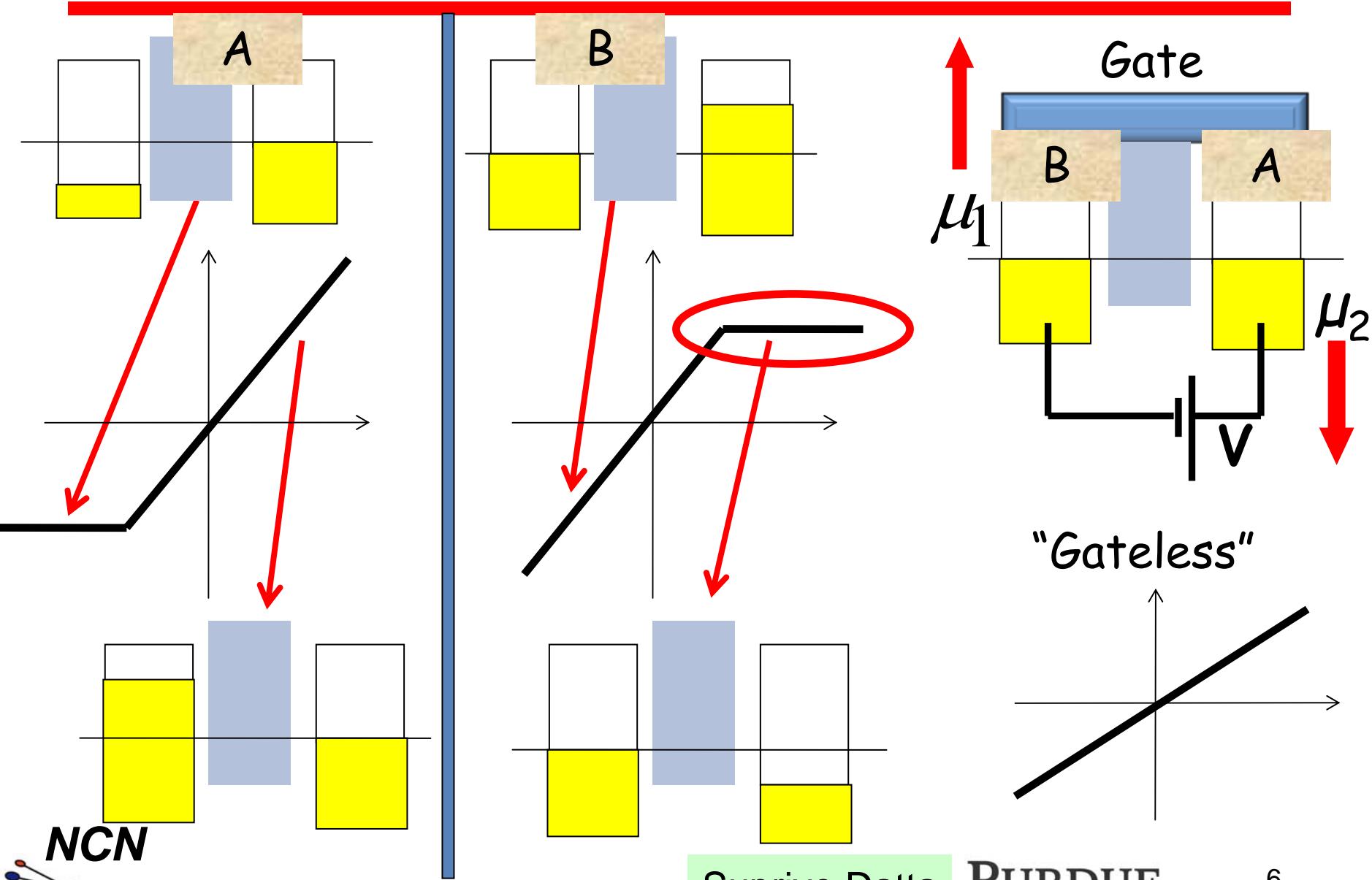
# Current through ONE level



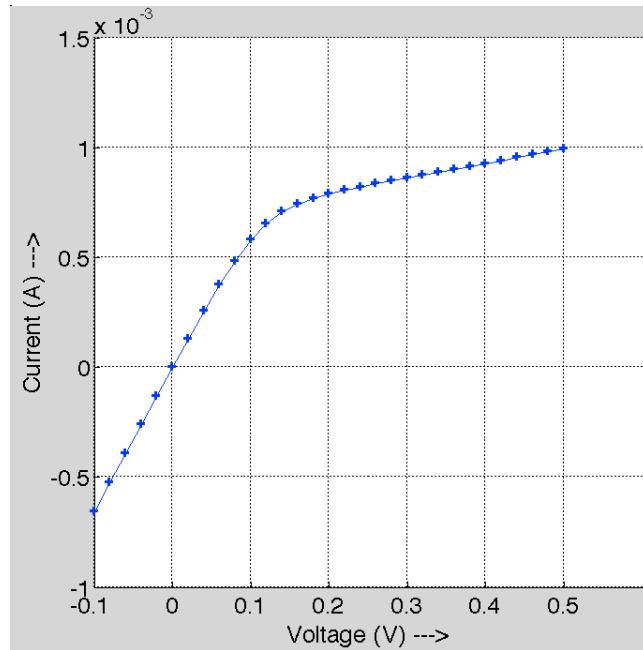
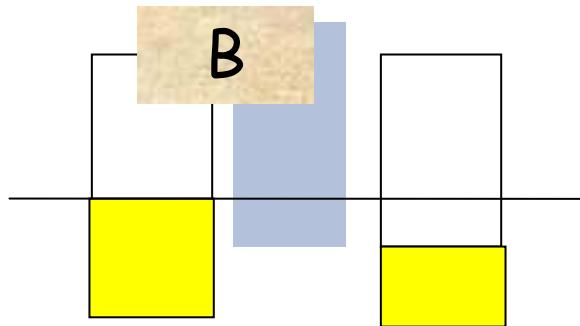
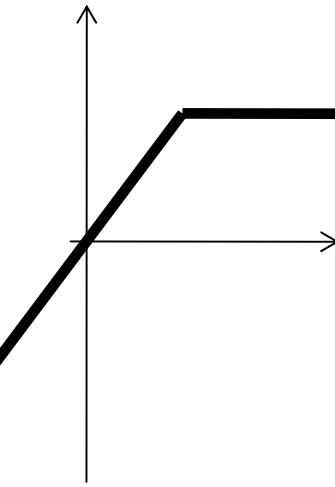
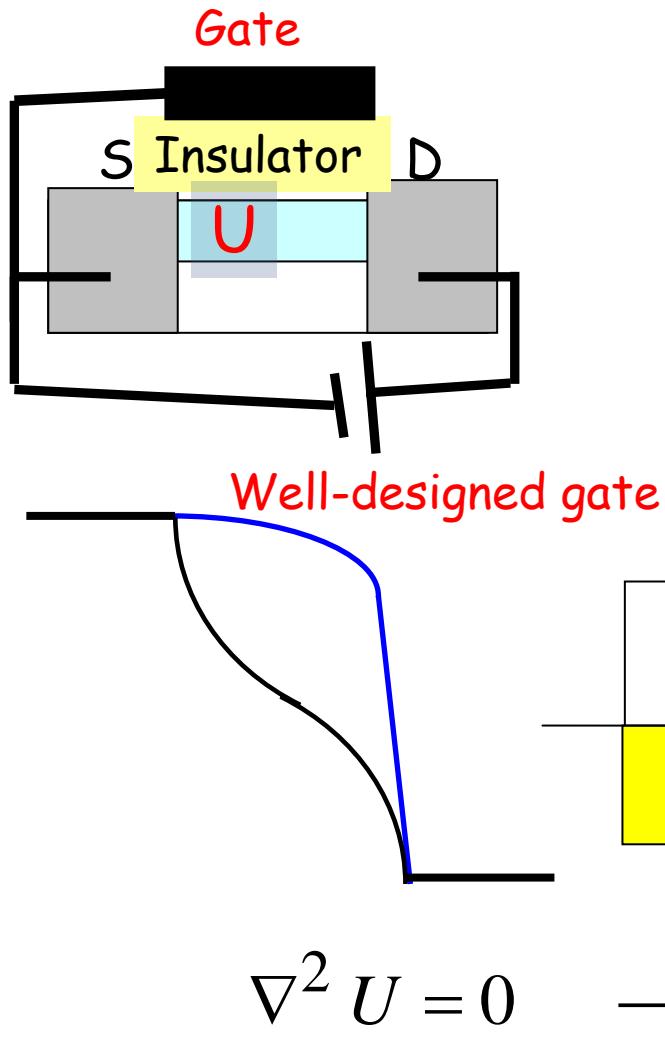
# "Gateless" conduction through ONE level



