

and the meaning of resistance

1a,b: What and where is the resistance?

2a,b: Quantum transport

3a,b: Spins and magnets

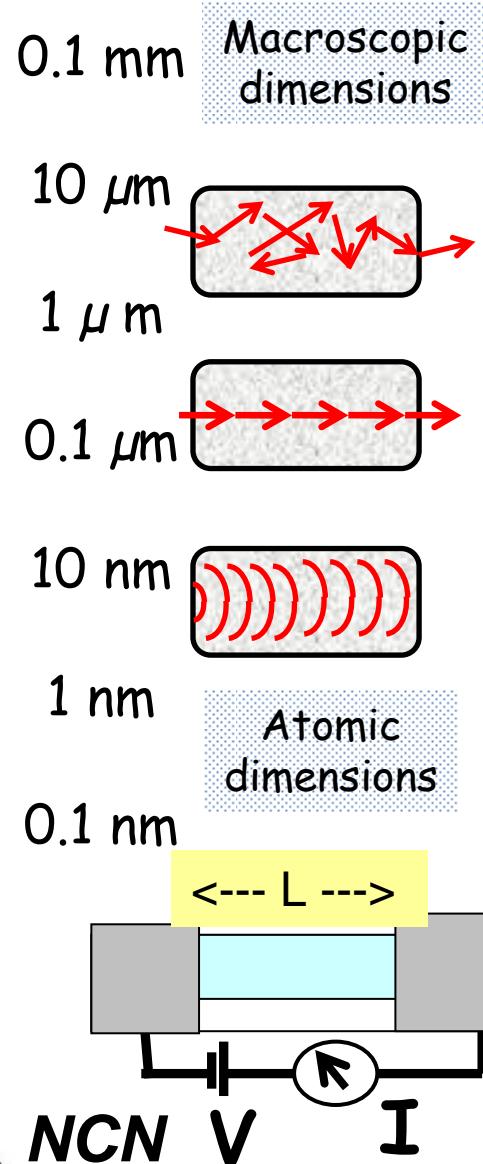
4a,b: Maxwell's demon

5a,b: Correlations and entanglement

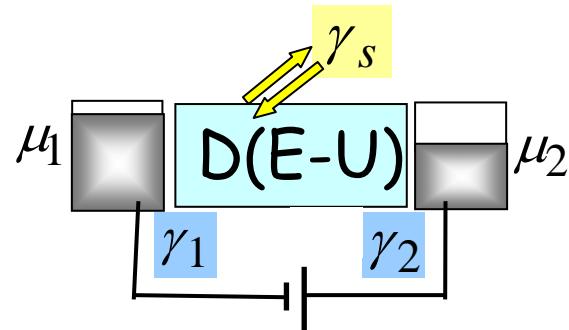
$$R = V/I, G = I/V$$

$$1 \mu\text{m} = .001 \text{ mm}$$
$$1 \text{ nm} = .001 \mu\text{m}$$

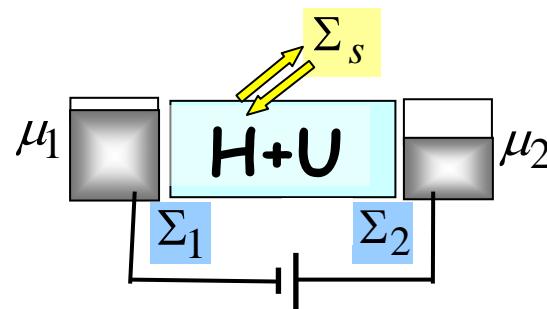
Nanoelectronics and the meaning of Resistance



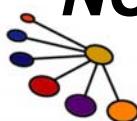
Lectures 1a,b:
Simple model



Lectures 3a,b:
Add spin



Lectures 2a,b:
Microscopic model



Differential → Matrix equation

$$\psi(\vec{r}) = \sum_m \psi_m \Phi_m(\vec{r})$$

$$E\psi(\vec{r}) = \left(-\frac{\hbar^2}{2m} \nabla^2 + U(\vec{r}) \right) \psi(\vec{r})$$

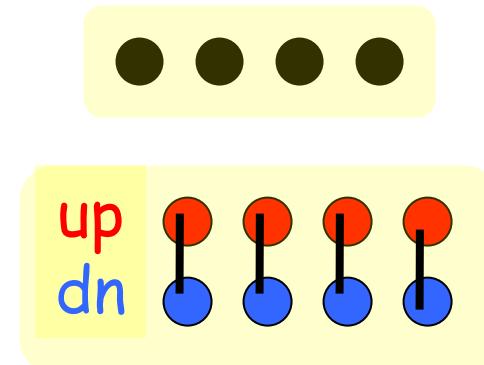
N = number of
"basis functions"

[H]: N × N

→ N eigenvalues

$$E \begin{Bmatrix} \psi_1 \\ \psi_2 \\ \dots \\ \dots \\ \psi_N \end{Bmatrix} = \begin{bmatrix} \dots & \dots & \dots \\ \dots & H & \dots \\ \dots & \dots & \dots \end{bmatrix} \begin{Bmatrix} \psi_1 \\ \psi_2 \\ \dots \\ \dots \\ \psi_N \end{Bmatrix}$$

All matrices double
From N × N
To 2N × 2N



NCN

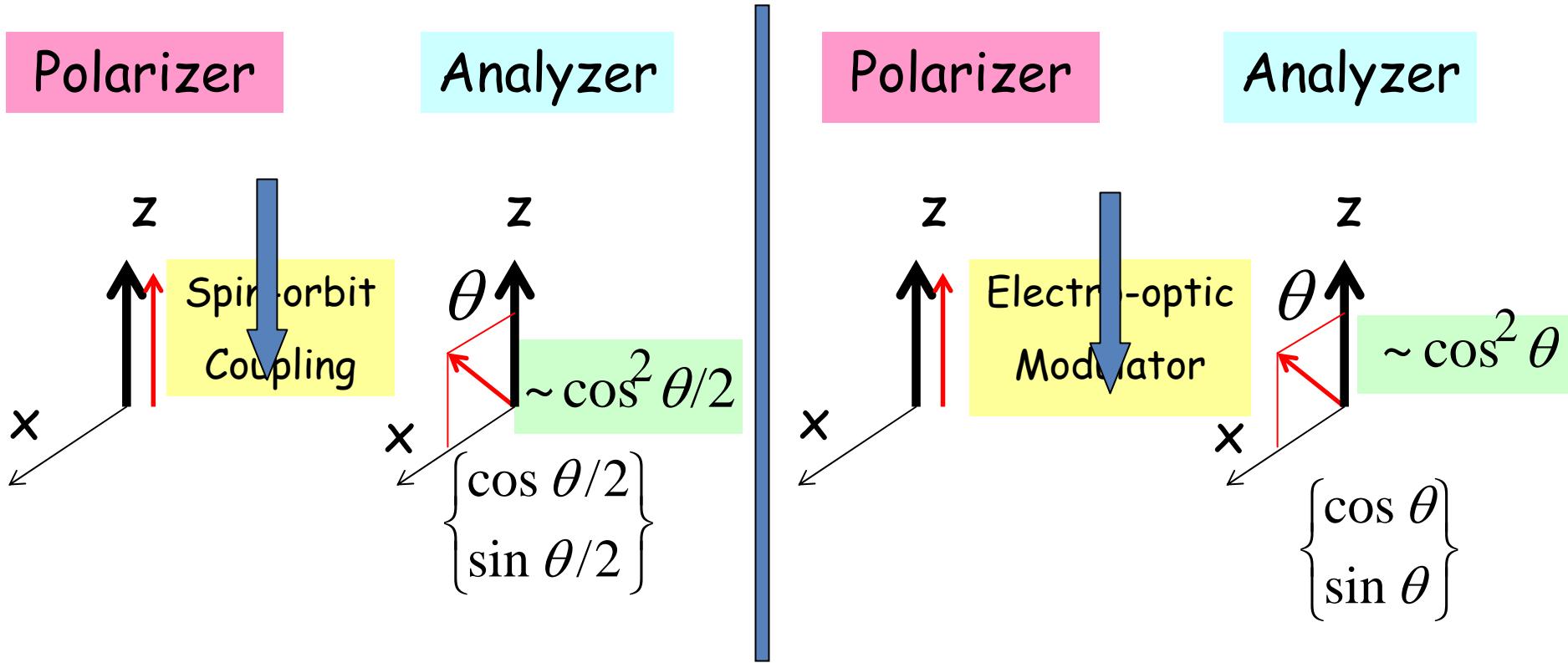


<http://www.nanohub.org/courses/cqt>

Supriyo Datta

PURDUE
UNIVERSITY

** Electron spin <--> Photon polarization



Appl. Phys. Lett.
1990

Electronic analog of the electro-optic modulator

Supriyo Datta and Biswajit Das

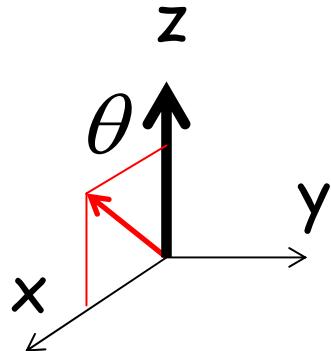
School of Electrical Engineering, Purdue University, West Lafayette, Indiana 47907

