

NCN@Purdue - Intel Summer School: July 14-25, 2008

***Physics of Nanoscale Transistors: Lecture 2:***

***Elementary Theory  
of the Nanoscale MOSFET***

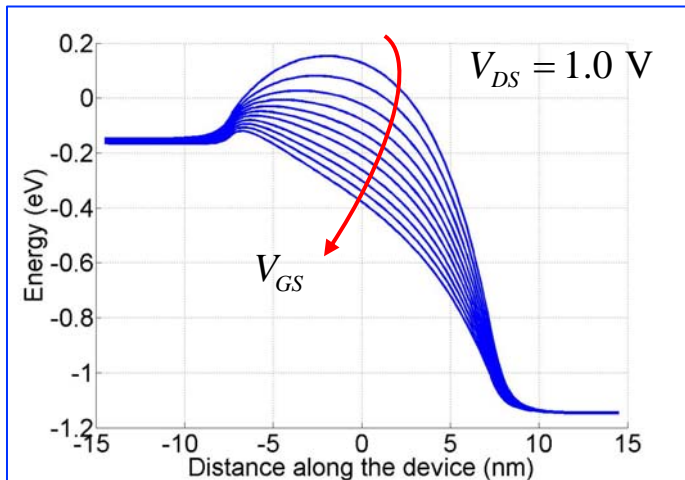
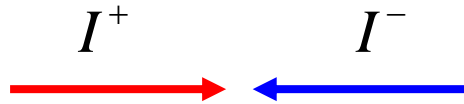
***Mark Lundstrom***

Network for Computational Nanotechnology

Discovery Park, Purdue University

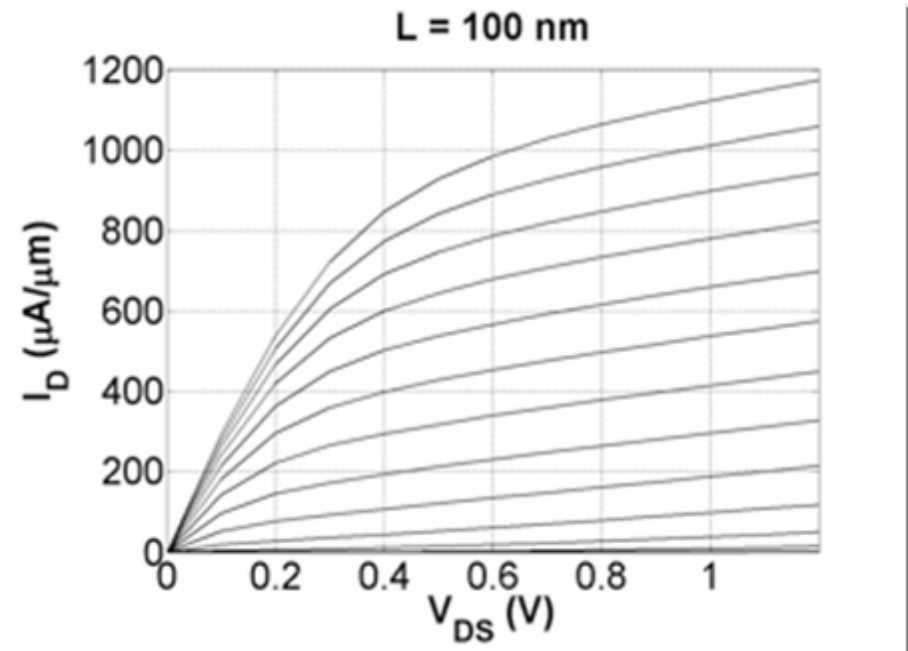
West Lafayette, Indiana USA

# transistors



$$I_D \propto I^+ - I^-$$

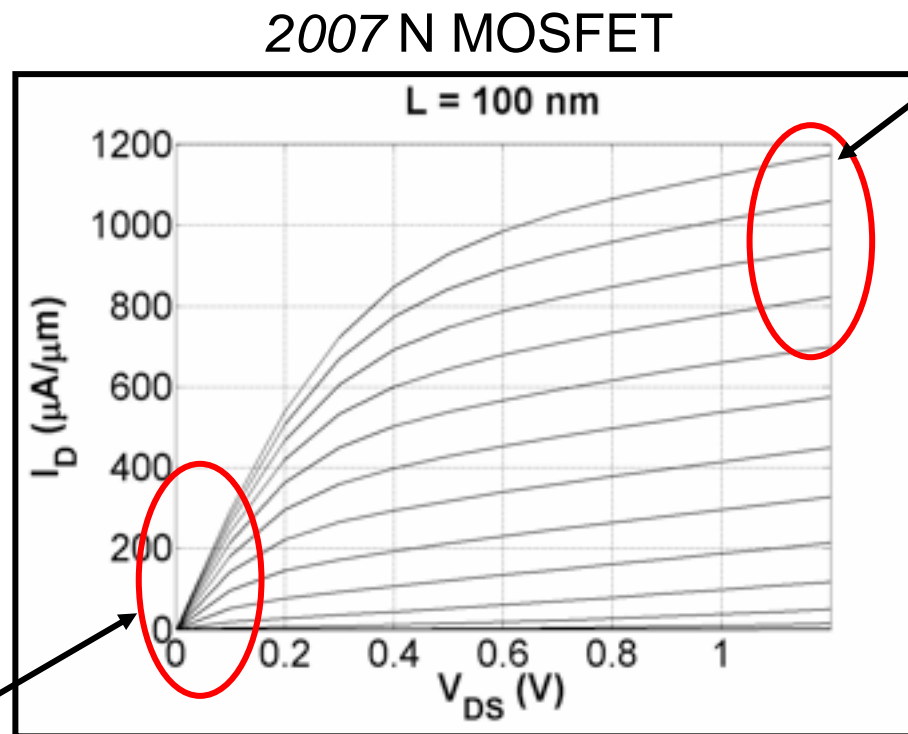
## 2007 N-MOSFET



(Courtesy, Shuji Ikeda, ATDF, Dec. 2007)

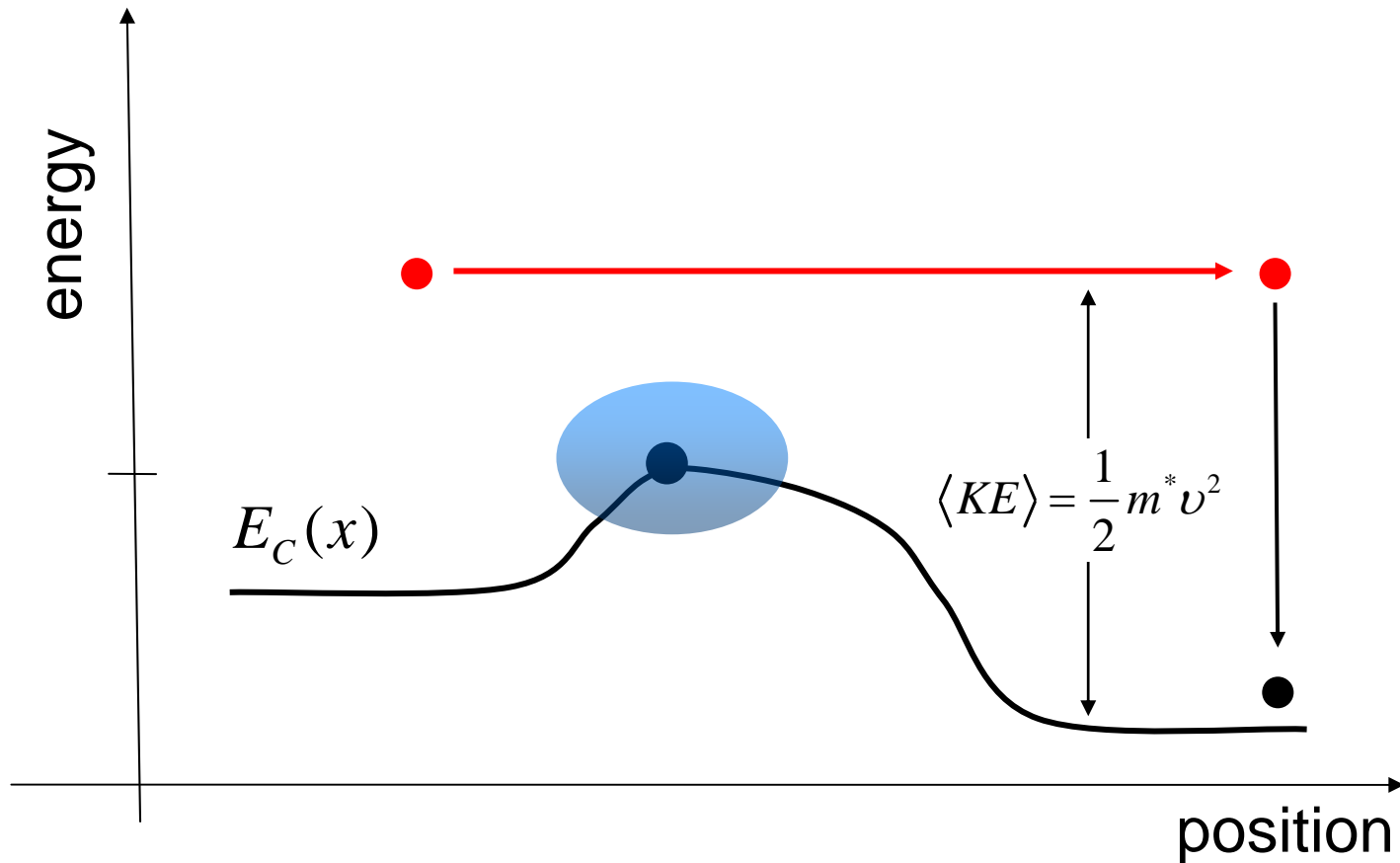
# diffusive transport ( $L \gg \lambda$ )

