

NCN@Purdue-Intel Summer School: July 14-25, 2008

***Physics of Nanoscale Transistors: Lecture 4:***

***Carrier Scattering  
in  
Nanoscale MOSFETs***

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# outline

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- 1) **Review and introduction**
- 2) Scattering theory of the MOSFET
- 3) Transmission under low  $V_{DS}$
- 4) Transmission under high  $V_{DS}$
- 5) Discussion
- 6) Summary

## review: ballistic I-V

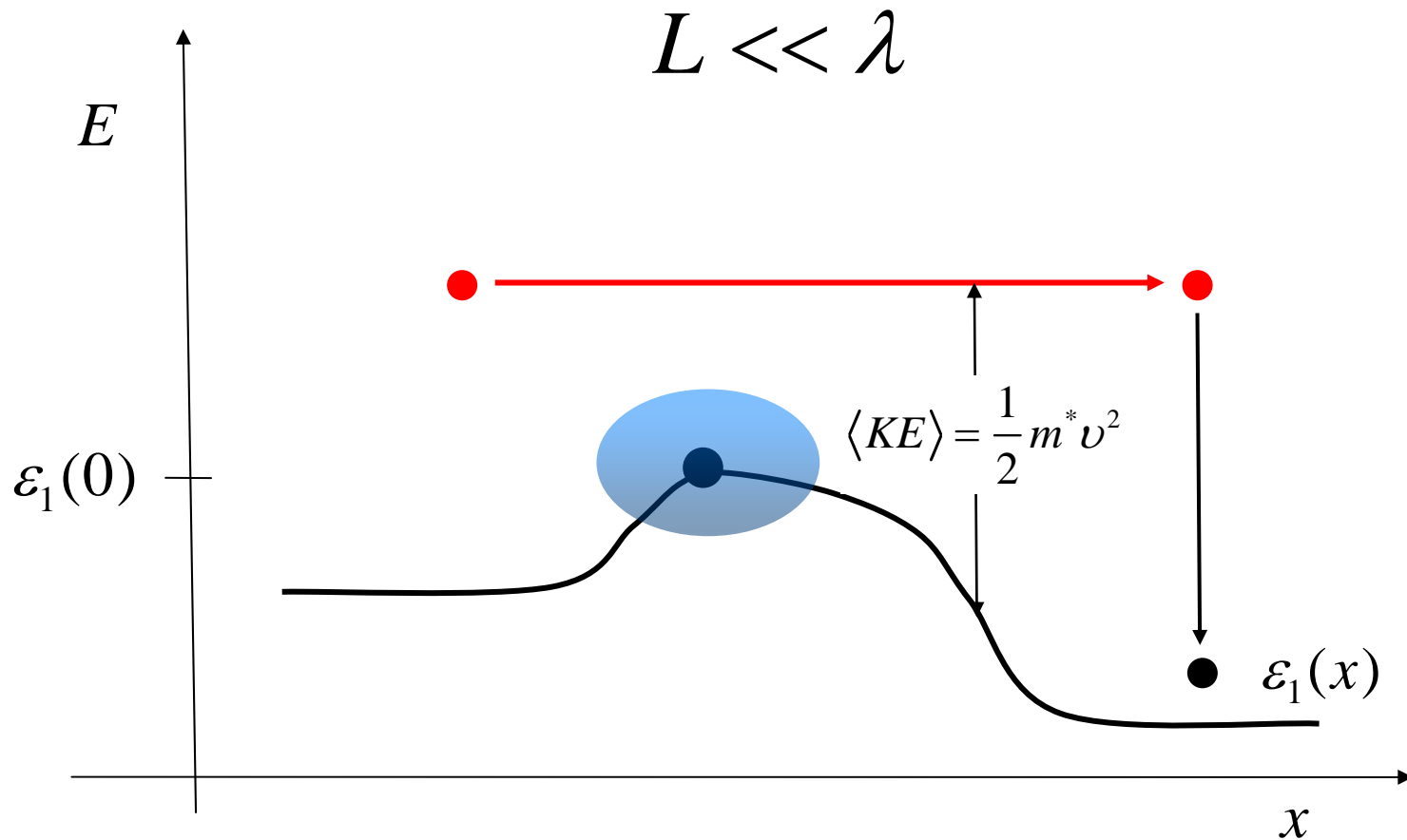
$$I_D = \frac{2q}{h} \int_{-\infty}^{\infty} T(E)M(E)(f_1 - f_2)dE$$

$$N = \int_{-\infty}^{\infty} [f_1(E)D_1(E) + f_2(E)D_2(E)]dE$$

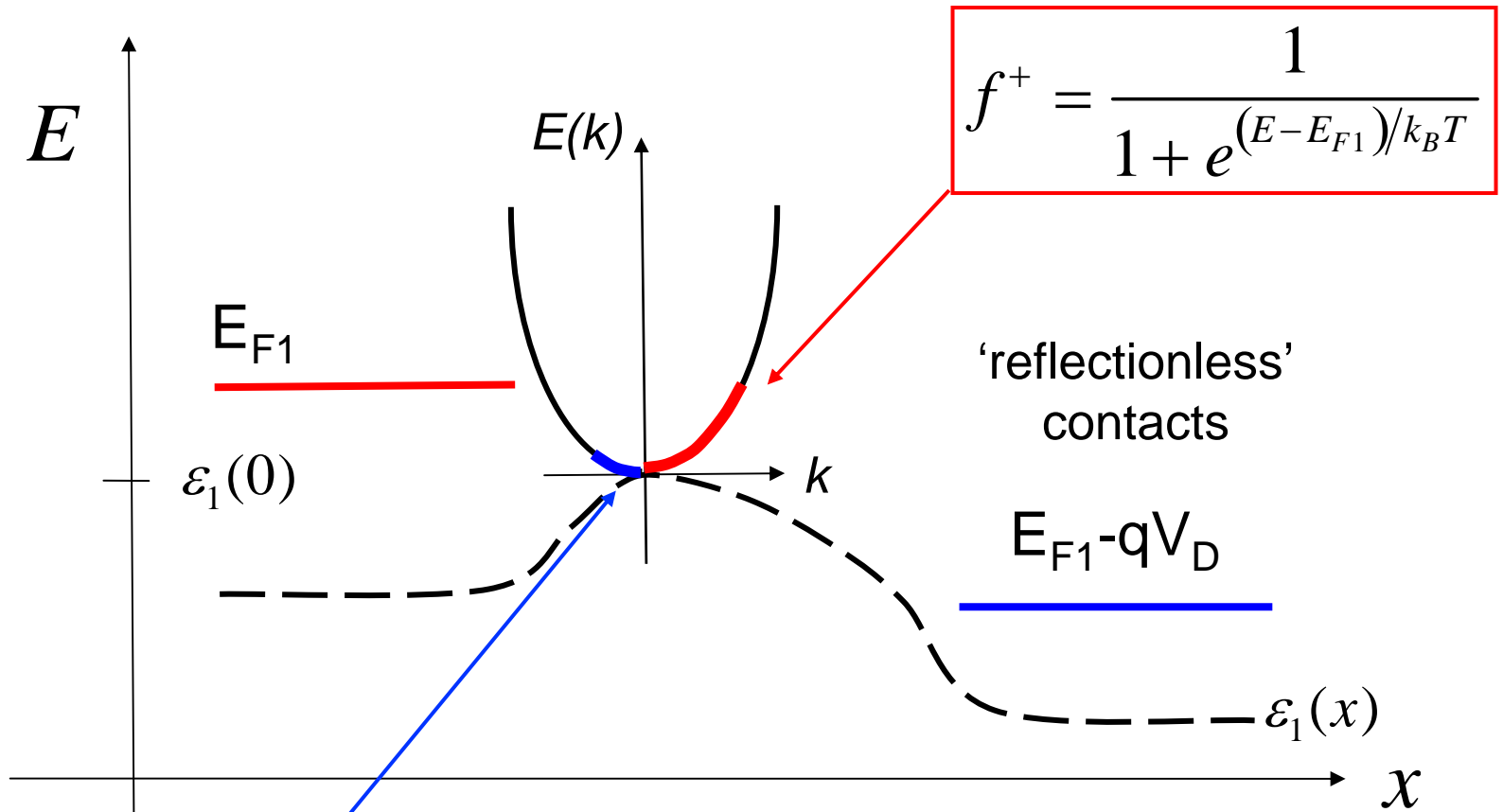
$$I_D = WC_{ox} (V_{GS} - V_T) \mathcal{V}_F \left[ \frac{1 - \mathcal{F}_{1/2}(\eta_{F2})/\mathcal{F}_{1/2}(\eta_{F1})}{1 + \mathcal{F}_0(\eta_{F2})/\mathcal{F}_0(\eta_{F1})} \right]$$