(Received 3 October 1989; accepted for publication 5 December 1989)



Spinor <--> Vector

$$\begin{cases} up : \cos \theta/2 \\ dn : \sin \theta/2 \end{cases}$$



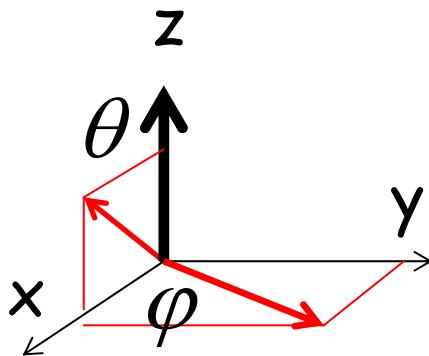
$$\begin{cases} z : \cos \theta \\ x : \sin \theta \end{cases}$$

$\theta = \pi$: "Orthogonal"

$\theta = \pi/2$: "Orthogonal"

$$\begin{cases} up : \cos \theta/2 e^{-i\varphi/2} \\ dn : \sin \theta/2 e^{+i\varphi/2} \end{cases}$$

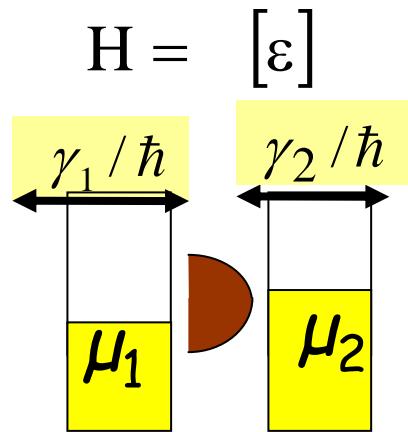
$$\begin{cases} x : \sin \theta \cos \varphi \\ y : \sin \theta \sin \varphi \\ z : \cos \theta \end{cases}$$



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NEGF Model: 1 level

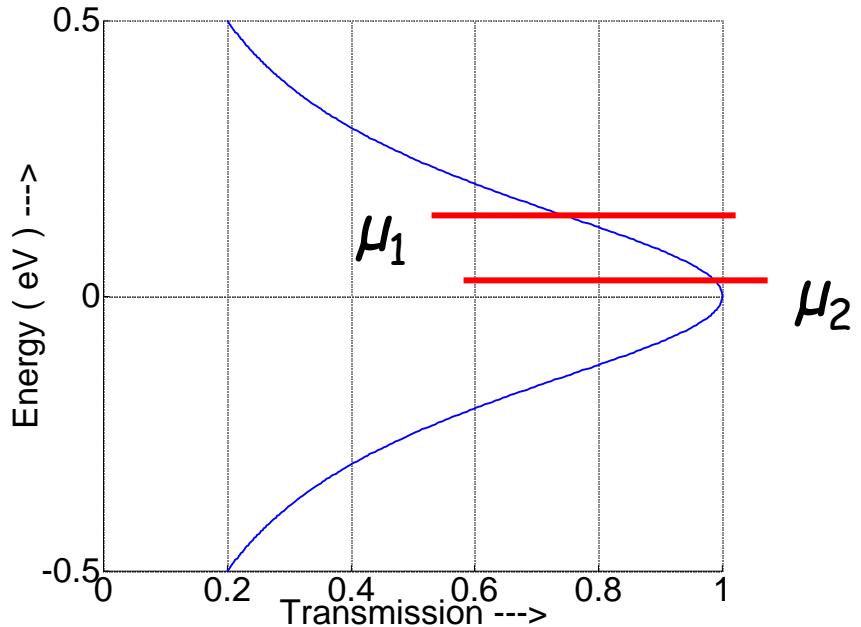


$$\Sigma_1 = -\frac{i}{2}[\gamma_1] \quad \Sigma_2 = -\frac{i}{2}[\gamma_2]$$

$$\Gamma_1 = i[\Sigma_1 - \Sigma_1^+] \quad \Gamma_2 = i[\Sigma_2 - \Sigma_2^+]$$

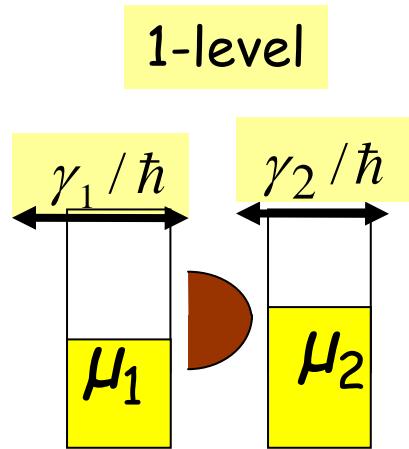
$$\rightarrow \gamma_1 \qquad \qquad \rightarrow \gamma_2$$

$$Conductance = \underbrace{\left(\frac{q^2}{h}\right)}_{1/25.8\text{ K}\Omega} \underbrace{2\pi D \frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2}}_{Transmission}$$



Maximum
Transmission $\rightarrow \frac{4\gamma_1 \gamma_2}{(\gamma_1 + \gamma_2)^2}$

Spin: 1 --> 2 levels



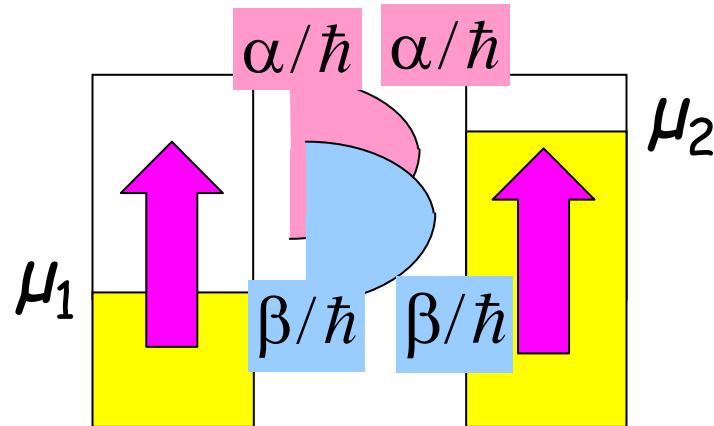
$$H = [\varepsilon]$$

$$\Sigma_1 = -\frac{i}{2}[\gamma_1]$$

$$\Sigma_2 = -\frac{i}{2}[\gamma_2]$$

$$\Gamma_1 = [\gamma_1] \quad , \quad \Gamma_2 = [\gamma_2]$$

2-level



$$H = \begin{bmatrix} \varepsilon & 0 \\ 0 & \varepsilon \end{bmatrix}$$

$$\Sigma_1 = -\frac{i}{2} \begin{bmatrix} \alpha & 0 \\ 0 & \beta \end{bmatrix}$$

$$\Gamma_1 = \begin{bmatrix} \alpha & 0 \\ 0 & \beta \end{bmatrix}$$

$$\Sigma_2 = -\frac{i}{2} \begin{bmatrix} \alpha & 0 \\ 0 & \beta \end{bmatrix}$$

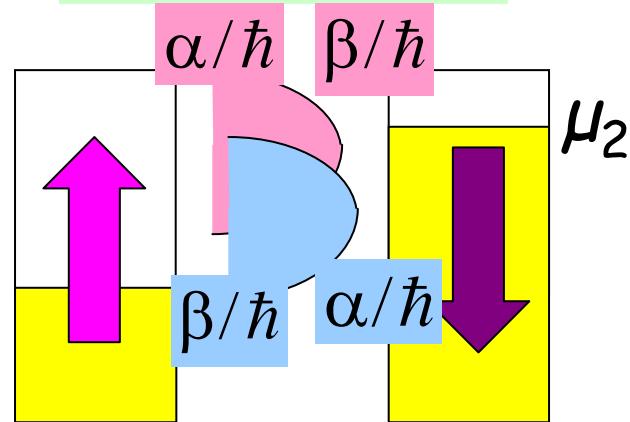
$$\Gamma_2 = \begin{bmatrix} \alpha & 0 \\ 0 & \beta \end{bmatrix}$$

