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Physics of Nanoscale Transistors: Lecture 2:

Elementary Theory of the Nanoscale MOSFET

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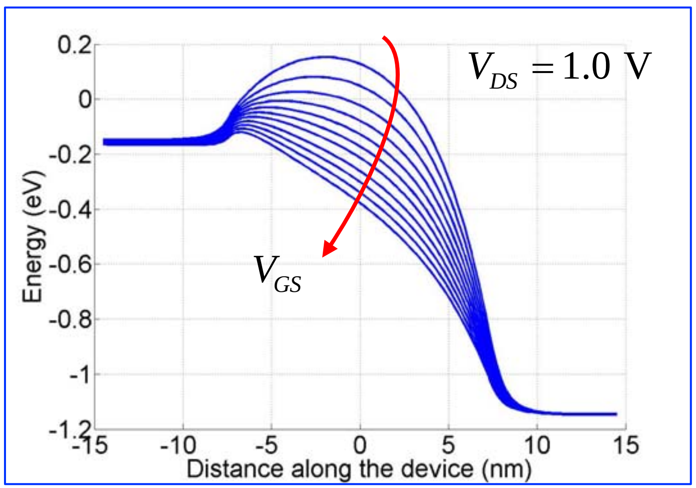
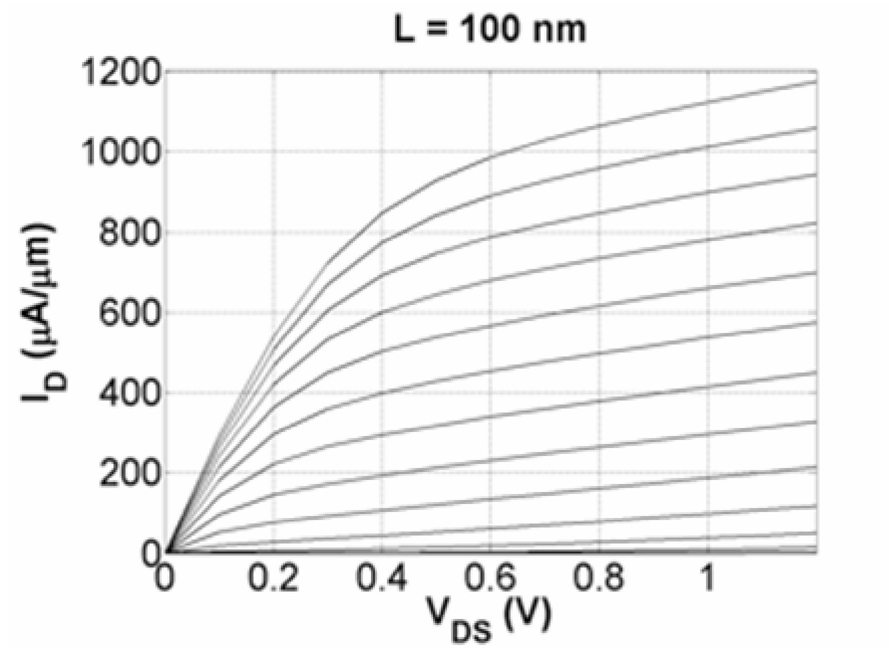
transistors

$$I^{+} \longrightarrow \quad \longleftarrow I^{-}$$



$$I_D \propto I^{+} - I^{-}$$

2007 N-MOSFET

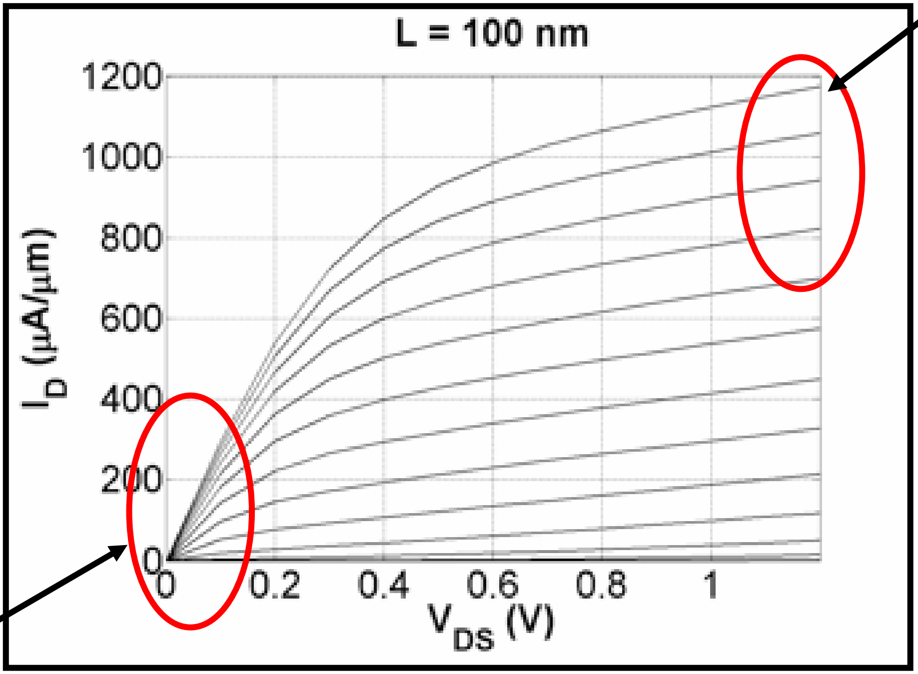


(Courtesy, Shuji Ikeda, ATDF, Dec. 2007)

diffusive transport ($L >> \lambda$)

$$I_D = W C_{ox} \upsilon_{sat} \left(V_{GS} - V_T\right)$$

2007 N MOSFET



$$I_D = \frac{W}{L} \mu_{eff} C_{ox} \left(V_{GS} - V_T\right) V_{DS}$$

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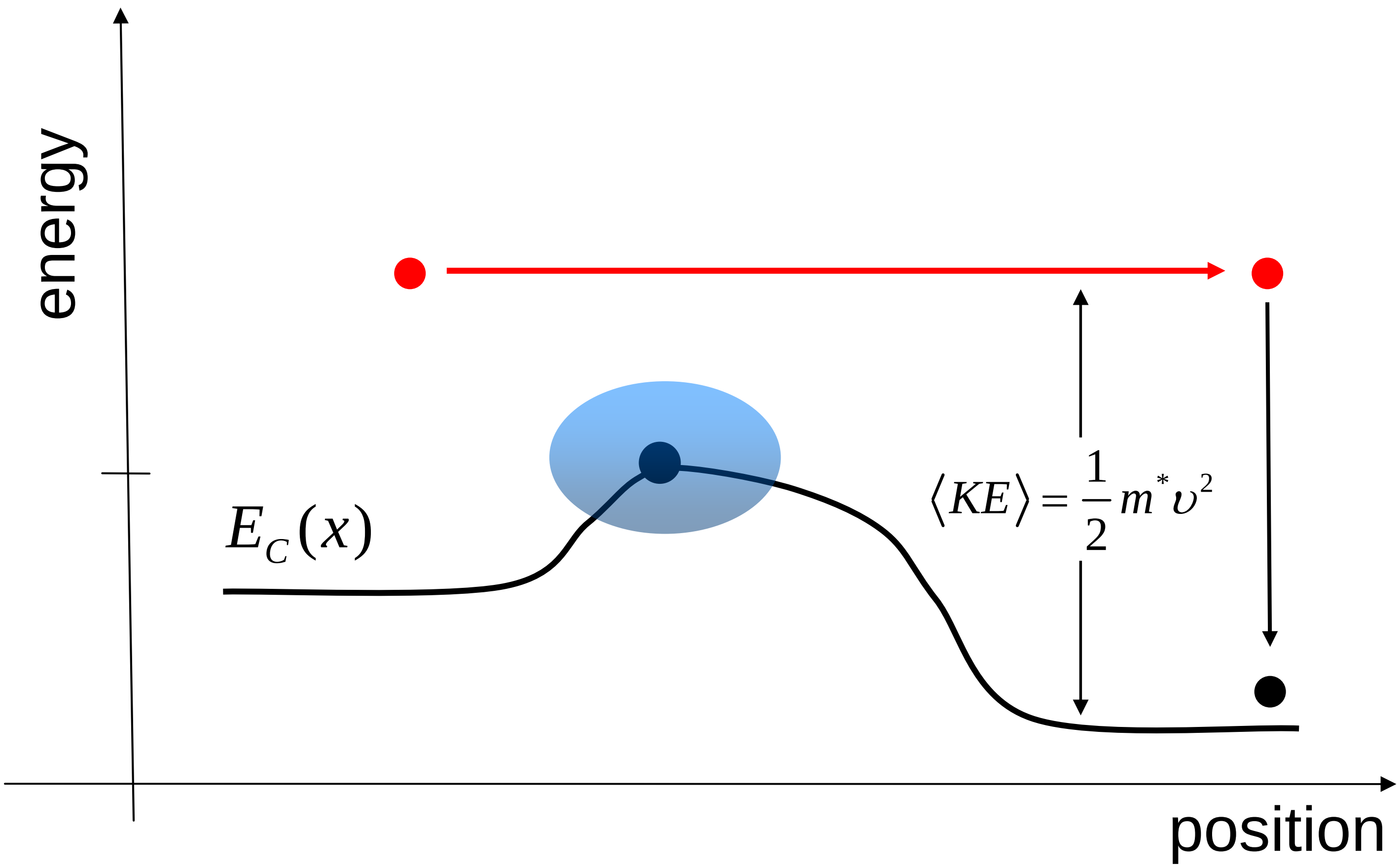
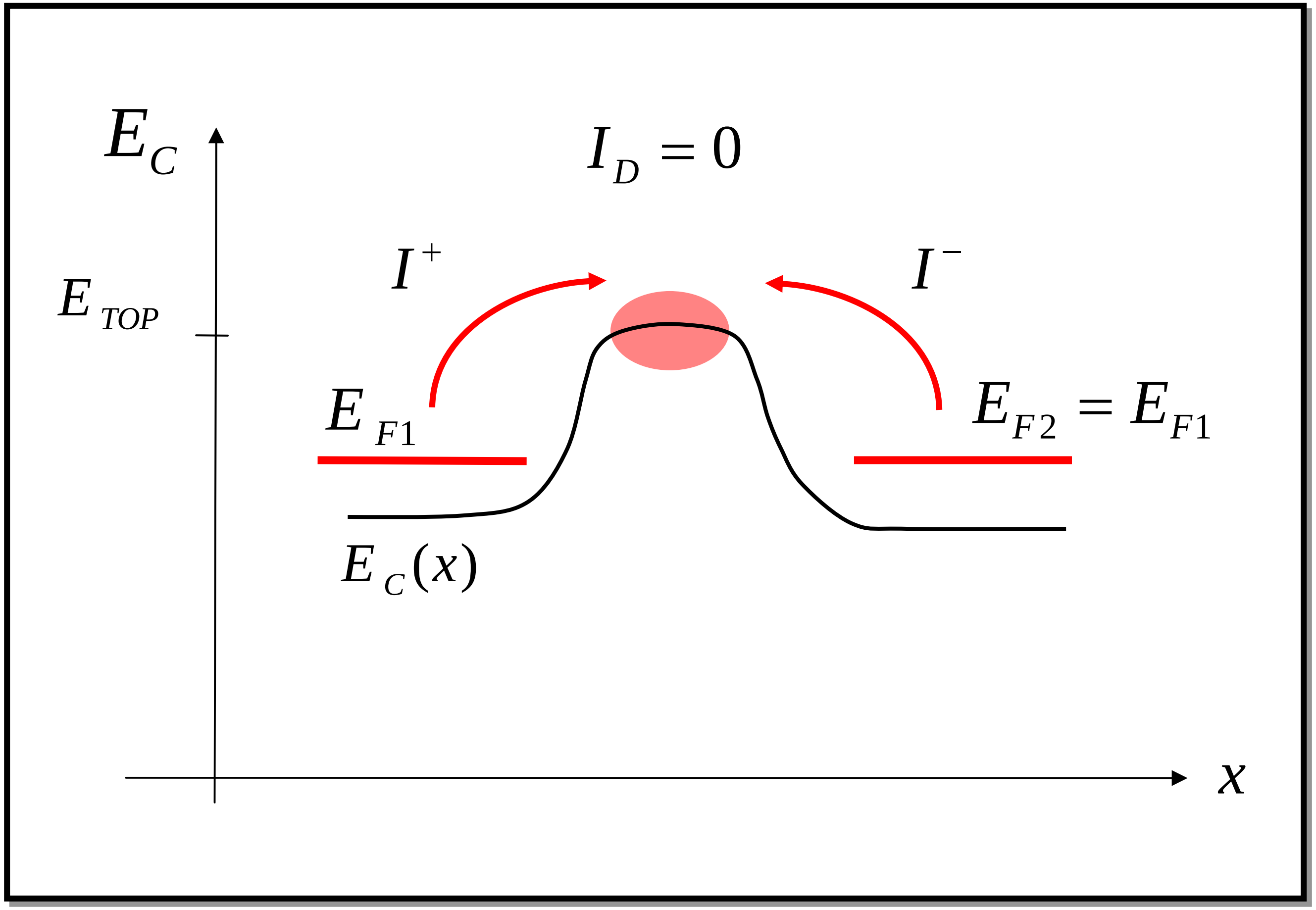
ballistic transport ($L << \lambda$)



