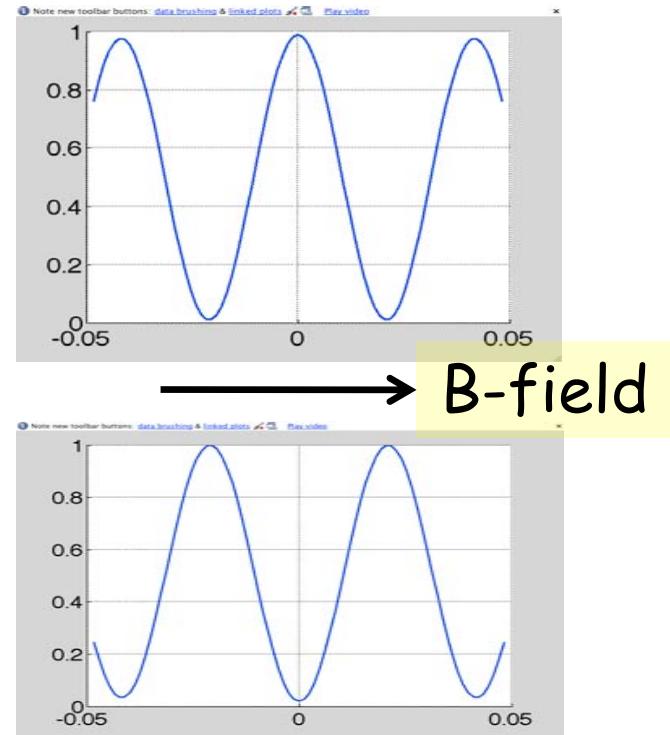
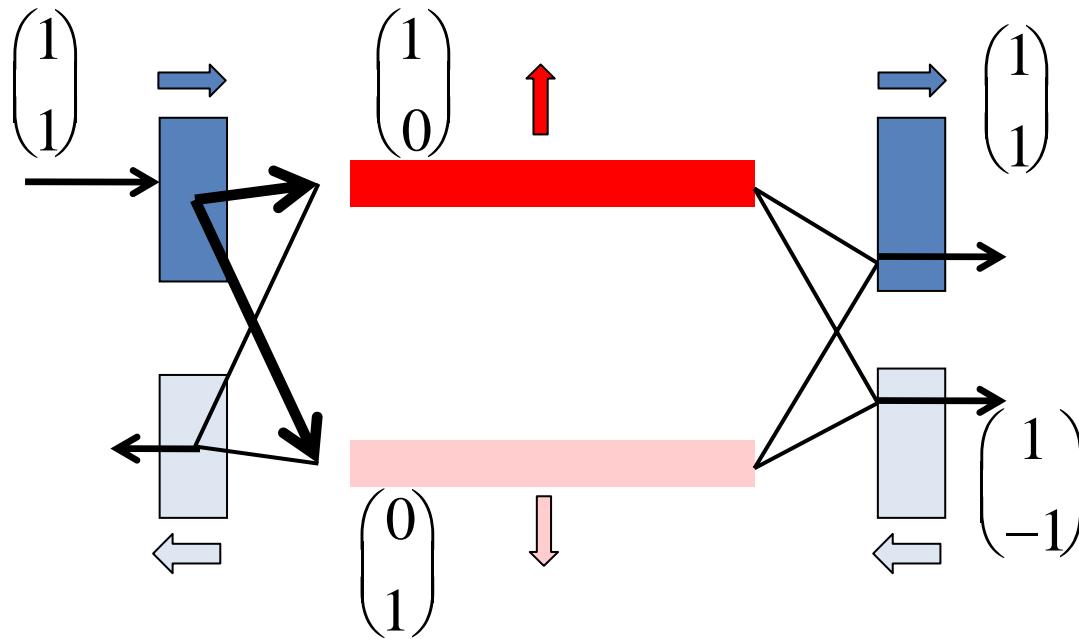
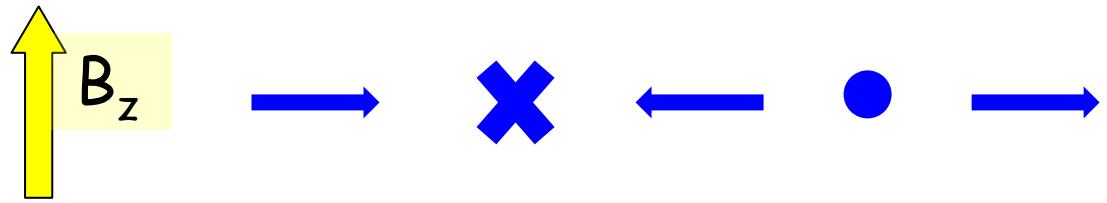


Spin rotation

$$\begin{cases} up : \cos \theta/2 e^{-i\phi/2} \\ dn : \sin \theta/2 e^{+i\phi/2} \end{cases}$$



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Spinors and Vectors **

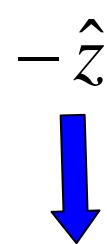
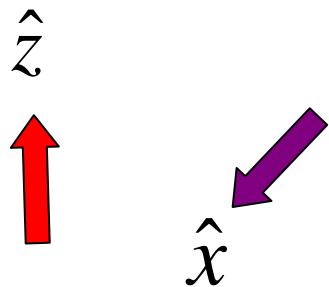
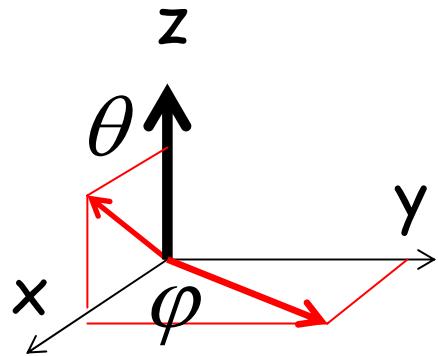
$$\begin{cases} up : \cos \theta/2 e^{-i\varphi/2} \\ dn : \sin \theta/2 e^{+i\varphi/2} \end{cases}$$

$$\begin{Bmatrix} 1 \\ 0 \end{Bmatrix}$$

$$\begin{Bmatrix} 1 \\ 1 \end{Bmatrix}$$

$$\begin{Bmatrix} 0 \\ 1 \end{Bmatrix}$$

$$\begin{Bmatrix} -1 \\ 0 \end{Bmatrix}$$



$$\begin{cases} x : \sin \theta \cos \varphi \\ y : \sin \theta \sin \varphi \\ z : \cos \theta \end{cases}$$

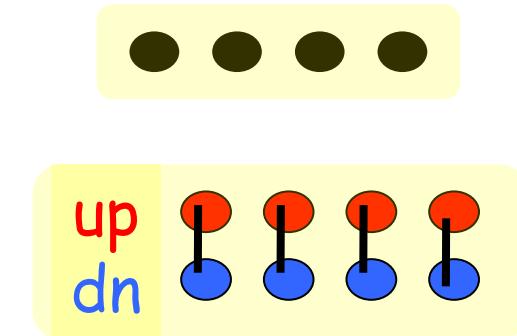
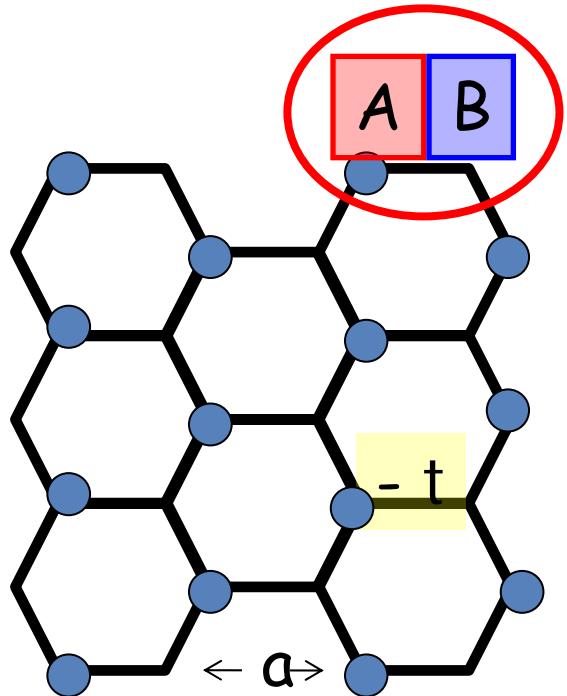
$$\begin{Bmatrix} 0 \\ 0 \\ 1 \end{Bmatrix}$$

$$\begin{Bmatrix} 1 \\ 0 \\ 0 \end{Bmatrix}$$

$$\begin{Bmatrix} 0 \\ 0 \\ -1 \end{Bmatrix}$$

$$\begin{Bmatrix} 0 \\ 0 \\ 1 \end{Bmatrix}$$

Spin-like entities



$$\begin{array}{c} \text{A} \\ \text{B} \end{array} \begin{cases} 1 \\ 0 \end{cases} \quad \begin{cases} 1 \\ 1 \end{cases} \quad \begin{cases} 0 \\ 1 \end{cases}$$