$$I_D = W C_{ox} v_{sat} (V_{GS} - V_T)$$



$$I_D = \frac{W}{L} \mu_{eff} C_{ox} (V_{GS} - V_T) V_{DS}$$

# ballistic transport ( $L \ll \lambda$ )

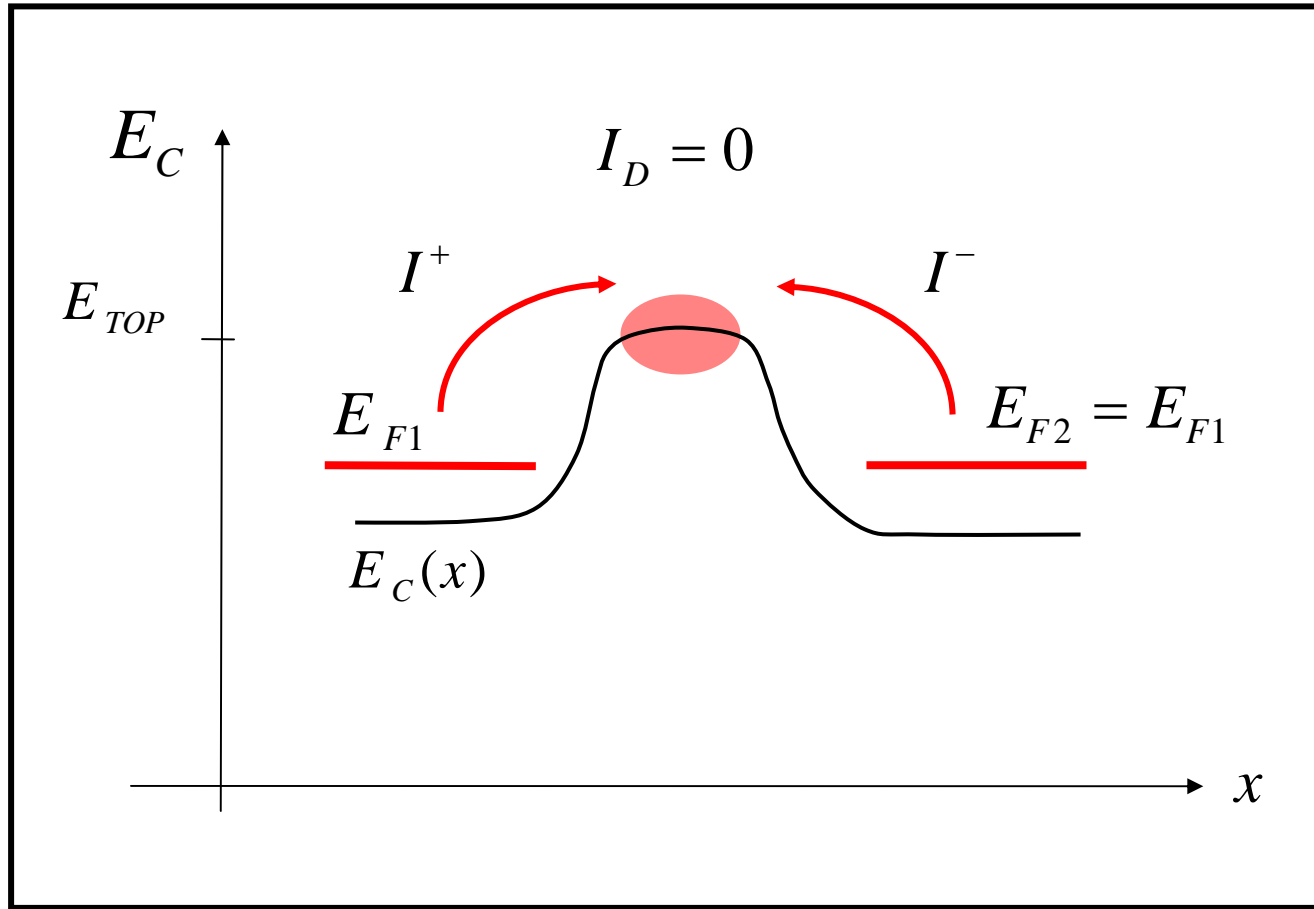


# outline

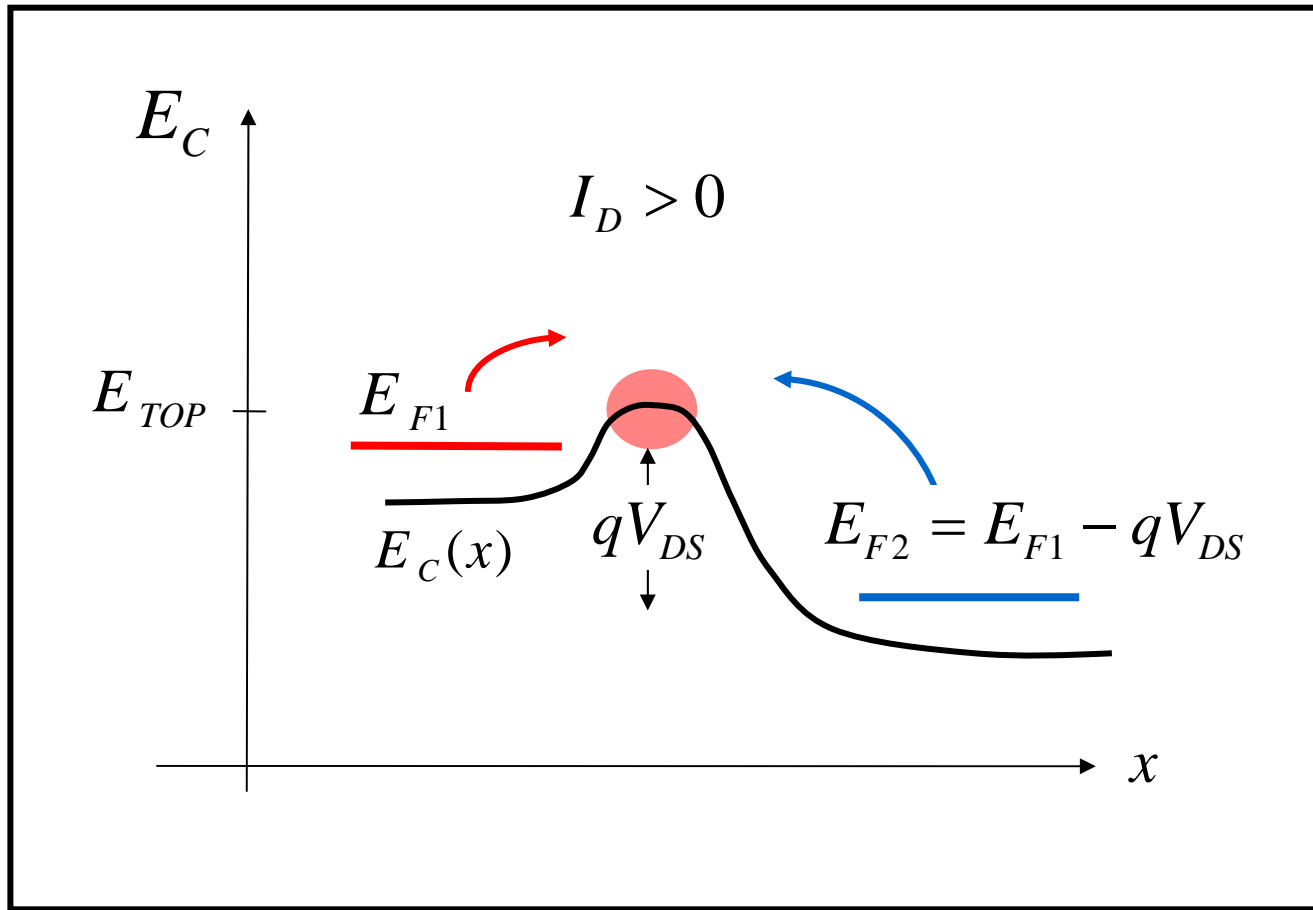
---

- 1) Introduction
- 2) Ballistic theory of the MOSFET**
- 3) Discussion: ballistic MOSFETs
- 4) Scattering in nano-MOSFETs
- 5) Summary