outline

1) Introduction
2) **Ballistic theory of the MOSFET**
3) Discussion: ballistic MOSFETs
4) Scattering in nano-MOSFETs
5) Summary

MOSFET in equilibrium



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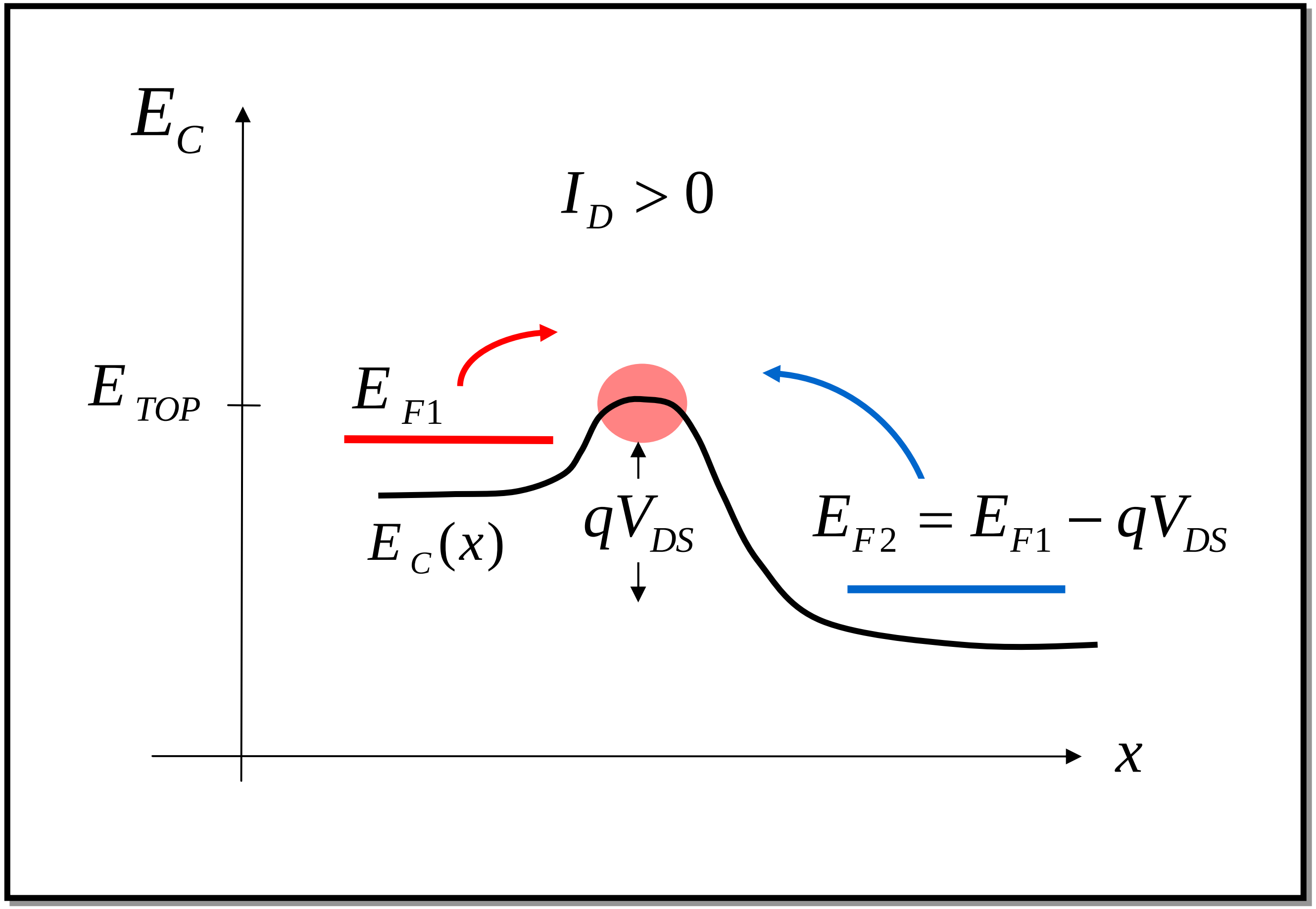
high gate and drain bias



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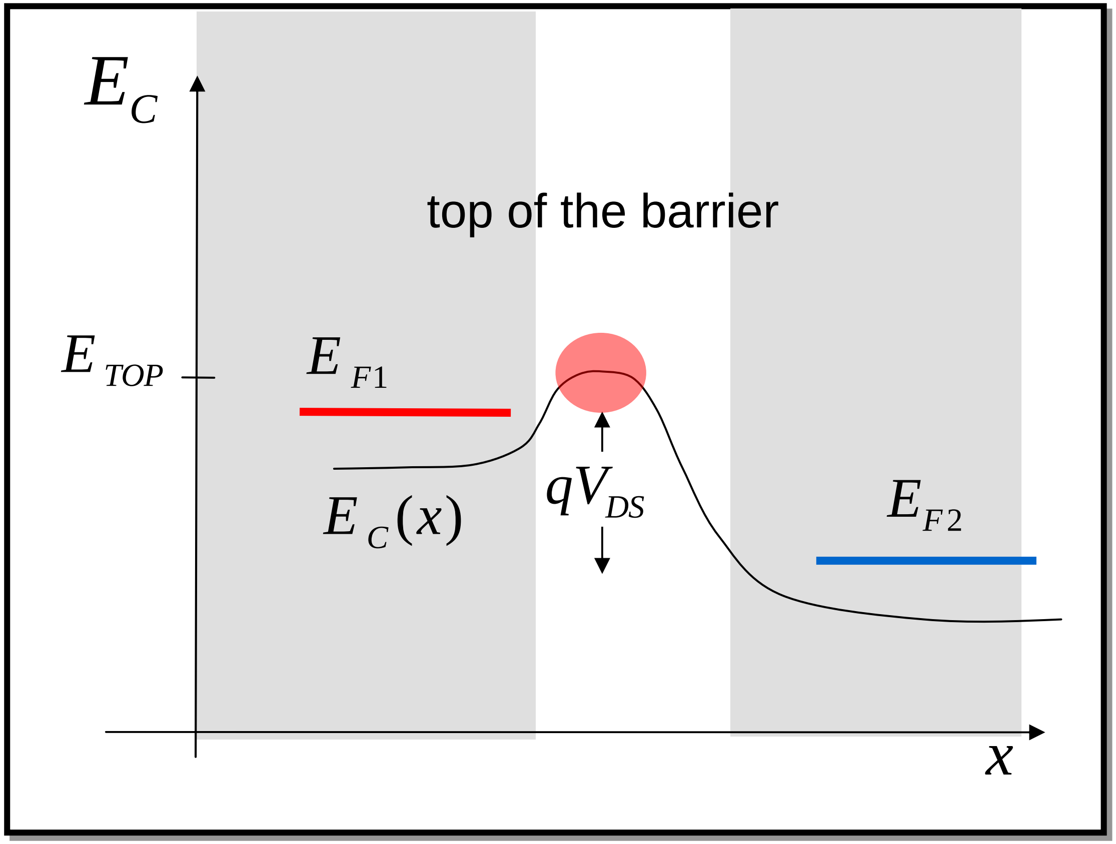
the ballistic MOSFET



ballistic channel

We will evaluate the current at the top of the barrier (x = 0) under the following **assumptions**.

1) ballistic transport in the channel

2) source and drain are reservoirs of thermal equilibrium carriers.