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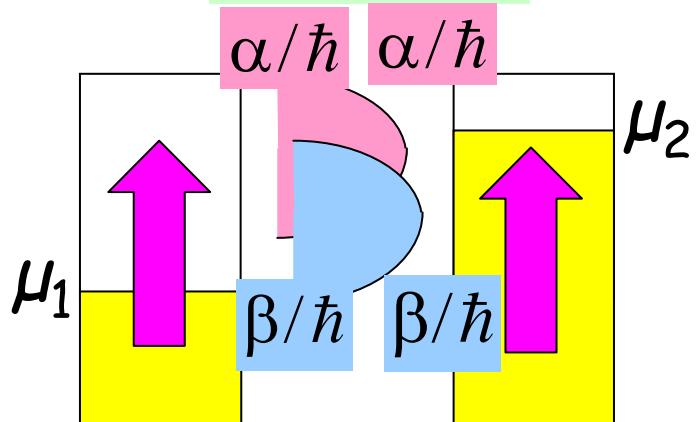
"GMR" Device

Anti-Parallel

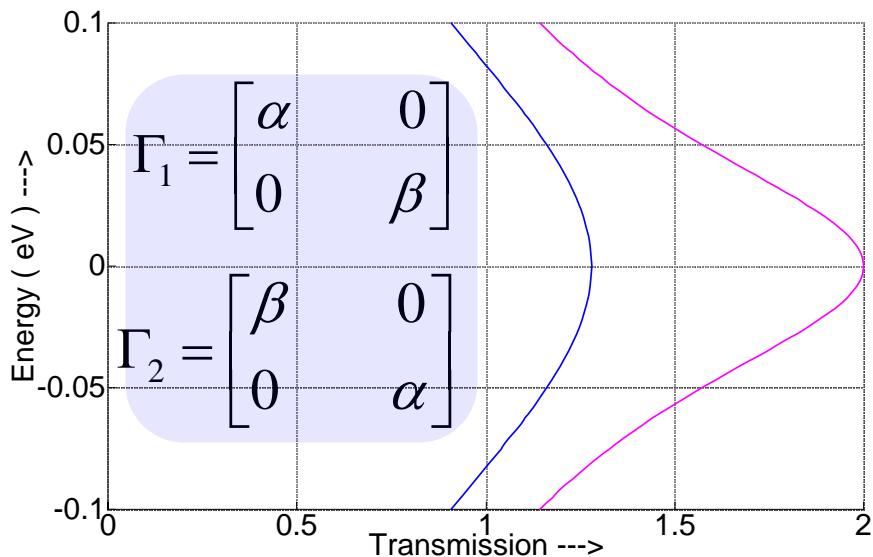


$$H = \begin{bmatrix} \varepsilon & 0 \\ 0 & \varepsilon \end{bmatrix} \quad \begin{array}{c} \uparrow \\ \downarrow \end{array}$$

Parallel



$$\begin{aligned} \varepsilon &= 0 \\ \alpha &= 0.25 \text{ eV} \\ \beta &= 0.25 \alpha \end{aligned}$$



$$\Gamma_1 = \begin{bmatrix} \alpha & 0 \\ 0 & \beta \end{bmatrix}$$

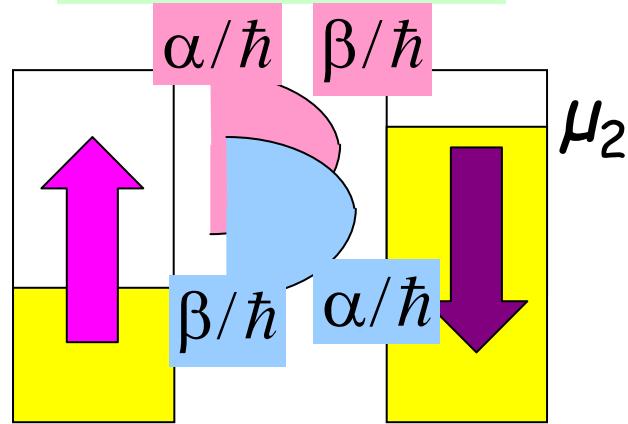
$$\Gamma_2 = \begin{bmatrix} \beta & 0 \\ 0 & \alpha \end{bmatrix}$$

NCN



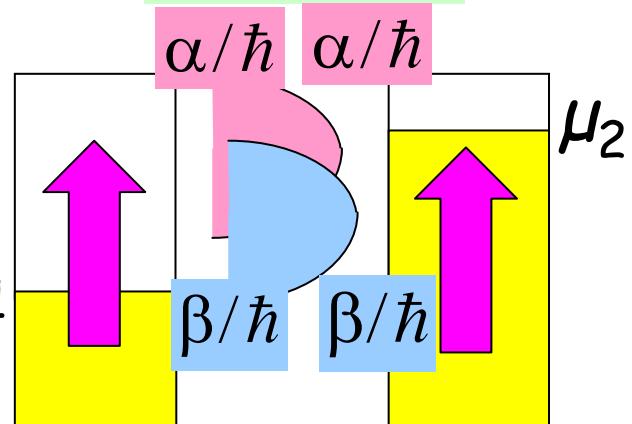
"GMR" Device

Anti-Parallel



$$2 * \frac{4\alpha\beta}{(\alpha + \beta)^2}$$

Parallel

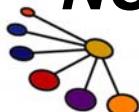


$$\frac{4\alpha^2}{4\alpha^2} + \frac{4\beta^2}{4\beta^2} = 2$$

$$\begin{aligned} & \text{Maximum} \\ & \text{Transmission} \\ & = \frac{4\gamma_1\gamma_2}{(\gamma_1 + \gamma_2)^2} \end{aligned}$$

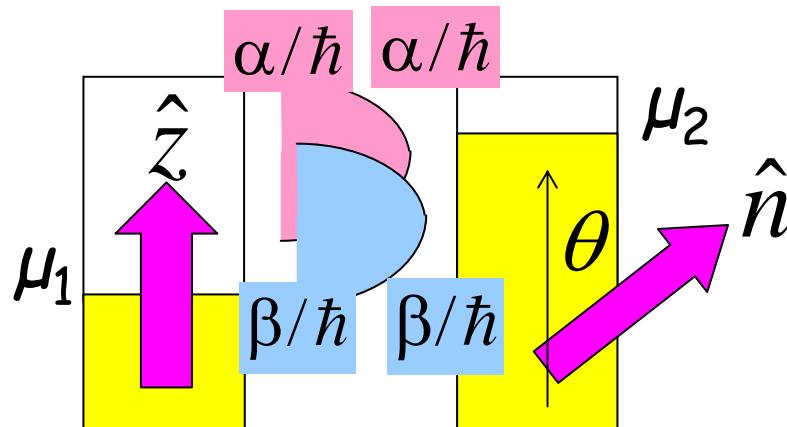
$$\frac{AP}{P} = \frac{4\alpha\beta}{(\alpha + \beta)^2} = 1 - \left(\frac{\alpha - \beta}{\alpha + \beta} \right)^2 < 1$$