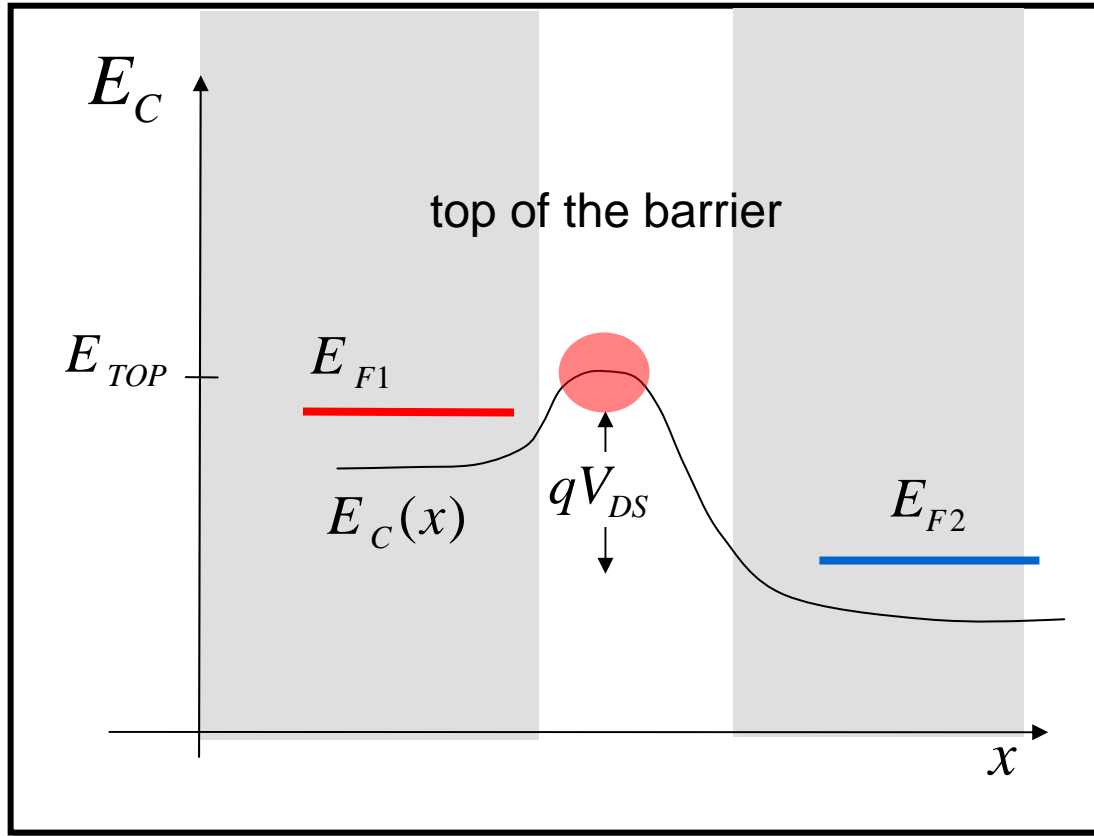
# MOSFET in equilibrium



# high gate and drain bias



# the ballistic MOSFET



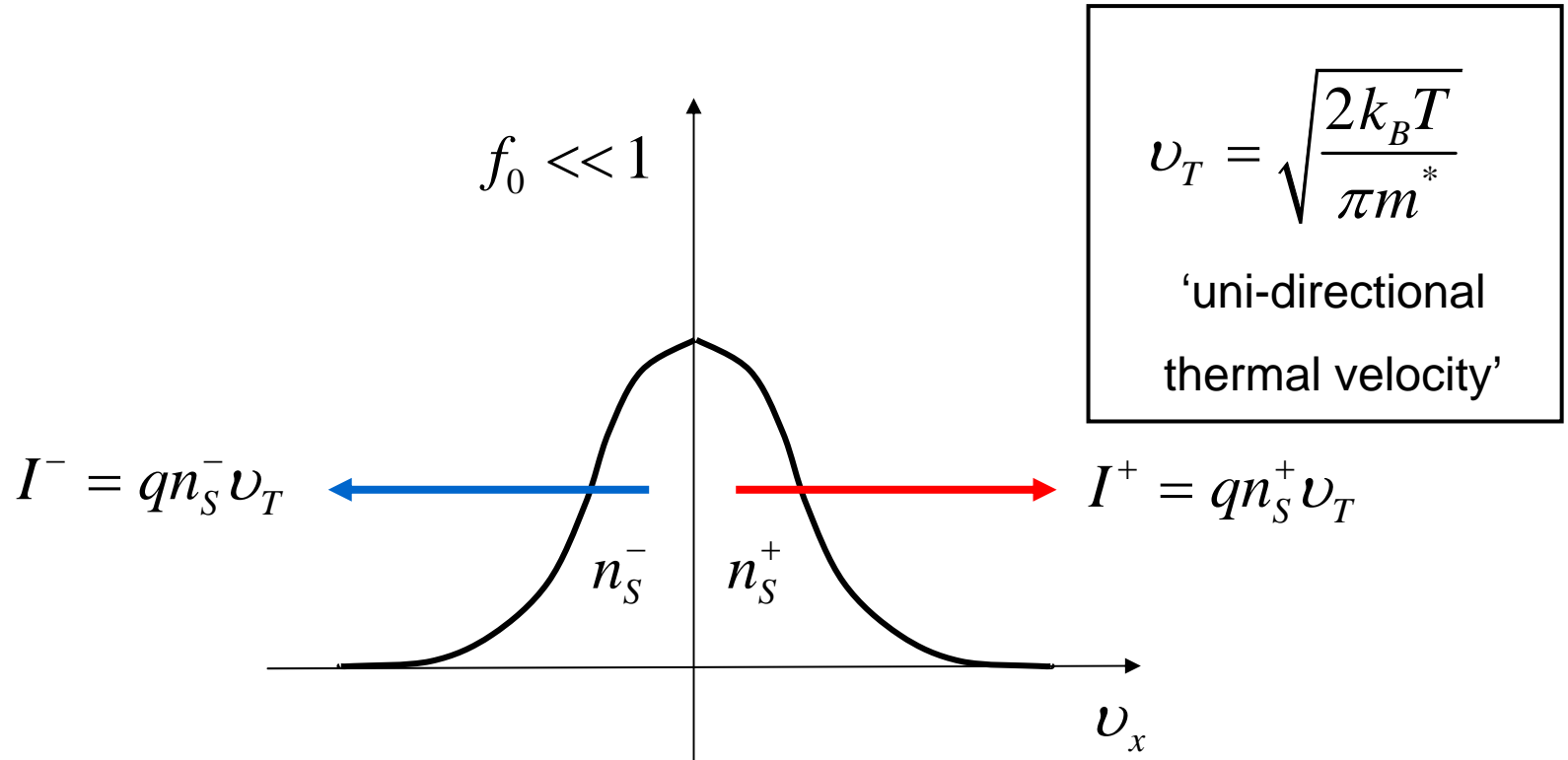
ballistic channel

We will evaluate the current at the top of the barrier ( $x = 0$ ) under the following **assumptions**.

- 1) ballistic transport in the channel
- 2) source and drain are reservoirs of thermal equilibrium carriers.
- 3) nondegenerate carriers
- 4)  $V_{GS} > V_T$

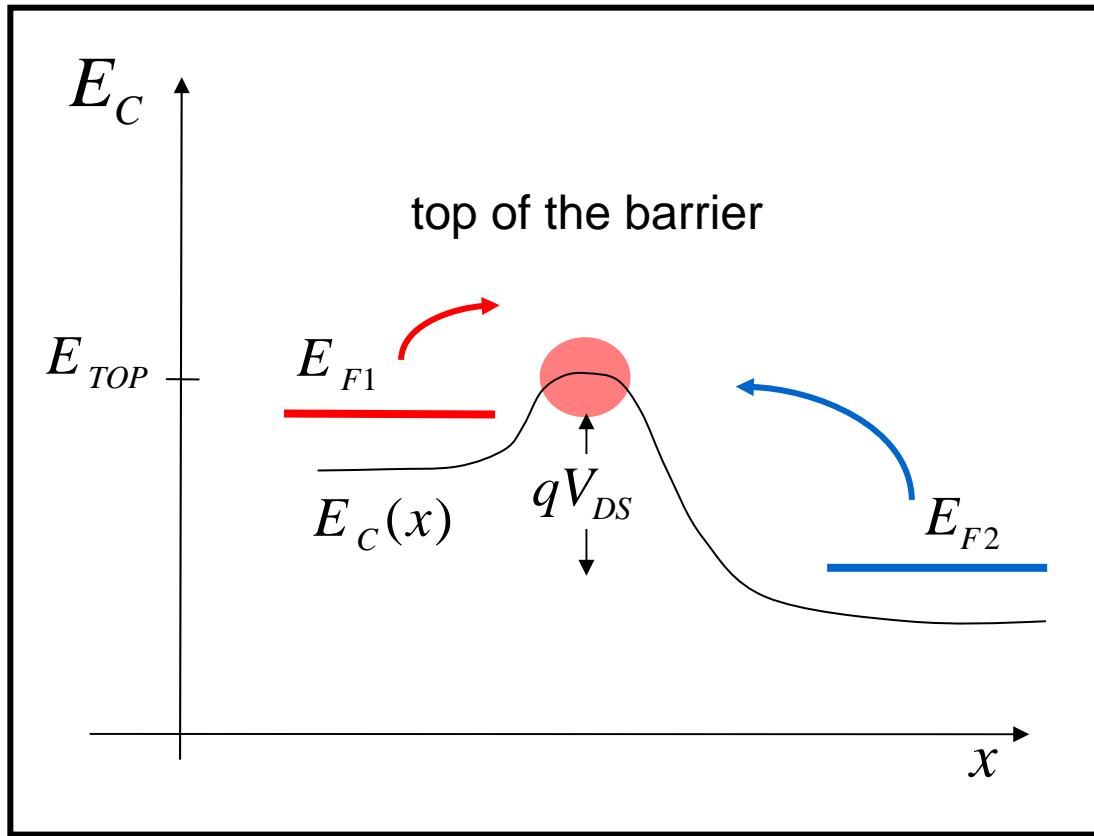


# equilibrium velocity distribution of carriers



$$f_0 = \frac{1}{1 + e^{(E - E_F)/k_B T}} \approx e^{(E_F - E)/k_B T} \propto e^{-m^* v_x^2 / 2k_B T}$$

# the ballistic MOSFET



- 1) ballistic transport
- 2) nondegenerate carriers

$$Q_I(0) \text{ C/cm}^2$$

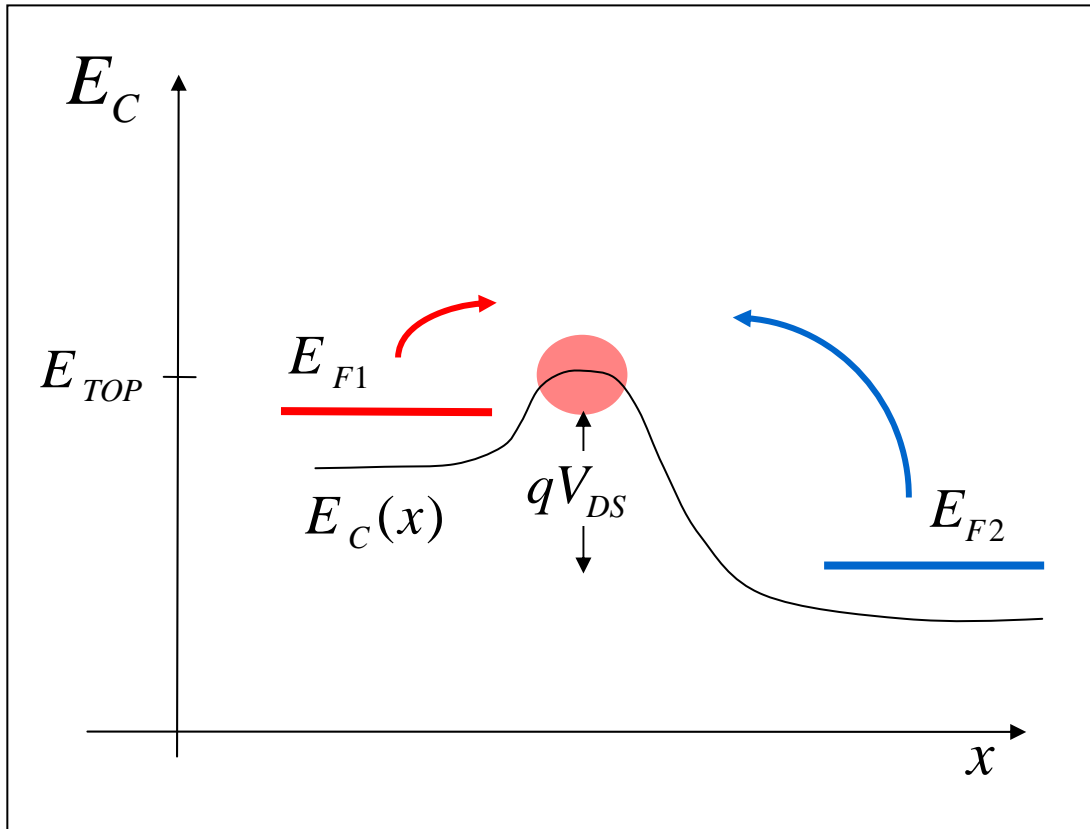
$$Q_I = -q(n_S^+ + n_S^-) \text{ C/cm}^2$$

$$n_S^- / n_S^+ = e^{-qV_{DS}/k_B T}$$

$$Q_I = -qn_S^+ (1 + e^{-qV_{DS}/k_B T})$$

$$qn_S^+ = \frac{-Q_I}{1 + e^{-qV_{DS}/k_B T}}$$

# the ballistic MOSFET: IV



- 1) ballistic
- 2) nondegenerate carriers
- 3)  $V_{GS} > V_T$

$$I_D = q(n_S^+ v_T - n_S^- v_T)$$

$$I_D = qn_S^+ v_T (1 - e^{-qV_{DS}/k_B T})$$

$$qn_S^+ = \frac{-Q_I}{1 + e^{-qV_{DS}/k_B T}}$$

$$I_D = -Q_I v_T \left( \frac{1 - e^{-qV_{DS}/k_B T}}{1 + e^{-qV_{DS}/k_B T}} \right)$$

$$Q_I = -C_{ox} (V_{GS} - V_T)$$

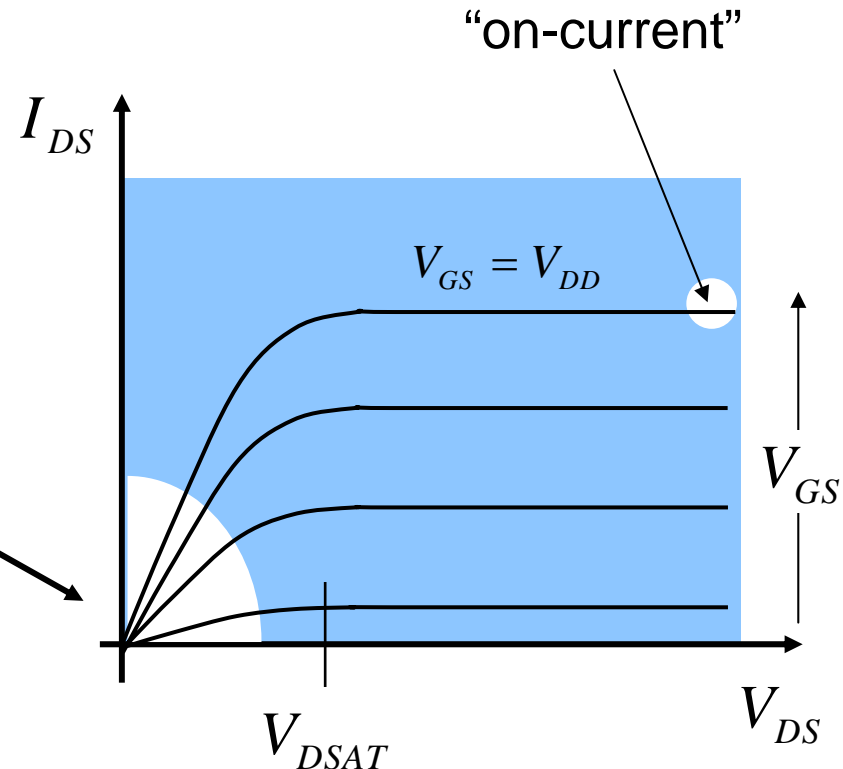
# the ballistic MOSFET: IV

$$I_{DS} = C_{ox} (V_{GS} - V_T) v_T \left( \frac{1 - e^{-qV_{DS}/k_B T}}{1 + e^{-qV_{DS}/k_B T}} \right)$$

$$I_{ON} = WC_{ox} v_T (V_{DD} - V_T)$$

$$\left\{ \begin{array}{l} V_{DS} < k_B T / q \\ I_{DS} = WC_{ox} \frac{v_T}{2k_B T / q} (V_{GS} - V_T) V_{DS} \\ I_{DS} = V_{DS} / R_{CH} \end{array} \right.$$

$$V_{DSAT} \approx k_B T / q$$



# outline

---

- 1) Introduction
- 2) Ballistic theory of the MOSFET
- 3) Discussion: ballistic MOSFETs**
- 4) Scattering in nano-MOSFETs
- 5) Summary