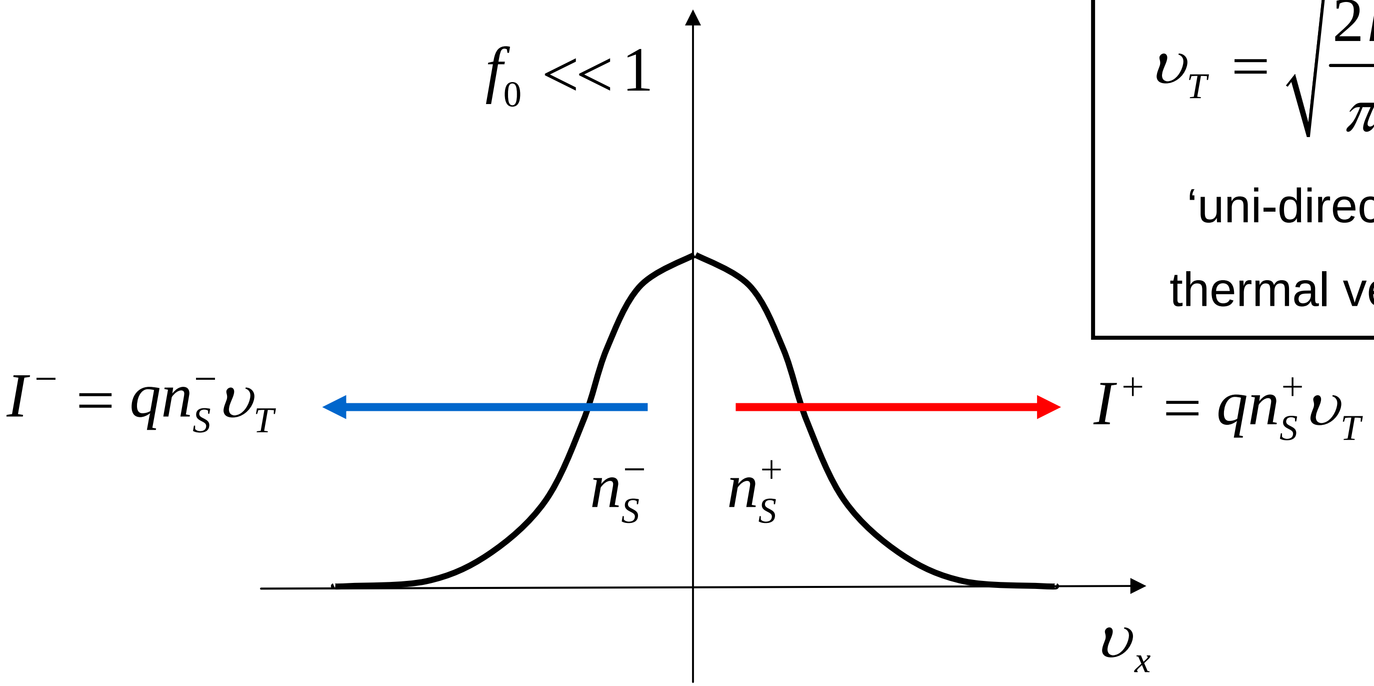
3) nondegenerate carriers

4) $V_{GS} > V_T$

equilibrium velocity distribution of carriers

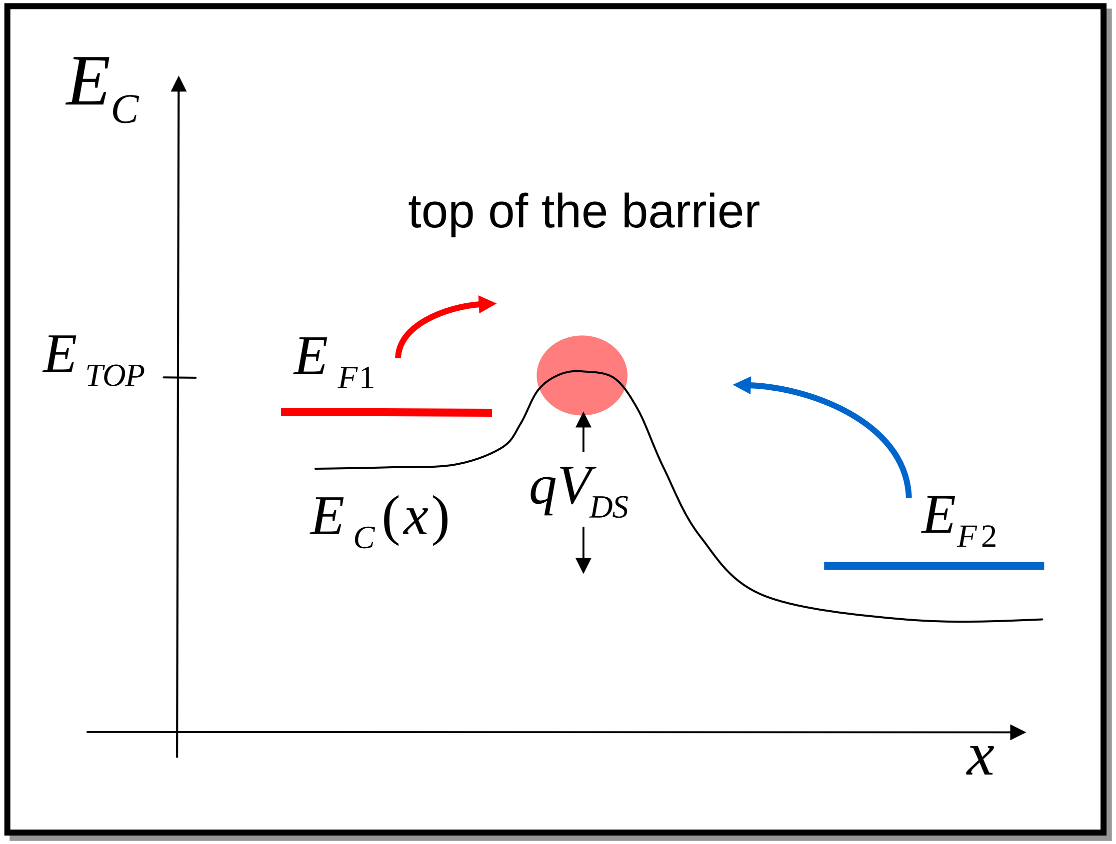
$$\upsilon_T = \sqrt{\frac{2k_B T}{\pi m^*}}$$

'uni-directional thermal velocity'



$$f_0 = \frac{1}{1 + e^{\left(E - E_F / k_B T\right)}} \approx e^{\left(E_F - E\right)/k_B T} \propto e^{-m^* \upsilon_x^2 / 2k_B T}$$

the ballistic MOSFET



$$Q_I(0) \;\; \mathrm{C/cm^2}$$

$$Q_I = -q\left(n_S^+ + n_S^-\right) \;\; \mathrm{C/cm^2}$$

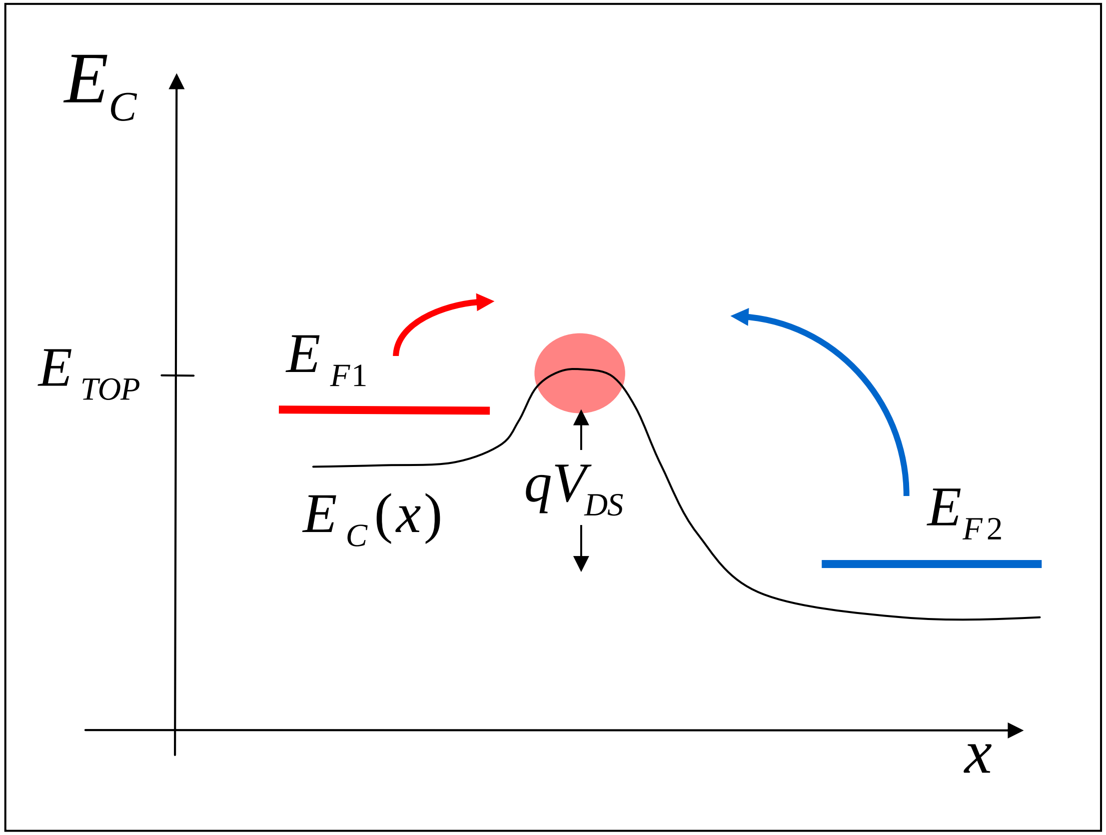
$$n_S^- / n_S^+ = e^{-qV_{DS}/k_BT}$$

$$Q_I = -qn_S^+\left(1 + e^{-qV_{DS}/k_BT}\right)$$

$$qn_S^+ = \frac{-Q_I}{1 + e^{-qV_{DS}/k_BT}}$$

1) ballistic transport
2) nondegenerate carriers

the ballistic MOSFET: IV



$$I_D = q\left(n_S^+ \upsilon_T - n_S^- \upsilon_T\right)$$

$$I_D = qn_S^+ \upsilon_T \left(1 - e^{-qV_{DS}/k_BT}\right)$$

$$qn_S^+ = \frac{-Q_I}{1 + e^{-qV_{DS}/k_BT}}$$

$$I_D = -Q_I \upsilon_T \left(\frac{1 - e^{-qV_{DS}/k_BT}}{1 + e^{-qV_{DS}/k_BT}}\right)$$

$$Q_I = -C_{ox}\left(V_{GS} - V_T\right)$$

1) ballistic
2) nondegenerate carriers
3) ***$V_{GS} > V_T$***

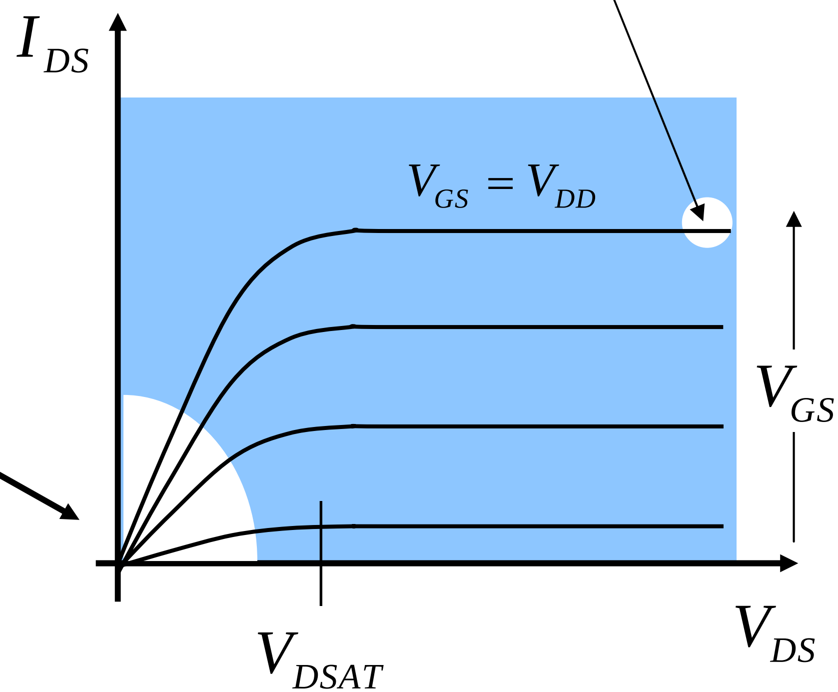
the ballistic MOSFET: IV

$$I_{DS} = C_{ox}\left(V_{GS} - V_T\right)\upsilon_T\left(\frac{1 - e^{-qV_{DS}/k_BT}}{1 + e^{-qV_{DS}/k_BT}}\right)$$