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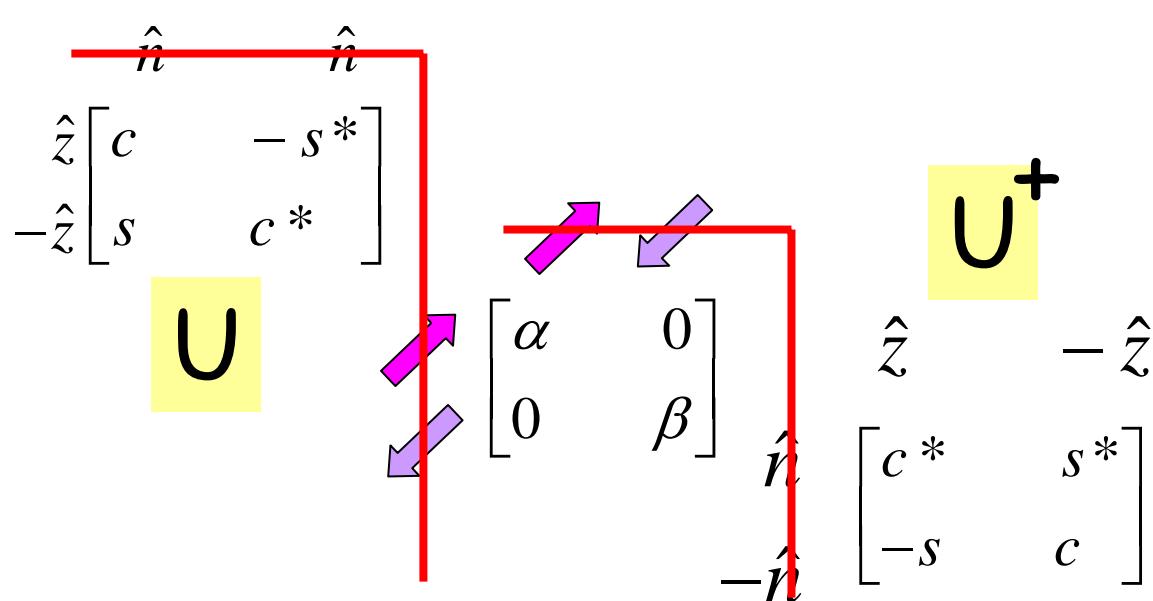


** Basis transformation

$$\Gamma_1 = \begin{bmatrix} \alpha & 0 \\ 0 & \beta \end{bmatrix}$$



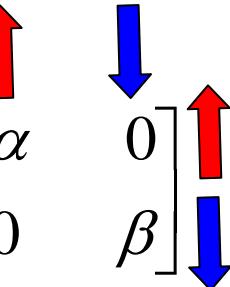
$$\Gamma_2 = \begin{bmatrix} \alpha & 0 \\ 0 & \beta \end{bmatrix}$$

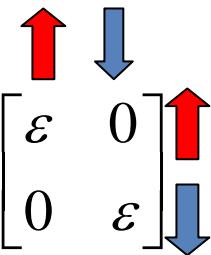


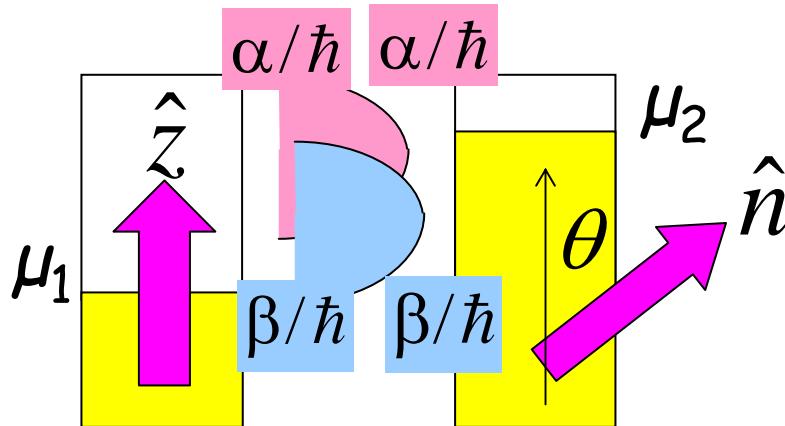
$$\begin{aligned} \hat{z} &\quad \left\{ \cos \theta/2 e^{-i\varphi/2} \equiv c \right\} \\ -\hat{z} &\quad \left\{ \sin \theta/2 e^{+i\varphi/2} \equiv s \right\} \end{aligned}$$

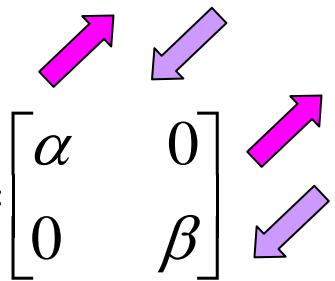
$$= \begin{bmatrix} \uparrow & \downarrow & \uparrow \\ * & * & * \\ * & * & * \end{bmatrix}$$

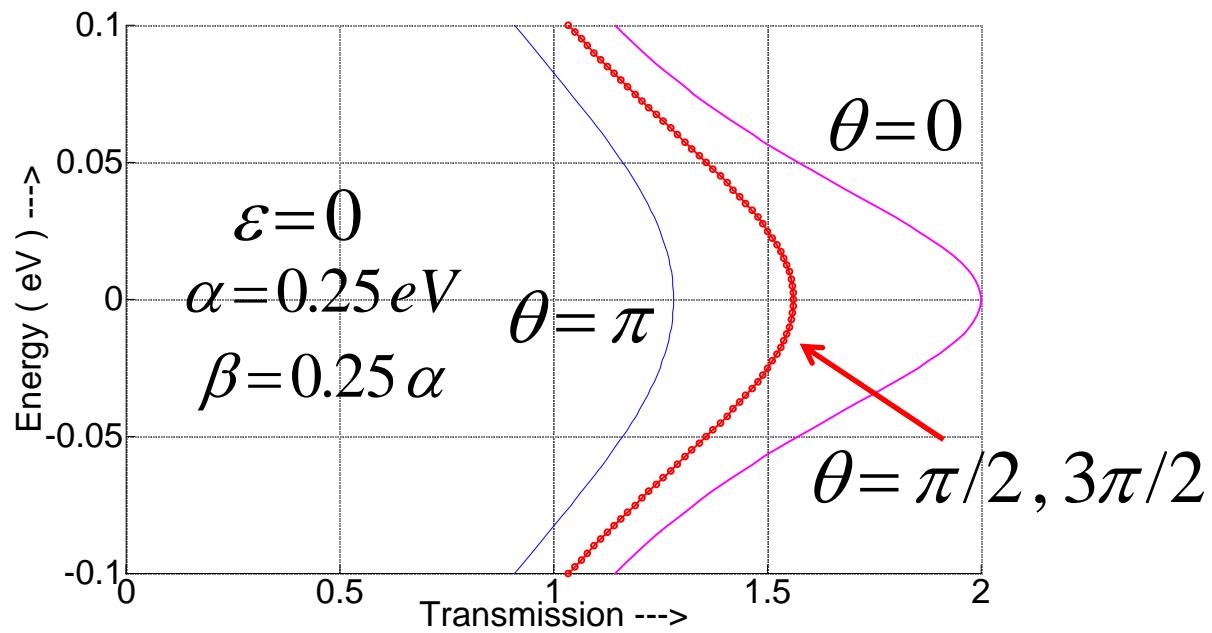
Generalized "GMR" Device

$$\Gamma_1 = \begin{bmatrix} \alpha & 0 \\ 0 & \beta \end{bmatrix}$$


$$H = \begin{bmatrix} \varepsilon & 0 \\ 0 & \varepsilon \end{bmatrix}$$




$$\Gamma_2 = \begin{bmatrix} \alpha & 0 \\ 0 & \beta \end{bmatrix}$$


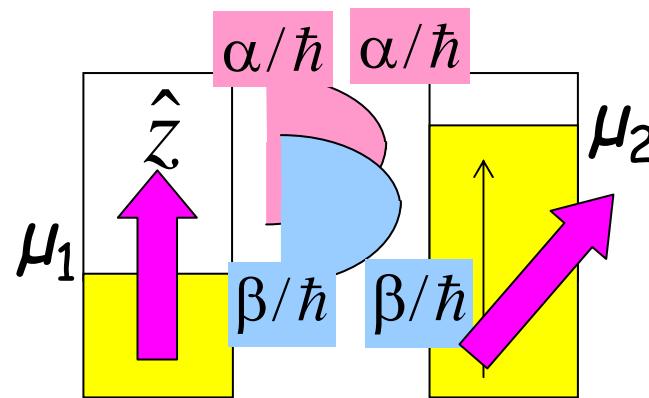


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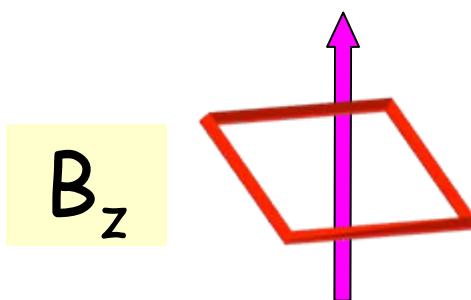