# low $V_{DS}$ : diffusive vs. ballistic

**diffusive:**

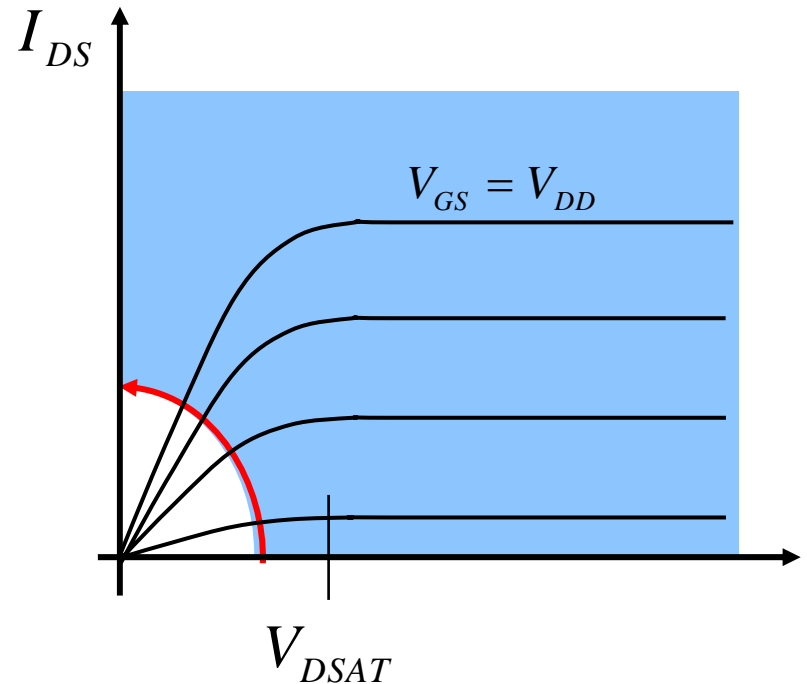
$$I_D = \frac{W}{L} \mu_{eff} C_{ox} (V_{GS} - V_T) V_{DS}$$

$$I_D = V_{DS} / R_{CH}$$

$$R_{CH} = \frac{1}{\mu_{eff} C_{ox} (V_{GS} - V_T)} \left( \frac{L}{W} \right)$$

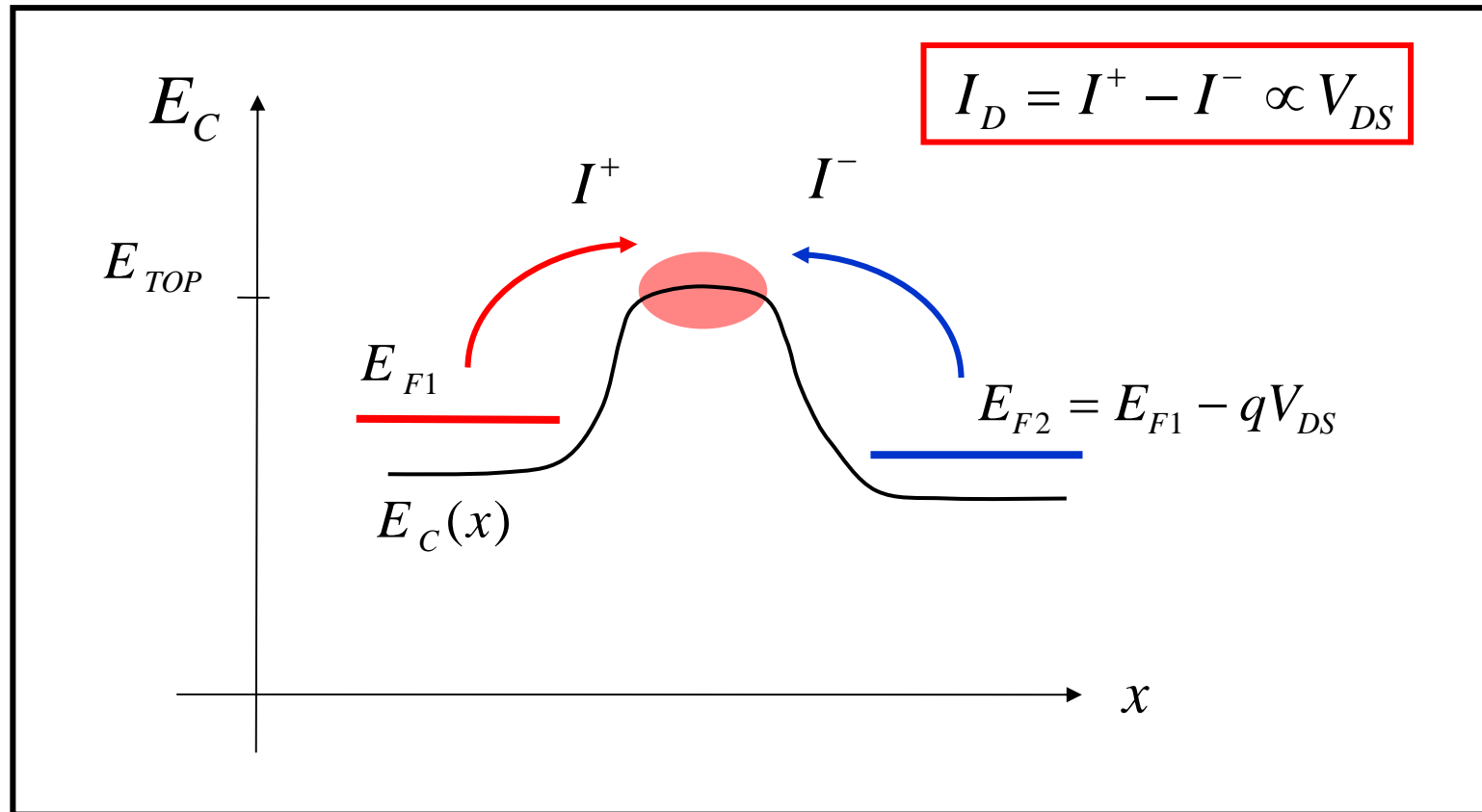
**ballistic:**

$$I_D = WC_{ox} \frac{v_T}{2k_B T / q} (V_{GS} - V_T) V_{DS}$$



- a ballistic MOSFET has a finite channel resistance - independent of  $L$
- the channel resistance cannot be lower than the ballistic limit

# why is there a finite ballistic channel resistance?



# relation between diffusive and ballistic models

diffusive:

$$I_D = \frac{W}{L} \mu_{eff} C_{ox} (V_{GS} - V_T) V_{DS}$$

ballistic:

$$I_D = WC_{ox} \frac{v_T}{2k_B T / q} (V_{GS} - V_T) V_{DS}$$

$$I_D = \frac{W}{L} C_{ox} \left[ \frac{v_T L}{2k_B T / q} \right] (V_{GS} - V_T) V_{DS}$$

$$\mu_B \equiv \left[ \frac{v_T L}{2k_B T / q} \right]$$

‘ballistic mobility’

(M.S. Shur, *IEEE Elect. Dev. Lett.*,  
3, 511, 2002.)

$$I_D = \frac{W}{L} \mu_B C_{ox} (V_{GS} - V_T) V_{DS}$$



# the ballistic mobility

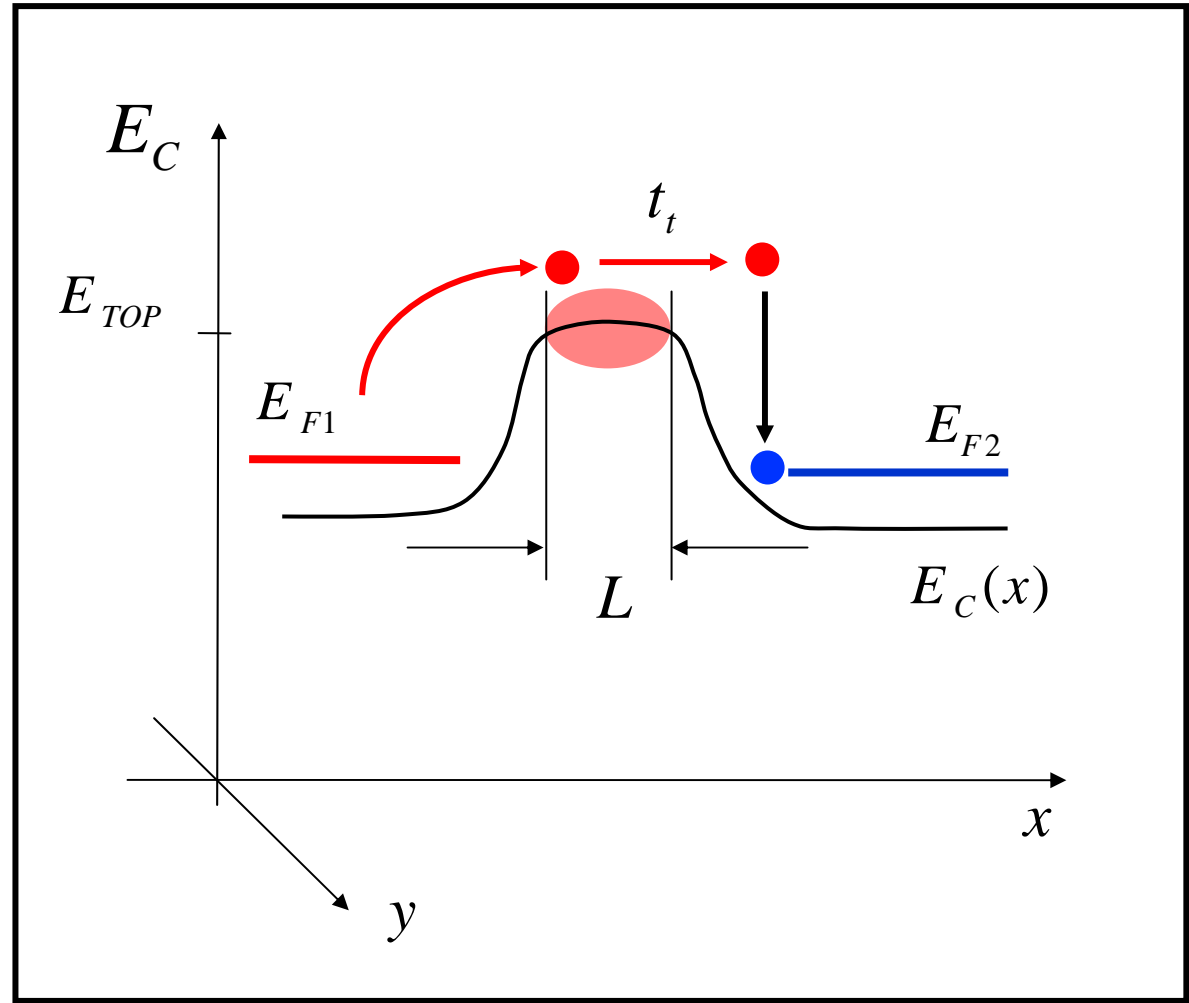
$$\mu_B \equiv \left[ \frac{v_T L}{2k_B T / q} \right]$$

$$v_T = \sqrt{2k_B T / \pi m^*}$$

$$\mu_B = \left[ \frac{q(L / \pi v_T)}{m^*} \right]$$

$$= \left[ \frac{q \tau_t}{m^*} \right]$$

$$t_t = L / (\pi v_T)$$



# quasi-ballistic MOSFET: low $V_{DS}$

---

**ballistic:**

$$I_D = \frac{W}{L} \mu_B C_{ox} (V_{GS} - V_T) V_{DS} \quad \mu_B \equiv \left[ \frac{v_T L}{2k_B T / q} \right]$$