$$I_{ON} = WC_{ox}\upsilon_T\left(V_{DD} - V_T\right)$$

“on-current”

$$\begin{cases} V_{DS} < k_BT/q \\ I_{DS} = WC_{OX}\dfrac{\upsilon_T}{2k_BT/q}\left(V_{GS} - V_T\right)V_{DS} \\ I_{DS} = V_{DS}/R_{CH} \end{cases}$$



$$V_{DSAT} \approx k_BT/q$$

outline

1) Introduction
2) Ballistic theory of the MOSFET
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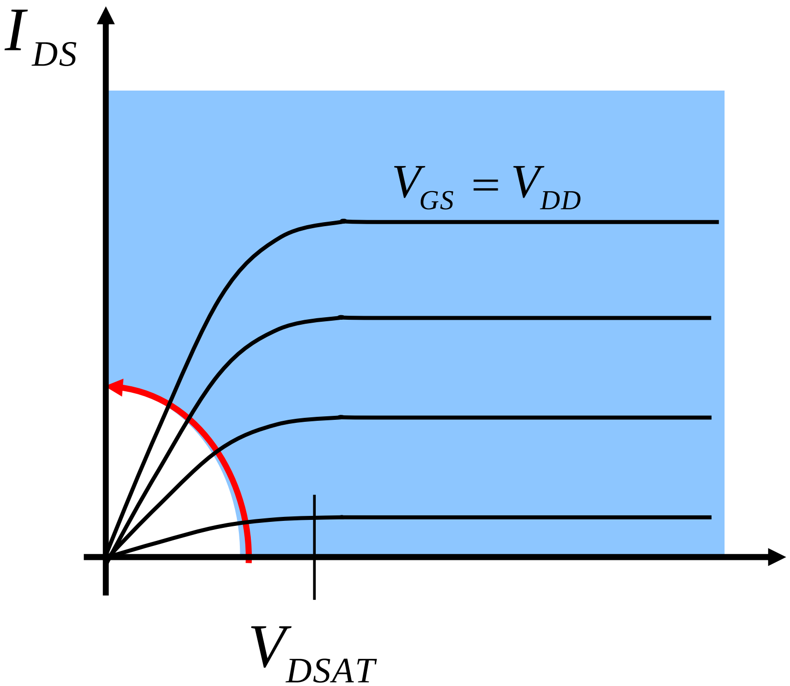
low V_{DS}: diffusive vs. ballistic

diffusive:

$$I_D = \frac{W}{L}\mu_{eff}C_{ox}\left(V_{GS} - V_T\right)V_{DS}$$

$$I_D = V_{DS}/R_{CH}$$

$$R_{CH} = \frac{1}{\mu_{eff}C_{ox}\left(V_{GS} - V_T\right)}\left(\frac{L}{W}\right)$$



ballistic:

$$I_D = WC_{ox}\frac{\upsilon_T}{2k_BT/q}\left(V_{GS} - V_T\right)V_{DS}$$

- a ballistic MOSFET has a finite channel resistance - independent of *L*

- the channel resistance cannot be lower than the ballistic limit

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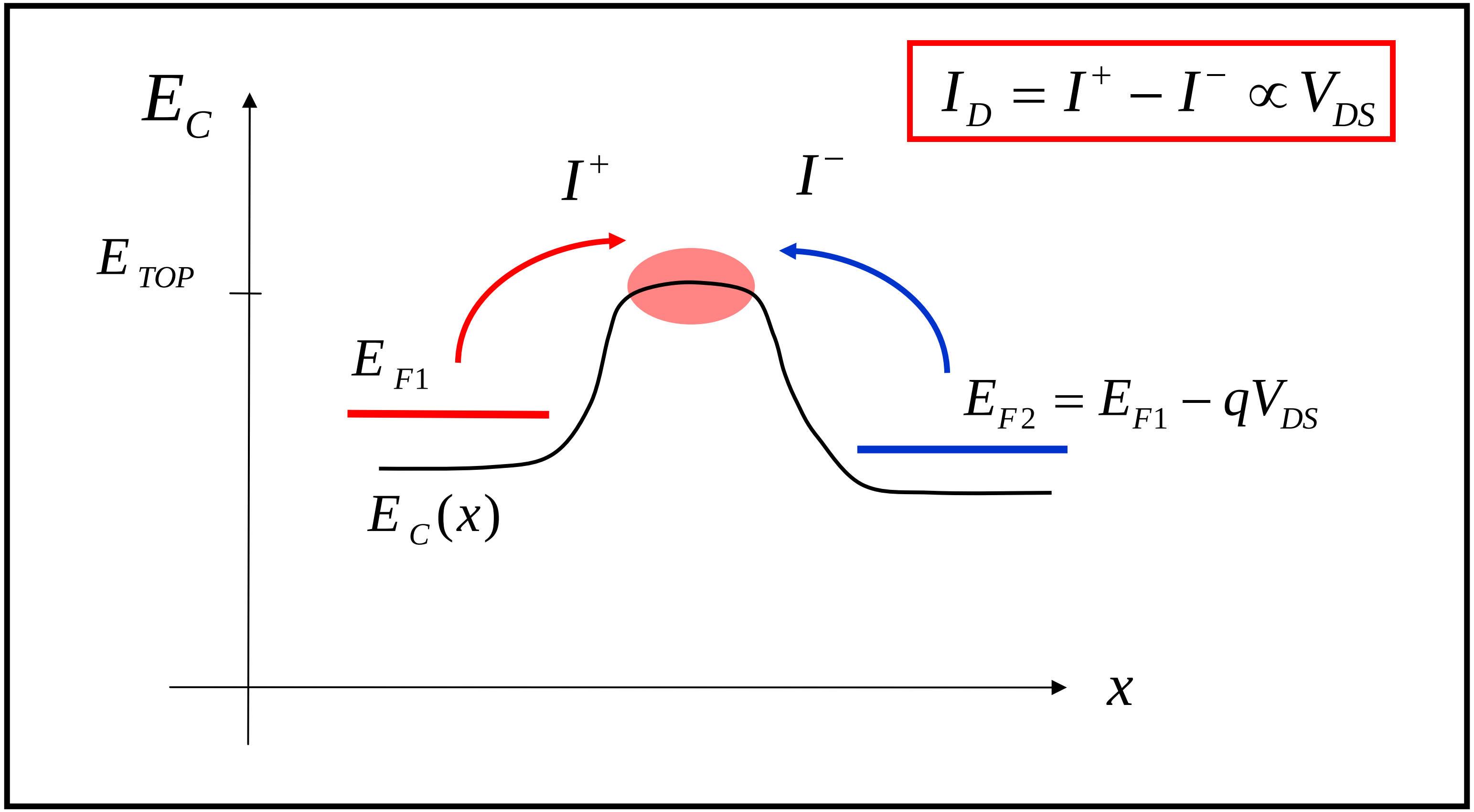
why is there a finite ballistic channel resistance?



relation between diffusive and ballistic models

diffusive:

$$\boxed{I_D = \frac{W}{L}\mu_{eff} C_{ox}\left(V_{GS} - V_T\right)V_{DS}}$$

ballistic:

$$I_D = WC_{ox}\frac{\upsilon_T}{2k_BT/q}\left(V_{GS} - V_T\right)V_{DS}$$

$$I_D = \frac{W}{L}C_{ox}\left[\frac{\upsilon_T L}{2k_BT/q}\right]\left(V_{GS} - V_T\right)V_{DS}$$

$$\mu_B \equiv \left[\frac{\upsilon_T L}{2k_BT/q}\right]$$

'ballistic mobility'

(M.S. Shur, *IEEE Elect. Dev. Lett.*, **3**, 511, 2002.)

$$\boxed{I_D = \frac{W}{L}\mu_B C_{ox}\left(V_{GS} - V_T\right)V_{DS}}$$

the ballistic mobility

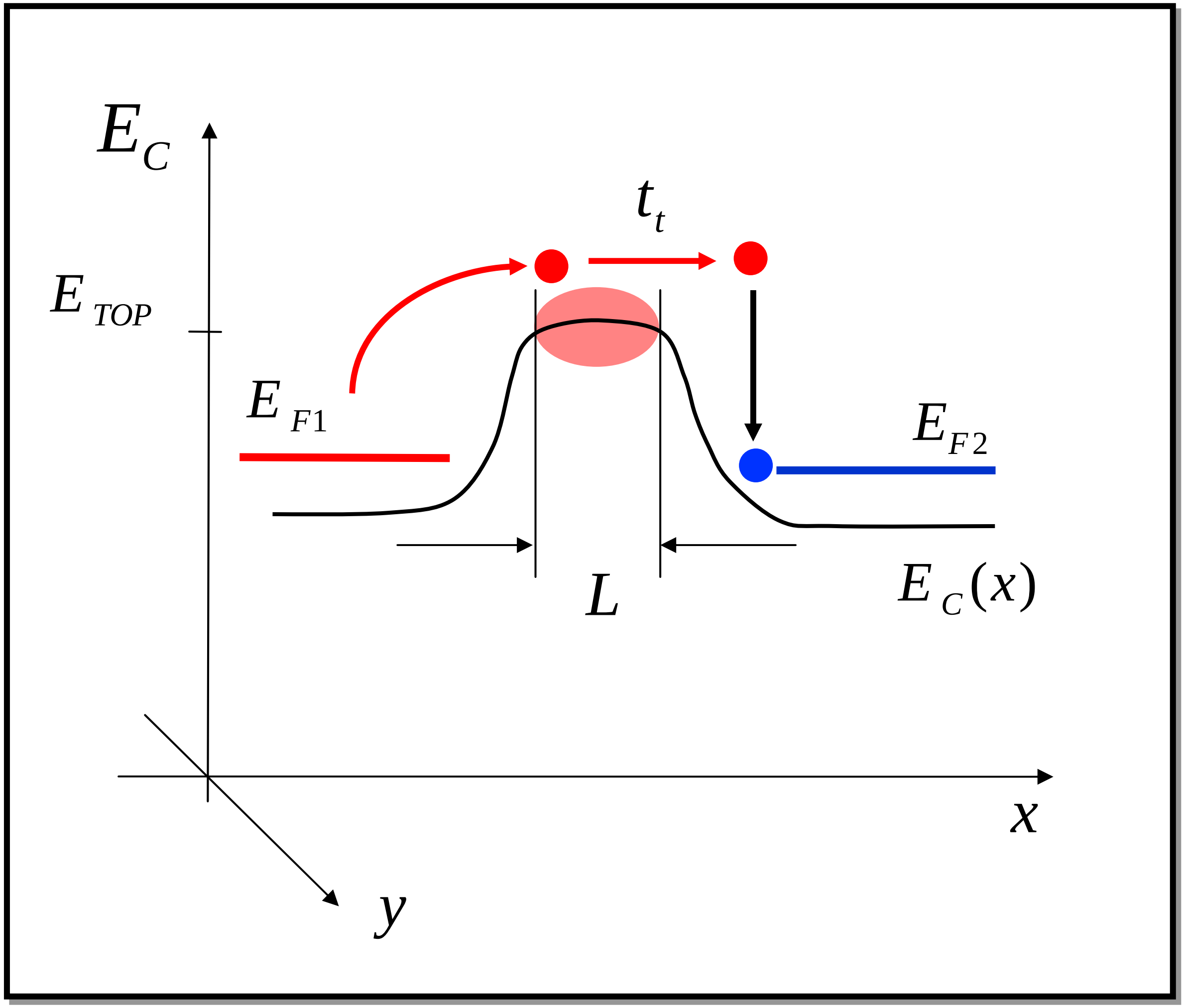
$$\mu_B \equiv \left[\frac{\upsilon_T L}{2k_B T/q}\right]$$