\hat{z}
 \uparrow
 \hat{x}
 \downarrow
 $-\hat{z}$

Spin Rotation in B_z

$$i\hbar \frac{d}{dt} \begin{Bmatrix} u \\ d \end{Bmatrix} = \mu_B B_z \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{Bmatrix} u \\ d \end{Bmatrix}$$

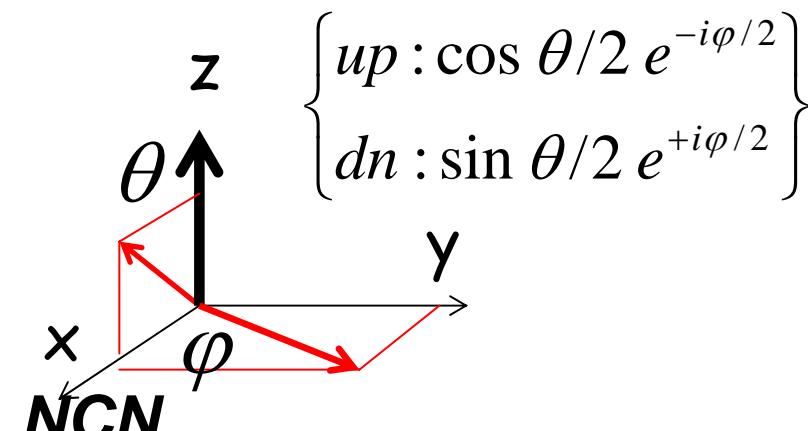
$$S_x = \cos \varphi(t),$$

$$S_y = \sin \varphi(t)$$

$$u(t) = \exp \left(-i \frac{\mu_B B_z t}{\hbar} \right) u(0)$$

$$d(t) = \exp \left(+i \frac{\mu_B B_z t}{\hbar} \right) d(0)$$

$$\varphi(t) = \varphi(0) + \frac{2\mu_B B_z}{\hbar} t$$



Spin Rotation in B_z

$$\frac{d}{dt} \begin{Bmatrix} S_x \\ S_y \\ S_z \end{Bmatrix} = \frac{2\mu_B B_z}{\hbar} \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} S_x \\ S_y \\ S_z \end{Bmatrix} \quad \frac{dS_x}{dt} = -S_y \frac{d\varphi}{dt} = -S_y \frac{2\mu_B B_z}{\hbar}$$

$$\frac{dS_y}{dt} = S_x \frac{d\varphi}{dt} = S_x \frac{2\mu_B B_z}{\hbar}$$

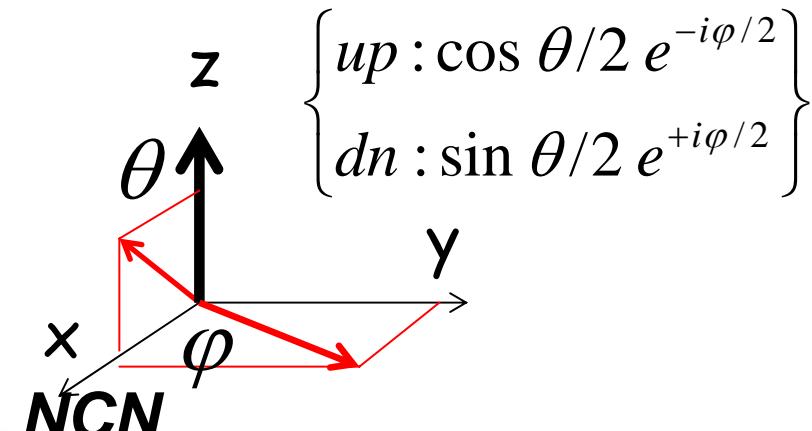
$$S_x = \cos \varphi(t),$$

$$S_y = \sin \varphi(t)$$

$$\frac{dS_z}{dt} = 0$$

$$\varphi(t) = \varphi(0) + \frac{2\mu_B B_z}{\hbar} t$$

$$\frac{d\varphi}{dt} = \frac{2\mu_B B_z}{\hbar} = \frac{q}{m} B_z$$



Spin Rotation in B_z

$$\frac{d}{dt} \begin{Bmatrix} S_x \\ S_y \\ S_z \end{Bmatrix} = \frac{2\mu_B B_z}{\hbar} \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} S_x \\ S_y \\ S_z \end{Bmatrix}$$

$$i\hbar \frac{d}{dt} \begin{Bmatrix} u \\ d \end{Bmatrix} = \mu_B B_z \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{Bmatrix} u \\ d \end{Bmatrix}$$

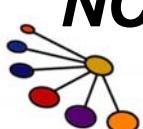
$$\frac{d}{dt} \vec{S} = \frac{2\mu_B \vec{B}}{\hbar} \times \vec{S}$$

$$\frac{d}{dt} \vec{\mu} = - \underbrace{\frac{q}{m}}_{\gamma} \vec{\mu} \times \vec{B}$$

$\left. \begin{array}{l} up : \cos \theta/2 e^{-i\varphi/2} \\ dn : \sin \theta/2 e^{+i\varphi/2} \end{array} \right\}$

$$\varphi(t) = \varphi(0) + \frac{2\mu_B B_z}{\hbar} t$$

$$\frac{d\varphi}{dt} = \frac{2\mu_B B_z}{\hbar} = \frac{q}{m} B_z$$



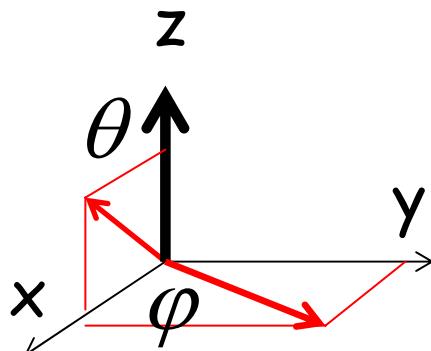
Spin Rotation in B_x

$$\frac{d}{dt} \begin{Bmatrix} S_x \\ S_y \\ S_z \end{Bmatrix} = \frac{2\mu_B B_z}{\hbar} \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} S_x \\ S_y \\ S_z \end{Bmatrix}$$

$$\frac{d}{dt} \begin{Bmatrix} S_x \\ S_y \\ S_z \end{Bmatrix} = \frac{2\mu_B B_x}{\hbar} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \begin{Bmatrix} S_x \\ S_y \\ S_z \end{Bmatrix}$$

$$\frac{d}{dt} \begin{Bmatrix} u \\ d \end{Bmatrix} = \frac{\mu_B B_z}{i\hbar} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{Bmatrix} u \\ d \end{Bmatrix}$$

$$\frac{d}{dt} \begin{Bmatrix} u \\ d \end{Bmatrix} = \frac{\mu_B B_x}{i\hbar} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{Bmatrix} u \\ d \end{Bmatrix}$$