**diffusive:**

$$I_D = \frac{W}{L} \mu_{eff} C_{ox} (V_{GS} - V_T) V_{DS}$$

**general:**

$$I_D = \frac{W}{L} \left( \frac{1}{\mu_{eff}} + \frac{1}{\mu_B} \right)^{-1} C_{ox} (V_{GS} - V_T) V_{DS}$$

## the ballistic mobility: example

---

For  $L = 100$  nm NMOS Si technology:  $\mu_{eff} \approx 200 \text{ cm}^2/\text{V-s}$

$$v_T = \sqrt{2k_B T / \pi m^*} \approx 1 \times 10^7 \text{ cm/s}$$

$$\mu_B \equiv \left[ \frac{v_T L}{2k_B T / q} \right] \approx 2000 \text{ cm}^2/\text{V-s}$$

$$\mu_{eff} \ll \mu_B$$

so the real mobility limits the current; the actual current is roughly 10% of the ballistic limit.

But what would happen for a high-mobility (e.g. III-V) FET?

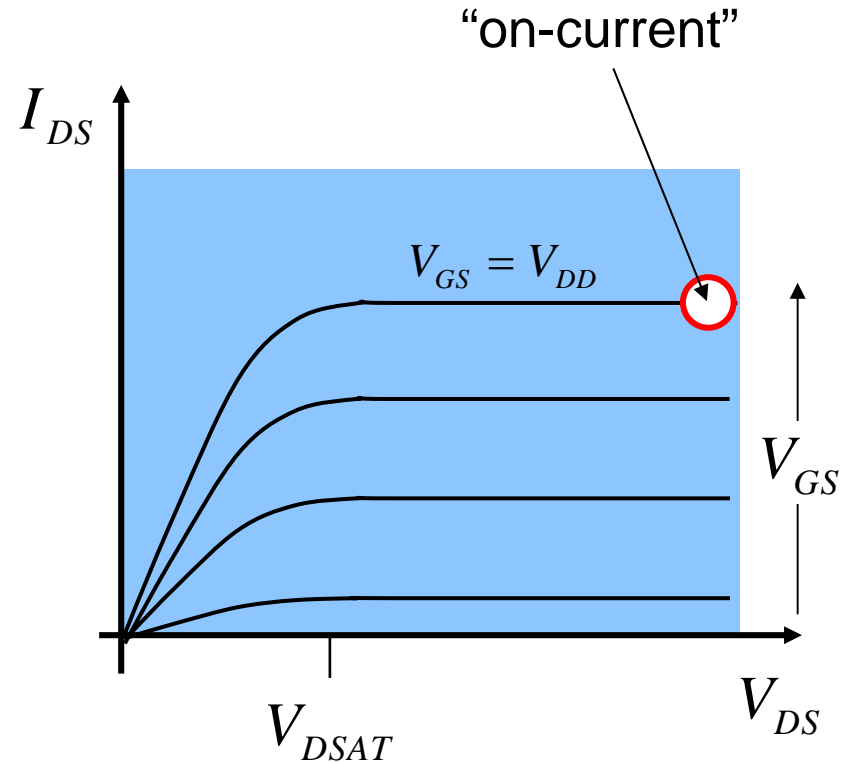
# high $V_{DS}$ : diffusive vs. ballistic

**diffusive:**

$$I_D = WC_{ox} v_{sat} (V_{GS} - V_T)$$

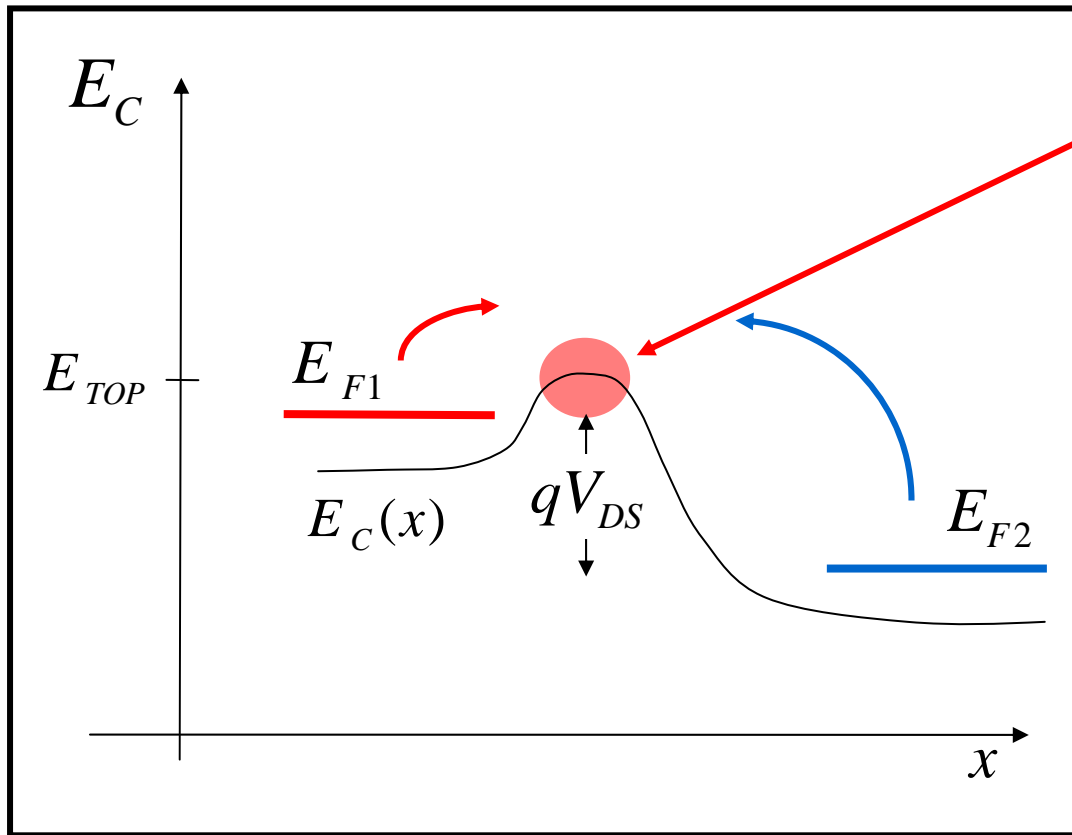
**ballistic:**

$$I_{ON} = WC_{ox} v_T (V_{DD} - V_T)$$



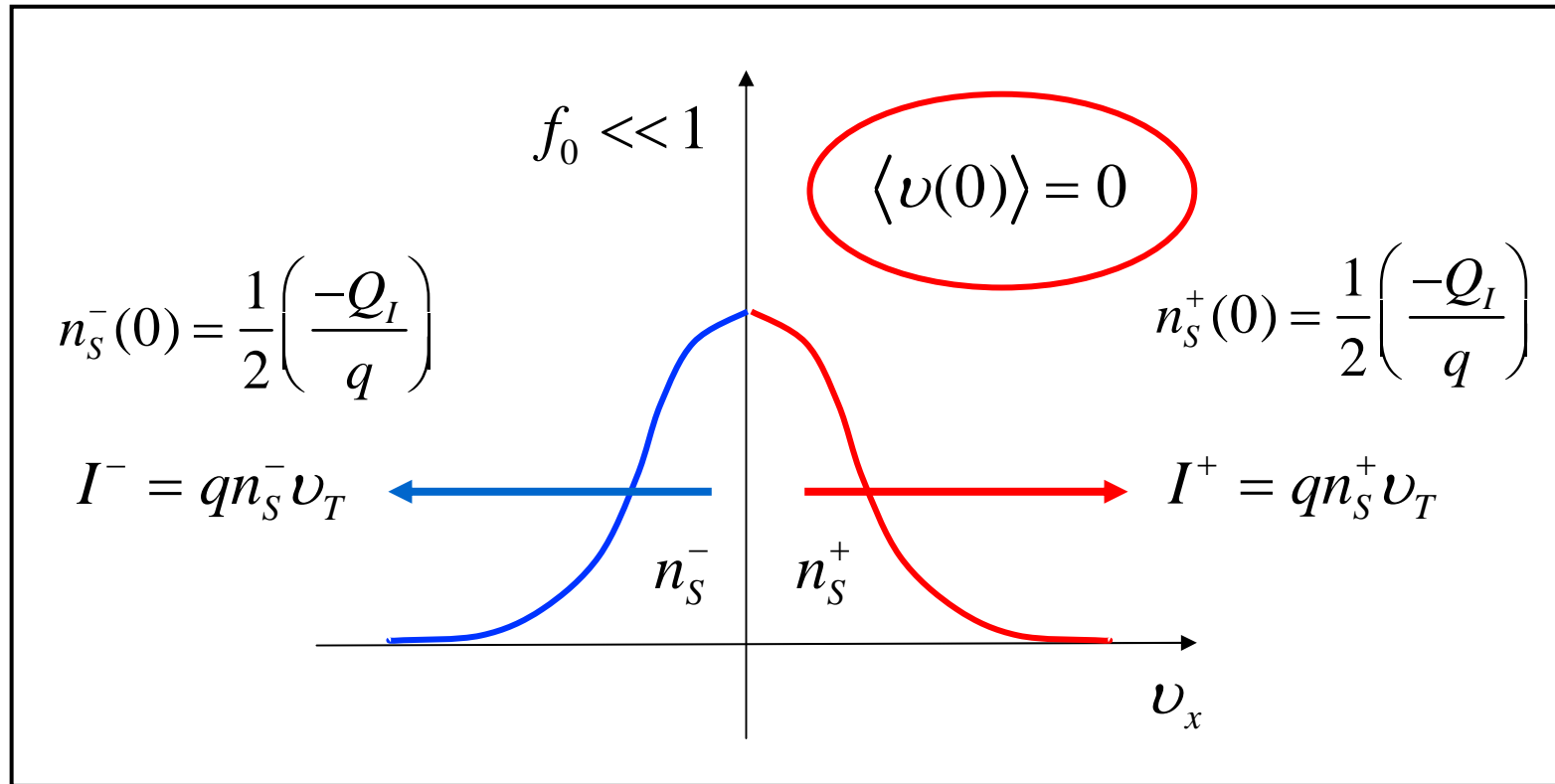
How does the velocity saturate in a ballistic MOSFET?