$$\upsilon_T = \sqrt{2k_B T/\pi m^*}$$

$$\mu_B = \left[\frac{q\left(L/\pi\upsilon_T\right)}{m^*}\right]$$

$$= \left[\frac{q\tau_t}{m^*}\right]$$

$$t_t = L/\left(\pi\upsilon_T\right)$$



quasi-ballistic MOSFET: low V_{DS}

ballistic:

$$I_D = \frac{W}{L}\mu_B C_{ox}\left(V_{GS} - V_T\right)V_{DS} \qquad \mu_B \equiv \left[\frac{\upsilon_T L}{2k_B T/q}\right]$$

diffusive:

$$I_D = \frac{W}{L}\mu_{eff} C_{ox}\left(V_{GS} - V_T\right)V_{DS}$$

general:

$$I_D = \frac{W}{L}\left(\frac{1}{\mu_{eff}} + \frac{1}{\mu_B}\right)^{-1} C_{ox}\left(V_{GS} - V_T\right)V_{DS}$$

the ballistic mobility: example

For L = 100 nm NMOS Si technology: $\mu_{eff} \approx 200\ \mathrm{cm}^2/\mathrm{V\text{-}s}$

$$\upsilon_T = \sqrt{2k_BT/\pi m^*} \approx 1\times 10^7\ \mathrm{cm/s}$$

$$\mu_B \equiv \left[\frac{\upsilon_T L}{2k_BT/q}\right] \approx 2000\ \mathrm{cm}^2/\mathrm{V\text{-}s}$$

$\mu_{eff} << \mu_B$ so the real mobility limits the current; the actual current is roughly 10% of the ballistic limit.

But what would happen for a high-mobility (e.g. III-V) FET?

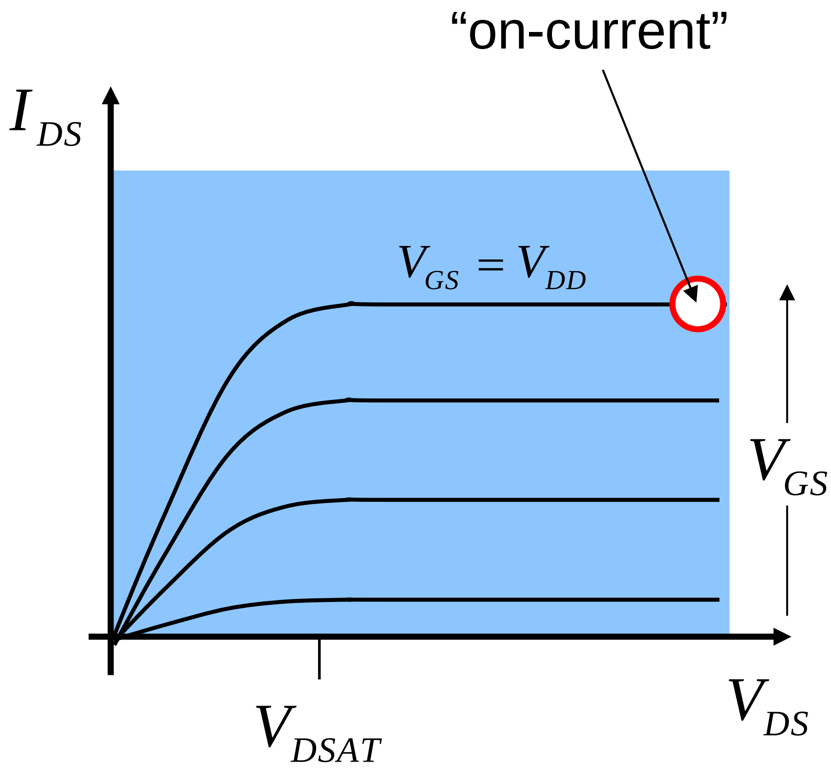
high V_{DS}: diffusive vs. ballistic

diffusive:

$$I_D = W C_{ox} \upsilon_{sat} \left(V_{GS} - V_T\right)$$

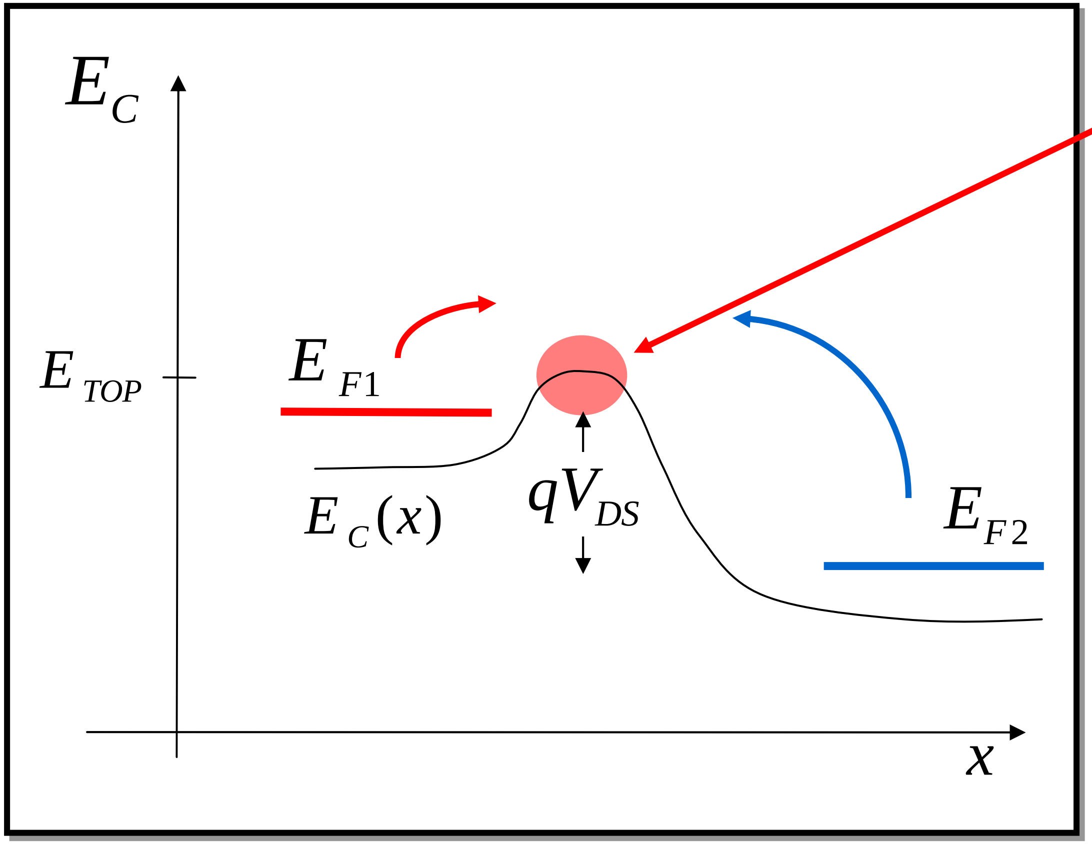
ballistic:

$$I_{ON} = W C_{ox} \upsilon_T \left(V_{DD} - V_T\right)$$



How does the velocity saturate in a ballistic MOSFET?

velocity vs. drain bias in a ballistic MOSFET



What is the velocity vs. drain bias at the top of the barrier?

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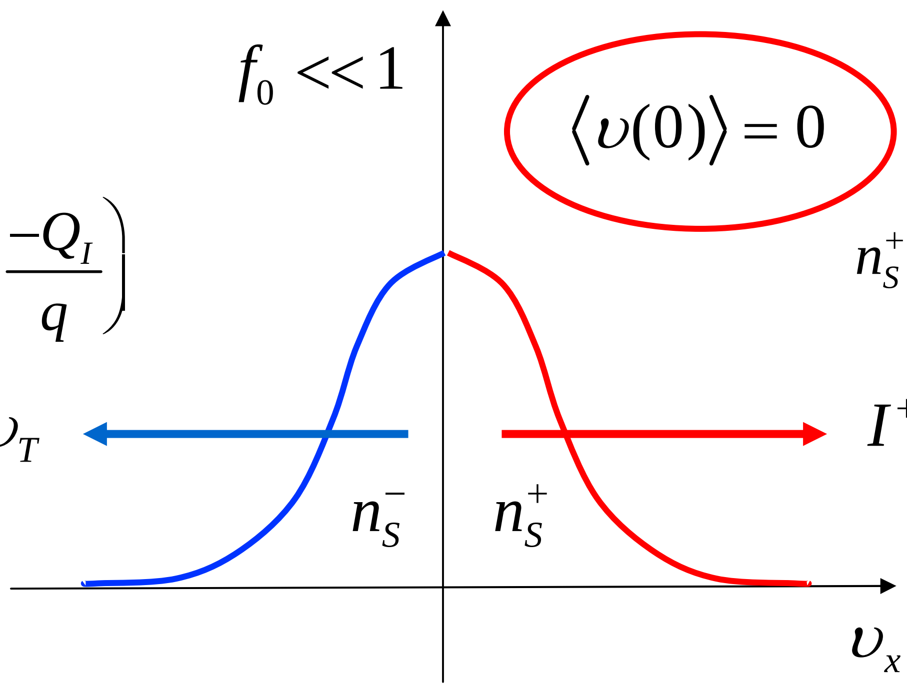
the ballistic MOSFET: $V_{DS} = 0$

$$n_S^-(0) = \frac{1}{2}\left(\frac{-Q_I}{q}\right) \qquad n_S^+(0) = \frac{1}{2}\left(\frac{-Q_I}{q}\right)$$