$$\varphi(t) = \varphi(0) + \frac{2\mu_B B}{\hbar} t$$

Rotation operators

$$\frac{d}{dt} \begin{Bmatrix} S_x \\ S_y \\ S_z \end{Bmatrix} = \frac{2\mu_B B_z}{\hbar} \underbrace{\begin{bmatrix} 0-10 \\ 100 \\ 000 \end{bmatrix}}_{R_z} \begin{Bmatrix} S_x \\ S_y \\ S_z \end{Bmatrix}$$

$$\frac{d}{dt} \begin{Bmatrix} u \\ d \end{Bmatrix} = \frac{\mu_B B_z}{i\hbar} \underbrace{\begin{bmatrix} 10 \\ 0-1 \end{bmatrix}}_{\sigma_z} \begin{Bmatrix} u \\ d \end{Bmatrix}$$

$$\frac{d}{dt} \begin{Bmatrix} S_x \\ S_y \\ S_z \end{Bmatrix} = \frac{2\mu_B B_x}{\hbar} \underbrace{\begin{bmatrix} 000 \\ 00-1 \\ 010 \end{bmatrix}}_{R_x} \begin{Bmatrix} S_x \\ S_y \\ S_z \end{Bmatrix}$$

$$\frac{d}{dt} \begin{Bmatrix} u \\ d \end{Bmatrix} = \frac{\mu_B B_x}{i\hbar} \underbrace{\begin{bmatrix} 01 \\ 10 \end{bmatrix}}_{\sigma_x} \begin{Bmatrix} u \\ d \end{Bmatrix}$$

$$R_x R_y - R_y R_x = R_z$$

$$\frac{\sigma_x}{2i} \frac{\sigma_y}{2i} - \frac{\sigma_y}{2i} \frac{\sigma_x}{2i} = \frac{\sigma_z}{2i}$$

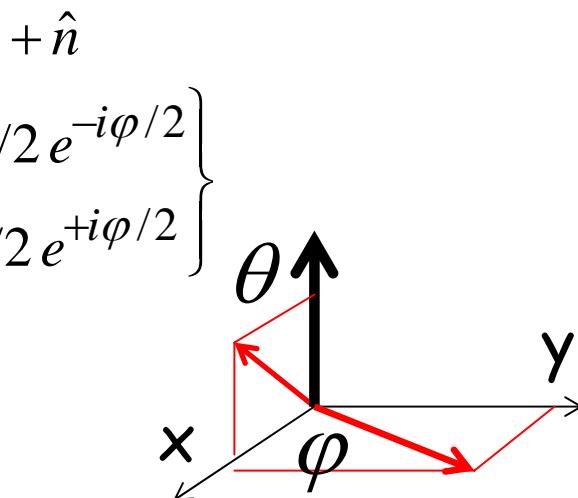
$$\sigma_x \sigma_y - \sigma_y \sigma_x = 2i \sigma_z$$

Rotation operators

Eigenvectors

$$\begin{array}{ll} +\hat{z} & -\hat{z} \\ \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} & \begin{Bmatrix} 0 \\ 1 \end{Bmatrix} \end{array}$$

$$\frac{d}{dt} \begin{Bmatrix} u \\ d \end{Bmatrix} = \frac{\mu_B B_z}{i\hbar} \underbrace{\begin{bmatrix} 10 \\ 0-1 \end{bmatrix}}_{\sigma_z} \begin{Bmatrix} u \\ d \end{Bmatrix}$$



$$\frac{d}{dt} \begin{Bmatrix} u \\ d \end{Bmatrix} = \frac{\mu_B B_x}{i\hbar} \underbrace{\begin{bmatrix} 01 \\ 10 \end{bmatrix}}_{\sigma_x} \begin{Bmatrix} u \\ d \end{Bmatrix}$$

$$\frac{d}{dt} \begin{Bmatrix} u \\ d \end{Bmatrix} = \frac{\mu_B B_y}{i\hbar} \underbrace{\begin{bmatrix} 0-i \\ i0 \end{bmatrix}}_{\sigma_y} \begin{Bmatrix} u \\ d \end{Bmatrix}$$

$$\sigma_x \sigma_y - \sigma_y \sigma_x = 2i\sigma_z$$

Rotation operators

Eigenvectors

$$\begin{array}{cc} +\hat{x} & -\hat{x} \\ \begin{Bmatrix} 1 \\ 1 \end{Bmatrix} & \begin{Bmatrix} -1 \\ 1 \end{Bmatrix} \end{array}$$

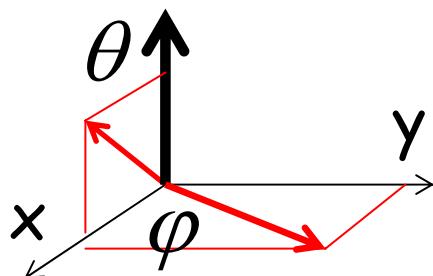
$$\frac{d}{dt} \begin{Bmatrix} u \\ d \end{Bmatrix} = \frac{\mu_B B_x}{i\hbar} \underbrace{\begin{Bmatrix} 01 \\ 10 \end{Bmatrix}}_{\sigma_x} \begin{Bmatrix} u \\ d \end{Bmatrix}$$

$+\hat{n}$

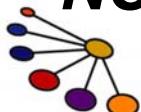
$$\begin{Bmatrix} \cos \theta/2 e^{-i\varphi/2} \\ \sin \theta/2 e^{+i\varphi/2} \end{Bmatrix}$$

$$\frac{d}{dt} \begin{Bmatrix} u \\ d \end{Bmatrix} = -i K \begin{Bmatrix} u \\ d \end{Bmatrix}$$

$$\begin{Bmatrix} u \\ d \end{Bmatrix} = e^{-iKt} \begin{Bmatrix} u \\ d \end{Bmatrix}$$



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Rotation operators

Eigenvectors

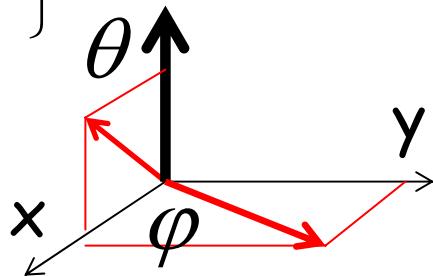
$+\hat{n}$

$$\begin{Bmatrix} \cos\theta/2 e^{-i\phi/2} \\ \sin\theta/2 e^{+i\phi/2} \end{Bmatrix}$$

$-\hat{n}$

$$[\vec{\sigma} \cdot \hat{n}]$$

$$\begin{Bmatrix} -\sin\theta/2 e^{-i\phi/2} \\ \cos\theta/2 e^{+i\phi/2} \end{Bmatrix}$$



$$\begin{array}{ll} +\hat{z} & -\hat{z} \\ \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} & \begin{Bmatrix} 0 \\ 1 \end{Bmatrix} \end{array}$$

$$\begin{array}{ll} +\hat{x} & -\hat{x} \\ \begin{Bmatrix} 1 \\ 1 \end{Bmatrix} & \begin{Bmatrix} -1 \\ 1 \end{Bmatrix} \end{array}$$

$$\begin{array}{ll} +\hat{y} & -\hat{y} \\ \begin{Bmatrix} 1 \\ i \end{Bmatrix} & \begin{Bmatrix} 1 \\ -i \end{Bmatrix} \end{array}$$

