Notes on the
Solution of the Poisson-Boltzmann Equation
for MOS Capacitors and MOSFETs
2nd Edition

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1. Introduction

These notes are intended to complement discussions in standard textbook such as *Fundamentals of Modern VLSI Devices* by Yuan Taur and Tak H. Ning [1] and *Semiconductor Device Fundamentals* by R.F. Pierret [2].) The objective here is to understand how to treat MOS electrostatics without making the so-called δ-depletion approximation. We assume a bulk MOS structure, but a similar approach could be used for SOI structures.

2. MOS Electrostatics: Electric Field vs. Position

We begin with Poisson's equation:

$$\frac{d^2\psi}{dx^2} = \frac{-q}{\varepsilon_{Si}}\left[p(x) - n(x) + N_D^+ - N_A^- \right] \qquad (0)$$

and assume a p-type semiconductor for which $N_D = 0$. Complete ionization of dopants will be assumed ($N_A^- = N_A$). In the bulk, we have space charge neutrality, so $p_B - n_B - N_A = 0$, which means

$$N_A = p_B - n_B, \tag{1}$$

so Poisson's equation becomes

$$\frac{d^2\psi}{dx^2} = \frac{-q}{\varepsilon_{Si}}\left[p(x) - n(x) + n_B - p_B \right], \tag{2}$$

where

$$\begin{aligned} p_B &\cong N_A \\ n_B &\cong n_i^2 / N_A \end{aligned}. \tag{3}$$

Using eqn. (3), we can express eqn. (2) as

$$\frac{d^2\psi}{dx^2} = \frac{-q}{\varepsilon_{S_i}}\left[p(x) - N_A - n(x) + n_i^2 / N_A \right]. \tag{4}$$

The energy bands vary with position when the electrostatic potential varies with position. For example, a positive gate voltage bends the bands down, and a negative gate voltage bends the bands up (because a positive potential lowers the electron energy). If we assume equilibrium, then the equilibrium carrier densities in the semiconductor can be related to the electrostatic potential by

$$p(x) = N_A e^{-q\psi(x)/k_B T_L} \tag{5}$$

and

$$n(x) = \frac{n_i^2}{N_A} e^{+q\psi(x)/k_B T_L}. \tag{6}$$

Finally, using eqns. (5) and (6) in (4), we find the *Poisson-Boltzmann equation*,

$$\boxed{\frac{d^2\psi}{dx^2} = \frac{-q}{\varepsilon_{Si}}\left[N_A\left(e^{-q\psi/k_B T_L} - 1\right) - \frac{n_i^2}{N_A}\left(e^{q\psi/k_B T_L} - 1\right)\right]} \tag{7}$$

which we need to integrate twice to find $\psi(x)$.

To integrate eqn. (7), we begin with

$$\frac{d^2\psi}{dx^2} = \frac{d}{dx} \times \left(\frac{d\psi}{dx} \right)$$

and use the chain rule,

$$\frac{d^2\psi}{dx^2} = \frac{d}{d\psi} \times \left(\frac{d\psi}{dx} \right) \times \frac{d\psi}{dx}.$$

If we let $p = \dfrac{d\psi}{dx}$, then

$$\frac{d^2\psi}{dx^2} = p\frac{dp}{d\psi},$$

and eqn. (7) becomes

$$p\frac{dp}{d\psi} = \frac{-q}{\varepsilon_{Si}}\left[N_A(e^{-q\psi/k_BT_L} - 1) - \frac{n_i^2}{N_A}(e^{q\psi/k_BT_L} - 1) \right]. \tag{8}$$