# velocity vs. drain bias in a ballistic MOSFET



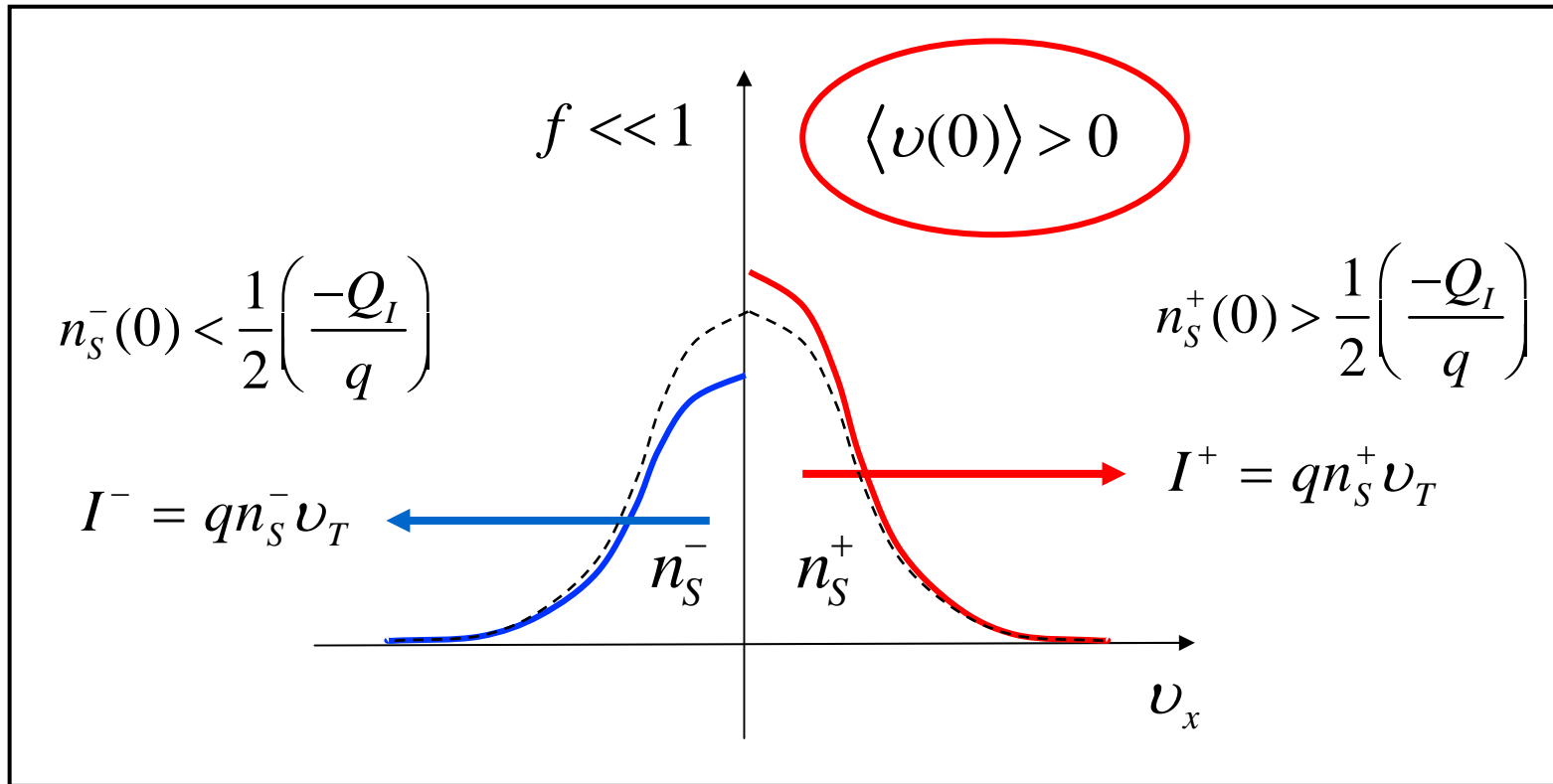
What is the velocity vs. drain bias at the top of the barrier?

# the ballistic MOSFET: $V_{DS} = 0$



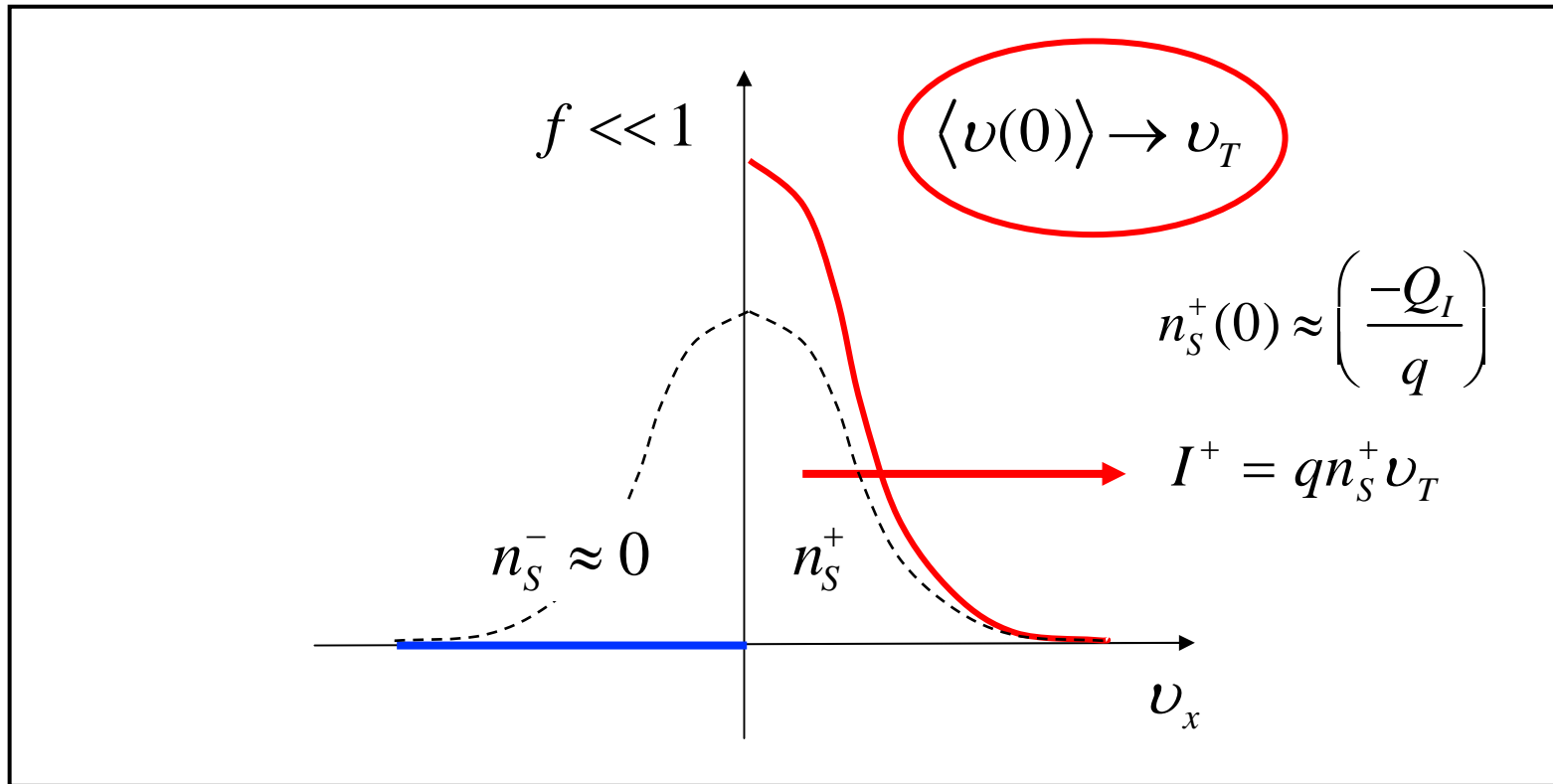
$$Q_I(0) = -q \left[ n_s^+(0) + n_s^-(0) \right] = Q_G$$