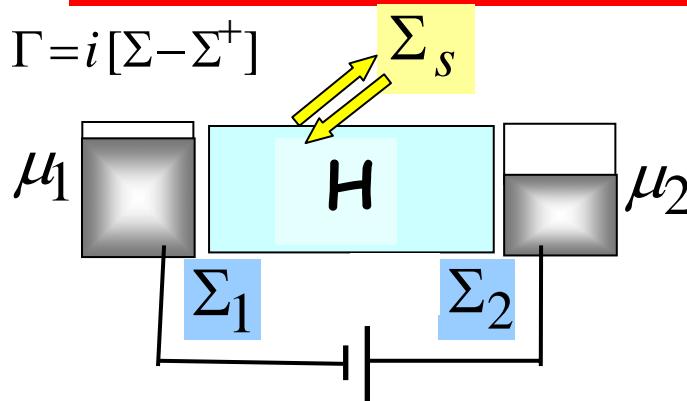
$$\frac{d}{dt} \begin{Bmatrix} u \\ d \end{Bmatrix} = \frac{\mu_B B_z}{i\hbar} \underbrace{\begin{bmatrix} 10 \\ 0-1 \end{bmatrix}}_{\sigma_z} \begin{Bmatrix} u \\ d \end{Bmatrix}$$

$$\frac{d}{dt} \begin{Bmatrix} u \\ d \end{Bmatrix} = \frac{\mu_B B_x}{i\hbar} \underbrace{\begin{bmatrix} 01 \\ 10 \end{bmatrix}}_{\sigma_x} \begin{Bmatrix} u \\ d \end{Bmatrix}$$

$$\frac{d}{dt} \begin{Bmatrix} u \\ d \end{Bmatrix} = \frac{\mu_B B_y}{i\hbar} \underbrace{\begin{bmatrix} 0-i \\ i0 \end{bmatrix}}_{\sigma_y} \begin{Bmatrix} u \\ d \end{Bmatrix}$$

$$\sigma_x \sigma_y - \sigma_y \sigma_x = 2i\sigma_z$$

** NEGF-Landauer equations



Green
function

$$[G] = [EI - H - \Sigma_1 - \Sigma_2 - \Sigma_s]^{-1}$$

"Density of states" $A = i[G - G^+]$

"Electron density"

$$G^n = G\Gamma_2 G^+ f_2 + G\Gamma_1 G^+ f_1 + G\Sigma_s^{in} G^+$$

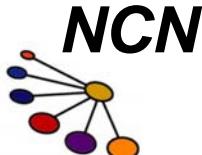
Current

$$\frac{I_1}{q/\hbar} = \text{Trace} \left([\Gamma_1 A] f_1 - [\Gamma_1 G^n] \right)$$

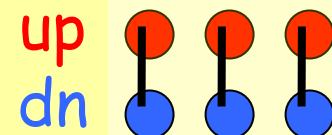
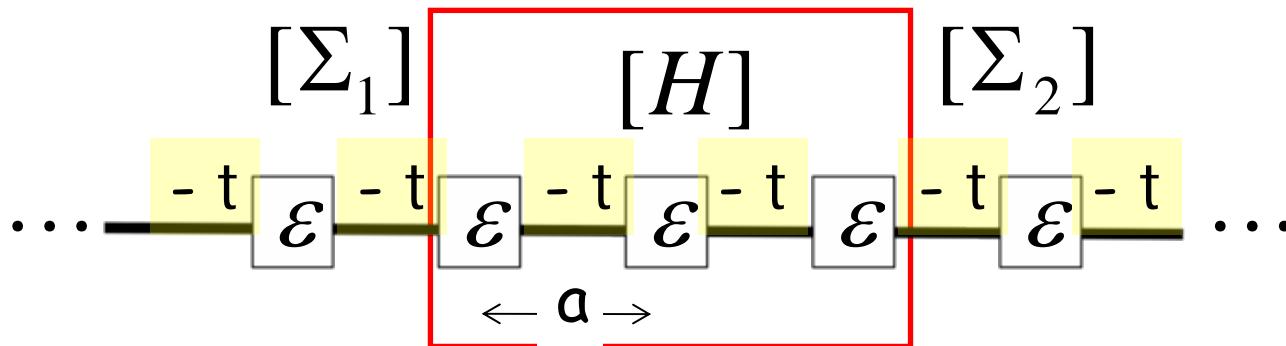
Dephasing model:

$$[\Sigma_s^{in}] = D[G^n]$$

All matrices double
From $N \times N$
To $2N \times 2N$

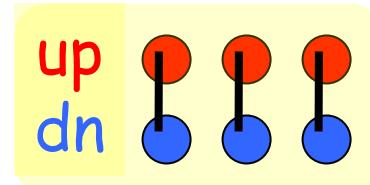
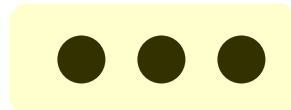
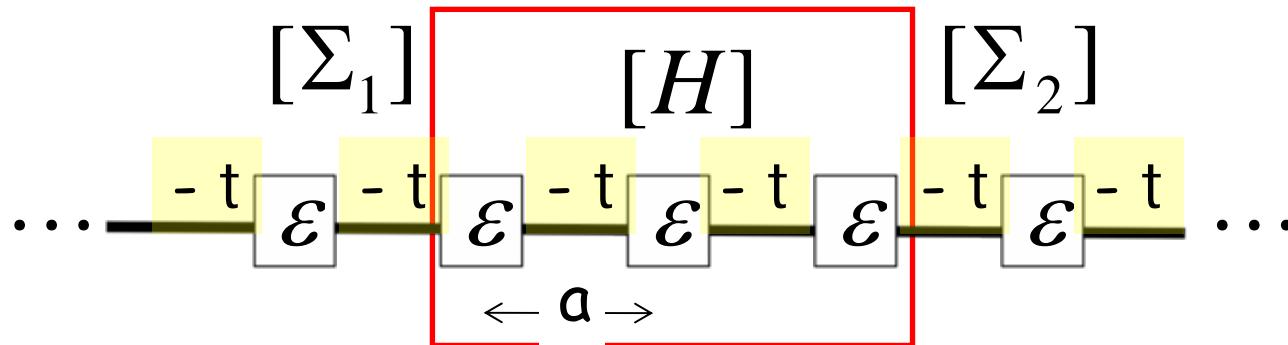


Extended devices (1-D)



$$H = \begin{bmatrix} \varepsilon & t & 0 \\ t & \varepsilon & t \\ 0 & t & \varepsilon \end{bmatrix} \quad \begin{aligned} \varepsilon &\rightarrow \varepsilon I + \mu_B \vec{\sigma} \cdot \vec{B} \\ -t &\rightarrow -t I \end{aligned}$$

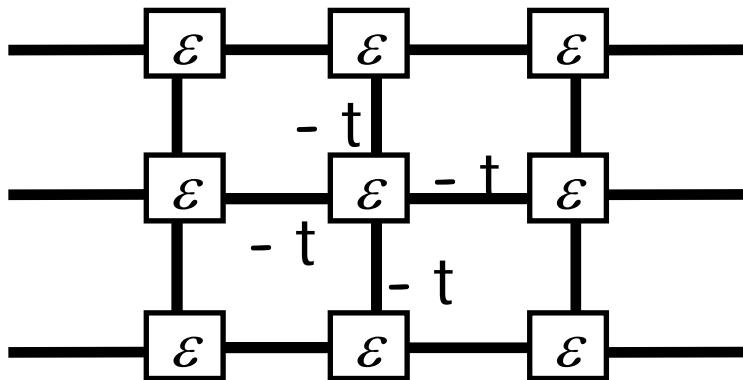
Contact self-energy for extended devices (1-D)



$$-te^{ika} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} - te^{ika} \rightarrow -te^{ika} I \equiv -te^{ika} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$
$$-te^{ika} \begin{bmatrix} \hat{z} \\ \uparrow \end{bmatrix} - te^{ika} \begin{bmatrix} \alpha & 0 \\ 0 & \beta \end{bmatrix}$$