Now integrate eqn. (8) with respect to $d\psi$,

$$pdp = \frac{-q}{\varepsilon_{Si}}\left[N_A(e^{-q\psi/k_BT_L} - 1) - \frac{n_i^2}{N_A}(e^{q\psi/k_BT_L} - 1) \right] d\psi$$

$$\int_0^p p'\,dp' = \frac{-q}{\varepsilon_{Si}}\int_o^\psi \left[N_A(e^{-q\psi/k_BT_L} - 1) - \frac{n_i^2}{N_A}(e^{+q\psi/k_BT_L} - 1) \right] d\psi$$

$$\frac{p^2}{2} = \frac{-q}{\varepsilon_{Si}}\left[N_A\left\{ \left(\frac{-k_BT_L}{q} \right) e^{-q\psi/k_BT_L} \Big|_o^\psi - \psi \right\} - \frac{n_i^2}{N_A}\left\{ \left(\frac{k_BT_L}{q} \right) e^{q\psi/k_BT_L} \Big|_o^\psi - \psi \right\} \right]$$

$$p^2 = \frac{2k_BT_LN_A}{\varepsilon_{Si}}\left[\left(e^{-q\psi/k_BT_L} + \frac{q\psi}{k_BT_L} - 1 \right) + \frac{n_i^2}{N_A^2}\left(e^{q\psi/k_BT_L} - \frac{q\psi}{k_BT_L} - 1 \right) \right].$$

Recall that $p = \frac{d\psi}{dx}$, so we can take the square root of the final result to find

$$\mathcal{E}(x) = -\frac{d\psi(x)}{dx} = \pm\sqrt{\frac{2k_B T_L N_A}{\varepsilon_{Si}}}\, F(\psi), \tag{9}$$

where

$$F(\psi) = \sqrt{\left(e^{-q\psi/k_B T_L} + \frac{q\psi}{k_B T_L} - 1\right) + \frac{n_i^2}{N_A^2}\left(e^{+q\psi/k_B T_L} - \frac{q\psi}{k_B T_L} - 1\right)}\,. \tag{10}$$

If $\psi > 0$, choose the "+" sign (for a p-type semiconductor), and if $\psi < 0$, choose the "-" sign. Equations (9) and (10) give the position-dependent electric field within the semiconductor *if* we know the position-dependent electrostatic potential.

The surface electric field is an important quantity in MOS electrostatics. From eqn. (9) evaluated at the surface (x = 0) where the electrostatic potential, $\psi(x=0)$, is the surface potential, ψ_S, we find the surface electric field as

$$\mathcal{E}_S = \pm\sqrt{\frac{2k_B T_L N_A}{\varepsilon_{Si}}}\, F(\psi_S)\ . \tag{11}$$

From eqn. (11) and Gauss's Law, we find the total charge in the semiconductor as a function of the surface potential as

$$\boxed{Q_S(\psi_S) = -\varepsilon_{Si}\mathcal{E}_S = \mp\sqrt{2\varepsilon_{Si} k_B T_L N_A}\, F(\psi_s)}\ , \tag{12}$$

which is **an important and frequently-used result**. For the assumed p-type semiconductor, the - sign in front of the square root is used when $\psi_S > 0$ and the + sign when $\psi_S < 0$.

Example 1: Semiconductor Charge vs. Surface Potential

Consider a p-type, silicon MOS capacitor at room temperature with $N_A = 5\times10^{17}\,\text{cm}^{-3}$. Plot $|Q_S/q|$ vs. surface potential, ψ_S.

A Matlab script to perform this calculation is included in the appendix. The result is shown below in Fig. 1.