# Current through a 2-D conductor



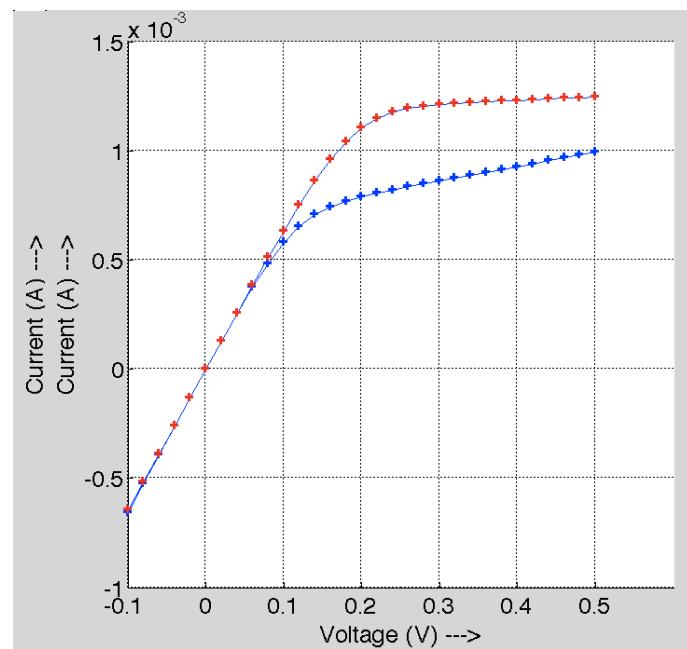
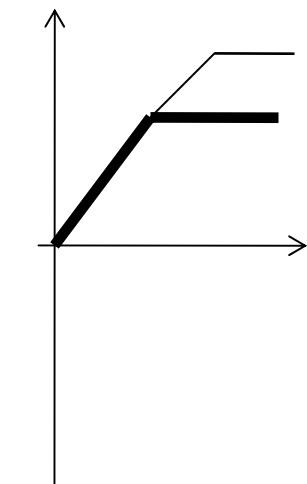
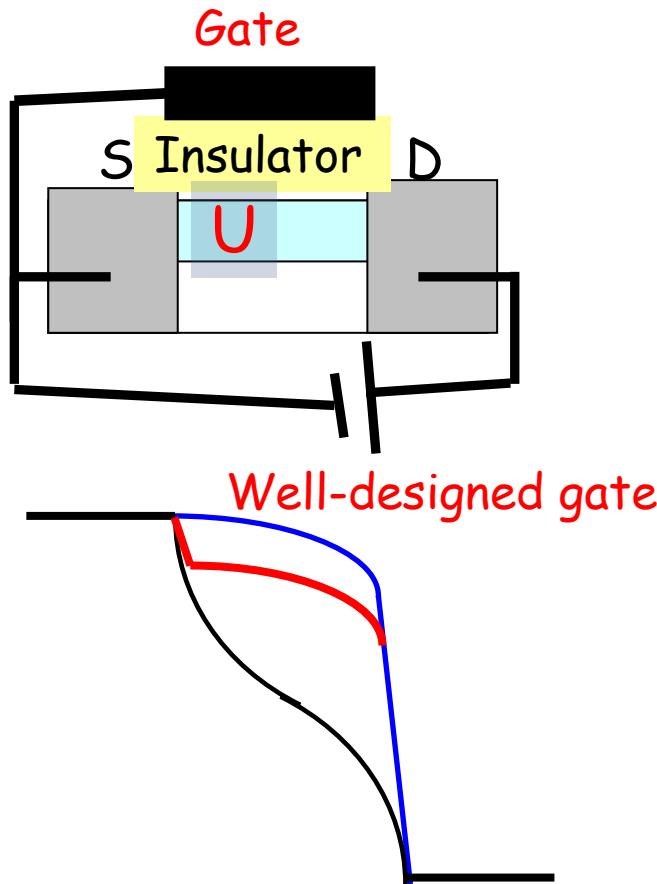
# Electrostatics of nanotransistors



$$n(E) = D(E - U) \left[ \frac{\gamma_1 f_1 + \gamma_2 f_2}{\gamma_1 + \gamma_2} \right]$$

$$I = \frac{q}{\hbar} D(E - U) \frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2} [f_1 - f_2]$$

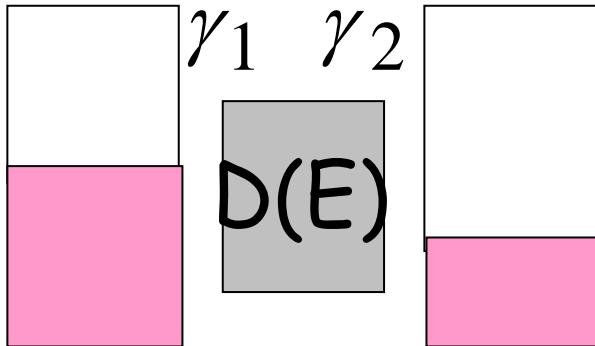
# Electrostatics of nanotransistors



Semi-depleted  
of electrons

$$\nabla^2 U \sim n \rightarrow U = U_L + U_0(n - n_0)$$

# Self-consistent field (scf) method

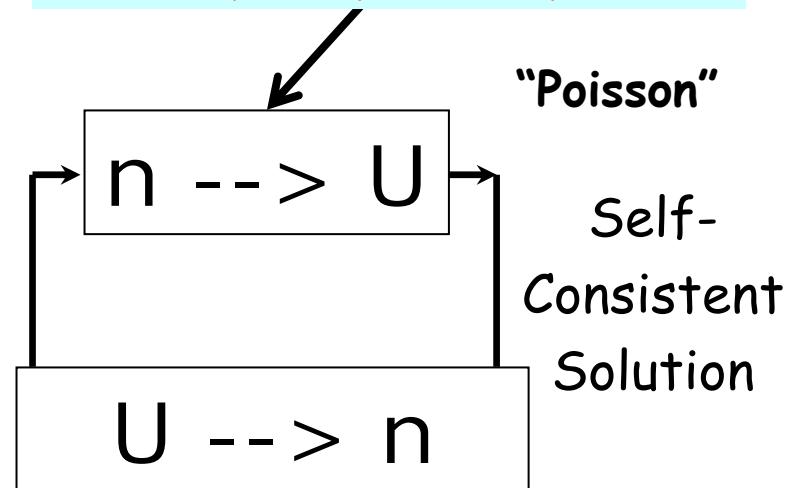


$$U = U_L + U_0(n - n_0)$$

$$n = D(E - U) \left[ \frac{\gamma_1 f_1 + \gamma_2 f_2}{\gamma_1 + \gamma_2} \right]$$

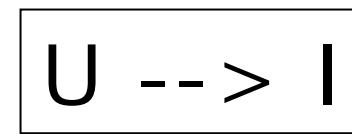
$$I = \frac{q}{\hbar} D(E - U) \frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2} [f_1 - f_2]$$

Simplified treatment  
of a very complicated problem



"Poisson"

Self-  
Consistent  
Solution



"Schrodinger"