Spin-orbit (Rashba) Hamiltonian

2-D model



$$E = \varepsilon - te^{+ik_x a} - te^{-ik_x a}$$

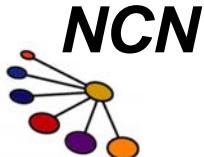
$$- te^{+ik_y a} - te^{-ik_y a}$$

$$= \varepsilon - 2t(\cos k_x a + \cos k_y a)$$

$$E(k_x, k_y)$$

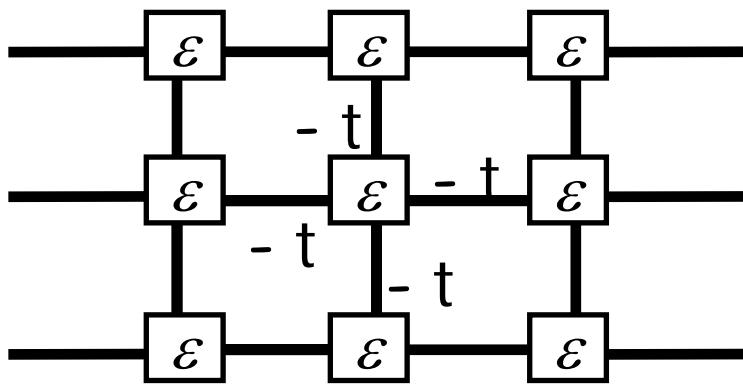
$$\rightarrow (\varepsilon - 4t) + \frac{\hbar^2}{2m}(k_x^2 + k_y^2)$$

$$= E_c + \frac{\hbar^2}{2m}(k_x^2 + k_y^2)$$



Spin-orbit (Rashba) Hamiltonian

2-D model



$$H = \mu_B \vec{\sigma} \cdot \vec{B}$$

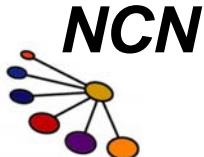
$$\vec{B}_{eff} \sim \frac{\vec{E} \times \vec{p}}{2mc^2}$$

$$H \sim \vec{\sigma} \cdot (\vec{E} \times \vec{k})$$

$$= -\vec{E} \cdot (\vec{\sigma} \times \vec{k})$$

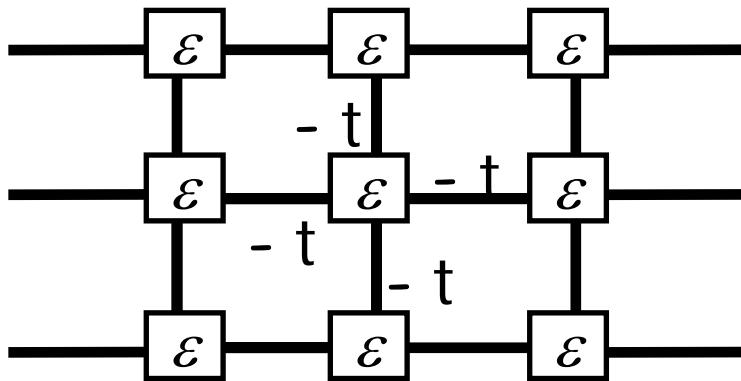
$$E(k_x, k_y)$$

$$= E_c + \frac{\hbar^2}{2m} (k_x^2 + k_y^2) + \alpha (\sigma_x k_y - \sigma_y k_x)$$



Spin-orbit (Rashba) Hamiltonian

2-D devices



$$E = \varepsilon - t e^{+ik_x a} - t e^{-ik_x a}$$

$$- t e^{+ik_y a} - t e^{-ik_y a}$$

$$= \varepsilon - 2t(\cos k_x a + \cos k_y a)$$

$$\rightarrow (\varepsilon - 4t) + \frac{\hbar^2}{2m} (k_x^2 + k_y^2)$$

$$- t_x \rightarrow \frac{i\alpha}{2a} [\sigma_y]$$

$$- t_y \rightarrow - \frac{i\alpha}{2a} [\sigma_x]$$

$$E = \frac{i\alpha}{2a} \begin{pmatrix} \sigma_y e^{+ik_x a} - \sigma_y e^{-ik_x a} \\ - \sigma_x e^{+ik_y a} + \sigma_x e^{-ik_y a} \end{pmatrix}$$

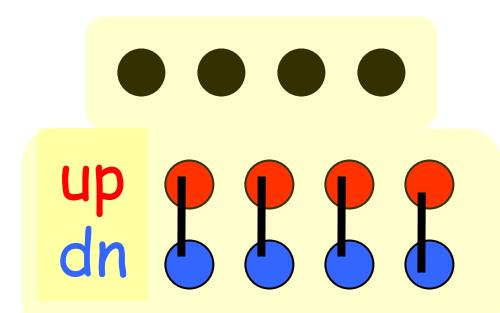
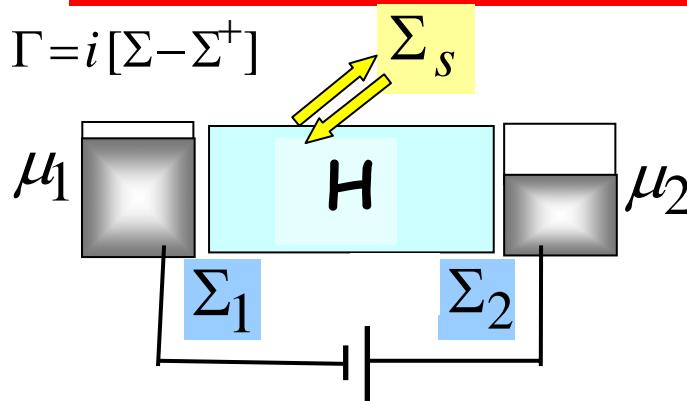
$$\frac{\alpha}{a} (\sigma_x \sin k_y a - \sigma_y \sin k_x a)$$

$$+ \alpha (\sigma_x k_y - \sigma_y k_x)$$

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** NEGF-Landauer equations



All matrices double
From $N \times N$
To $2N \times 2N$

Green
function

$$[G] = [EI - H - \Sigma_1 - \Sigma_2 - \Sigma_s]^{-1}$$

"Density of states" $A = i[G - G^+]$

"Electron density"

$$G^n = G\Gamma_2 G^+ f_2 + G\Gamma_1 G^+ f_1 + G\Sigma_s^{in} G^+$$

Current

$$\frac{I_1}{q/\hbar} = \text{Trace} \left([\Gamma_1 A] f_1 - [\Gamma_1 G^n] \right)$$

Dephasing model:

$$[\Sigma_s^{in}] = D[G^n]$$

Wavefunction <--> Correlation function

Wavefunction
↓

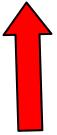
$$\begin{Bmatrix} u \\ d \end{Bmatrix} \{u^* \quad d^*\}$$

$$= \begin{bmatrix} uu^* & ud^* \\ du^* & dd^* \end{bmatrix}$$

Correlation
Function, G^n

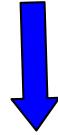
$$\begin{Bmatrix} 1 \\ 0 \end{Bmatrix}$$

$$\hat{z}$$



$$\begin{Bmatrix} 0 \\ 1 \end{Bmatrix}$$

$$-\hat{z}$$

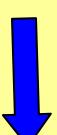


$$\frac{1}{\sqrt{2}} \begin{Bmatrix} 1 \\ 1 \end{Bmatrix}$$

$$\hat{x}$$

??

$$\hat{z} - \hat{z}$$



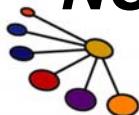
$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} .5 & .5 \\ .5 & .5 \end{bmatrix}$$

$$\begin{bmatrix} .5 & 0 \\ 0 & .5 \end{bmatrix}^* {}^2$$

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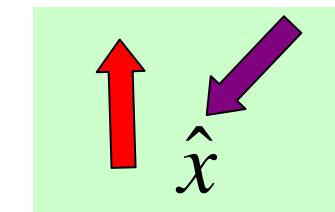
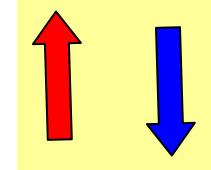
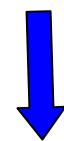
Spin density

$$S_z = \text{Trace} [G^n \sigma_z] = \text{Trace} \begin{bmatrix} a & c \\ c^* & b \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = a - b$$

$$S_x = \text{Trace} [G^n \sigma_x] = \text{Trace} \begin{bmatrix} a & c \\ c^* & b \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = c + c^*$$

$$n = \text{Trace} [G^n I] = a + b$$

$$S_z = 1 \quad S_z = -1 \quad S_x = 1 \quad S_z = S_x = 0 \quad S_z = S_x = 1$$



$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

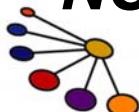
$$\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} .5 & .5 \\ .5 & .5 \end{bmatrix}$$