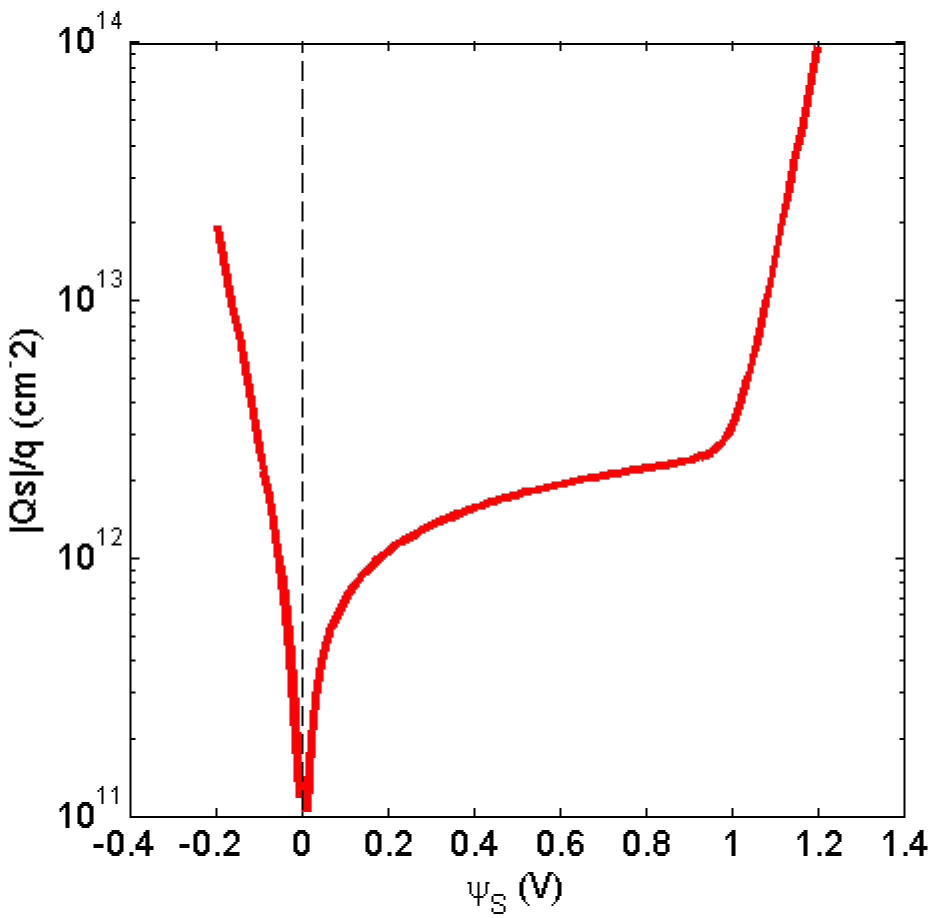

Fig. 1 Magnitude of the charge per square cm (divided by q) vs. surface potential in volts. (See the appendix for the Matlab® script that produced this plot.)

Figure 1 shows the accumulation of holes for $\psi_S < 0$, the depletion of the p-type semiconductor for $\psi_S > 0$, and inversion for $\psi_S > 2\psi_B$. To understand the result, let's examine eqn. (12) more closely. Consider accumulation ($\psi_S < 0$) first. In this case, we have from eqn. (12)

$$Q_S = +\sqrt{2\varepsilon_{Si} k_B T_L N_A}\; F(\psi_s)$$

and according to eqn. (10), for strong accumulation

$$F(\psi_S) \to e^{-\psi_S/2k_B T_L},$$

so

$$Q_S \approx +\sqrt{2\varepsilon_{Si} k_B T_L N_A}\; e^{-q\psi_S/2k_B T_L} \quad .$$

Next, consider inversion ($\psi_s > 2\psi_B$) next. In this case, we have from eqn. (12)

$$Q_S = -\sqrt{2\varepsilon_{Si} k_B T_L N_A}\; F(\psi_S),$$

and according to eqn. (1), for strong inversion

$$F(\psi_S) \to \frac{n_i}{N_A} e^{+q\psi_S/2k_B T_L},$$

so

$$Q_S \approx Q_n \approx +\sqrt{\frac{2\varepsilon_{Si} k_B T_L n_i^2}{N_A}}\; e^{+q\psi_S/2k_B T_L} \quad .$$

The electron density at the surface is

$$n(0) = \frac{n_i^2}{N_A} e^{+q\psi_S/k_B T_L} \ \text{cm}^{-3},$$

so we can write the charge density per cm^2 as

$$Q_S \approx +\sqrt{2\varepsilon_{Si} k_B T_L\, n(0)} \quad .$$

Taur and Ning [1] point out that this gives us a simple way to estimate the thickness of the inversion layer. Since

$$Q_n \approx qn(0)t_{inv},$$

we get the following estimate for the thickness of the inversion layer,

$$t_{inv} \approx \frac{2\varepsilon_{Si} k_B T_L n(0)}{q|Q_n|}.$$

Finally, consider the case of depletion ($0 < \psi_S < 2\psi_B$). In this case, we can ignore electrons and holes to find from eqn. (10),

$$F(\psi_S) \cong \sqrt{\frac{q\psi_S}{k_B T_L} - 1},$$

which, according to eqn. (12) gives

$$\mathcal{E}_S \cong \sqrt{\frac{2qN_A}{\varepsilon_{Si}}(\psi_S - k_B T_L / q)} \ .$$

The "exact" result is just like depletion approximation (eqn. (2.161) in [1]) except for $k_B T_L / q$ correction.