$$\mathcal{V}_F \equiv \sqrt{\frac{2k_B T}{\pi m^*}} \frac{\mathcal{F}_{1/2}(\eta_{F1})}{\mathcal{F}_0(\eta_{F1})} = v_T \frac{\mathcal{F}_{1/2}(\eta_{F1})}{\mathcal{F}_0(\eta_{F1})}$$

# review: ballistic transport in a MOSFET



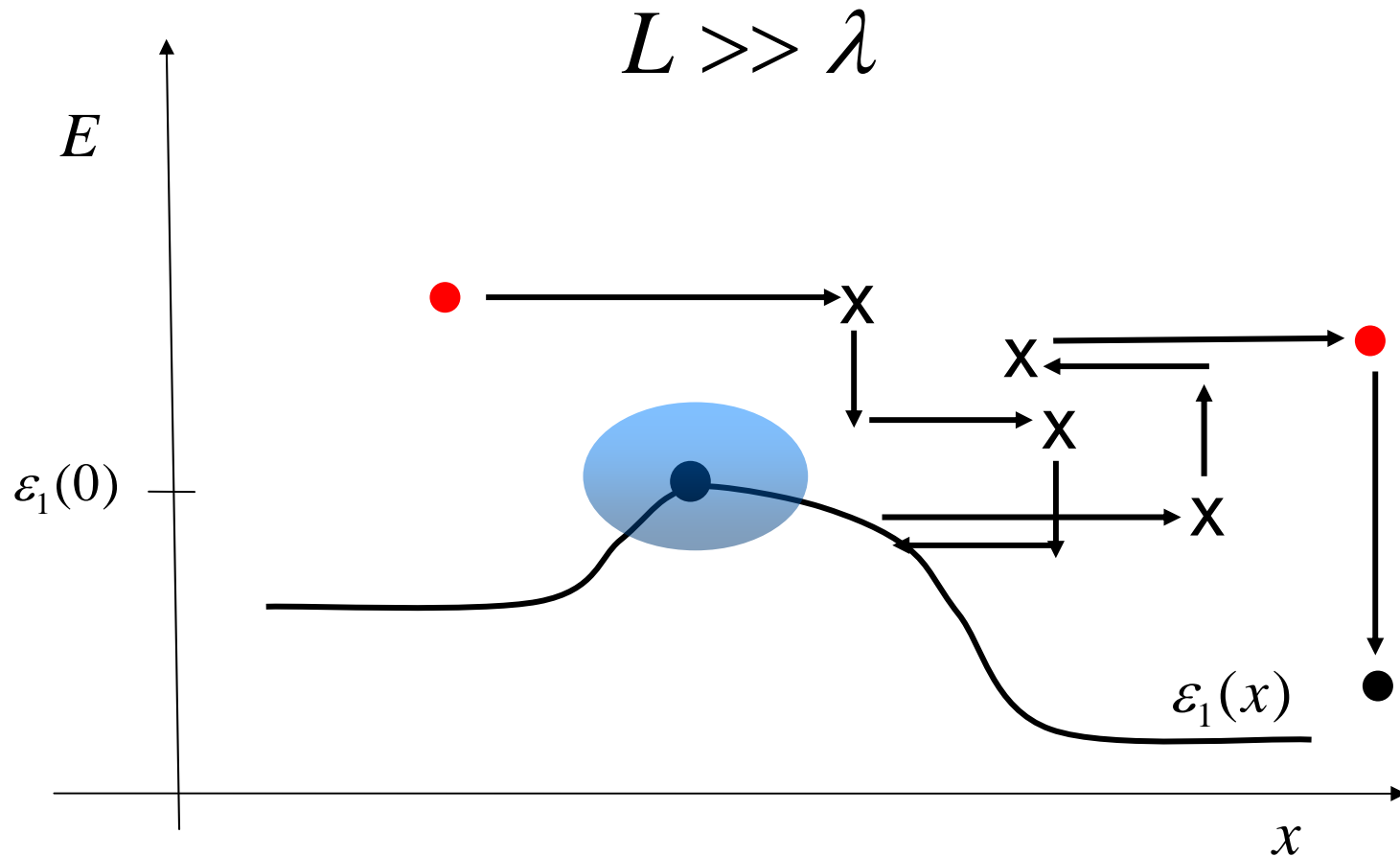
# review: filling states in a ballistic MOSFET