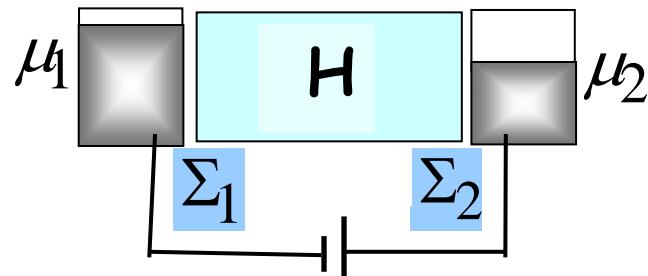
$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1.5 & .5 \\ .5 & .5 \end{bmatrix}$$

Correlation Function, G^n

NCN





$$\Gamma = i[\Sigma - \Sigma^+]$$

$$[EI - H - \Sigma_1 - \Sigma_2] \{\psi\} = \{S_1\}$$

$$\{\psi\} = [G] \{S_1\}$$

$$A = i[G - G^+]$$

$$\begin{aligned} i\hbar \frac{\partial}{\partial t} \{\psi\} \{\psi\}^+ &= [HG^n - G^n H] \\ &+ [\Sigma_1^{in} G^+ - G \Sigma_1^{in} + \Sigma_1 G^n - G^n \Sigma_1^+] \\ &+ [\Sigma_2^{in} G^+ - G \Sigma_2^{in} + \Sigma_2 G^n - G^n \Sigma_2^+] \end{aligned}$$

Correlation function

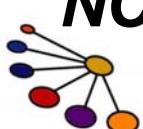
$$G^n = \{\psi\} \{\psi^+\} =$$

$$\begin{bmatrix} \psi_1 \psi_1^* & \vdash & \psi_1 \psi_N^* \\ \vdash & & \vdash \\ \psi_N \psi_1^* & \vdash & \psi_N \psi_N^* \end{bmatrix}$$

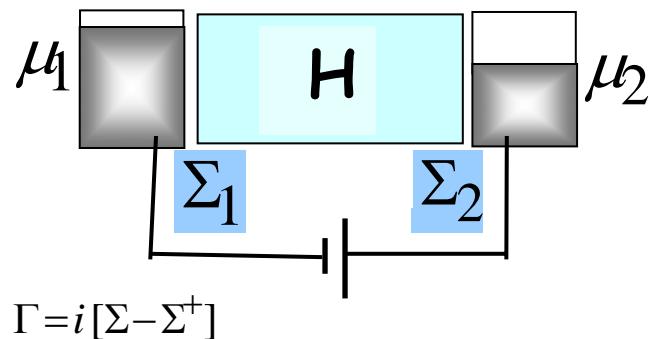
$$\begin{aligned} \hbar \frac{\partial N}{\partial t} &= Trace [\Sigma_1^{in} A - \Gamma_1 G^n] \\ &+ Trace [\Sigma_2^{in} A - \Gamma_2 G^n] \end{aligned}$$

$$\frac{I_1}{q/\hbar} = Trace \left(\overbrace{[\Sigma_1^{in} A]}^{[\Gamma_1] f_1} - [\Gamma_1 G^n] \right)$$

NCN



Spin current



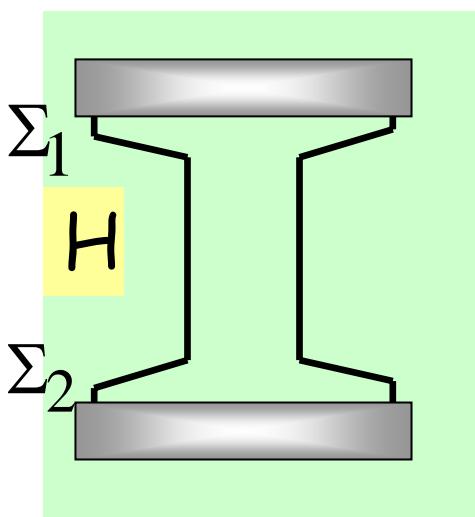
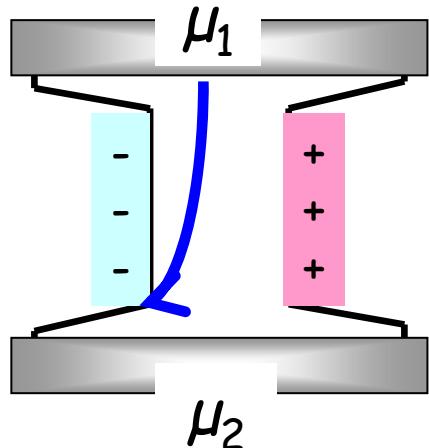
$$\begin{aligned}
 i\hbar \frac{\partial}{\partial t} \{\psi\} \{\psi\}^+ &= [HG^n - G^n H] \\
 &+ [\Sigma_1^{in} G^+ - G \Sigma_1^{in} + \Sigma_1 G^n - G^n \Sigma_1^+] \\
 &+ [\Sigma_2^{in} G^+ - G \Sigma_2^{in} + \Sigma_2 G^n - G^n \Sigma_2^+]
 \end{aligned}$$

$$A = i[G - G^+]$$

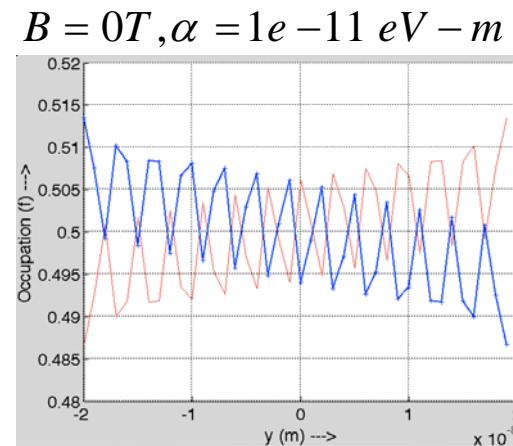
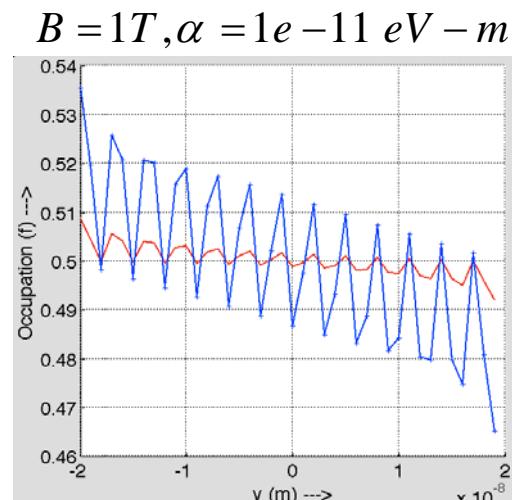
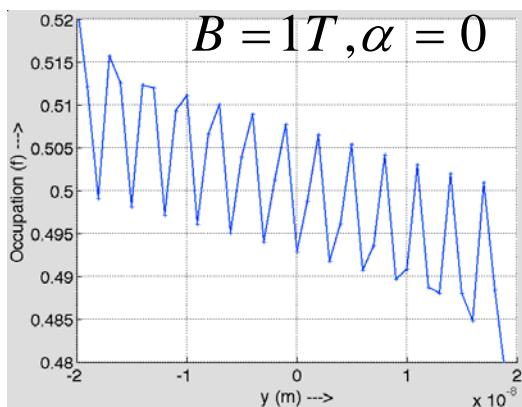
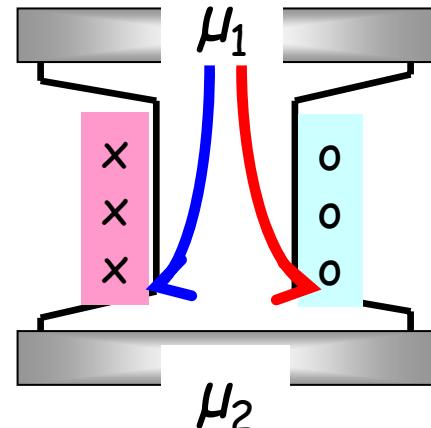
$$\begin{aligned}
 \hbar \frac{\partial S_i}{\partial t} &= \text{Trace } \sigma_i [HG^n - G^n H] \quad \sim \text{ Spin transfer} \\
 &+ \text{Trace } \sigma_i [\Sigma_1^{in} G^+ - G \Sigma_1^{in} + \Sigma_1 G^n - G^n \Sigma_1^+] \quad \sim I_{1,spin} \\
 &+ \text{Trace } \sigma_i [\Sigma_2^{in} G^+ - G \Sigma_2^{in} + \Sigma_2 G^n - G^n \Sigma_2^+] \quad + I_{2,spin}
 \end{aligned}$$