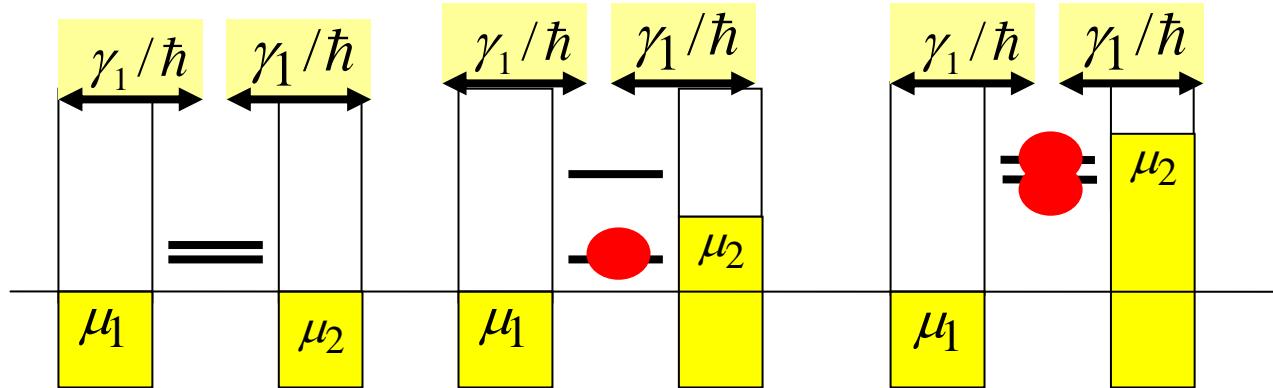
Nanowires /  
Nanotubes / Molecules

## \*\* Single-electron charging

$U_0$  : Increase in potential due to SINGLE electron

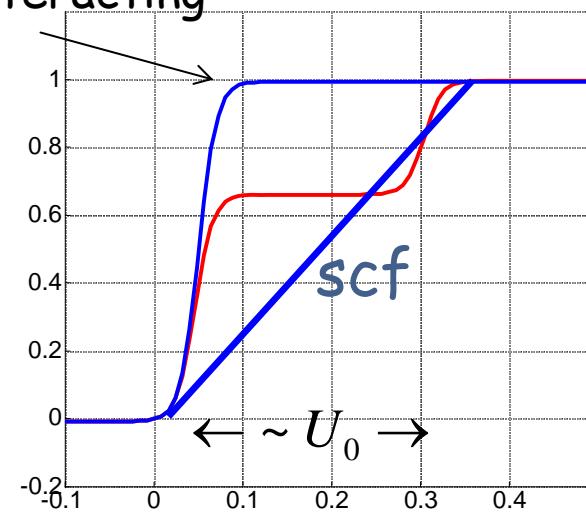
$$\gg \gamma, kT$$

"Self-interaction Correction"



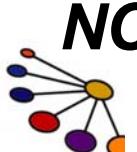
Non-interacting

Normalized Current



$$\frac{q\gamma_1}{2\hbar} * 2$$

$V \Rightarrow$



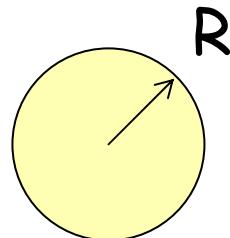
<http://www.nanohub.org/courses/cqt>

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# Single-electron charging energy