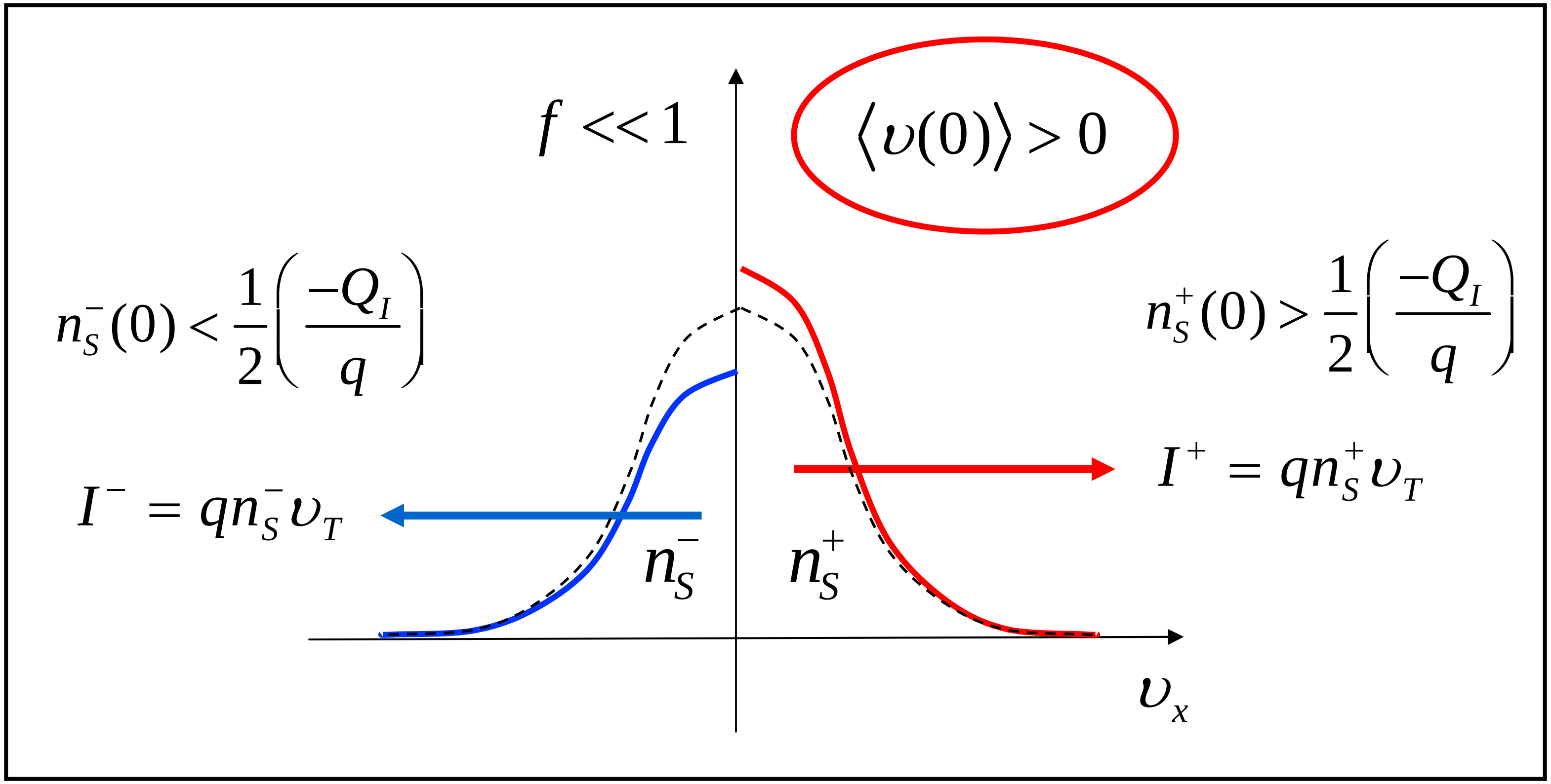
$$\langle \upsilon(0) \rangle = 0$$

$$I^- = q n_S^- \upsilon_T \qquad I^+ = q n_S^+ \upsilon_T$$



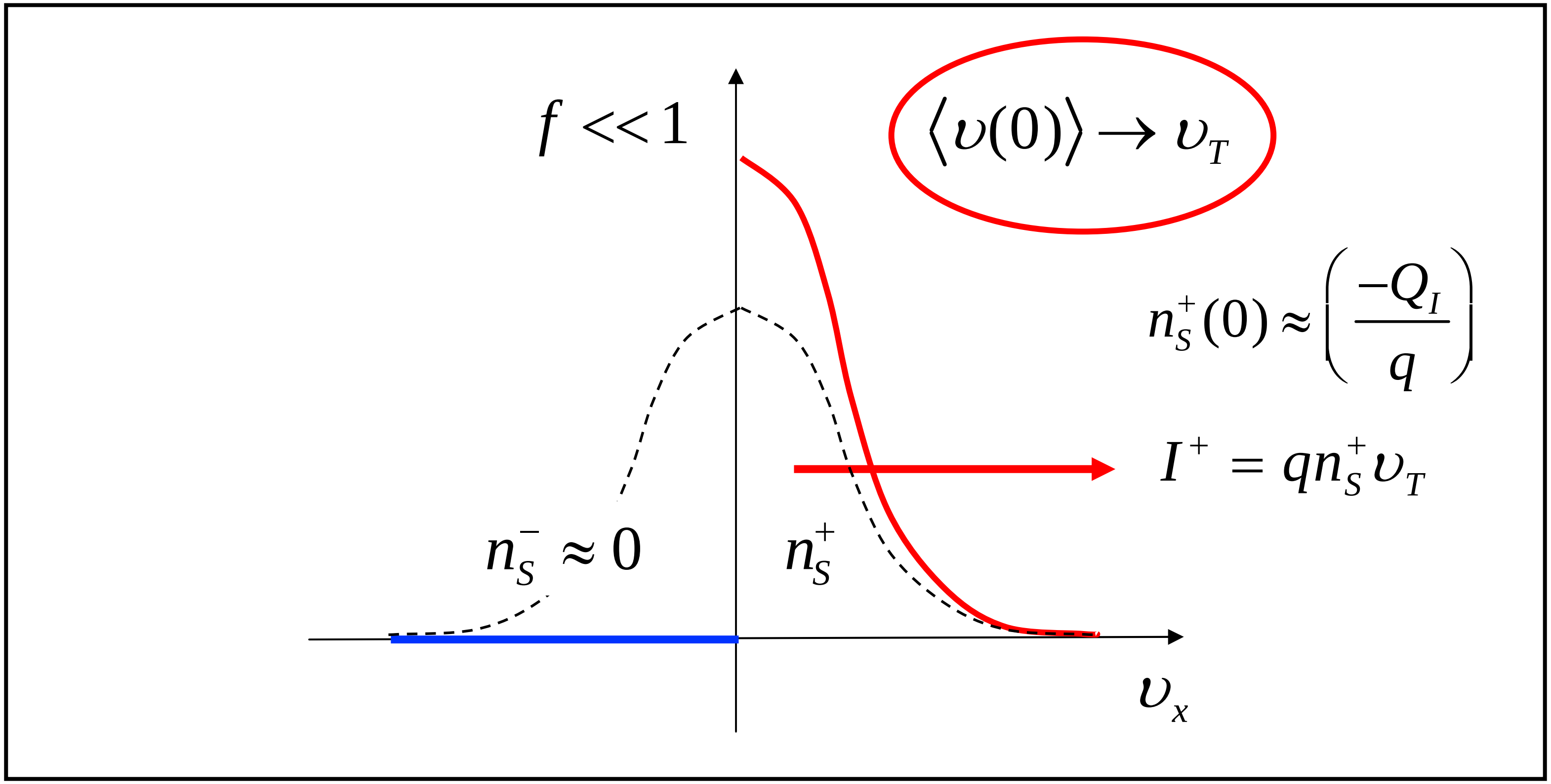
$$Q_I(0) = -q\left[n_S^+(0) + n_S^-(0)\right] = Q_G$$

the ballistic MOSFET: $V_{DS} > 0$



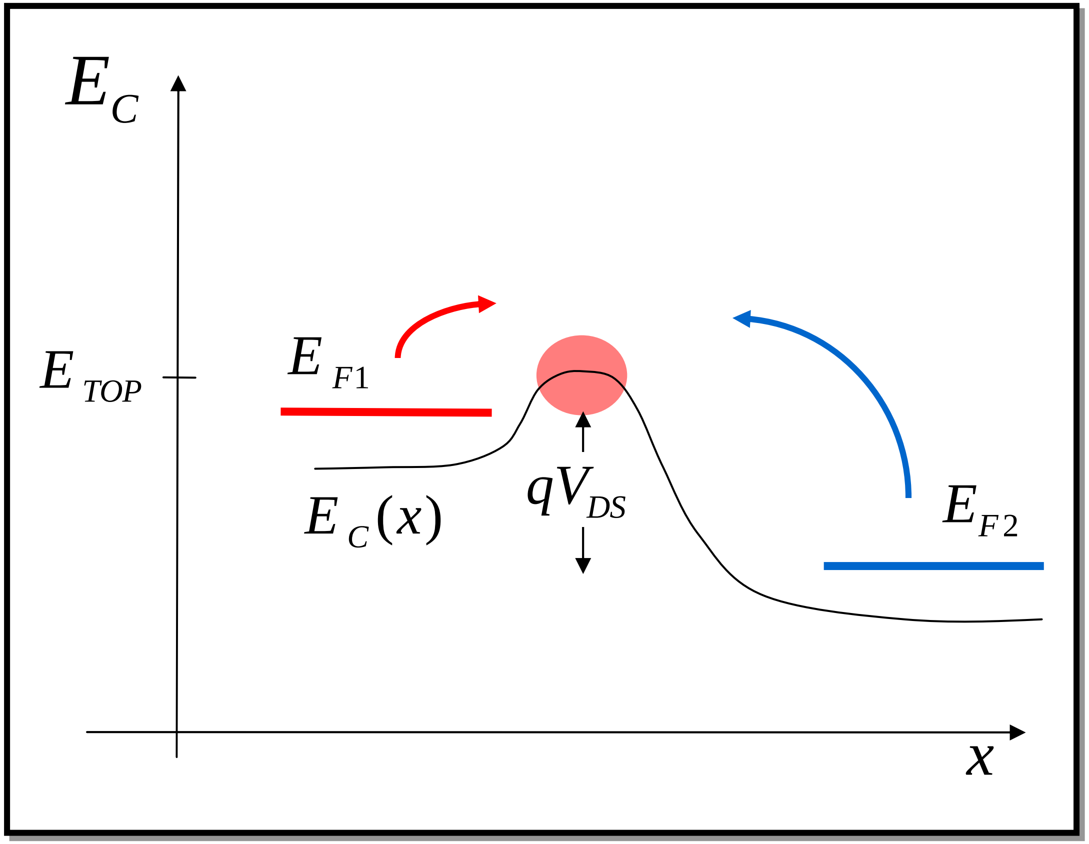
$$Q_I(0) = -q\left[n_S^+(0) + n_S^-(0)\right] = Q_G$$