Including B-field in Hamiltonian

$$\Gamma_1 = \begin{bmatrix} \alpha & 0 \\ 0 & \beta \end{bmatrix}$$



$$\Gamma_2 = \begin{bmatrix} \alpha & 0 \\ 0 & \beta \end{bmatrix}$$

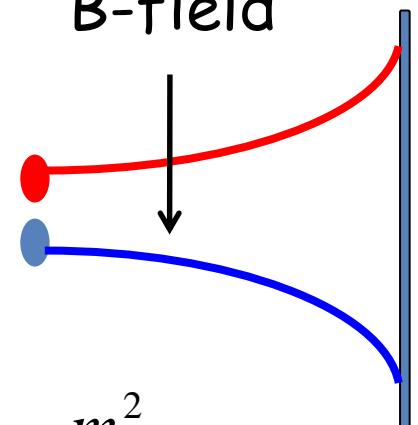


$$\begin{aligned} \mu_B &= q\hbar/2m \sim 10^{-23} A - m^2 \\ &= 1 mA - 1 Ang^2 \end{aligned}$$

$$H = \begin{bmatrix} \varepsilon + \mu_B B_z & 0 \\ 0 & \varepsilon - \mu_B B_z \end{bmatrix}$$

$$\begin{aligned} M &\sim N\mu_B \\ &\sim 10^{29}/m^3 \times 10^{-23} A - m^2 \\ &= 10^6 A/m \end{aligned}$$

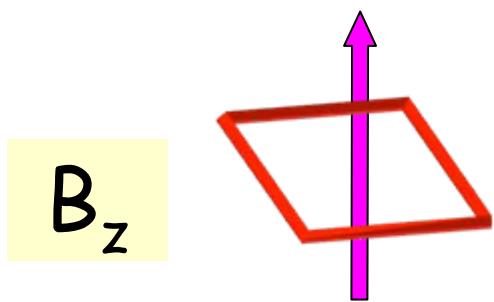
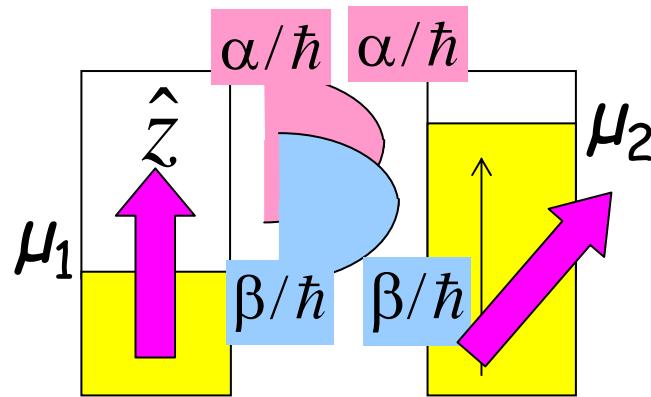
Increasing
B-field



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What if B-field were along x ?



$$\begin{aligned}\mu_B &\sim 10^{-23} A \cdot m^2 \\ &= 1 mA \cdot 1 Ang^2\end{aligned}$$

$$H = \begin{bmatrix} \varepsilon + \mu_B B_z & 0 \\ 0 & \varepsilon - \mu_B B_z \end{bmatrix}$$

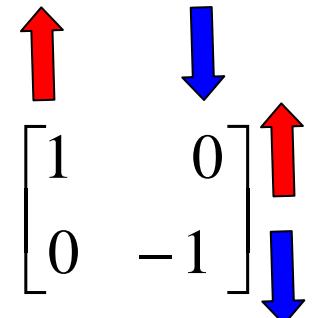
$$H = \begin{bmatrix} \varepsilon + \mu_B B_x & 0 \\ 0 & \varepsilon - \mu_B B_x \end{bmatrix}$$

Surrounding the matrix are several green and blue arrows indicating symmetry or transformation properties.

B in x-direction, transformed back to z-basis

$$\begin{matrix} \hat{n} \\ \hat{z} \\ -\hat{z} \end{matrix} \quad \left\{ \begin{array}{l} \cos \theta/2 e^{-i\varphi/2} \equiv c \\ \sin \theta/2 e^{+i\varphi/2} \equiv s \end{array} \right\}$$

$$\vec{B} = B_z \hat{z}: \uparrow \text{B}$$



$$\vec{B} = B_x \hat{x}: \rightarrow \text{B}$$

Diagram illustrating the transformation of the magnetic field vector \vec{B} from the z-basis to the x-basis. The z-basis is shown on the left with red up and blue down arrows. The x-basis is shown on the right with red up and blue down arrows. The transformation is represented by a sequence of unitary matrices and their inverses (conjugates). The first transformation is $\frac{1}{\sqrt{2}} \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$, followed by $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$, then $\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} \frac{1}{\sqrt{2}}$. The final result is equal to the x-basis representation, which is $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$.

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Pauli spin matrices

$$\vec{B} = B_x \hat{x} + B_y \hat{y} + B_z \hat{z}$$

$$H = \mu_B \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \sigma_x B_x$$

$$+ \begin{bmatrix} 0 & -i \\ +i & 0 \end{bmatrix} \sigma_y B_y$$

$$+ \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \sigma_z B_z$$

$$H = \mu_B \vec{\sigma} \cdot \vec{B}$$

$$\vec{B} = B_x \hat{x} :$$

$$\xrightarrow{\text{green}} \vec{B} =$$

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad \begin{array}{c} \uparrow \quad \downarrow \\ \text{red} \quad \text{blue} \end{array}$$

$$\vec{B} = B_y \hat{y} :$$

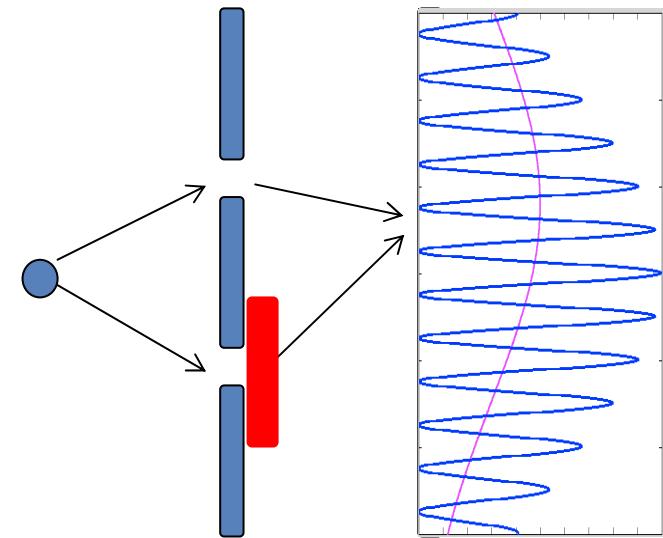
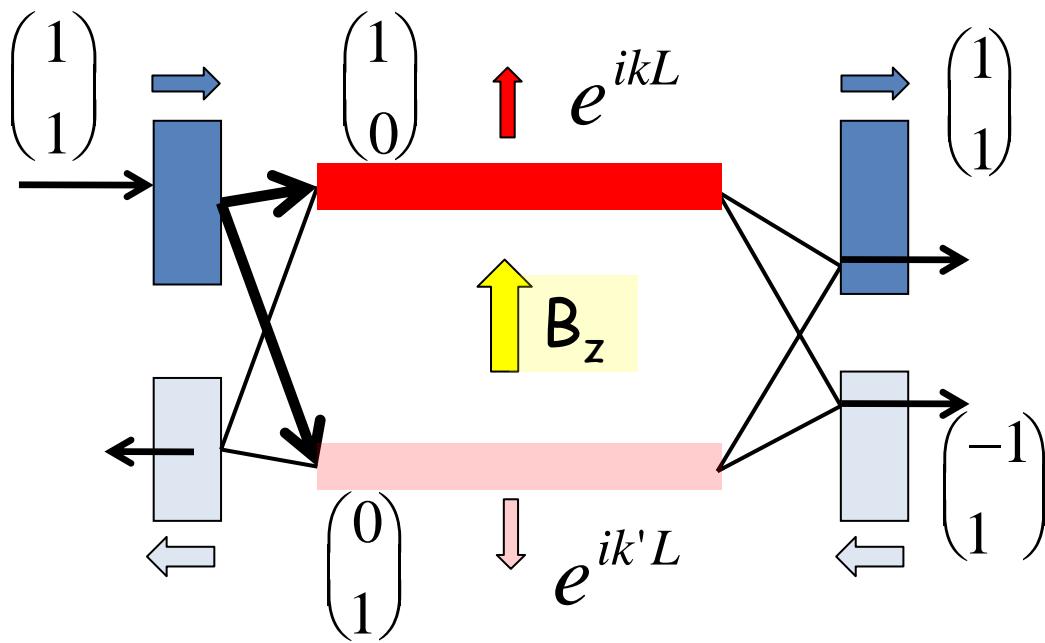
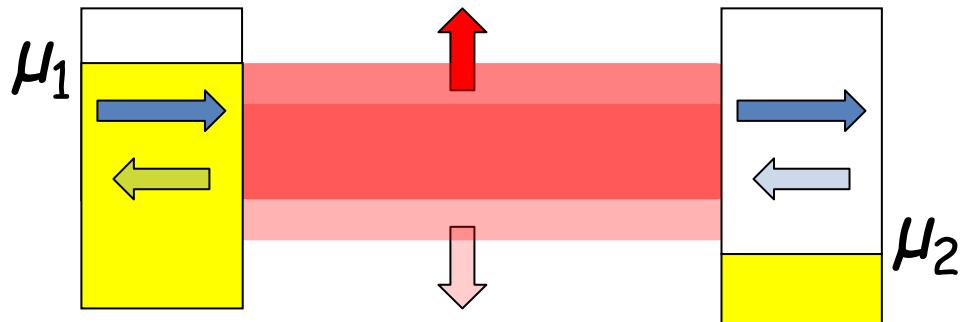
$$\xrightarrow{\text{yellow}} \vec{B} =$$