$$U_0 \sim \frac{q^2}{4\pi\epsilon_0 R}$$

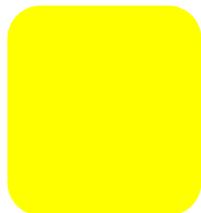


$\rightarrow 1.6 \text{ eV} \text{ for } R = 1 \text{ nm}$

Vacuum  
Level

13.6eV

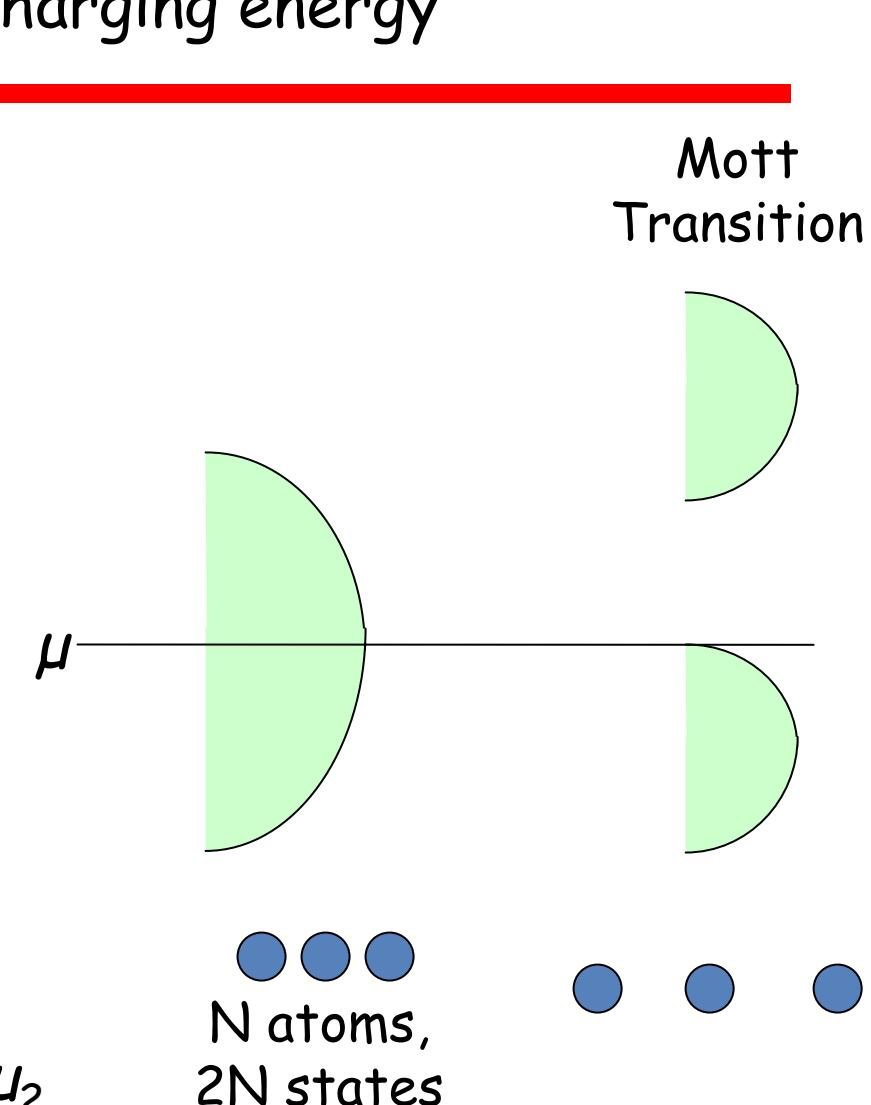
$\mu_1$



$\mu_2$



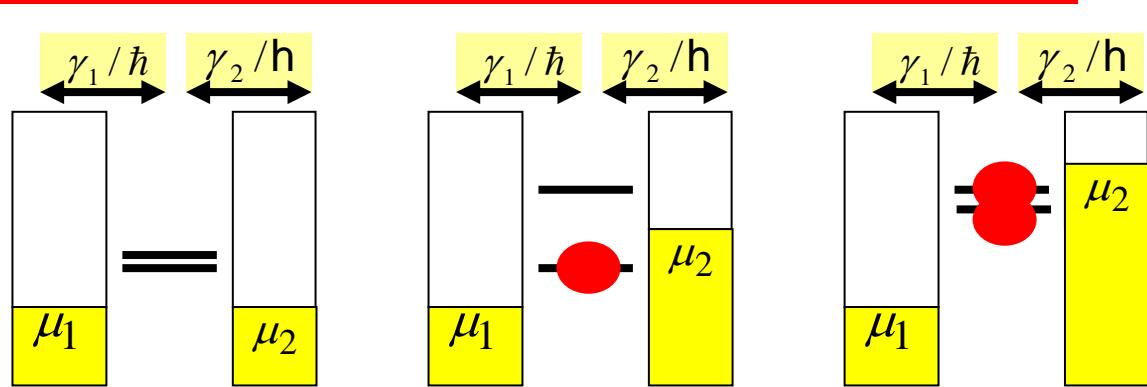
$\mu$



## 2 levels: SCF versus Exact

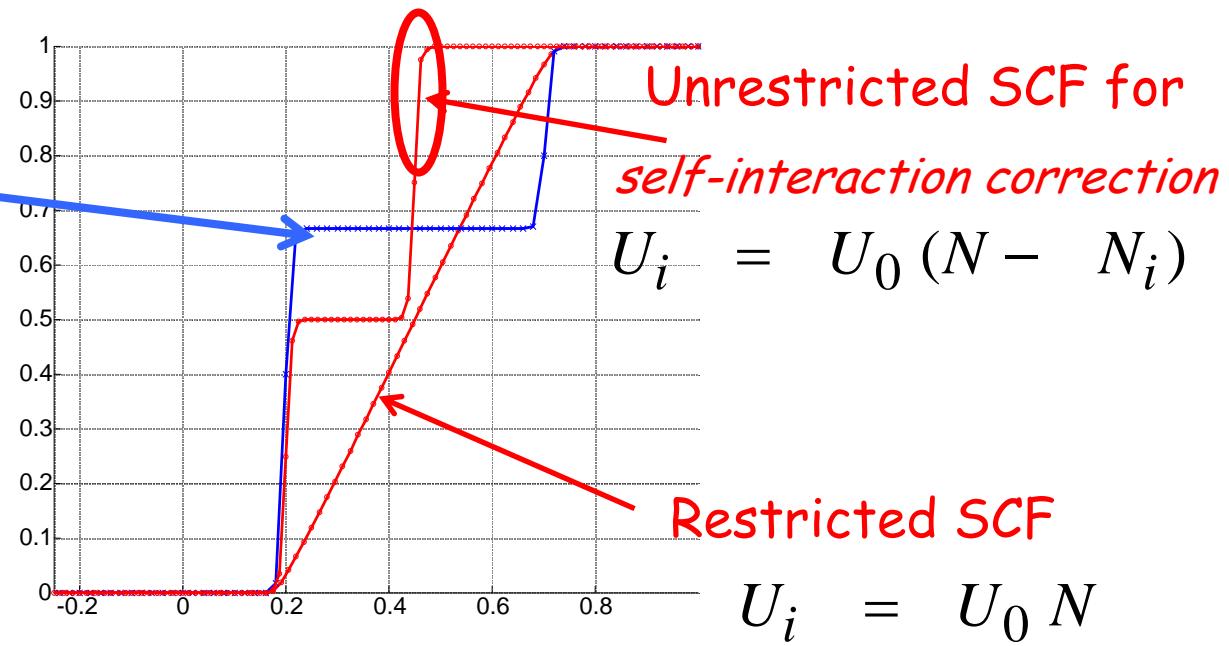
SCF works fine  
as long as

$U_0$ : Single electron  
charging energy  
 $\leq \gamma, kT$

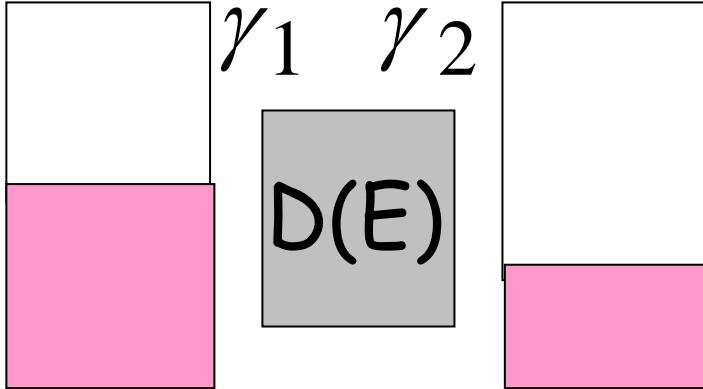


Exact  
Needs picture  
in "Fock" space:

Lectures 5



# Summary: Self-consistent field method

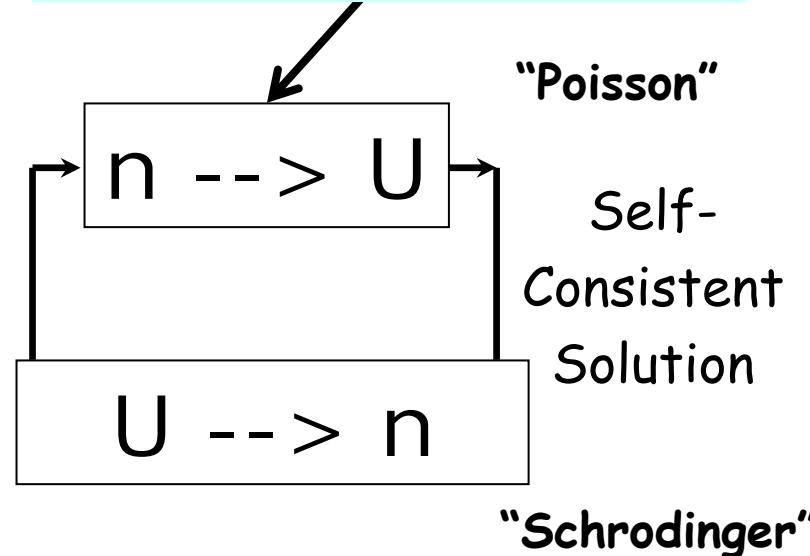


$$U = U_L + U_0(n - n_0)$$

$$n = D(E - U) \left[ \frac{\gamma_1 f_1 + \gamma_2 f_2}{\gamma_1 + \gamma_2} \right]$$

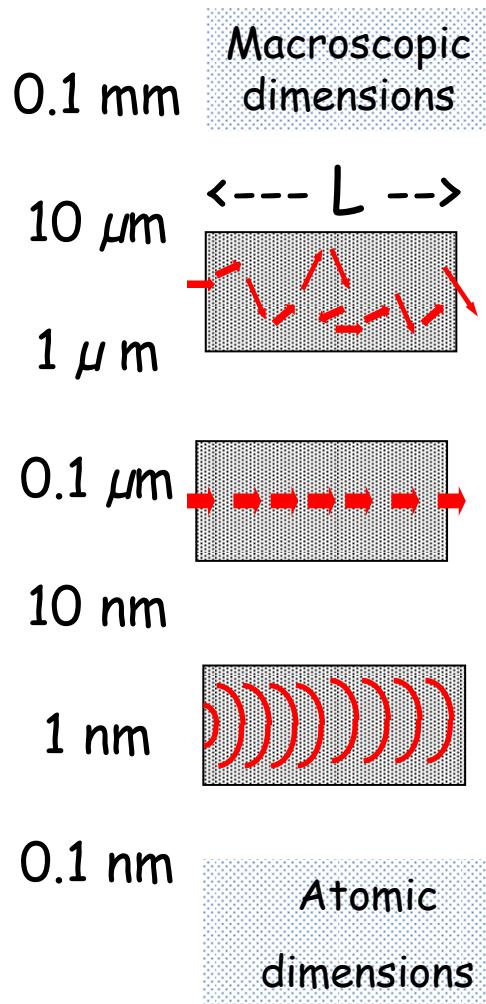
$$I = \frac{q}{\hbar} D(E - U) \frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2} [f_1 - f_2]$$