# the ballistic MOSFET: $V_{DS} > 0$



$$Q_I(0) = -q \left[ n_s^+(0) + n_s^-(0) \right] = Q_G$$

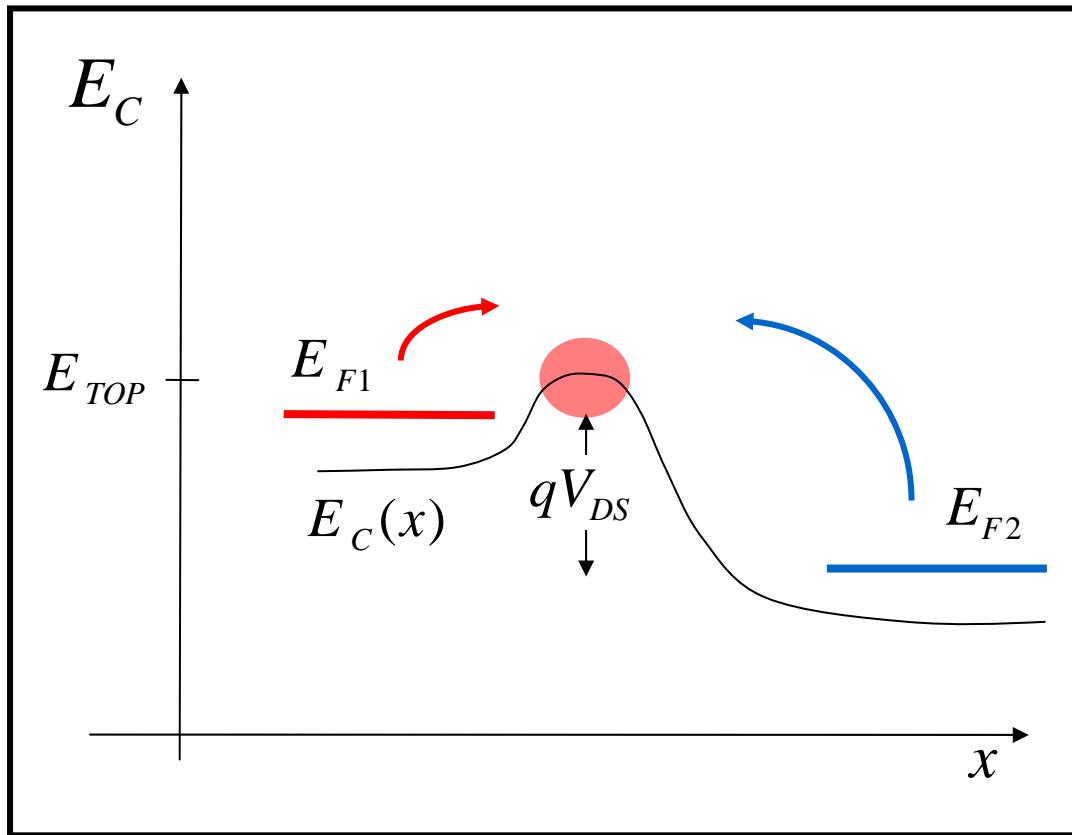
# the ballistic MOSFET: $V_{DS} \gg 0$



$$Q_I(0) = -q \left[ n_S^+(0) + n_S^-(0) \right] = Q_G$$

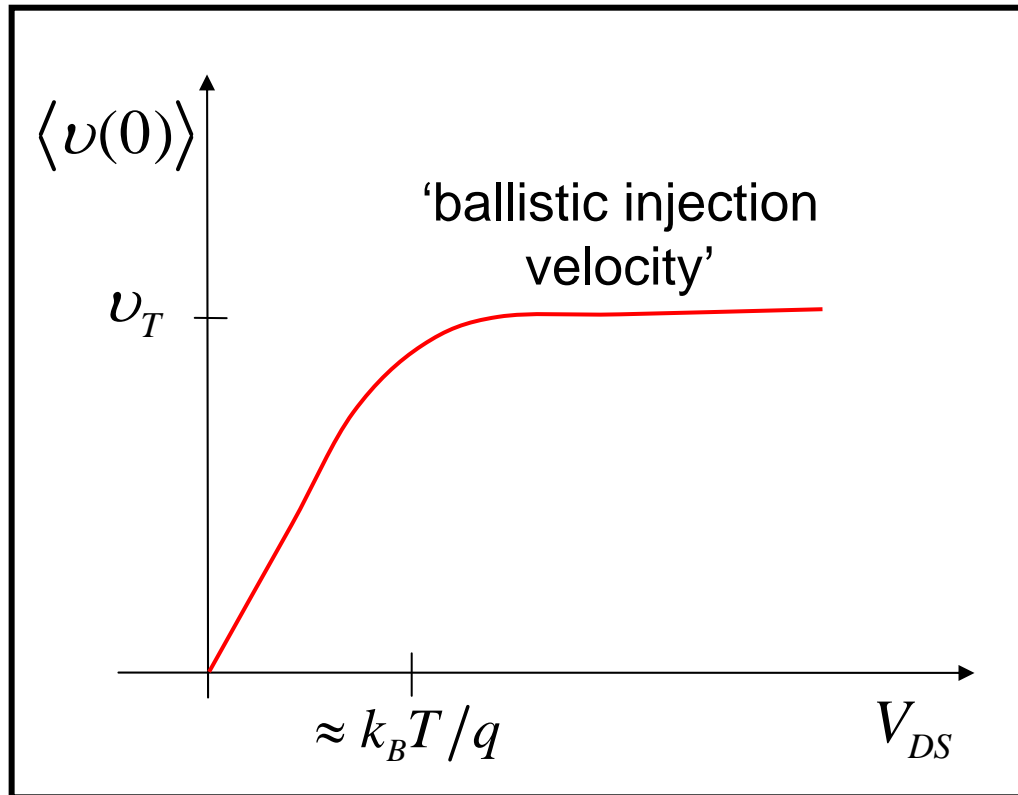


# velocity vs. drain bias in a ballistic MOSFET



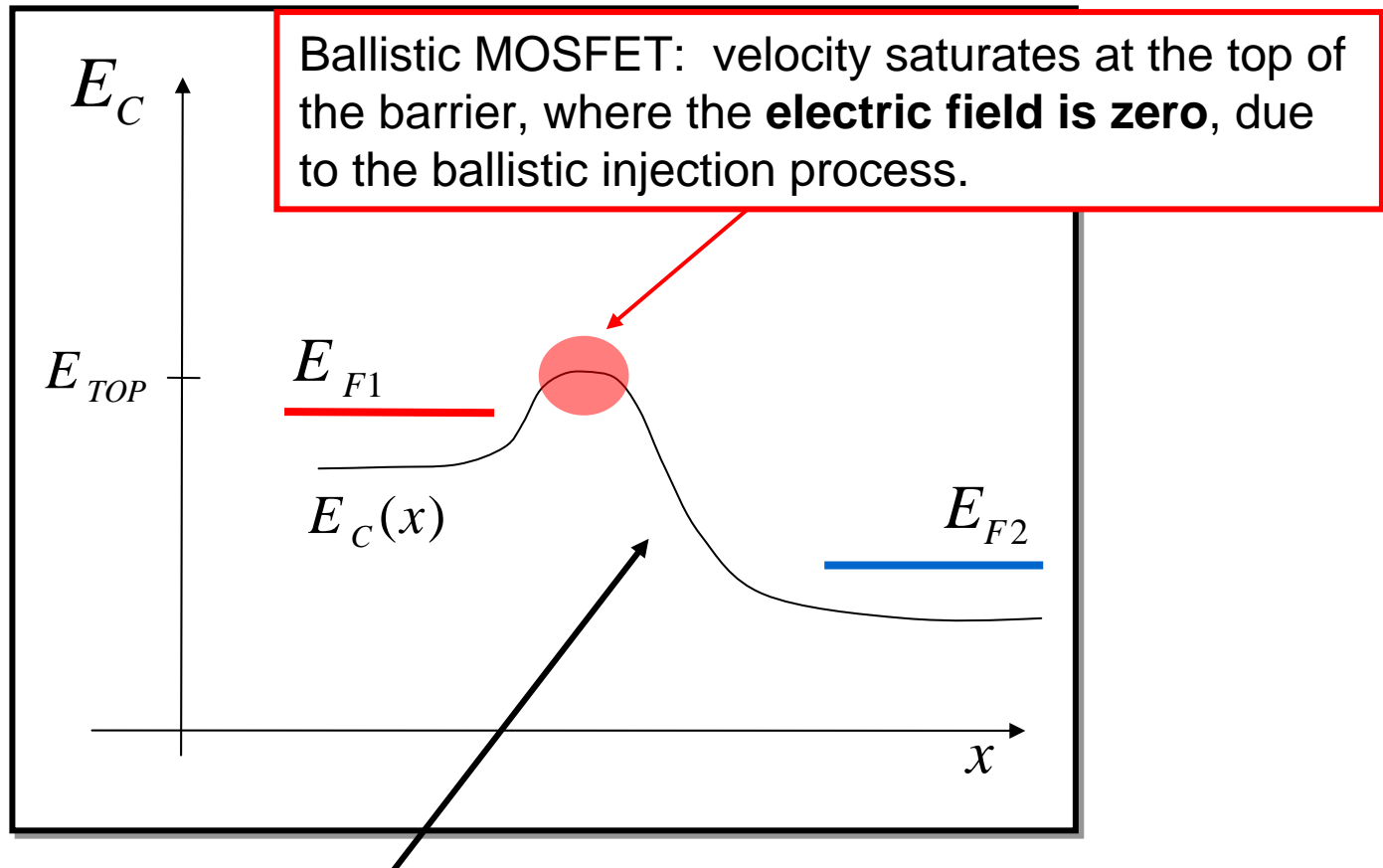
$$\begin{aligned}
 \langle v(0) \rangle &= \frac{n_S^+ v_T - n_S^- v_T}{n_S^+ + n_S^-} \\
 &= v_T \frac{1 - n_S^- / n_S^+}{1 + n_S^- / n_S^+} \\
 &= v_T \frac{1 - e^{-qV_{DS}/k_B T}}{1 + e^{-qV_{DS}/k_B T}}
 \end{aligned}$$

# velocity vs. drain bias in a ballistic MOSFET



$$\langle v(0) \rangle = v_T \left( \frac{1 - e^{-qV_{DS}/k_B T}}{1 + e^{-qV_{DS}/k_B T}} \right)$$

# velocity vs. drain bias in a ballistic MOSFET



Long channel MOSFET: velocity saturates near the drain due to scattering in the **high electric field** region.

# outline

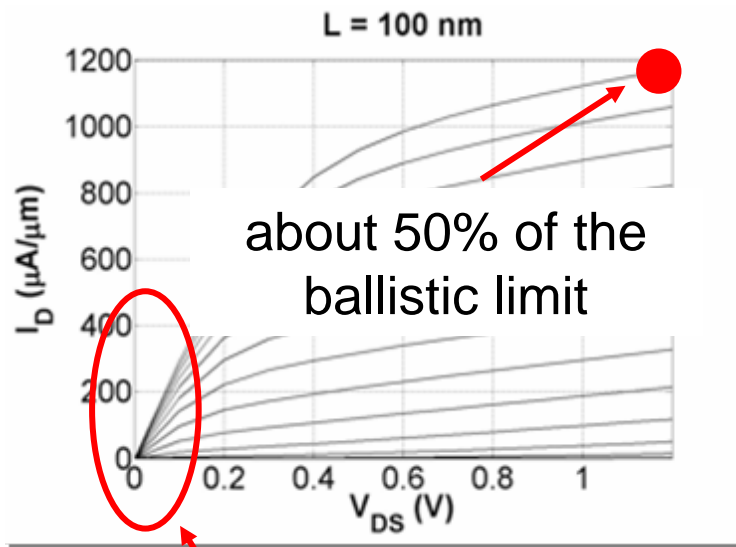
---

- 1) Introduction
- 2) Ballistic theory of the MOSFET
- 3) Discussion: ballistic MOSFETs
- 4) Scattering in nano-MOSFETs**
- 5) Summary

# Is a nanoscale MOSFET really ballistic?

Typical N-channel MOSFET:

$$I_{ON} \approx 1 \text{ mA}/\mu\text{m}$$



(Courtesy, Shuji Ikeda, ATDF, Dec. 2007)

about 10% of the ballistic limit.

$$I_{ON} (\text{ballistic}) = -WQ_I(0)v_T$$

$$v_T = \sqrt{2k_B T / \pi m^*}$$

$$\approx 1.2 \times 10^7 \text{ cm/s}$$

$$-Q_I(0)/q = C_{ox} (V_{DD} - V_T)$$

$$\approx 1 \times 10^{13} \text{ cm}^{-2}$$

$$I_{ON}/W (\text{ballistic}) \approx 2 \text{ mA}/\mu\text{m}$$

# mean-free-path for scattering

---

For  $L = 100$  nm NMOS Si technology:  $\mu_{eff} \approx 200 \text{ cm}^2/\text{V-s}$

$$D_{eff} = \frac{k_B T}{q} \mu_{eff} \approx 5 \text{ cm}^2/\text{s}$$

$$D_{eff} = \frac{v_T \lambda_0}{2} \Rightarrow \lambda \approx 8 \text{ nm}$$

$$\frac{L}{\lambda_0} \approx 11$$

explains why  $I_D(\text{lin})$  is  $\sim 10\%$  of the ballistic limit, but why is  $I_{ON} \sim 50\%$  of the ballistic limit?

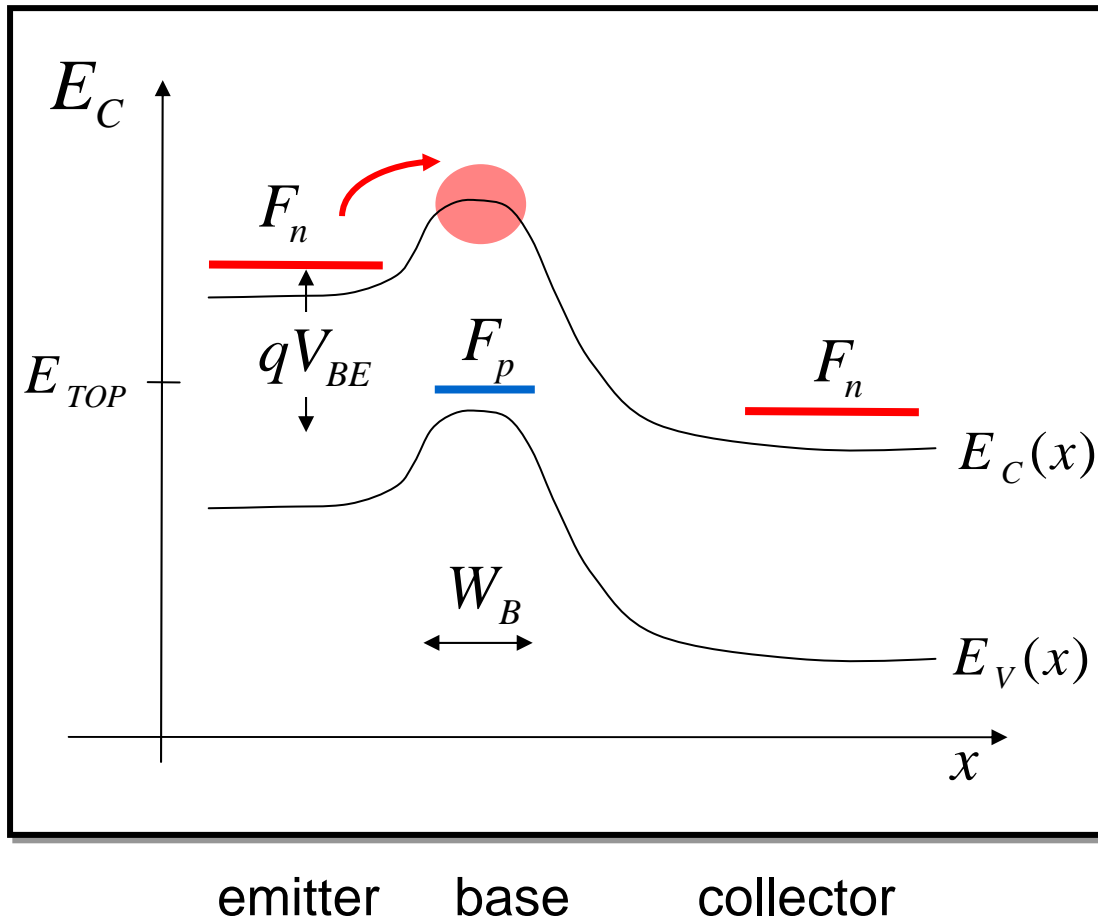
(Note that near the drain,  $\lambda$  is even shorter!)