$$f^+ = \frac{1}{1 + e^{(E - E_{F1})/k_B T}}$$

$$f^- = \frac{1}{1 + e^{(E - E_{F2})/k_B T}}$$

# review: diffusive transport in a MOSFET



# nanoscale MOSFETs

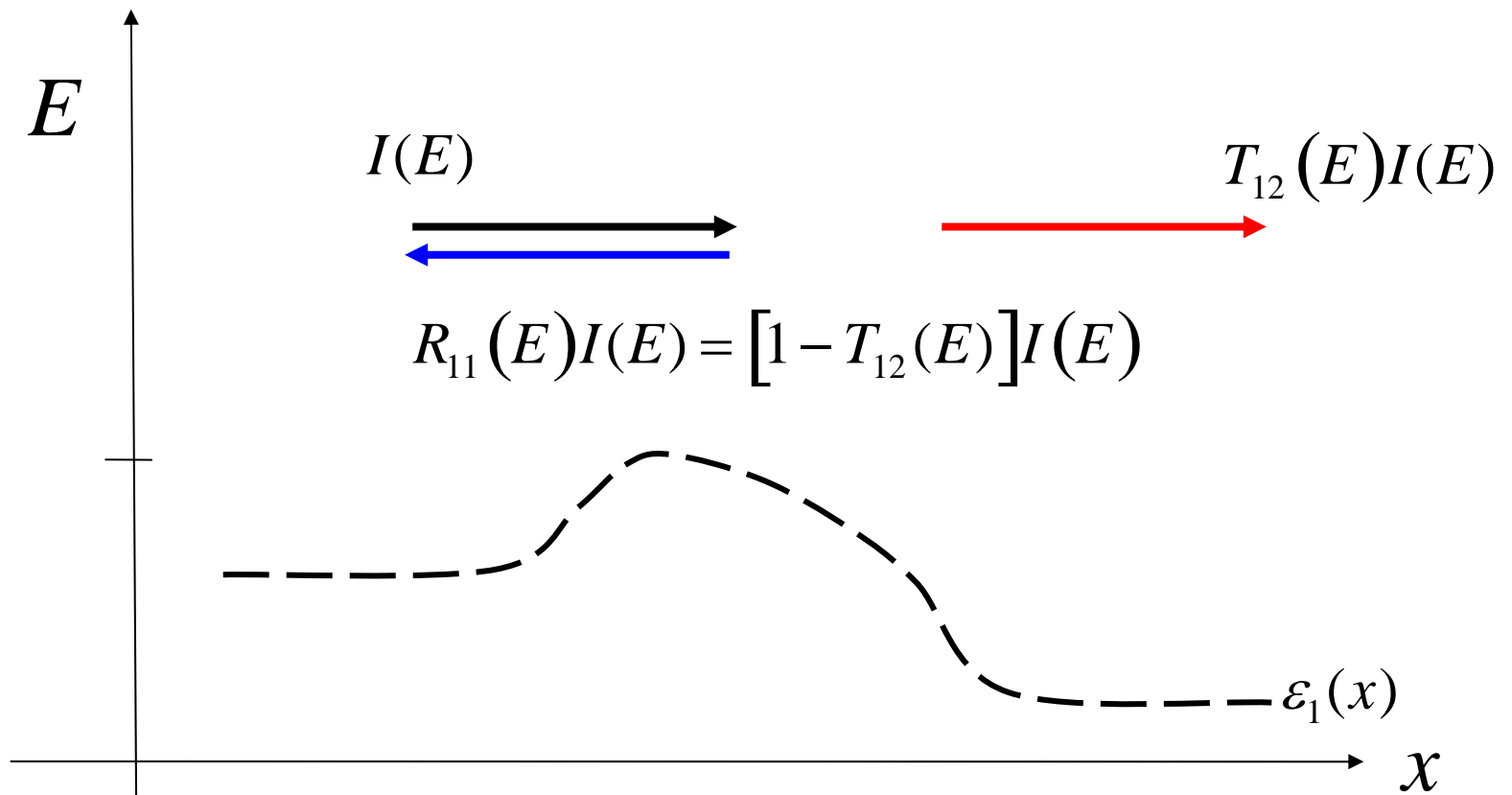
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Nanoscale MOSFETs are neither fully ballistic nor fully diffusive; they operate in a 'quasi-ballistic' regime.

How do we ***understand*** how carrier scattering affects the performance of a nanoscale MOSFET?