Example: Hall effect versus Spin-Hall effect

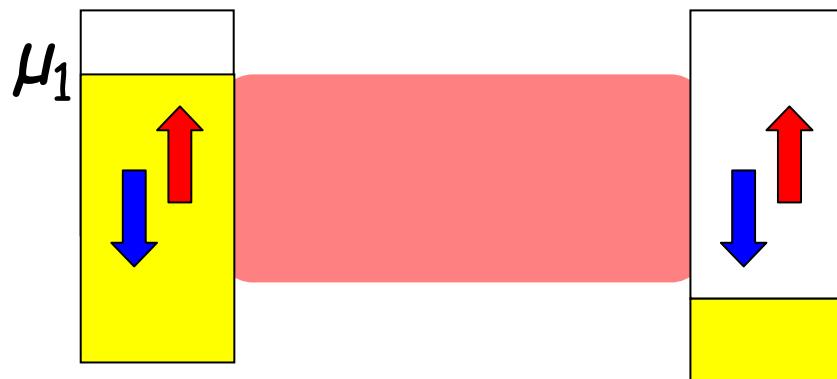
$$H = \frac{(p + eA)^2}{2m^*}, \quad \nabla \times A = B$$



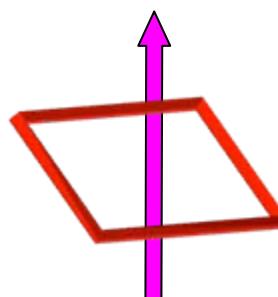
$$H = \frac{p^2}{2m^*} + \frac{\alpha}{h}(\sigma_x p_y - \sigma_y p_x)$$



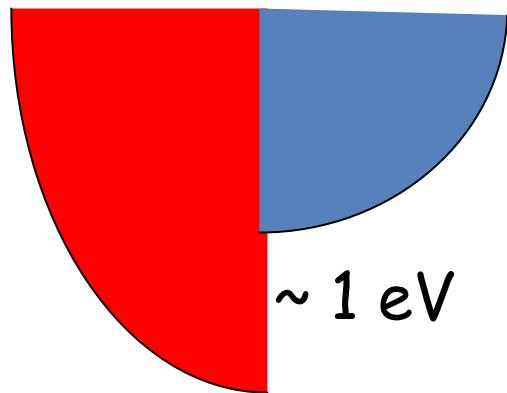
** Spins and magnets



$$N\mu_B \sim 10^6 A/m$$
$$\rightarrow 1 T$$



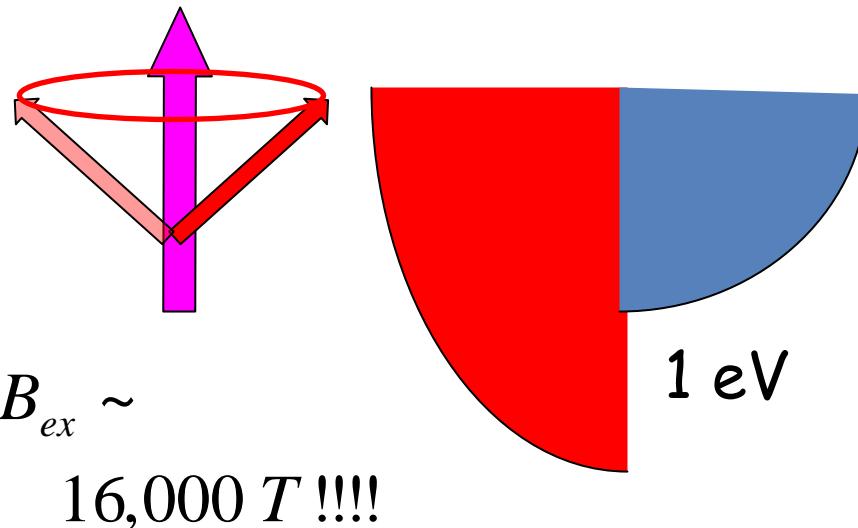
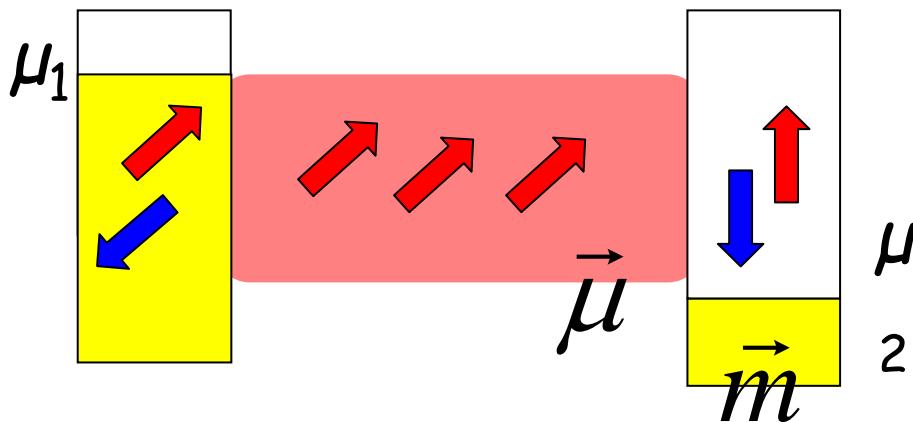
$$\mu_B \sim 10^{-23} A - m^2$$
$$= 1 mA - 1 Ang^2$$



$$\mu_B B = 1 eV$$

$$B = \frac{1.6e-19 J}{1e-23 A - m^2} = 16,000 T !!!!$$

Spin Torque



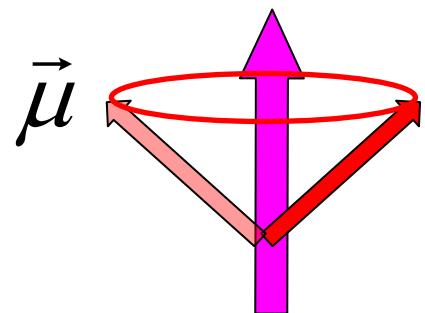
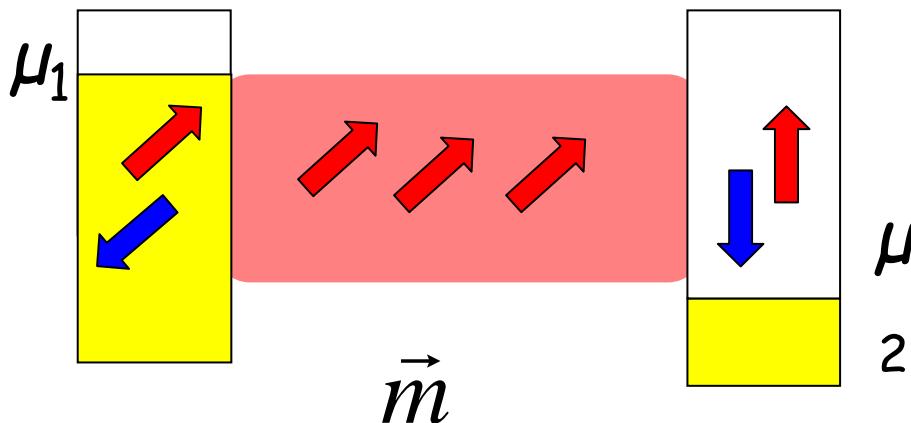
Torque due to
magnet on
moving electrons

$$\frac{d}{dt} \vec{\mu} = -\gamma (\vec{\mu} \times \vec{B}_{ex})$$

= - (Torque due to
moving electrons
on magnet)

$$= - \frac{d}{dt} \vec{m}$$

Spin Torque



$B_{ex} \sim$
16,000 T !!!