Example 2: Surface Potential vs. Gate Voltage

Figure 1 is in terms of the surface potential; we vary the surface potential with the gate voltage according to:

$$V_G = V_{FB} + \psi_S - \frac{Q_S}{C_{ox}}$$

where V_{FB} is the "flat band voltage". Consider a p-type, silicon MOS capacitor at room temperature with $N_A = 5\times10^{17}\,\text{cm}^{-3}$. Assume $V_{FB} = 0$, $t_{ox} = 2$ nm, and $\kappa_{ox} = 4$. Plot the surface potential, ψ_S vs. gate voltage on the x-axis.

A Matlab script to perform this calculation is included in the appendix. The result is shown below in Fig. 2.

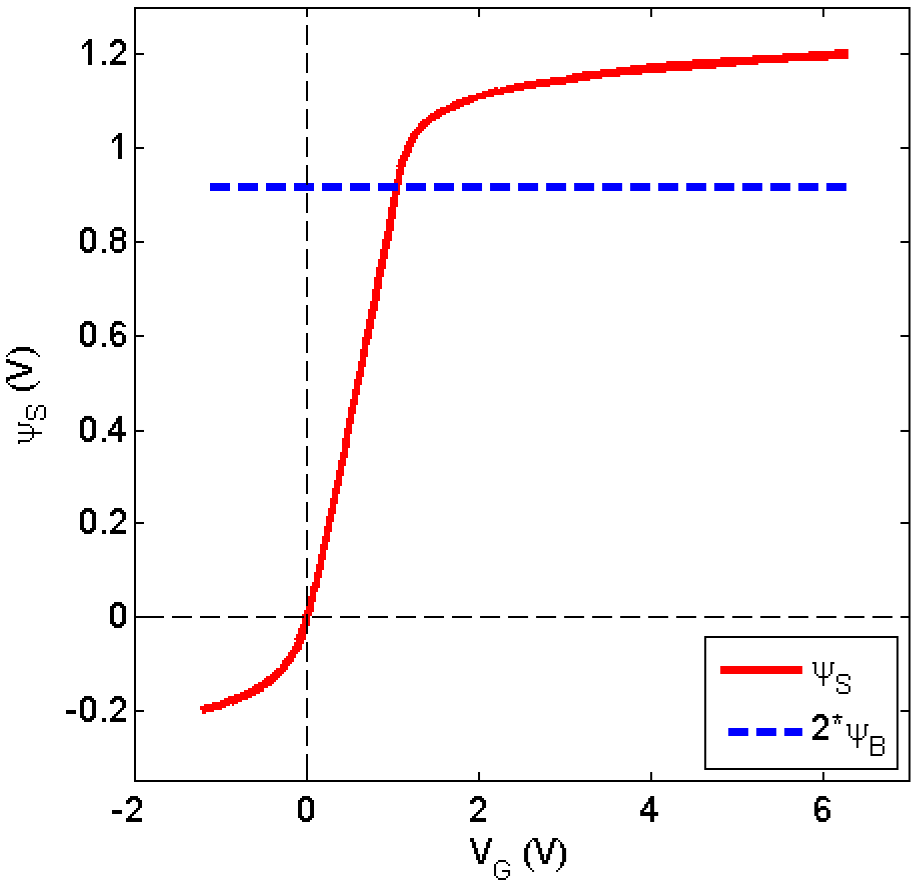

Fig. 2 Surface potential vs. gate voltage. (See the appendix for the Matlab® script that produced this plot.)

Figure 2 shows that no matter how large the gate voltage is, it is hard to push the surface potential very much beyond $2\psi_B$.