# current transmission in a MOSFET

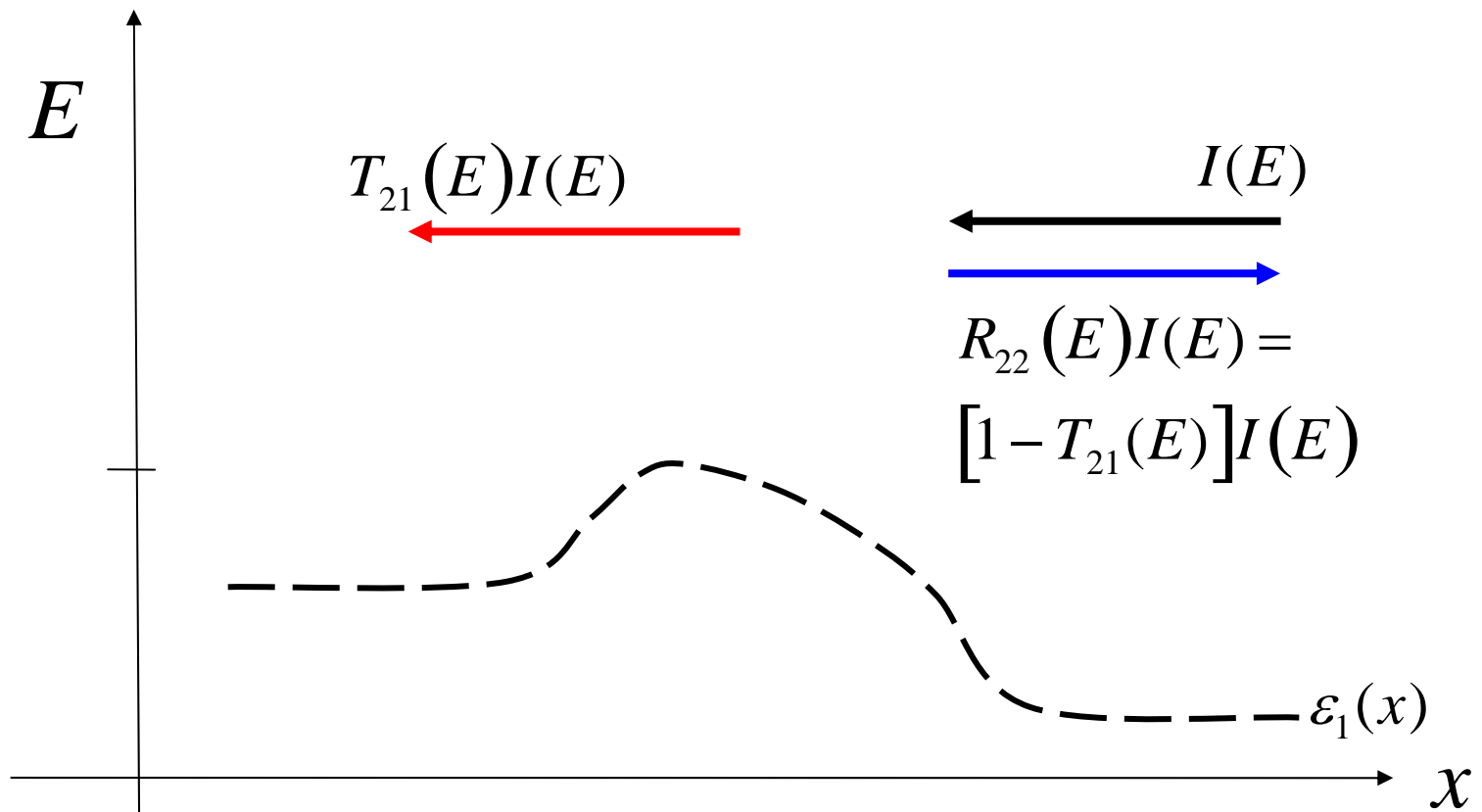
*elastic scattering....*





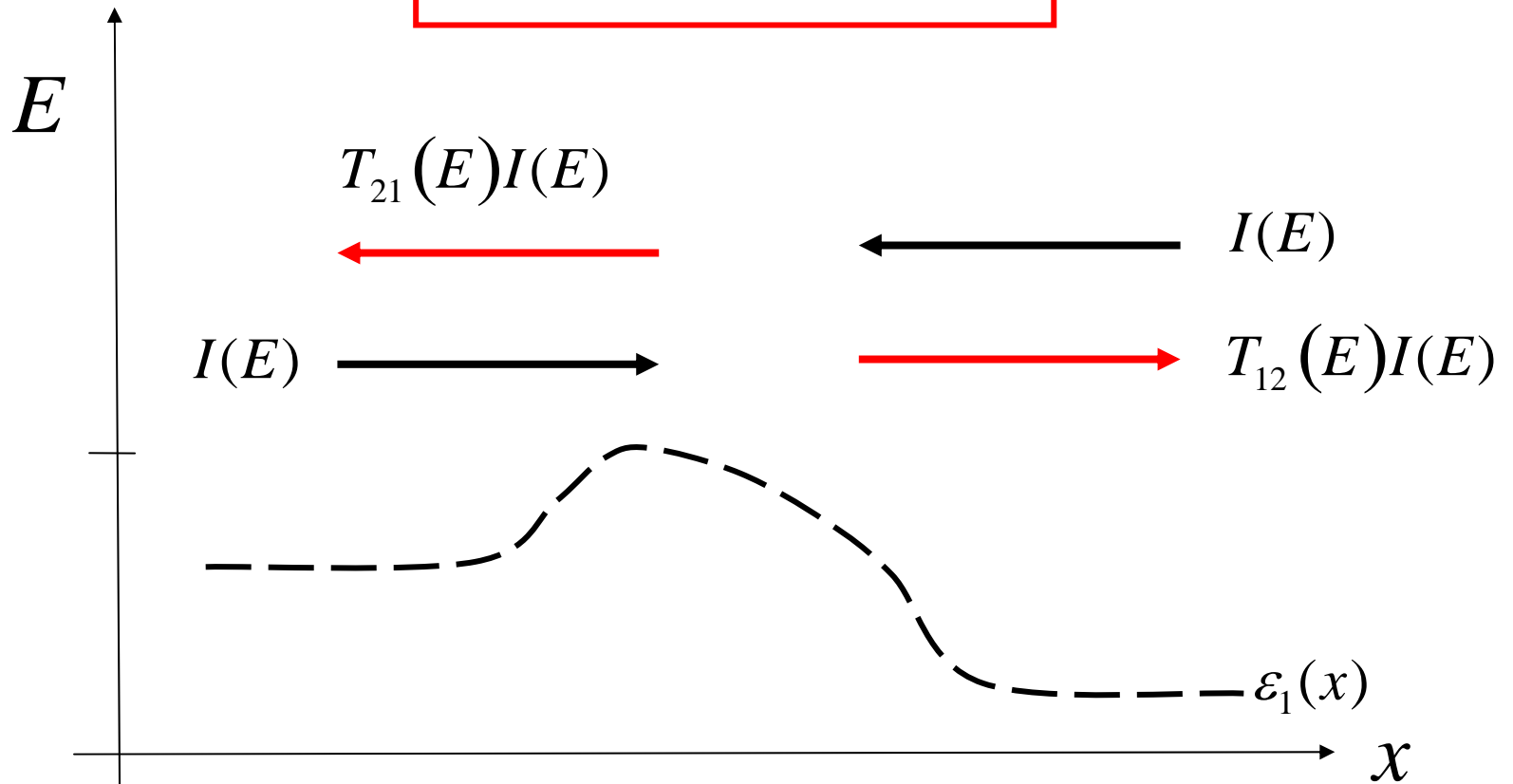
# current transmission in a MOSFET

*elastic scattering....*

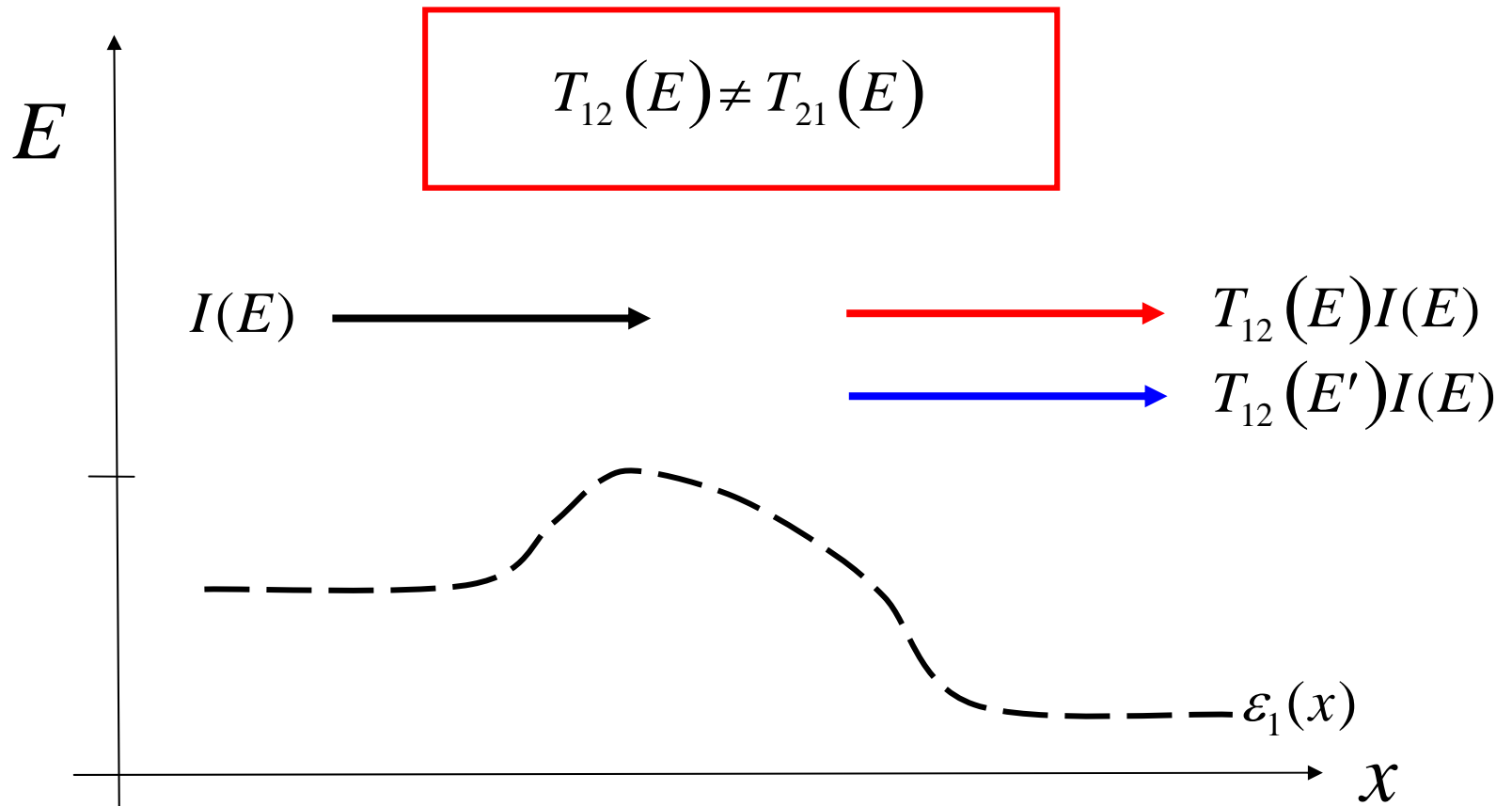


# transmission in the presence of **elastic** scattering

$$T_{12}(E) = T_{21}(E) = T(E)$$