Simplified treatment  
of a very complicated problem

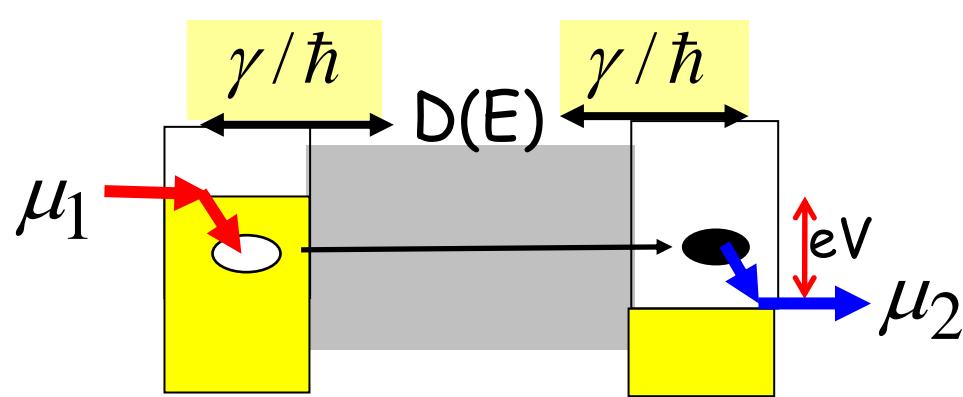


Nanowires /  
Nanotubes / Molecules

## \*\* Where is the heat ( $I^2R$ ) ?



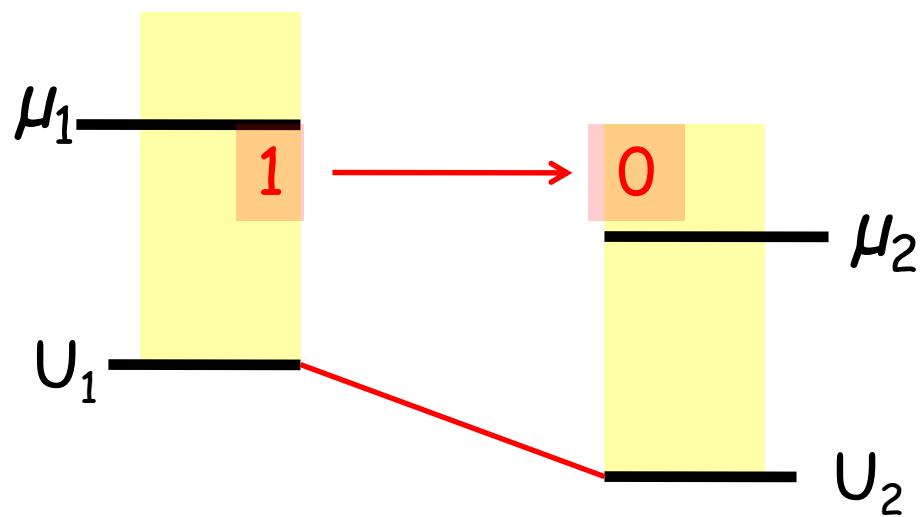
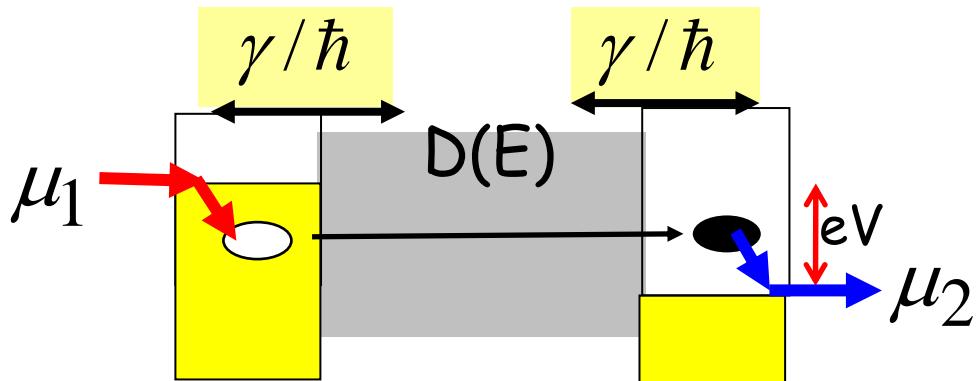
$$\frac{V}{I} = \frac{25.8 \text{ K}\Omega}{2} \frac{1}{M}$$



$$P = VI$$

$$= eV * \frac{N}{t}$$

# Where is the voltage ?

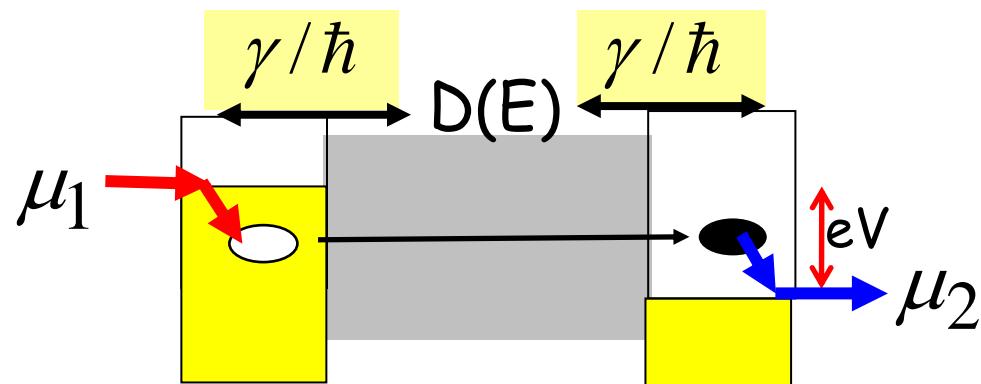


"Voltage"

$U$  or  $\mu$ ?

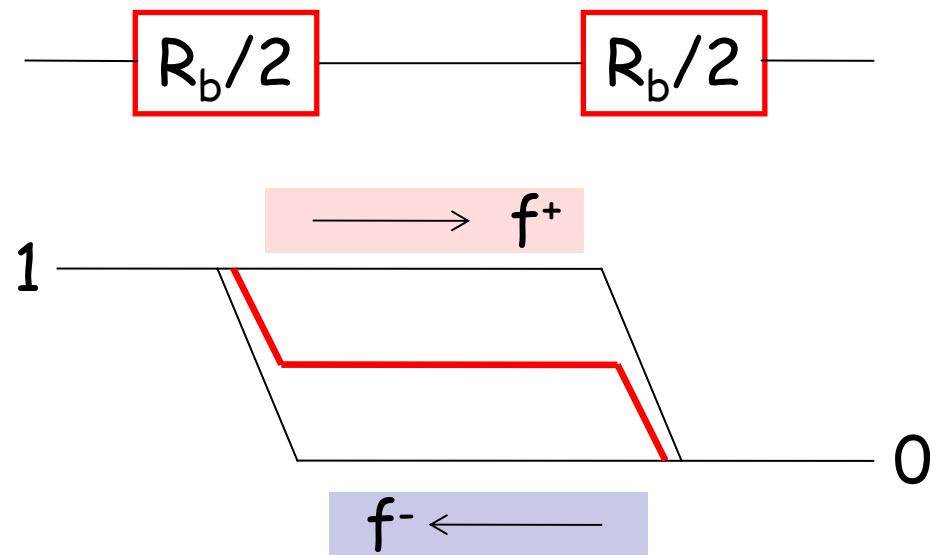
Answer: f

# Where is the voltage ?



$$R_b \equiv V/I_b$$

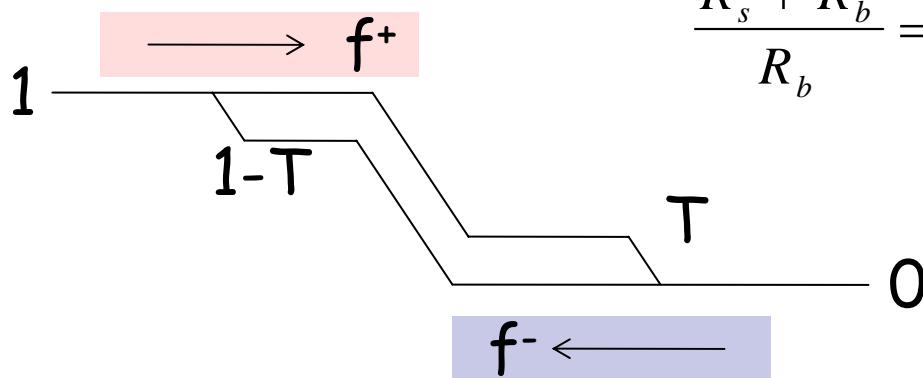
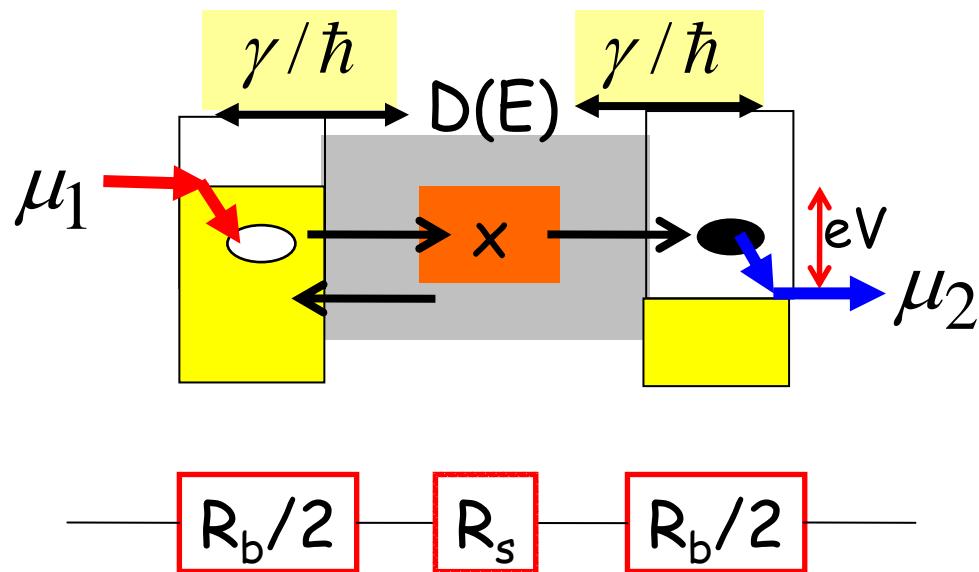
$$I_b = (2q^2V/h) M$$