3. The Semiconductor Capacitance

Capacitance is the derivative of charge with respect to voltage, so we find the semiconductor capacitance as

$$C_{Si} = \frac{-dQ_S}{d\psi_S} = \sqrt{2q\varepsilon_{Si} N_A} \times$$

$$\left[\frac{\left(1 - e^{-q\psi_S/kT_L}\right) + \frac{n_i^2}{N_A}(e^{q\psi_S/kT_L} - 1)}{2\sqrt{\left(\frac{k_B T_L}{q} e^{-q\psi_S/k_B T_L} + \psi_S - \frac{k_B T_L}{q}\right) + \left(\frac{n_i}{N_A}\right)^2 \left(\frac{k_B T_L}{q} e^{q\psi_S/k_B T_L} - \psi_S - \frac{k_B T_L}{q}\right)}} \right]. \quad (13)$$

Under depletion conditions, $0 < \psi_S < 2\psi_B$ and eqn. (13) simplifies to

$$C_{Si} = \frac{-dQ_S}{d\psi_S} = \sqrt{\frac{q\varepsilon_{Si} N_A}{2\left(\psi_S - k_B T_L / q\right)}} = \frac{\varepsilon_{Si}}{W_D},$$

which is the standard depletion approximation result (eqn. (2.174) of [1]) except for the $k_B T_L/q$ correction, which is often neglected.

We can also take the limit of eqn. (13) as $\psi_S \to 0$, to find the flat band capacitance as

$$C_{Si}(FB) = \frac{\varepsilon_{Si}}{L_D} \quad (14)$$

Equation (13) can be used to evaluate the low frequency MOS CV characteristic. With a little more care, it can also be adapted to evaluate the high frequency CV characteristic too.

4. MOS Electrostatics II (potential vs. position)

So far, we have only integrated Poisson's equation, eqn. (7), once. To get $\psi(x)$ for a given ψ_s, we need to integrate again. Recall that

$$\frac{d\psi}{dx} = \mp\sqrt{\frac{2k_B T_L N_A}{\varepsilon_{Si}}} F(\psi). \quad (9)$$

We can integrate eqn. (9) to find

$$\int_{\psi_S}^{\psi(x)} \frac{d\psi}{F(\psi)} = \mp\sqrt{\frac{2k_B T_L N_A}{\varepsilon_{Si}}}\, x$$

or

$$x=\sqrt{\frac{\varepsilon_{Si}}{2k_BT_L\ N_A}}\int_{\psi(x)}^{\psi_S}\frac{d\psi}{F(\psi)}.$$

Finally, it is convenient to write the result as

$$x=L_D\int_{\psi(x)}^{\psi_S}\frac{d\psi}{\frac{k_BT_L}{q}F(\psi)}, \tag{15}$$

where L_D is the extrinsic Debye length. Equation (15) cannot be integrated analytically, so, for a given ψ_S, select $0<\psi(x)<\psi_S$, then numerically integrate (15) to get x.

The next question is: "How do we compute $Q_n(\psi_s)$?" Well above threshold, $Q_S \approx Q_n$, so we can get it from eqn. (12), but how would we do it exactly for all bias conditions? First, recall that

$$n(x)=\frac{n_i^2}{N_A}e^{q\psi(x)/k_BT_L},$$

so

$$Q_n=q\int_0^\infty n(x)dx=q\int_{\psi_s}^{0}\frac{n_i^2}{N_A}e^{q\psi(x)/k_BT_L}\frac{dx}{d\psi}d\psi.$$

Since we know the electric field as a function of y from eqn. (9), we find

$$Q_n=q\frac{n_i^2}{N_A}\int_0^{\psi_S}\frac{e^{q\psi/k_BT_L}}{\sqrt{2k_BT_LN_A/\varepsilon_{Si}}\,F(\psi)}d\psi\ . \tag{16}$$

Consider another question: "How do we compute the depletion layer charge?" We know how to compute it using the depletion approximation, but how to we do it exactly. We begin with

$$Q_D = -q\int_0^\infty [N_A - p(x)]dx = -qN_A\int_0^\infty \left(1 - e^{-q\psi/k_BT_L}\right)dx \,,$$

which we can change the variable of integration to find.