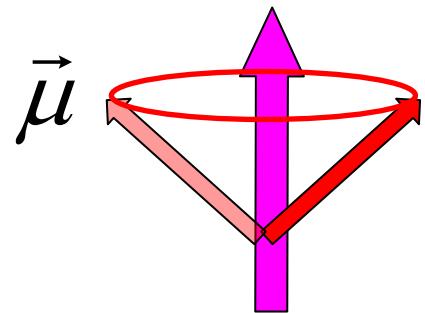
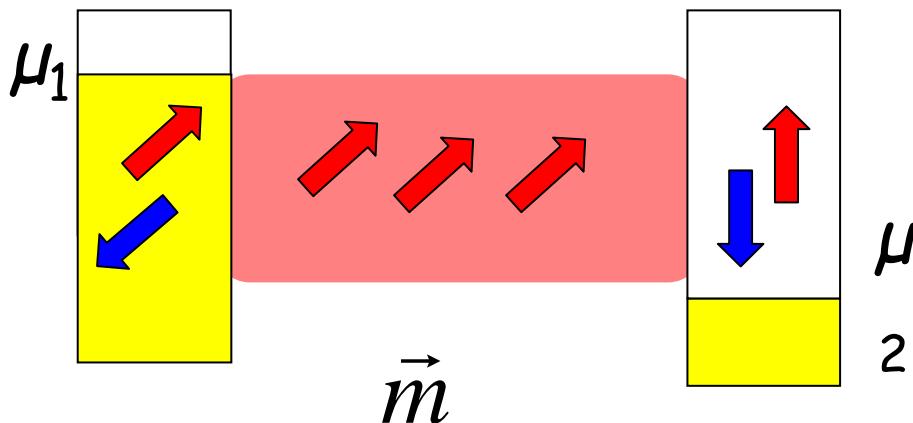
Torque due to
magnet on
moving electrons

$$\frac{d}{dt} \vec{\mu} = -\gamma (\vec{\mu} \times \vec{B}_{ex})$$

= - (Torque due to
moving electrons
on magnet)

$$= - \frac{d}{dt} \vec{m}$$

Spin Torque



$$\frac{d}{dt} \vec{m} = \gamma (\vec{\mu} \times \vec{B}_{ex})$$

$$= -\frac{\gamma B_{ex}}{m} (\vec{m} \times \vec{\mu})$$

$$\frac{d}{dt} \vec{m} = -\gamma (\vec{m} \times \vec{B}_{eff})$$

Magnet feels an effective field due to moving spins μ that is given by

$$\vec{B}_{eff} = B_{ex} \frac{\vec{\mu}}{m}$$

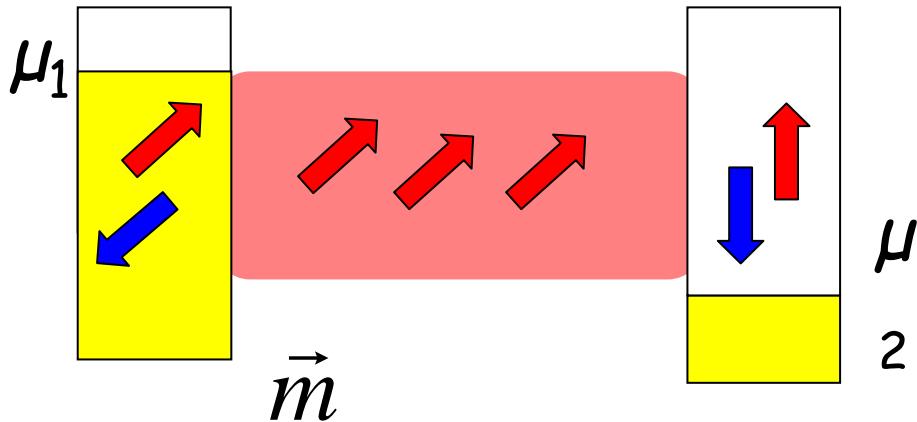
$$B_{ex} \sim 16,000 T !!!!$$

Moving spins of $\mu \sim 10^{-6} \text{ m}$ gives $B_{eff} \sim 160G$

NCN

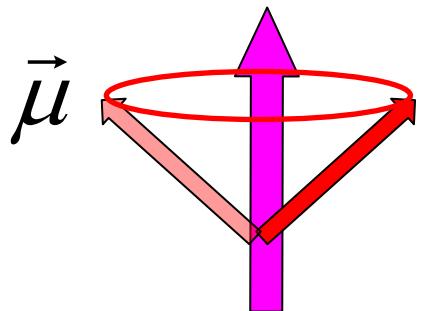


Spin Torque



Steady state : $\vec{m} \parallel \vec{B}_{eff}$

*But not necessarily
for spin-torque*



$$\vec{B}_{eff} = B_{ex} \frac{\vec{\mu}}{m}$$

$$\frac{d}{dt} \vec{m} = -\gamma (\vec{m} \times \vec{B}_{eff})$$

– Damping term

*Because of the nature of
 $\vec{\mu}(\vec{m})$:*

Salahuddin et al. 2008

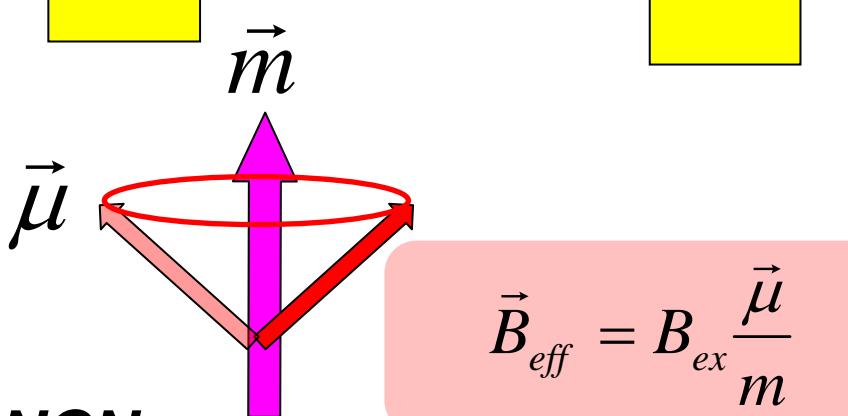
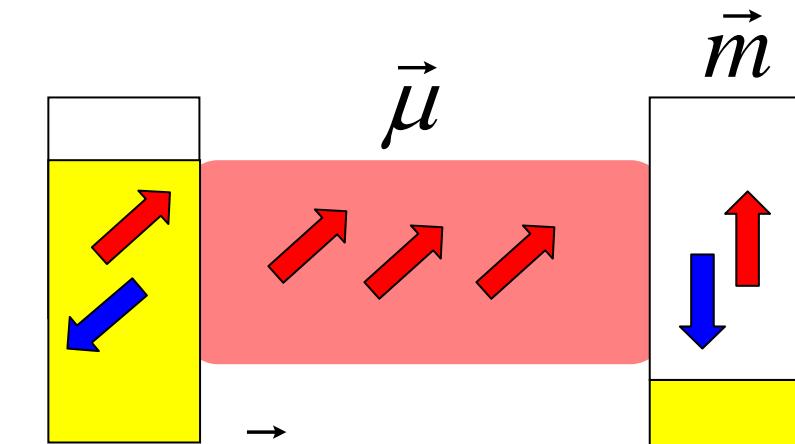
Spins and Magnets

SPINTRONICS

Injection, detection and manipulation
of spins: GMR/TMR, Spin-Hall

MAGNETICS:

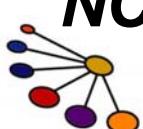
Controlling magnets
with currents



$$\vec{B}_{eff} = B_{ex} \frac{\vec{\mu}}{m}$$

Using spins to control
NANOMAGNETS,

which acts like a
digital non-volatile capacitor
for the spins.



Nanoelectronics and the meaning of Resistance