$$I^+ = I_b f^+$$

$$I^- = I_b f^-$$

# Voltage across a scatterer



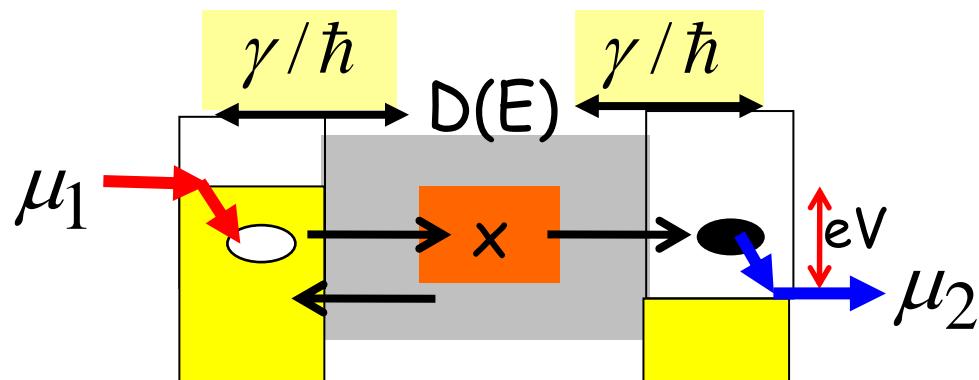
$$\frac{R_s + R_b}{R_b} = \frac{1}{T}$$

$$R_s + R_b = \frac{R_b}{T}$$

Landauer formula

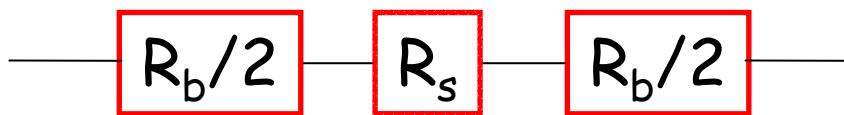
$$R_s = R_b \frac{1 - T}{T}$$

# Where is the resistance?

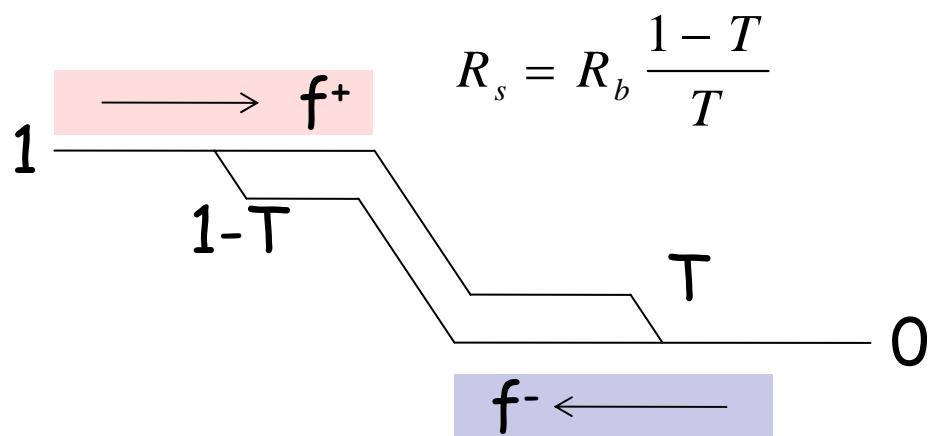


$$R_b \equiv V/I_b$$

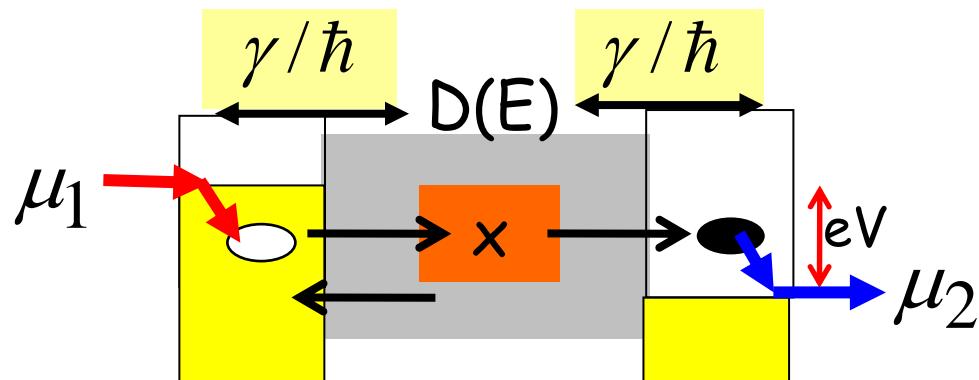
$$I_b = (2q^2V/h) M$$



" $I^2R$ " lost in the contacts,  
but "voltage" drops  
at scatterer too.



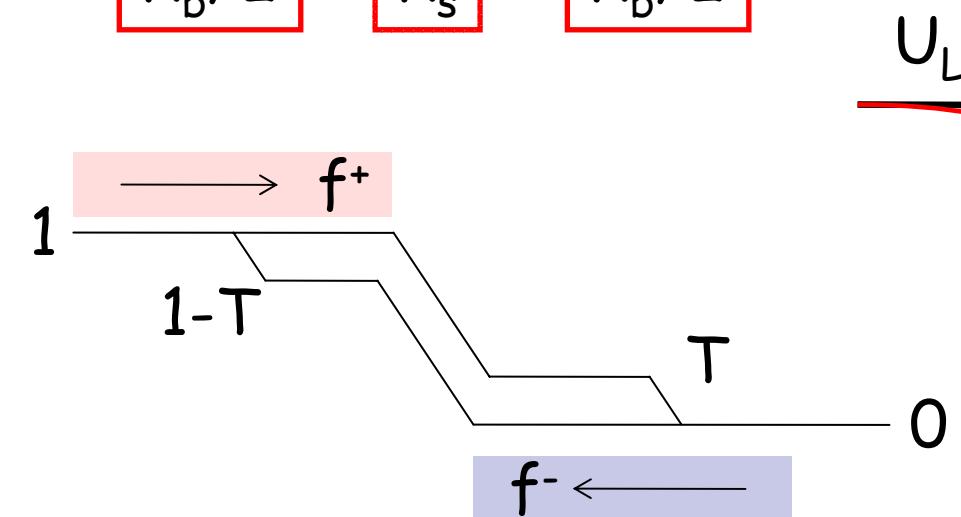
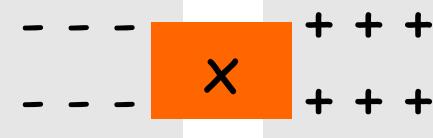
# Electrostatic Potential