$$Q_D = -qN_A \int_0^{\psi_s} \frac{1 - e^{-q\psi/k_BT_L}}{\sqrt{2k_BT_LN_A/\varepsilon_{Si}}\,F(\psi)} d\psi \;. \qquad (17)$$

5. "Exact" Solution of the MOSFET:

Taur and Ning discuss the "exact" solution for a long channel MOSFET [1]. These notes amplify on their discussion. Let's begin by reviewing the derivation of eqn. (6), which applies only equilibrium.

$$n = n_i e^{(E_F - E_i)/k_BT_L} \qquad (18)$$

let

$$E_i = -q\psi(x) - q\psi_{REF} \qquad (19)$$

and choose

$$q\psi_{REF} = -E_F \;. \qquad (20)$$

The result is eqn. (6),

$$n = n_i e^{q\psi(x)/k_BT_L} \,. \qquad (21)$$

Out of equilibrium, we have

$$n = n_i e^{\left[F_n(x) - E_i(x)\right]/k_BT_L} \qquad (22)$$

Using eqns. (19) and (20) in eqn. (22), we have

$$n(x) = n_i e^{q\left[\psi(x) + F_n(x) - E_F\right]/k_BT_L} \qquad (23)$$

If we define

$$q\phi_n \equiv E_F - F_n \qquad (24)$$

then

$$n(x) = n_i e^{q[\psi(x) - \phi_n(x)]/k_B T_L} . \tag{25}$$

In a MOSFET, a positive drain bias will pull F_n down by qV_D at the drain. If the source is grounded, $\phi_n(y=0)=0$ and $\phi_n(y=L)=V_D$. The quasi-Fermi potential will vary with distance, *y*, along the channel, but we assume that it is constant with *x*, at least across the inversion layer. In a MOSFET, the holes will stay in equilibrium, so eqn. (5) continues to apply.

Now, we make the gradual channel approximation, so that a 1D Poisson equation can be solved to find $E_x(x,y)$. Equation (7) becomes

$$\frac{d^2\psi}{dx^2} = \frac{-q}{\varepsilon_{Si}}\left[p_o\left(e^{-q\psi/k_B T_L} - 1\right) - \frac{n_i^2}{N_A}\left(e^{q(\psi-\phi_n)/k_B T_L} - 1\right)\right], \tag{26}$$

which can be integrated to find

$$\mathcal{E}(\psi,\phi_n) = \pm\sqrt{\frac{2k_B T_L\ N_A}{\varepsilon_{Si}}}\,F(\psi,\phi_n) \tag{27}$$

where

$$F(\psi,\phi_n) = \sqrt{\left(e^{-q\psi/k_B T_L} + \frac{q\psi}{k_B T_L} - 1\right) + \frac{n_i^2}{N_A}\left(e^{-q\phi_n/k_B T_L}\left(e^{q\psi/k_B T_L} - 1\right) - \frac{q\psi}{k_B T_L}\right)}. \tag{28}$$

To find the inversion layer density at any point along the channel,

$$Q_n = -q\int_o^\infty n(x)\,dx = -q\int n(x)\frac{dx}{d\psi}d\psi$$

$$= -q\int_{\psi_B}^{\psi_s}\frac{n(x)}{\mathcal{E}(\psi,\phi_n)}d\psi$$

or

$$Q_n(\phi_n) = -q\frac{n_i^2}{N_A}\int_{\psi_B}^{\psi_S}\frac{e^{q(\psi-\phi_n)/k_B T_L}}{\mathcal{E}(\psi,\phi_n)}d\psi \tag{29}$$

We should note ψ_S varies along the channel. We determine ψ_S from

$$V_G = V_{FB} + \psi_S - \frac{Q_S}{C_{ox}}$$

or

$$V_G = V_{FB} + \psi_S + \frac{\varepsilon_{Si}\mathcal{E}_s\left(\psi_S, \phi_n\right)}{C_{ox}} \;, \tag{30}$$

Where $\mathcal{E}_s$ is determined from eqn. (27) with $\psi = \psi_S$.