$$\begin{bmatrix} 0 & -i \\ +i & 0 \end{bmatrix} \quad \begin{array}{c} \uparrow \quad \downarrow \\ \text{red} \quad \text{blue} \end{array}$$

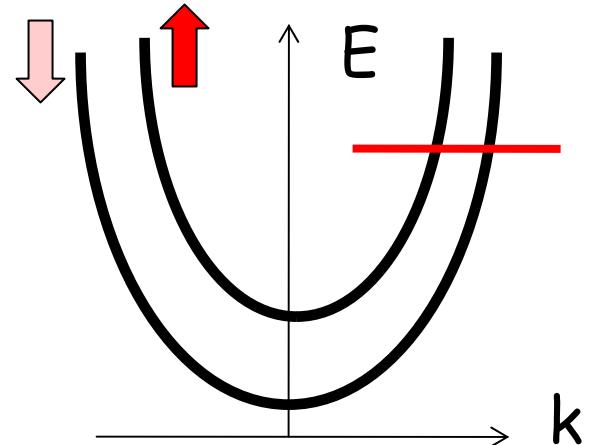
$$\vec{B} = B_z \hat{z} :$$

$$\xrightarrow{\text{magenta}} \vec{B} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \quad \begin{array}{c} \uparrow \quad \downarrow \\ \text{red} \quad \text{blue} \end{array}$$

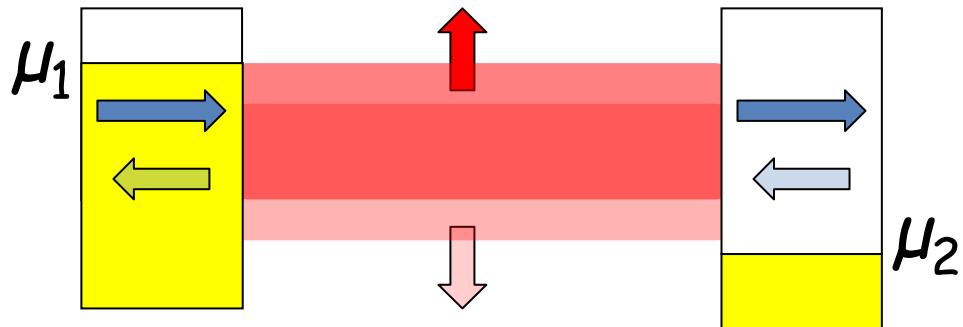
** "Two-slit" interference



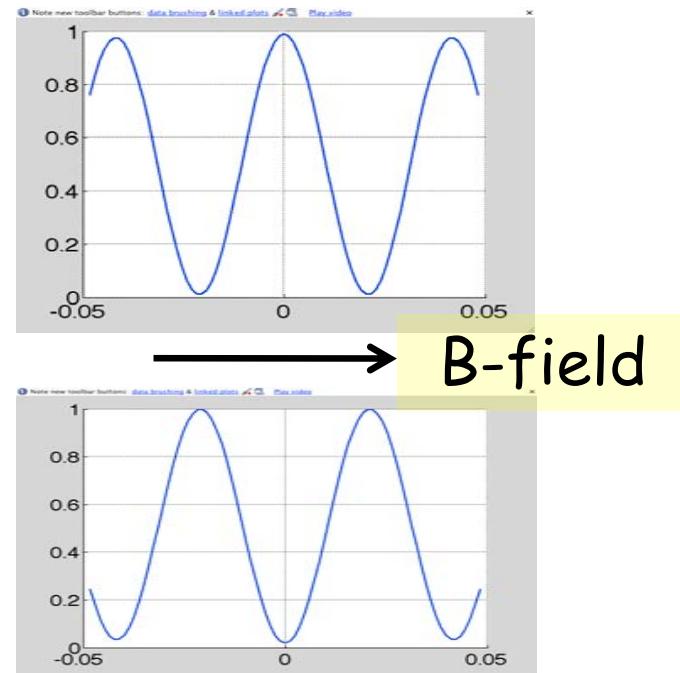
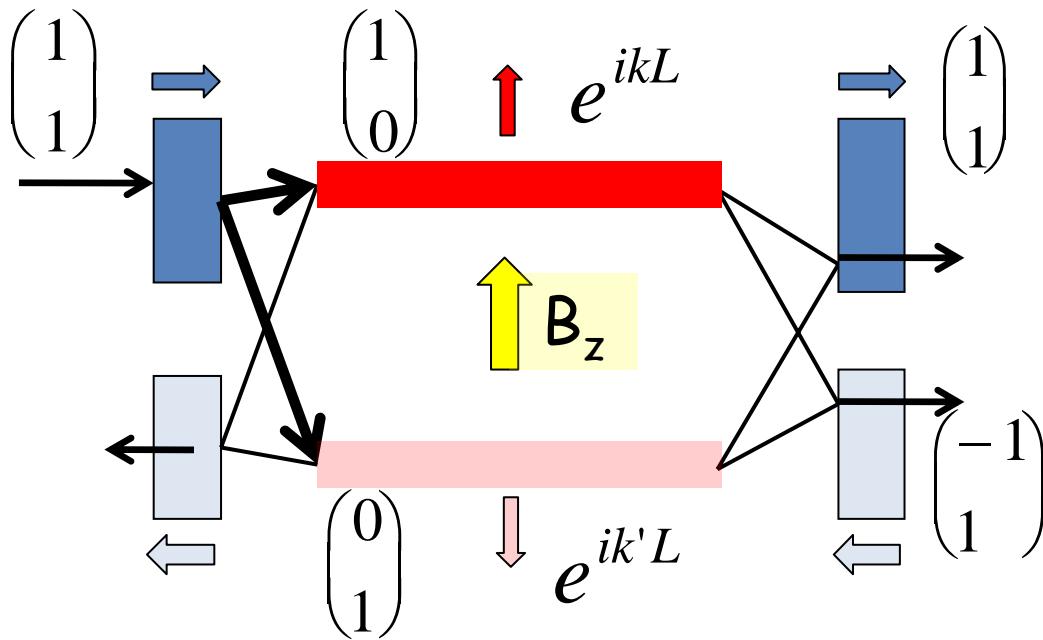
Feynman Lectures
vol.III, chapter 1