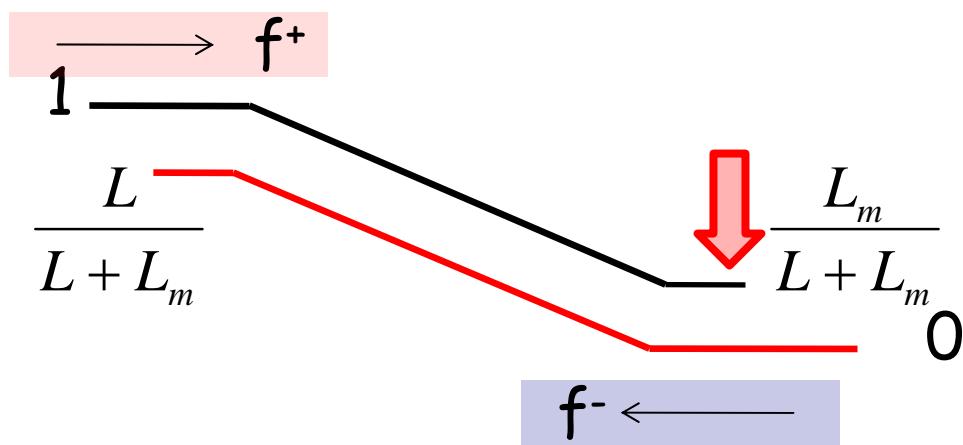
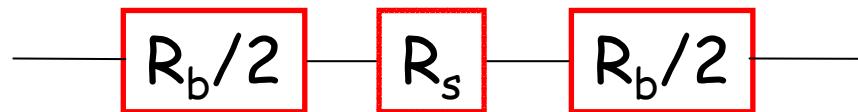
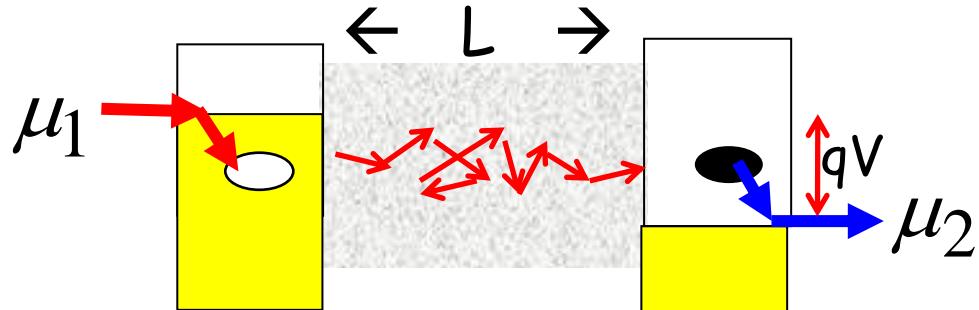
$$I / I_b \equiv f^+ - f^-$$

$$n \sim f^+ + f^-$$

Resistivity dipole



# Continuous scattering



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$$\begin{aligned} R_b &\equiv V/I_b \\ I_b &= (2q^2V/h) M \\ I/I_b &\equiv f^+ - f^- \end{aligned}$$

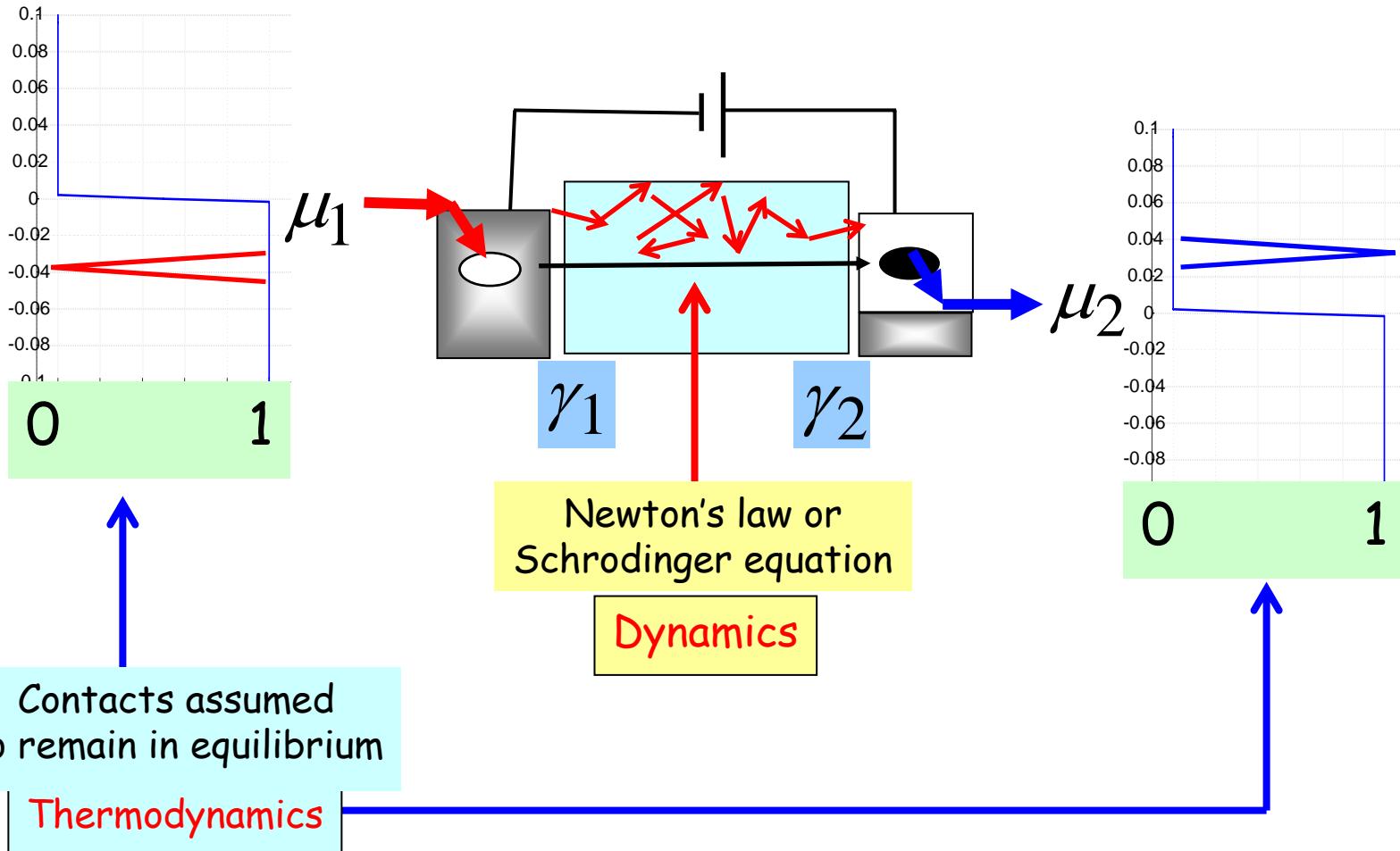
$$df^+ = -\frac{dx}{L_m} (f^+ - f^-)$$

$$\frac{R_s + R_b}{R_b} = \frac{1}{L_m / (L + L_m)}$$

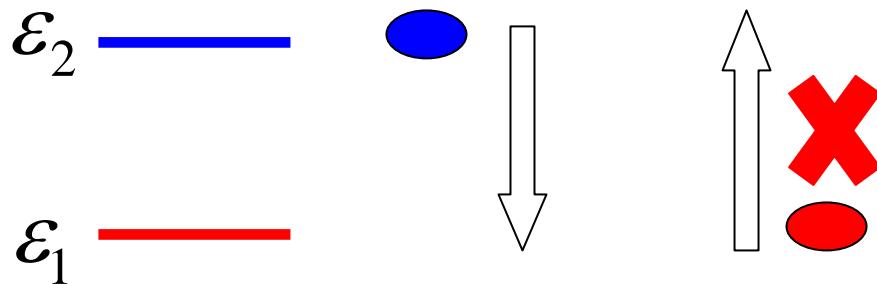
$$R_s = \frac{L}{L_m} R_b$$

Ohm's law

## \*\* "Landauer model"



# Pure dynamics is not enough



Isolated

$$i\hbar \frac{\partial}{\partial t} \begin{Bmatrix} \psi_1 \\ \psi_2 \end{Bmatrix} = \begin{bmatrix} \varepsilon_1 & 0 \\ 0 & \varepsilon_2 \end{bmatrix} \begin{Bmatrix} \psi_1 \\ \psi_2 \end{Bmatrix}$$