To summarize, at some location, *y*, along the channel there is a corresponding $\phi_n(y)$. Equation (30) can be solved iteratively for ψ_S, given the gate voltage, V_G. Knowing ψ_S, we determine the corresponding inversion layer charge by integrating eqn. (29) numerically.

The next question is: "How do we determine I_{DS}." From

$$J_n = n\mu_n \frac{dF_n}{dy} \tag{31}$$

we find

$$I_{DS} = \mu_{eff}\frac{W}{L}\int_o^{V_{DS}} [-Q_n(\phi_n)]\,d\phi_n \tag{32}$$

which is eqn. (3.10) in Taur and Ning (recall that Taur and Ning use *V* for ϕ_n). Using eq. (29) in eqn. (32), we find

$$I_{DS} = q\mu_{eff}\frac{W}{L}\int_0^{V_{DS}} \left(\int_{\psi_B}^{\psi_S} \frac{(n_i^2/N_A)e^{q(\psi-\phi_n)/k_BT_L}}{\mathcal{E}(\psi,\phi_n)}\,d\psi\right) d\phi_n \tag{33}$$

Equation (33) is the famous Pao-Sah double integral expression for I_{DS}.

Equation (33) can be integrated numerically to obtain $I_{DS}(V_{GS}, V_{DS})$. Note that we made the gradual channel approximation, so it applies only to long channel transistors. Note also, that it is valid for the entire V_{DS} range; the drain current saturates automatically without needing to worry about channel pinch-off. Finally, note that the double integral expression can be converted to a single integral as discussed by Pierret and Shields [2].

6. Summary

A few key things to remember (or look up when you need them) are listed below.

1) Surface electric field for a given surface potential:

$$\mathcal{E}_s = \pm\sqrt{\frac{2k_BT_LN_A}{\varepsilon_{Si}}}\,F(\psi_S) \tag{11}$$

where

$$F(\psi) = \sqrt{\left(e^{-q\psi/k_BT_L} + \frac{q\psi}{k_BT_L} - 1\right) + \frac{n_i^2}{N_A^2}\left(e^{+q\psi/k_BT_L} - \frac{q\psi}{k_BT_L} - 1\right)} \tag{10}$$

2) Semiconductor charge density for a given surface potential:

$$Q_S(\psi_S) = -\varepsilon_{Si}\mathcal{E}_S = \mp\sqrt{2\varepsilon_{Si}k_BT_LN_A}\,F(\psi_S) \tag{12}$$

In depletion, eqn. (12) reduces to:

$$Q_S \cong \sqrt{2q\varepsilon_{Si}N_A(\psi_S - k_BT_L/q)},$$

which is very close to the depletion approximation, except for the $-k_BT_L/q$ correction.

For strong accumulation, eqn. (12) reduces to:

$$Q_S \approx +\sqrt{2\varepsilon_{Si}k_BT_LN_A}\,e^{-q\psi_S/2k_BT_L} \quad \text{(strong accumulation)}$$

For strong inversion, eqn. (12) reduces to:

$$Q_S \approx Q_n \approx +\sqrt{\frac{2\varepsilon_{Si}k_BT_Ln_i^2}{N_A}}\,e^{+q\psi_S/2k_BT_L} \quad \text{(strong inversion)}$$

which can also be expressed as

$$Q_S \approx +\sqrt{2\varepsilon_{Si} k_B T_L \, n(0)}$$

The inversion layer thickness can be estimated as:

$$t_{inv} \approx \frac{2\varepsilon_{Si} k_B T_L}{q|Q_n|}$$

3) Capacitance:

An analytical expression for the semiconductor capacitance exists as given by eqn. (13). Under flatband conditions, the semiconductor capacitance becomes:

$$C_{Si}(FB) = \frac{\varepsilon_{Si}}{L_D} \qquad (14)$$

References

[1] Yuan Taur and Tak Ning, *Fundamentals of Modern VLSI Devices*, Cambridge University Press, Cambridge, UK, 1998.