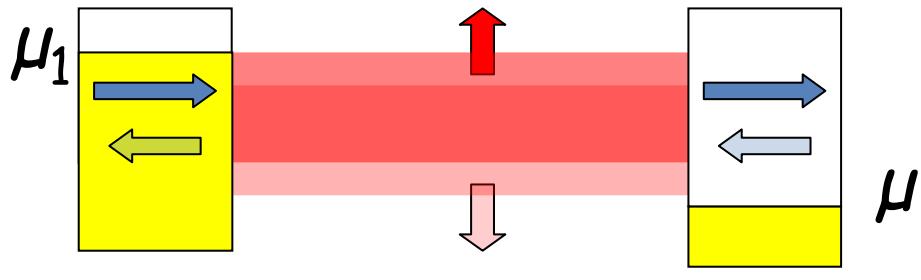
"Two-slit" interference



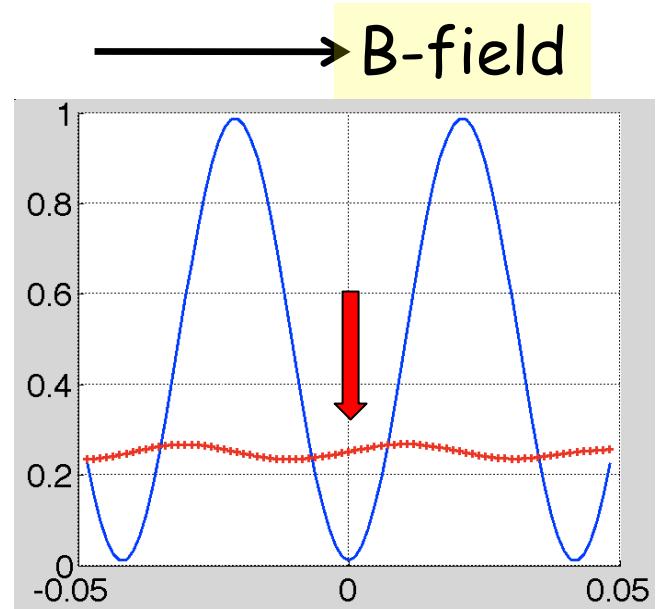
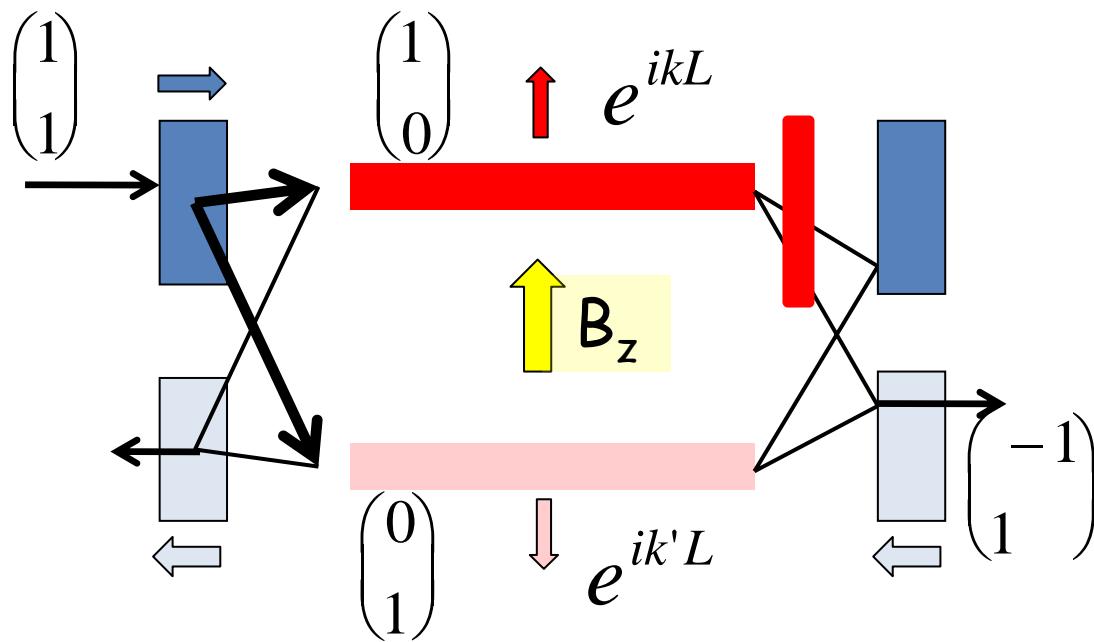
$$e^{i\delta \cdot L} \begin{pmatrix} 1 \\ 0 \end{pmatrix} + e^{-i\delta \cdot L} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \cos \delta \cdot L \begin{pmatrix} 1 \\ 1 \end{pmatrix} - i \sin \delta \cdot L \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$



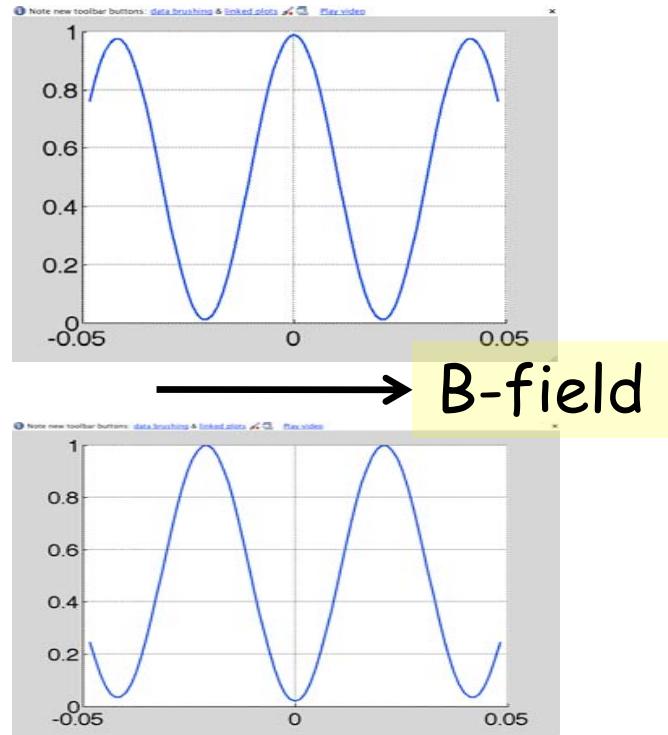
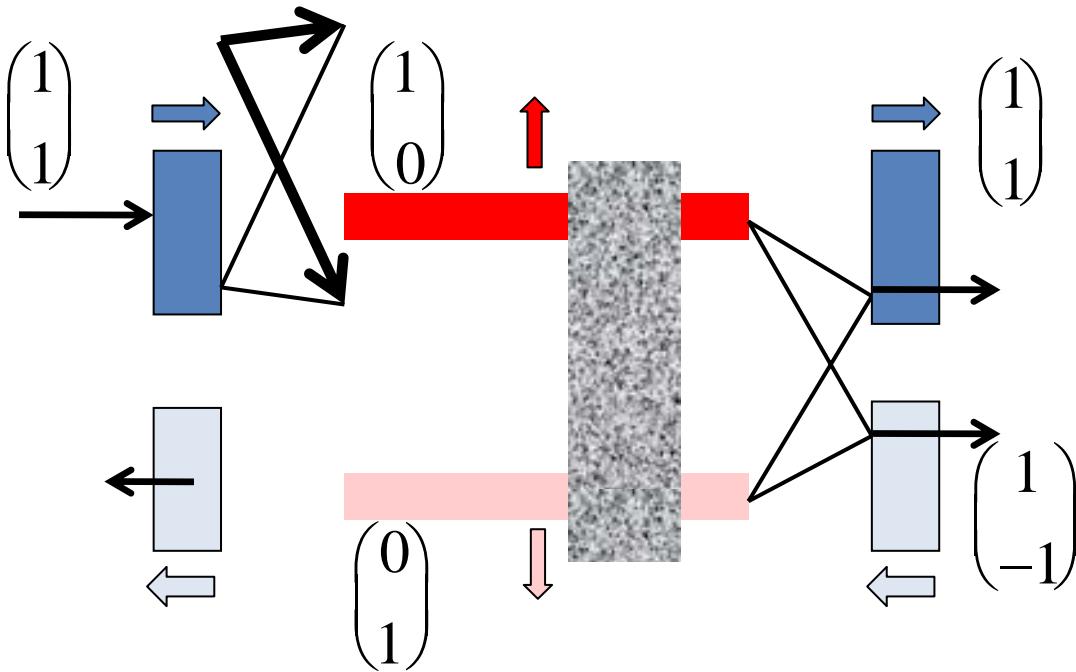
Closing a slit