the ballistic MOSFET: $V_{DS} >> 0$



$$Q_I(0) = -q\left[n_S^+(0) + n_S^-(0) \right] = Q_G$$

velocity vs. drain bias in a ballistic MOSFET

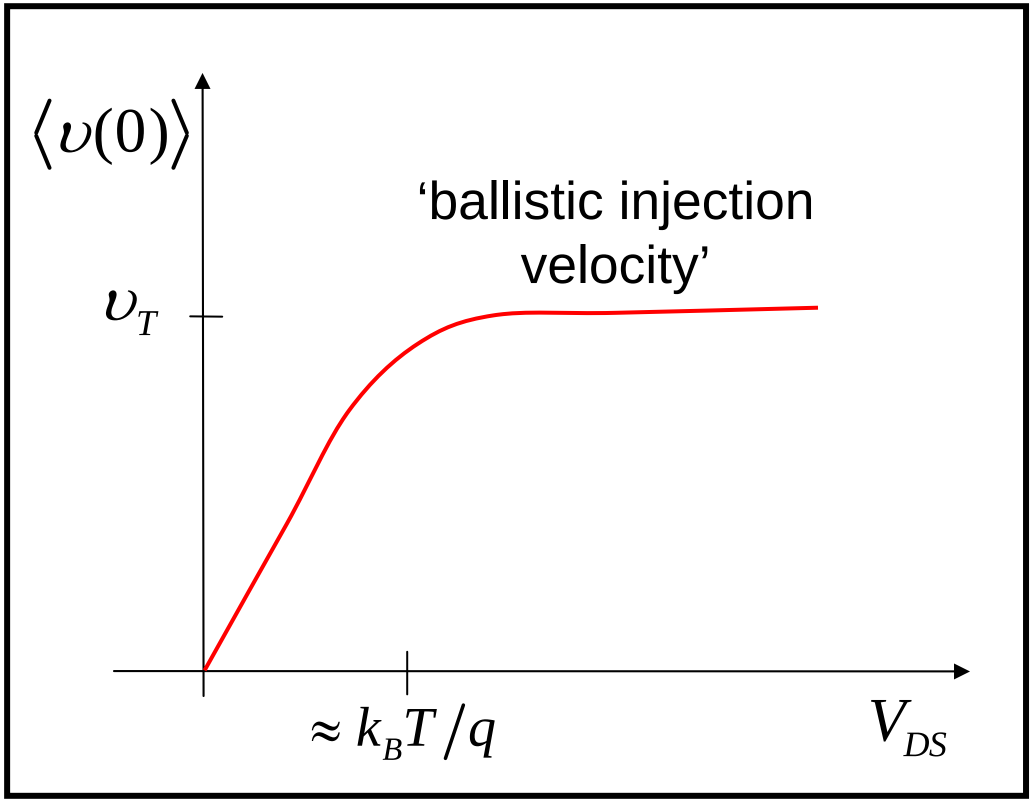


$$\langle \upsilon(0) \rangle = \frac{n_S^+ \upsilon_T - n_S^- \upsilon_T}{n_S^+ + n_S^-}$$

$$= \upsilon_T \frac{1 - n_S^- / n_S^+}{1 + n_S^- / n_S^+}$$

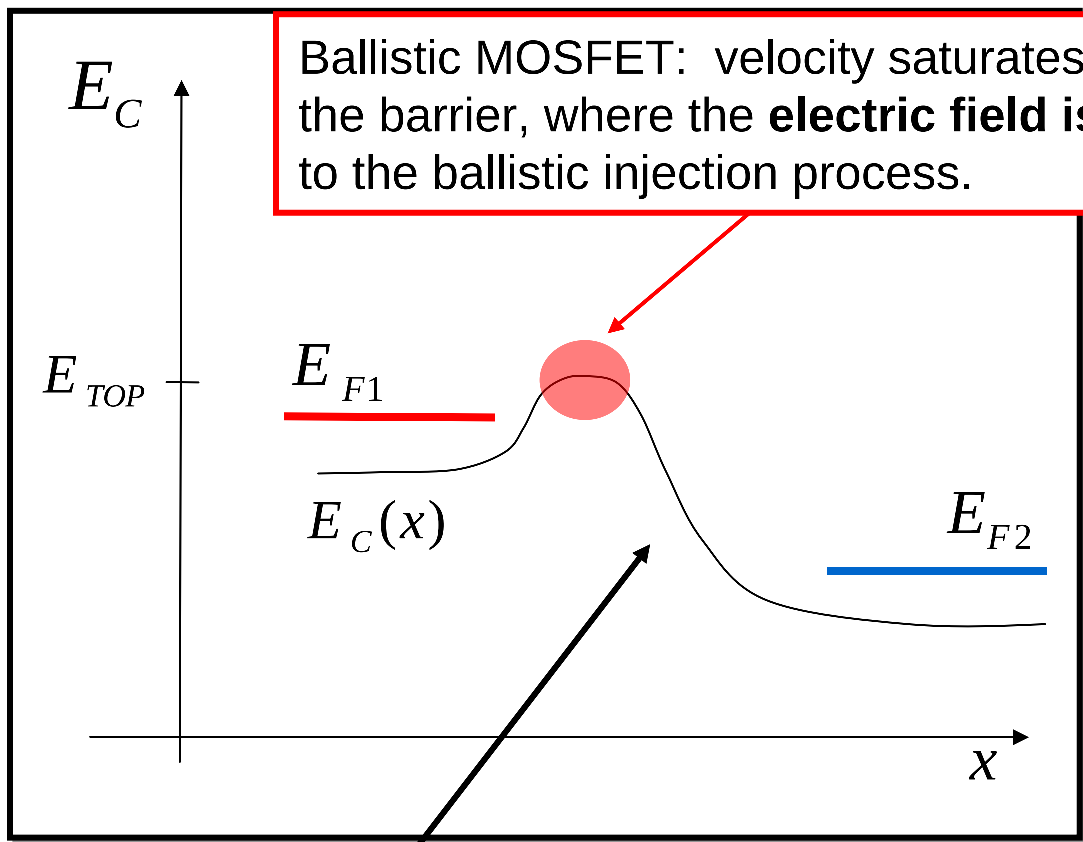
$$= \upsilon_T \frac{1 - e^{-qV_{DS}/k_B T}}{1 + e^{-qV_{DS}/k_B T}}$$

velocity vs. drain bias in a ballistic MOSFET



$$\langle \upsilon(0) \rangle = \upsilon_T \left(\frac{1 - e^{-qV_{DS}/k_B T}}{1 + e^{-qV_{DS}/k_B T}} \right)$$

velocity vs. drain bias in a ballistic MOSFET



Ballistic MOSFET: velocity saturates at the top of the barrier, where the **electric field is zero**, due to the ballistic injection process.

Long channel MOSFET: velocity saturates near the drain due to scattering in the **high electric field** region.

outline

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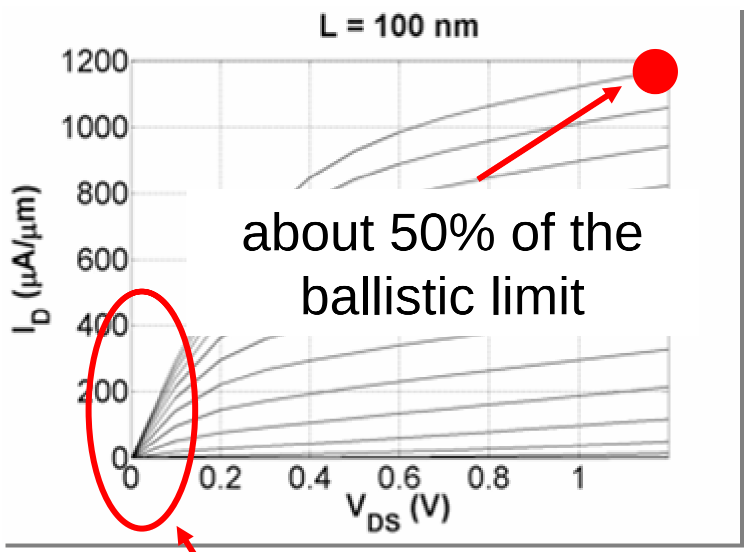
Is a nanoscale MOSFET really ballistic?

Typical N-channel MOSFET:

$$I_{ON} \approx 1\,\mathrm{mA}/\mu\mathrm{m}$$



(Courtesy, Shuji Ikeda, ATDF, Dec. 2007)

about 10% of the ballistic limit.

$$I_{ON}\left(\text{ballistic}\right) = -WQ_I(0)\upsilon_T$$

$$\upsilon_T = \sqrt{2k_BT/\pi m^*}$$

$$\approx 1.2\times 10^7 \text{ cm/s}$$

$$-Q_I(0)/q = C_{ox}\left(V_{DD} - V_T\right)$$

$$\approx 1\times 10^{13}\text{ cm}^{-2}$$

$$I_{ON}/W\left(\text{ballistic}\right) \approx 2\text{ mA}/\mu\text{m}$$

mean-free-path for scattering

For L = 100 nm NMOS Si technology: $\mu_{eff} \approx 200 \text{ cm}^2/\text{V-s}$

$$D_{eff} = \frac{k_B T}{q} \mu_{eff} \approx 5 \text{ cm}^2/\text{s}$$

$$D_{eff} = \frac{\upsilon_T \lambda_0}{2} \Rightarrow \lambda \approx 8 \text{ nm}$$

$$\frac{L}{\lambda_0} \approx 11$$

explains why I_D(lin) is ~ 10% of the ballistic limit, but why is I_{ON} ~ 50% of the ballistic limit?

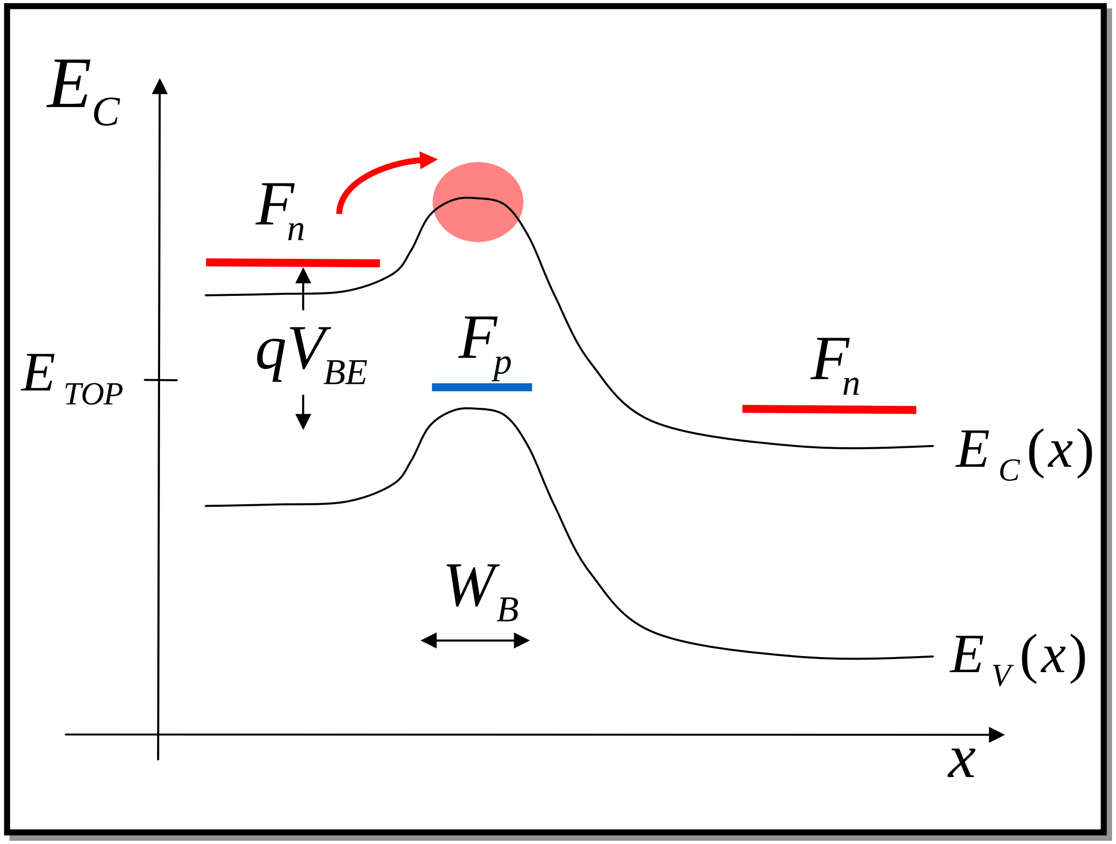
(Note that near the drain, λ is even shorter!)

scattering in a nano-MOSFET

1) Where in the channel does scattering matter the most? (for low V_{DS}, the answer is everywhere)

2) How can a MOSFET with a channel length many mean-free-paths long deliver an on-current that is within ~50% of the ballistic limit?