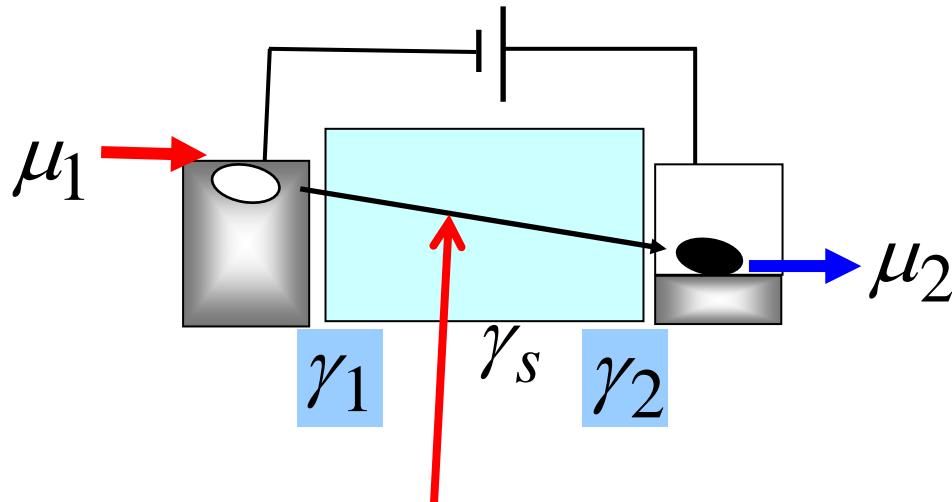
Down > Up

Interaction

$$i\hbar \frac{\partial}{\partial t} \begin{Bmatrix} \psi_1 \\ \psi_2 \end{Bmatrix} = \begin{bmatrix} \varepsilon_1 & a \\ a^* & \varepsilon_2 \end{bmatrix} \begin{Bmatrix} \psi_1 \\ \psi_2 \end{Bmatrix}$$

"Unidirectionality"  
does NOT follow from  
Schrodinger equation

# Combining dynamic and "entropic forces"

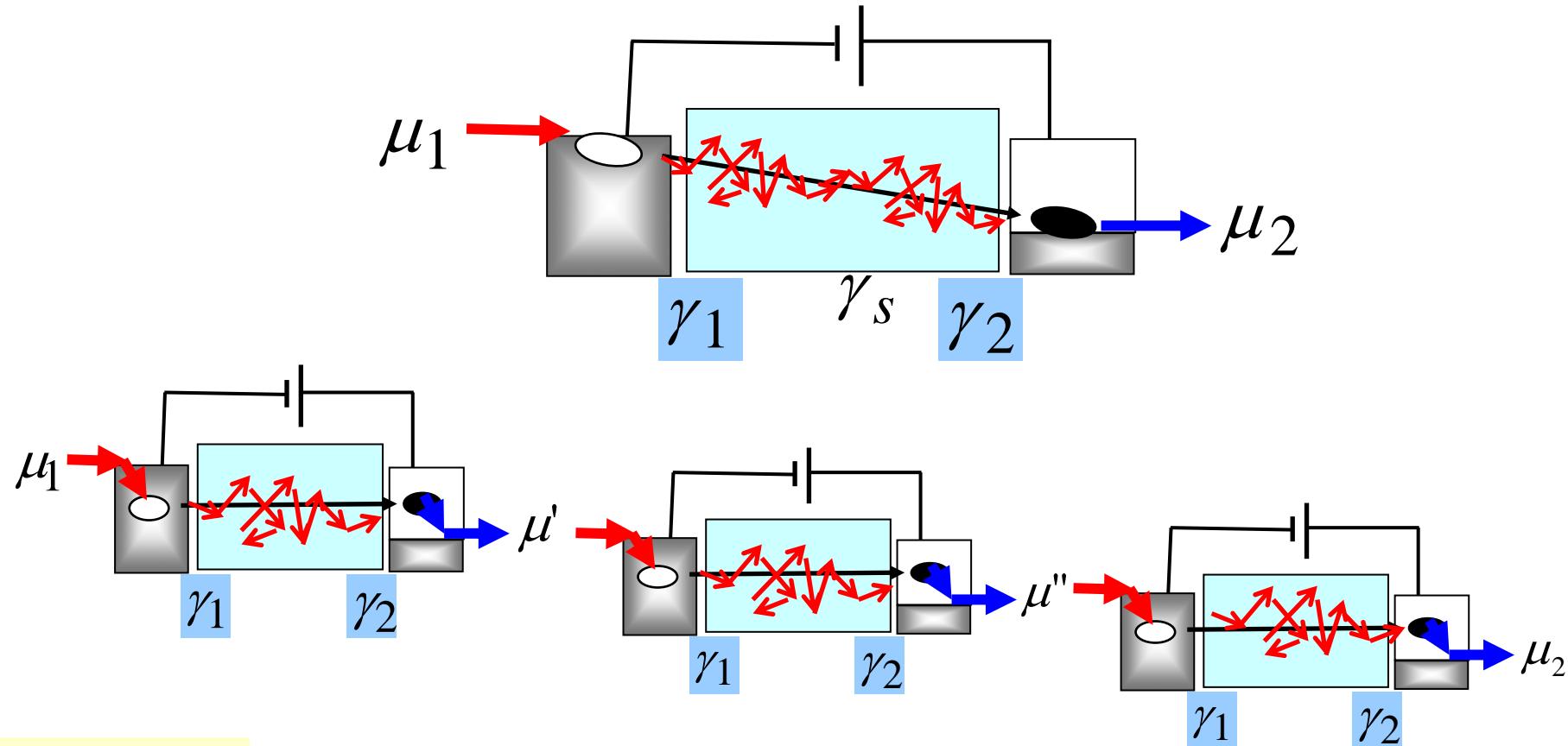


Newton's law + "Entropic forces"  
= Boltzmann Equation

Schrodinger equation  
+ "Entropic forces" = NEGF

Schwinger, Keldysh,  
Kadanoff, Baym (1960's)

# Long devices → "Landauer devices" in series



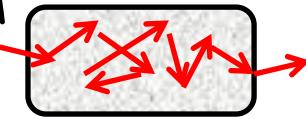
What drives the current?

$$\frac{\Delta f}{\Delta \mu} \rightarrow \frac{\partial f}{\partial \mu} \rightarrow \left( - \frac{\partial f}{\partial E} \right) \quad \text{or,} \quad \frac{\Delta f}{\Delta T} \rightarrow \frac{\partial f}{\partial T} \rightarrow \left( - \frac{\partial f}{\partial E} \right)$$

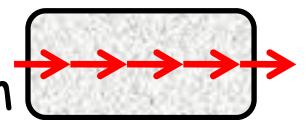
# Nanoelectronics and the meaning of Resistance :

0.1 mm Macroscopic dimensions

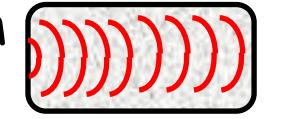
10  $\mu\text{m}$



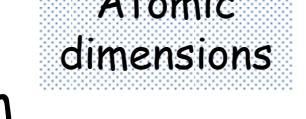
1  $\mu\text{m}$



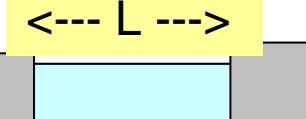
0.1  $\mu\text{m}$



10 nm



1 nm

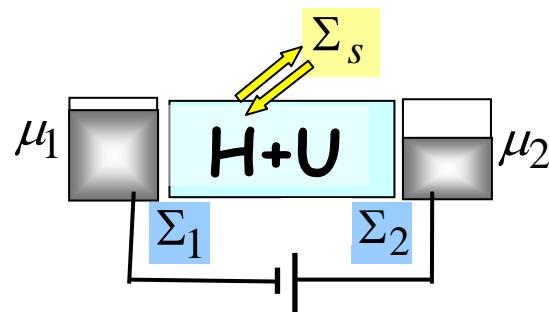
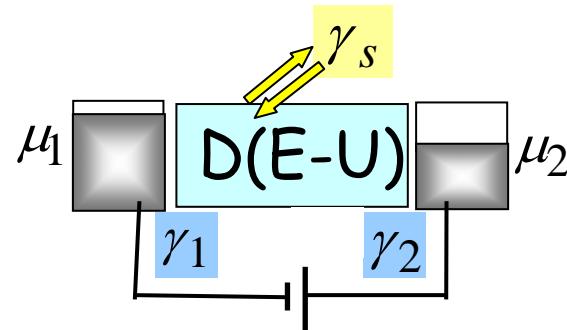


0.1 nm

NCN V I

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Lectures 1a,b:  
Simple model



Lectures 2a,b:  
Microscopic model

Materials

Semiconductors  
Graphene / CNT  
Molecules

...

1  $\mu\text{m} = .001 \text{ mm}$   
1 nm = .001  $\mu\text{m}$

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End of Lectures 1a,b