[2] R.F. Pierret, *Semiconductor Device Fundamentals*, Addison Wesley, Reading, MA, 1996.

[3] R. F. Pierret and J. A. Shields, "Simplified Long Channel MOSFET Theory," *Solid-State Electronics*, **26**, pp. 143-147, 1983.

Appendix

This appendix contains the Matlab® script that produces Figs. 1 and 2.

```
% Surface potential vs. Gate voltage for MOS Capacitor
% Date: Oct. 23, 2012
% Author: Xingshu Sun and Mark S. Lundstrom(Purdue University)
%
% References

% [1] Mark Lundstrom and Xingshu Sun (2012), "Notes on the Solution of the Poisson-Boltzmann Equation for MOS Capacitors and MOSFETs, 2nd Ed." http://nanohub.org/resources/5338.

%Initialize the range of the surface potential
psi = -0.2:0.01:1.2; %[V]
%Specify the physical constants

epsilon_siO2 = 4*8.854*1e-14; %Permittivity of SiO2 [F/cm]
```

```
epsilon_si = 11.68*8.854*1e-14; %Permittivity of Si [F/cm]
k_b = 1.380e-23; %Boltzmann constant [J/K]
q = 1.6e-19; %Elementary charge [C]

%Specify the environmental parameters
TL = 300; %Room temperature[K]
ni = 1e10; %Intrinsic semiconductor carrier density [cm-3]
N_A = 5e17; %Acceptor concentration [cm-3]
tox = 2e-7; %The thickness of SiO2 [cm]
V_FB = 0; %The flat-band voltage [V]

%Calculate the capacitance
cox = epsilon_siO2/tox; %[F/cm2]

%Calculate the F function (to calculate the total charge) with
respect to different surface potentials.  See: eqn. (10) in [1]

F_psi1 = exp(-psi*q/k_b/TL) + psi*q/k_b/TL -1;
F_psi2 = ni^2/N_A^2*(exp(psi*q/k_b/TL) - psi*q/k_b/TL -1);
F_psi = sqrt(F_psi1 + F_psi2);

%Initialize the vectors
psi_length = length(psi);
Qs = zeros(psi_length,1);
V_G = zeros(psi_length,1);
psi_B = zeros(psi_length,1);
```

```
%Calculate the total charges and the corresponding gate voltages.
See eqn. (12) in [1]


for i = 1:psi_length
    if psi(i) <= 0
        Qs(i) = sqrt(2*epsilon_si*k_b*TL*N_A)*F_psi(i);
%Calculate charge when the surface potential is negative


    else
        Qs(i) = -sqrt(2*epsilon_si*k_b*TL*N_A)*F_psi(i);
%Calculate charge when the surface potential is positive
    end


    psi_B(i) = k_b*TL/q*log(N_A/ni); %Get the bulk potential as a
reference and it is a constant with respect to VG
    V_G(i) = V_FB+psi(i)-Qs(i)/cox; %Calculate the gate voltage
end
%Plot total charge vs. surface potential (psi_S)
figure(1)
semilogy(psi,abs(Qs)/q,'r','linewidth',3);
hold on
set(gca, 'xlim', [-0.4 1.4], 'ylim', [1e11 1e14]);
set(gca,'fontsize',13);
xlabel('\psi_S (V)');
ylabel('|Qs|/q (cm^-2)');
%Plot x=0
plot(zeros(1,21),logspace(10,16,21),'k--')
```

```
%Plot surface potential (psi_S) vs. gate voltage with reference
potential 2*psi_B
figure(2)
h1=plot(V_G,psi,'r','linewidth',3);
hold on
h2=plot(V_G,2*psi_B,'--b','linewidth',3);
set(gca, 'xlim', [-3 10], 'ylim', [-0.4 1.4]);
set(gca,'fontsize',13);
legend('\psi_S','2*\psi_B','Location','SouthEast')
xlabel('V_G (V)');
ylabel('\psi_S (V)');

%Plot x=0 and y=0
plot(-10:10, zeros(1,21), 'k--')
plot(zeros(1,21),-10:10, 'k--')
```