# scattering in a nano-MOSFET

---

- 1) Where in the channel does scattering matter the most?  
(for low  $V_{DS}$ , the answer is everywhere)
- 2) How can a MOSFET with a channel length many mean-free-paths long deliver an on-current that is within ~50% of the ballistic limit?

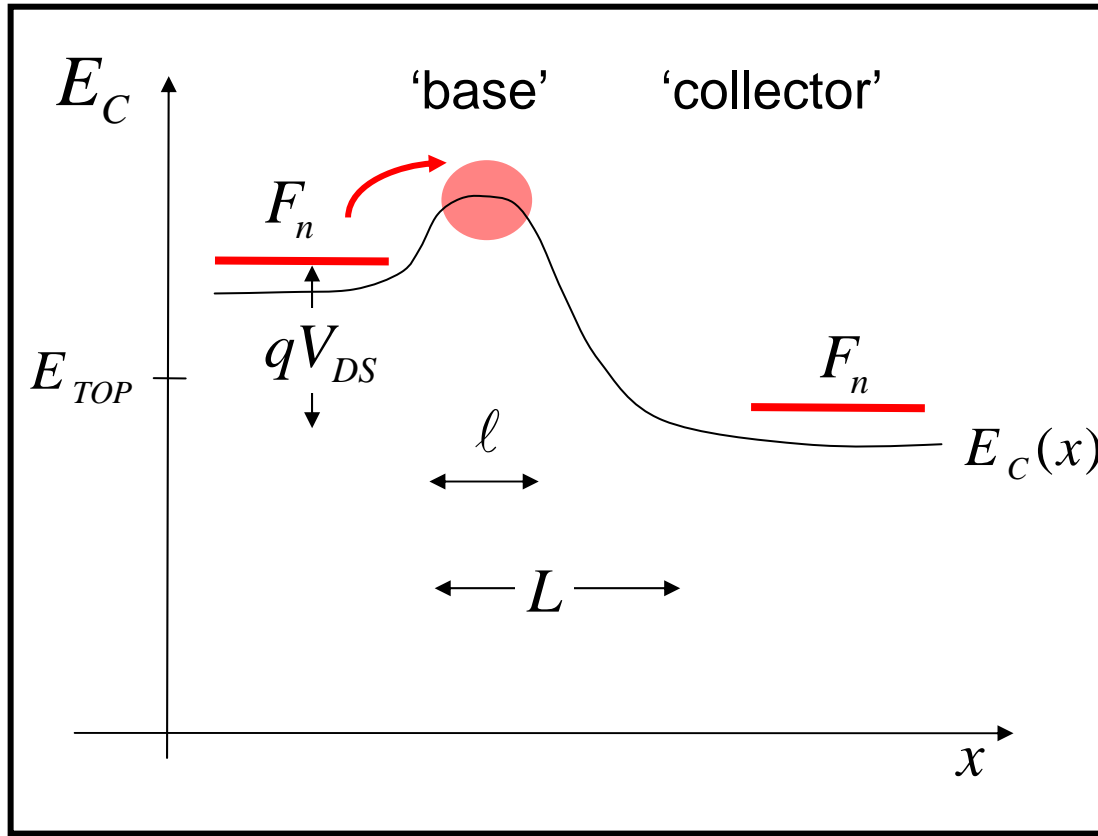
# scattering in a bipolar transistor



$$I_C = qA_E D_n \frac{\Delta n(0)}{W_B}$$



# scattering in a MOSFET



source channel drain

$$I_D = qWD_n \frac{n_s(0)}{l} \quad A/cm$$

$$I_D = -W \frac{D_n}{l} Q_I(0)$$

$$I_D = WC_{ox} \nu_D (V_{GS} - V_T)$$

$$\nu_D = \langle \nu(0) \rangle \equiv \frac{D_n}{l}$$

$$\langle \nu(0) \rangle \leq \nu_T$$

# the quasi-ballistic MOSFET

**ballistic:**

$$I_{ON} = WC_{ox} v_T (V_{DD} - V_T)$$

**diffusive:**

$$I_{ON} = WC_{ox} v_D (V_{GS} - V_T)$$

**general:**

$$I_{ON} = WC_{ox} \langle v(0) \rangle (V_{GS} - V_T)$$

$$\ell(V_{GS}, V_{DS}) = ?$$

$$\frac{1}{\langle v(0) \rangle} = \frac{1}{v_D} + \frac{1}{v_T} \quad v_T = \sqrt{\frac{2k_B T}{\pi m^*}} \quad v_D = D_n / l$$

recall: quasi-ballistic MOSFET: low  $V_{DS}$

---

$$I_D = \frac{W}{L} \left( \frac{1}{\mu_{eff}} + \frac{1}{\mu_B} \right)^{-1} C_{ox} (V_{GS} - V_T) V_{DS}$$

## the critical length: example

For  $L = 100$  nm NMOS Si technology:

$$\mu_{eff} \approx 200 \text{ cm}^2/\text{V-s}$$

$$D_{eff} = \frac{k_B T}{q} \mu_{eff} \approx 5 \text{ cm}^2/\text{s}$$

$$V_{DD} = 1.2 \text{ V}$$

$$V_T = 0.23 \text{ V}$$

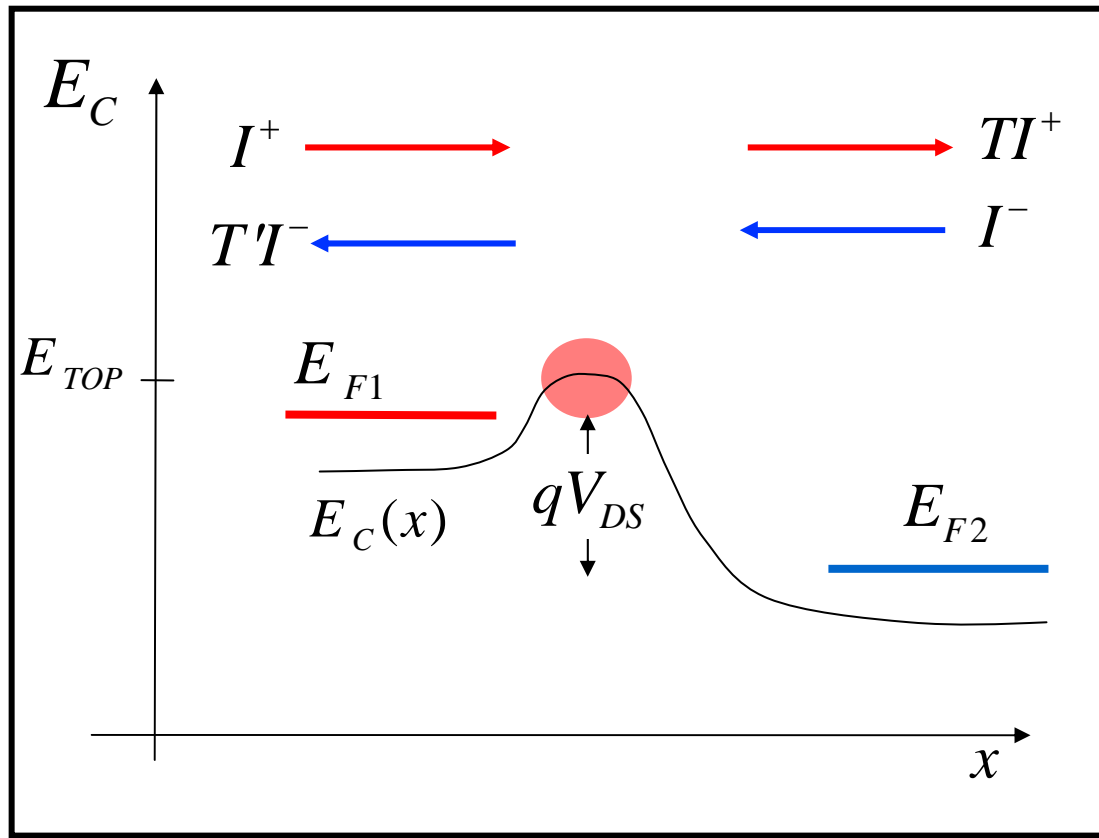
$$C_{ox} = 1.5 \times 10^{-6} \text{ F/cm}^2$$

$$I_{ON}/W = WC_{ox} \frac{D_n}{l} (V_{DD} - V_T) \approx 1.2 \text{ mA}/\mu\text{m} \Rightarrow l \approx 6 \text{ nm}$$

$$\frac{l}{\lambda_0} \approx 0.75$$

(Explains why on-current is ~ 50% of ballistic limit)

# another way of treating scattering....



$T$

'transmission'  
coefficient

$$R = 1 - T$$

'reflection' or  
'backscattering'  
coefficient

$$T = \frac{\lambda}{\lambda + L}$$

# outline

---

- 1) Introduction
- 2) Ballistic theory of the MOSFET
- 3) Discussion: ballistic MOSFETs
- 4) Scattering in nano-MOSFETs
- 5) **Summary**

# summary

---

1) MOSFETs are 'barrier-controlled' devices  
(just like bipolar transistors)

2) A simple, ballistic model is easy to derive  
(assuming Boltzmann statistics)

3) Connection to diffusive model is clear

$$\mu_{eff} \rightarrow \mu_B \quad v_{sat} \rightarrow v_T$$

4) Modern MOSFETs operate between the ballistic and diffusive limit where things get more complicated.

$$\ell(V_{GS}, V_{DS}) = ?$$

# what have we accomplished?

---

Developed a very simple, conceptual model that captures the essential physics of a nanoscale MOSFET

- not a replacement for numerical simulation, but helpful in interpreting simulation results.
- provides a ‘sanity check’ for numerical models and for compact circuit models.
- useful for interpreting experiments and guiding device design.



# what have we left out (of the ballistic model?)

---

- 1) Fermi-Dirac statistics (above threshold)
- 2) Treatment of sub-threshold conduction
- 3) Two-dimensional electrostatics
- 4) Multiple subbands and other details...

***These topics will be discussed in Lectures 3A and 3B.***