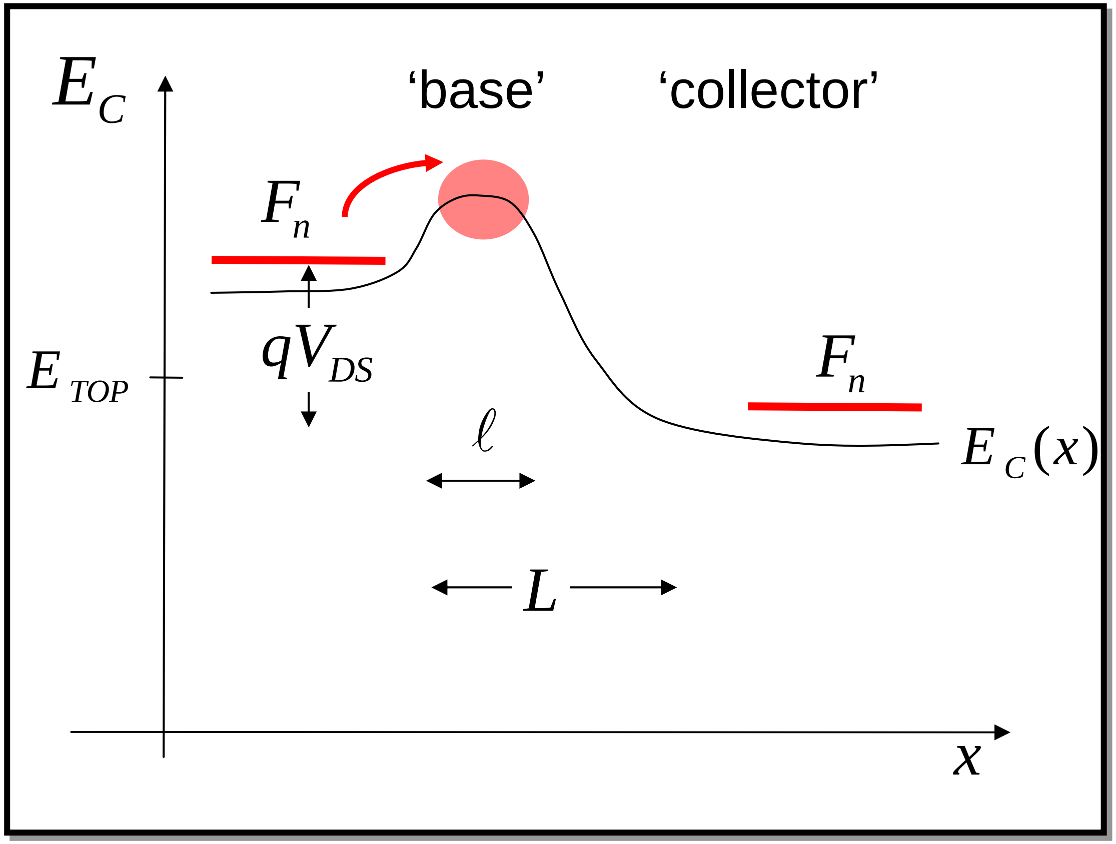
scattering in a bipolar transistor



emitter base collector

$$I_C = qA_E D_n \frac{\Delta n(0)}{W_B}$$

scattering in a MOSFET



source channel drain

$$I_D = qWD_n \frac{n_S(0)}{\mathrm{l}} \quad A/cm$$

$$I_D = -W\frac{D_n}{\mathrm{l}} Q_I(0)$$

$$I_D = WC_{ox}\upsilon_D\left(V_{GS} - V_T\right)$$

$$\upsilon_D = \langle \upsilon(0) \rangle \equiv \frac{D_n}{\mathrm{l}}$$

$$\langle \upsilon(0) \rangle \le \upsilon_T$$

the quasi-ballistic MOSFET

ballistic:

$$I_{ON} = WC_{ox}\upsilon_T\left(V_{DD} - V_T\right)$$

diffusive:

$$I_{ON} = WC_{ox}\upsilon_D\left(V_{GS} - V_T\right)$$

$$\ell\left(V_{GS}, V_{DS}\right) = ?$$

general:

$$I_{ON} = WC_{ox}\left\langle \upsilon(0)\right\rangle\left(V_{GS} - V_T\right)$$

$$\frac{1}{\left\langle \upsilon(0)\right\rangle} = \frac{1}{\upsilon_D} + \frac{1}{\upsilon_T} \qquad \upsilon_T = \sqrt{\frac{2k_BT}{\pi m^*}} \qquad \upsilon_D = D_n/\mathrm{l}$$

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recall: quasi-ballistic MOSFET: low V_{DS}

$$I_D = \frac{W}{L}\left(\frac{1}{\mu_{eff}} + \frac{1}{\mu_B}\right)^{-1} C_{ox}\left(V_{GS} - V_T\right)V_{DS}$$

the critical length: example

For *L* = 100 nm NMOS Si technology:

$$\mu_{eff} \approx 200 \text{ cm}^2/\text{V-s} \qquad D_{eff} = \frac{k_B T}{q}\mu_{eff} \approx 5 \text{ cm}^2/\text{s}$$

$$V_{DD} = 1.2 \text{ V} \qquad V_T = 0.23 \text{ V} \qquad C_{ox} = 1.5 \times 10^{-6} \text{ F}/\text{cm}^2$$

$$I_{ON}/W = WC_{ox}\frac{D_n}{l}\left(V_{DD} - V_T\right) \approx 1.2 \text{ mA}/\mu\text{m} \quad \Rightarrow l \approx 6 \text{ nm}$$

$$\frac{\ell}{\lambda_0} \approx 0.75$$

(Explains why on-current is ~ 50% of ballistic limit)

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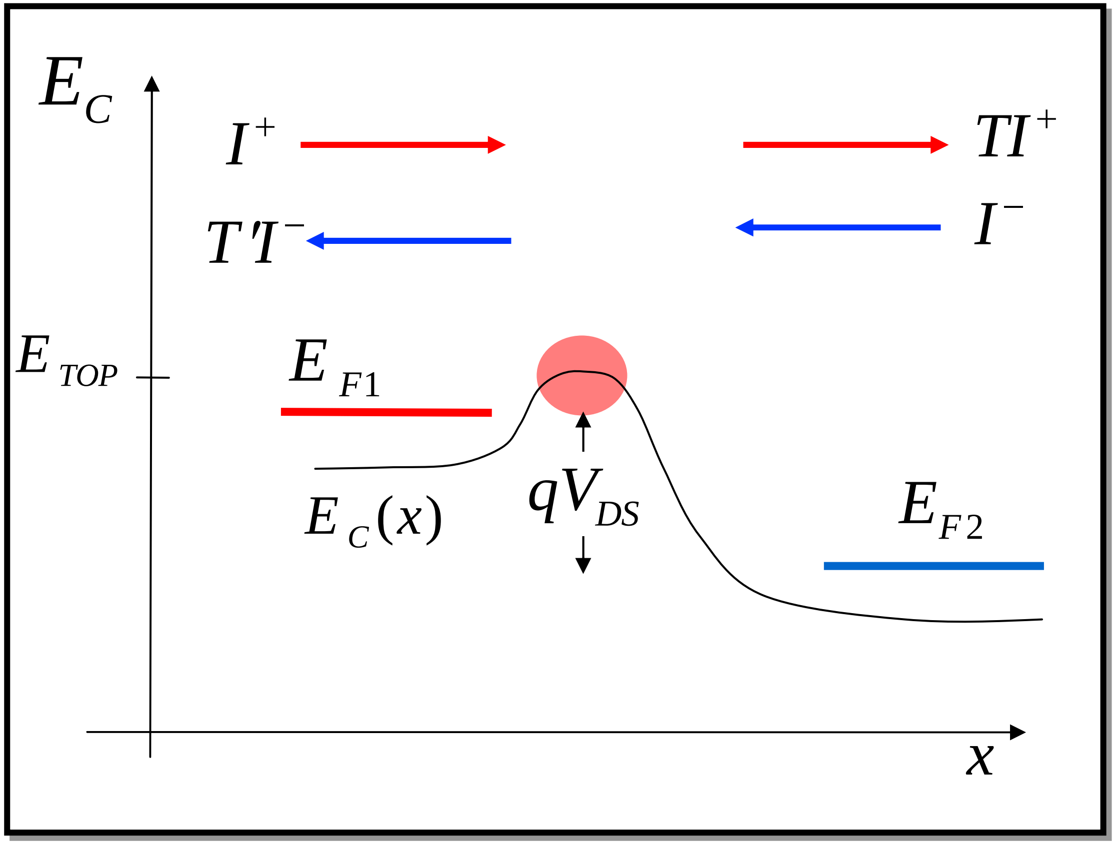
another way of treating scattering….



T

'transmission' coefficient

$$R = 1 - T$$

'reflection' or 'backscattering' coefficient

$$T = \frac{\lambda}{\lambda + L}$$

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summary

1) MOSFETs are 'barrier-controlled' devices (just like bipolar transistors)

2) A simple, ballistic model is easy to derive (assuming Boltzmann statistics)

3) Connection to diffusive model is clear

$$\mu_{eff} \rightarrow \mu_B \qquad \upsilon_{sat} \rightarrow \upsilon_T$$

4) Modern MOSFETs operate between the ballistic and diffusive limit where things get more complicated.

$$\ell\left(V_{GS}, V_{DS}\right) = ?$$

what have we accomplished?

Developed a very simple, conceptual model that captures the essential physics of a nanoscale MOSFET

- not a replacement for numerical simulation, but helpful in interpreting simulation results.

- provides a ‘sanity check’ for numerical models and for compact circuit models.

- useful for interpreting experiments and guiding device design.

what have we left out (of the ballistic model?)

1) Fermi-Dirac statistics (above threshold)

2) Treatment of sub-threshold conduction

3) Two-dimensional electrostatics

4) Multiple subbands and other details…

These topics will be discussed in Lectures 3A and 3B.