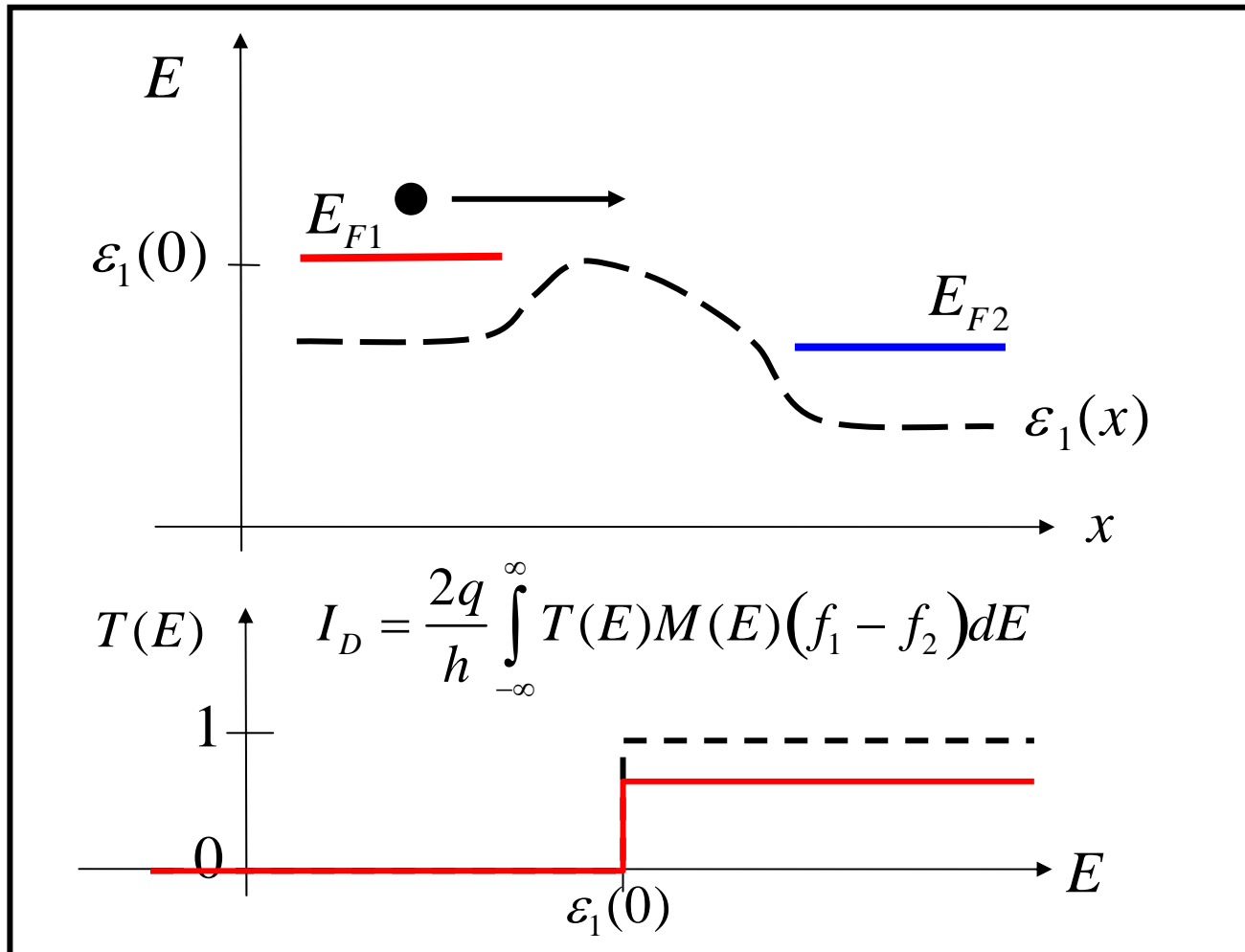


# inelastic scattering

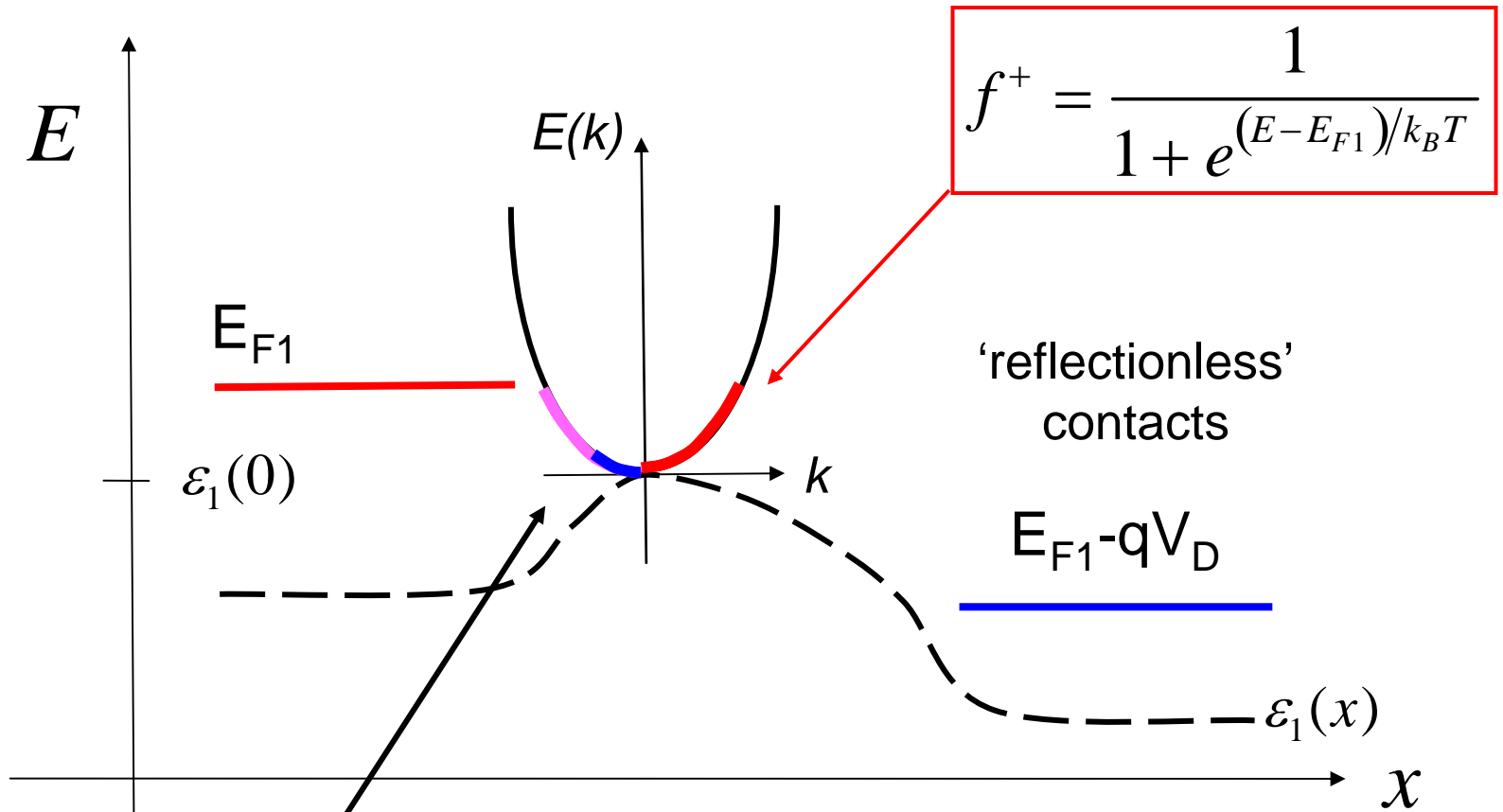


S. Datta, *Electronic Transport in Mesoscopic Systems*,  
Cambridge, 1995.

# transmission and the IV characteristic



# filling states in a quasi-ballistic MOSFET