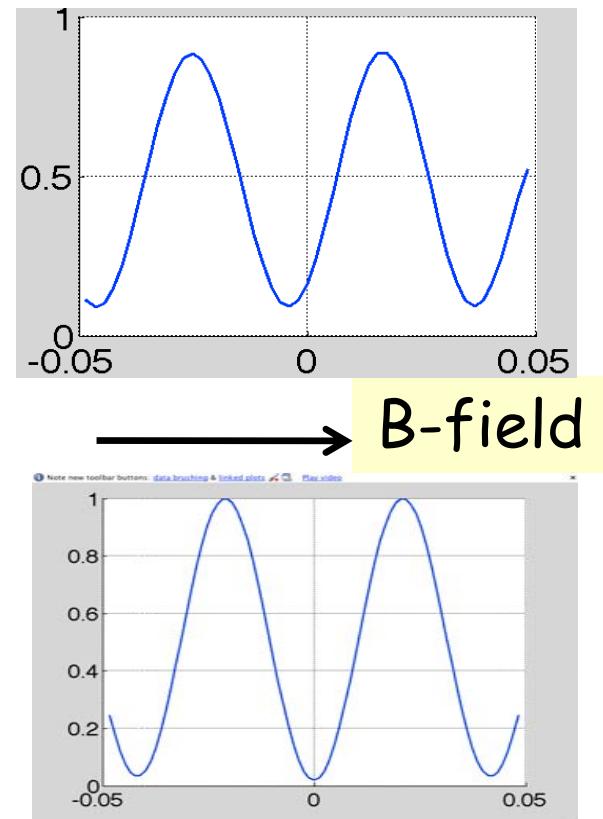
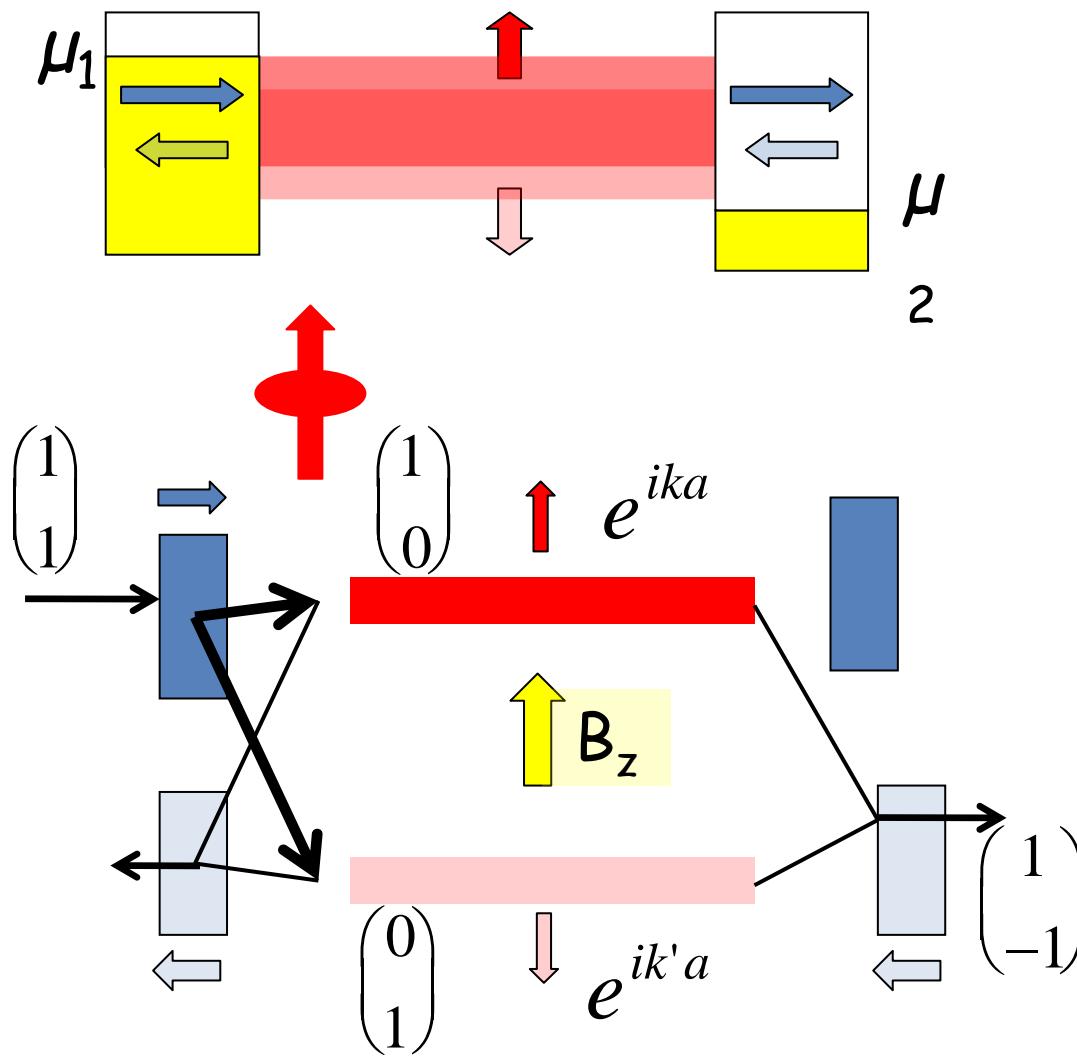
$$e^{ik'L} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \frac{e^{ik'L}}{2} \begin{pmatrix} 1 \\ 1 \end{pmatrix} + \frac{e^{ik'L}}{2} \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$



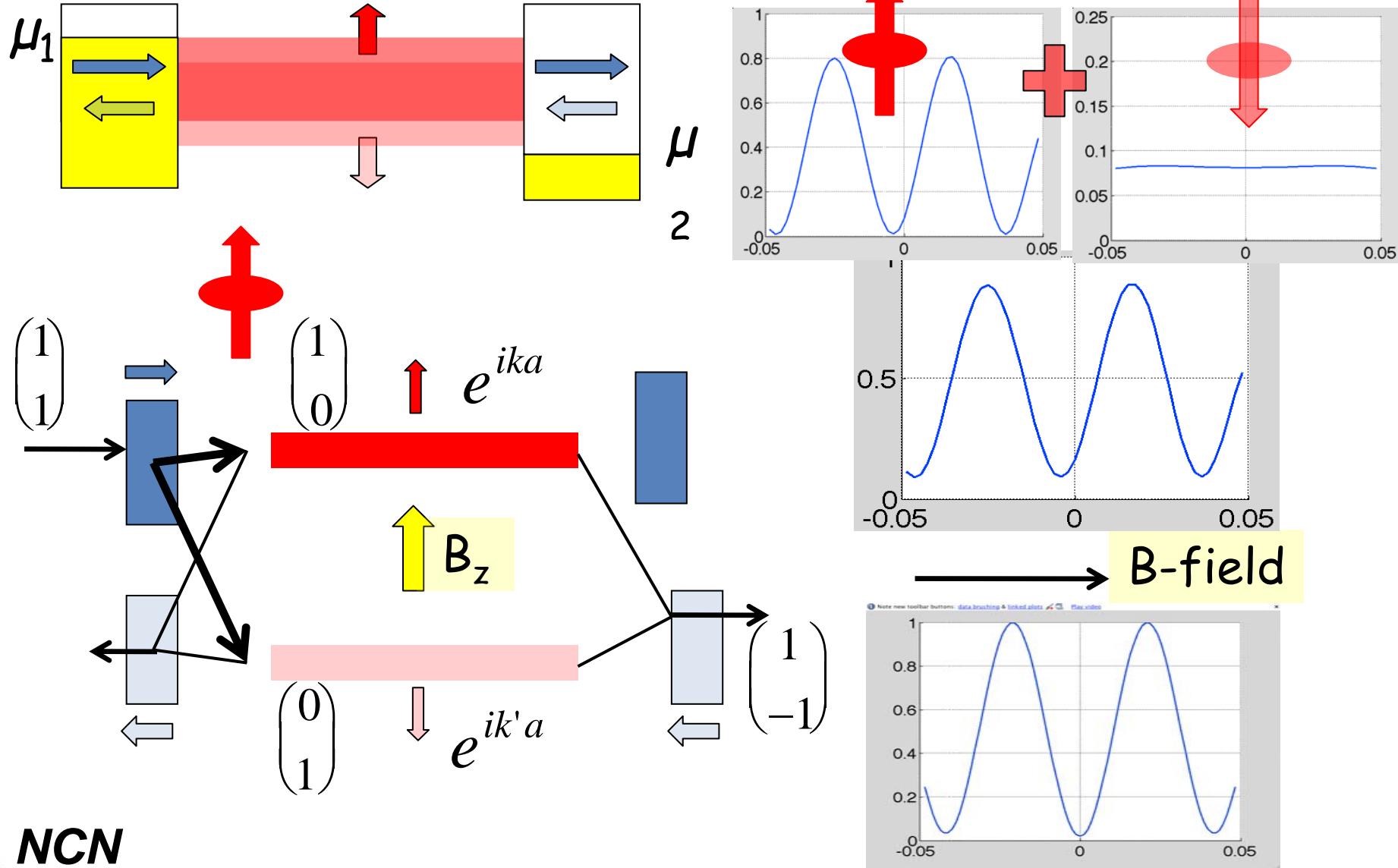
Spin interference



"Looking" at a slit



"Looking" at a slit .. more closely



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