some states are still filled from the drain, but some are now filled by backscattering from source-injected flux.

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# scattering theory of the MOSFET

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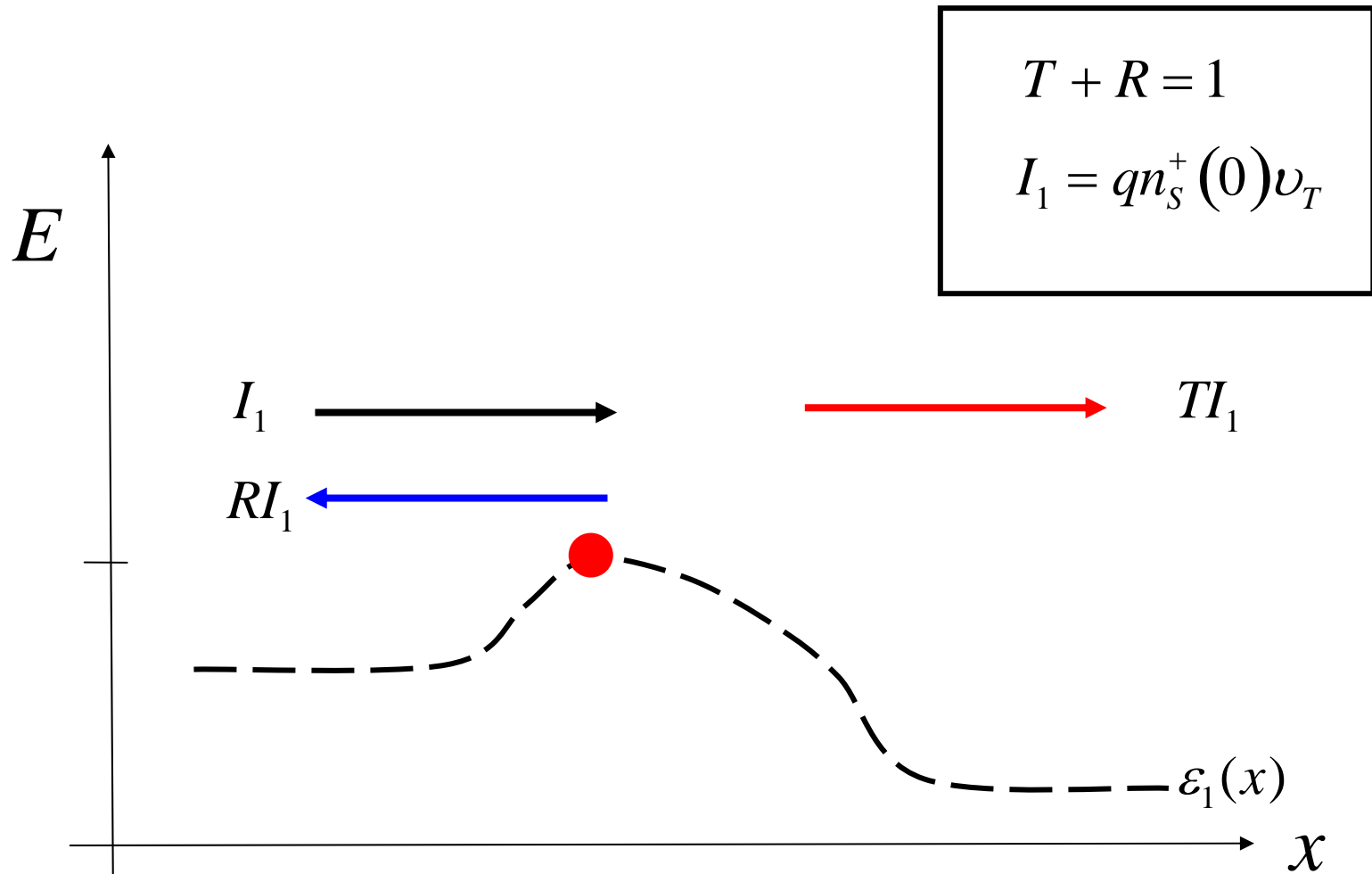
## Goal:

To illustrate the influence on scattering on the I-V characteristic of a MOSFET by developing a very simple theory.

## Assumptions:

- 1) Average quantities, not energy-resolved.
- 2) Boltzmann statistics for carriers
- 3)  $T_{12} = T_{21} = T$
- 4) *Average velocity of backscattered carriers equals that of the injected carriers.*

# scattering in a nano-MOSFET

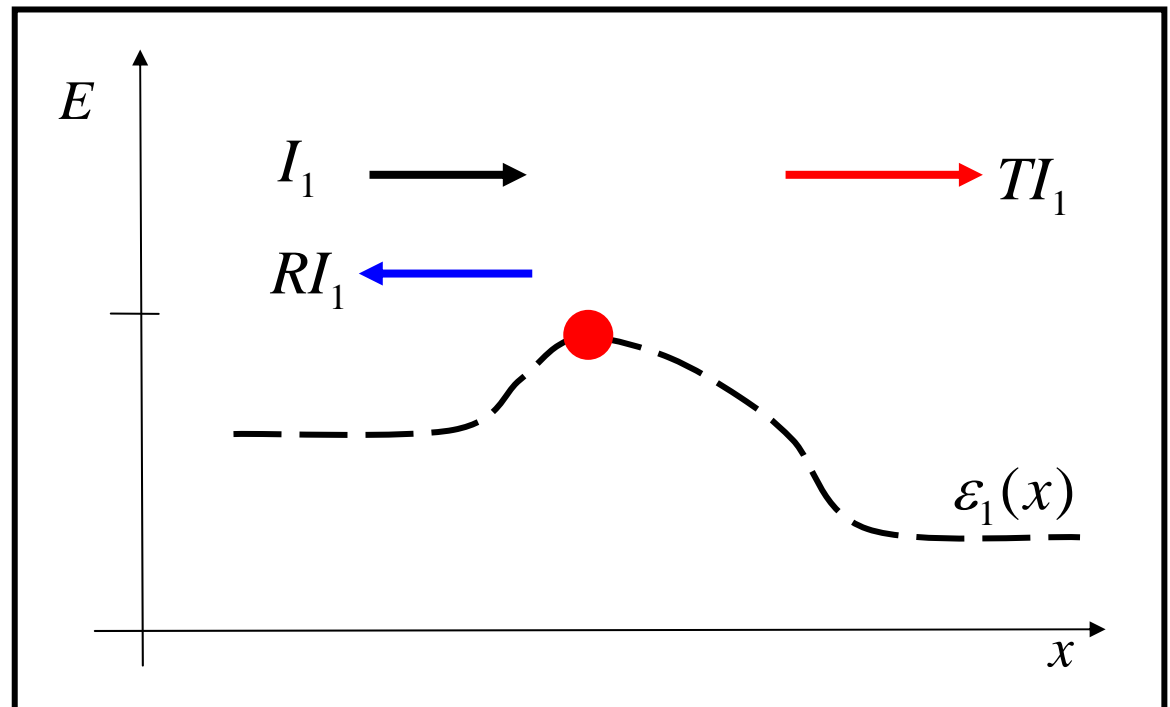




# current

$$I_D = W \left( qn_S^+(0)v_T - qn_S^-(0)v_T \right) = Wqn_S^+(0)v_T \left[ 1 - n_S^-(0)/n_S^+(0) \right] \quad (1)$$

$$n_S(0) = n_S^+(0) + n_S^-(0) = n_S^+(0) \left[ 1 + n_S^-(0)/n_S^+(0) \right] \quad (2)$$



## current

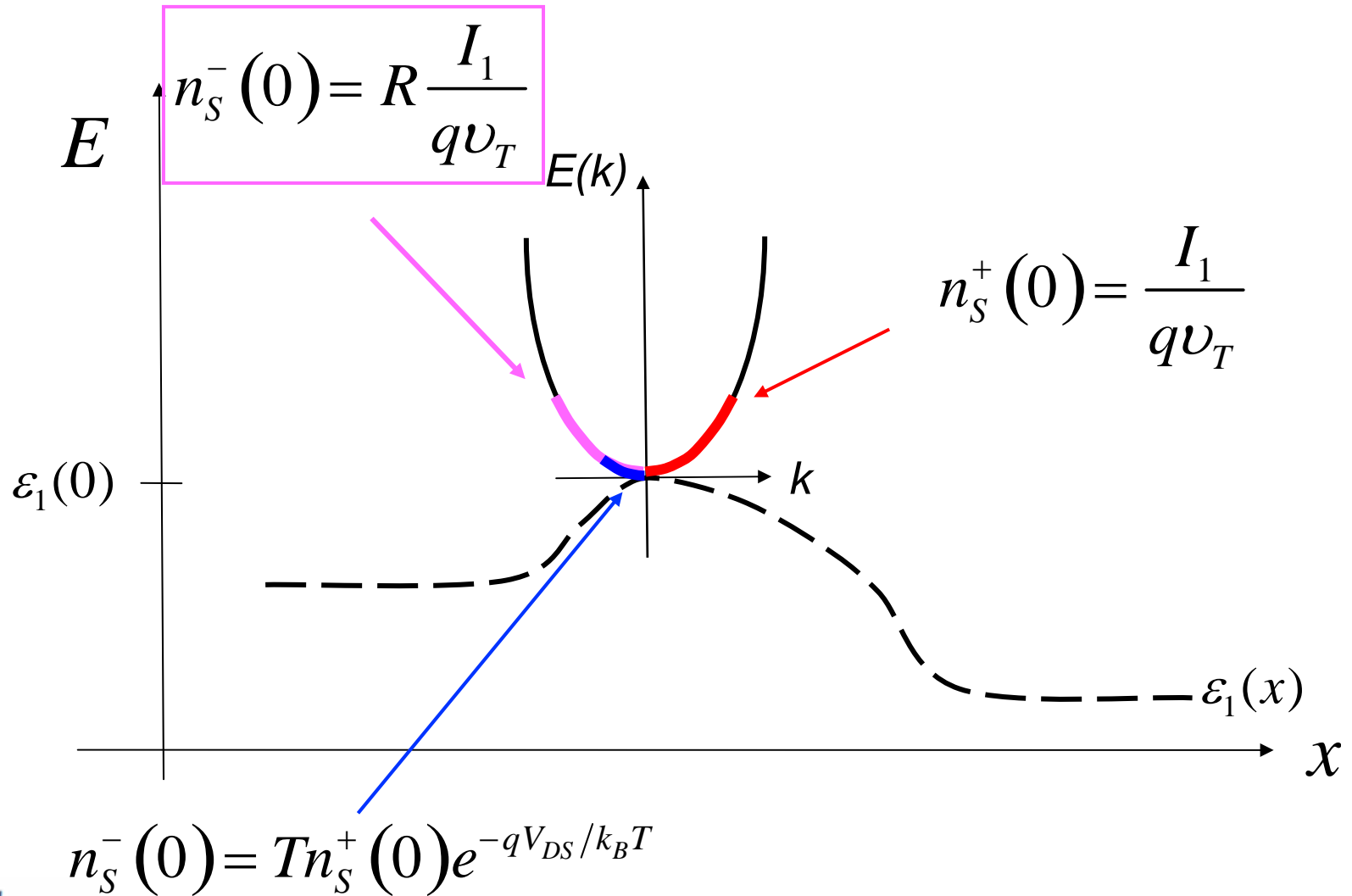
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$$I_D = Wqn_s(0)v_T \left( \frac{1 - n_s^-(0)/n_s^+(0)}{1 + n_s^-(0)/n_s^+(0)} \right)$$

$$I_D = WQ_I(0)v_T \left( \frac{1 - n_s^-(0)/n_s^+(0)}{1 + n_s^-(0)/n_s^+(0)} \right)$$

***Exactly the same result*** we had for the ballistic case, but the (- velocity) carrier density at the top of the barrier is altered by scattering.

# carrier densities at the top of the barrier



## from carrier densities to drain current

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$$n_S^-(0) = Rn_S^+(0) + Tn_S^+(0)e^{-qV_{DS}/k_B T} = n_S^+(0) \left[ R + (1-R)e^{-qV_{DS}/k_B T} \right]$$

$$\frac{n_S^-(0)}{n_S^+(0)} = R + (1-R)e^{-qV_{DS}/k_B T}$$

$$I_D = WQ_I(0)v_T \left( \frac{1 - n_S^-(0)/n_S^+(0)}{1 + n_S^-(0)/n_S^+(0)} \right)$$

$$I_D = WQ_I(0)v_T \left( \frac{(1-R) - (1-R)e^{-qV_{DS}/k_B T}}{(1+R) + (1-R)e^{-qV_{DS}/k_B T}} \right)$$

# the MOSFET I-V with scattering

$$I_{DS} = WC_{ox} (V_{GS} - V_T) v_T \left( \frac{1 - e^{qV_{DS}/k_B T}}{1 + e^{qV_{DS}/k_B T}} \right) \quad (\text{ballistic, Boltzmann statistics})$$

$$I_D = WC_{ox} (V_{GS} - V_T) v_T \left( \frac{(1 - R) - (1 - R)e^{-qV_{DS}/k_B T}}{(1 + R) + (1 - R)e^{-qV_{DS}/k_B T}} \right)$$

$$T = (1 - R)$$

$$I_D = WC_{ox} (V_{GS} - V_T) v_T T \left( \frac{1 - e^{-qV_{DS}/k_B T}}{(2 - T) + T e^{-qV_{DS}/k_B T}} \right)$$

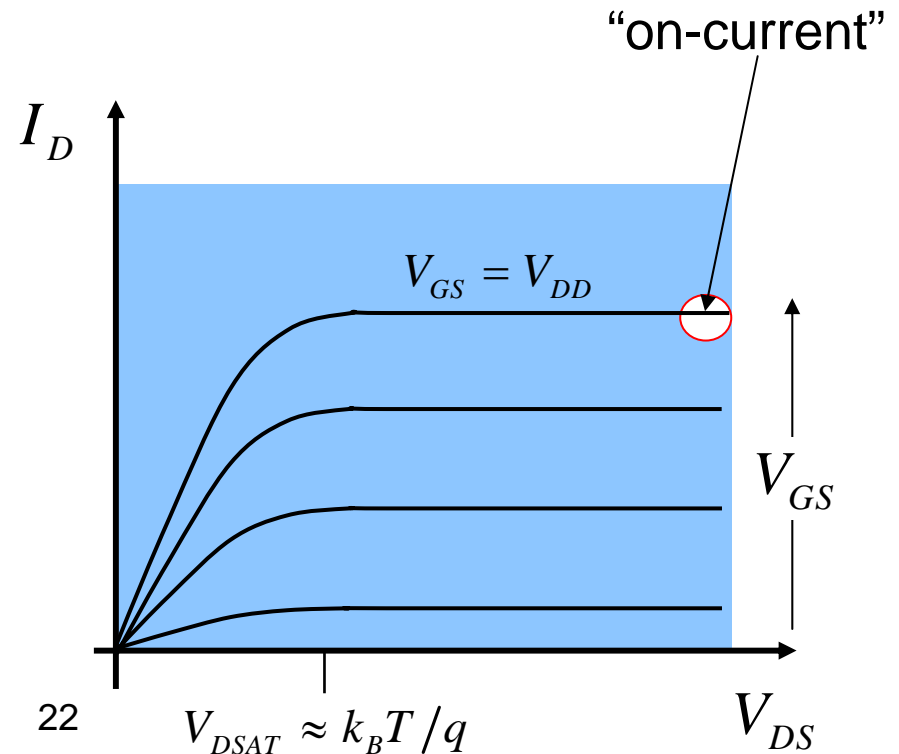
$$I_D (\text{scattering}) \neq T I_D (\text{ballistic})$$

# high drain bias

$$I_D = WC_{ox} (V_{GS} - V_T) v_T \left( \frac{(1-R) - (1-R)e^{-qV_{DS}/k_B T}}{(1+R) + (1-R)e^{-qV_{DS}/k_B T}} \right)$$

$$I_D = WC_{ox} (V_{GS} - V_T) v_T \frac{(1-R)}{(1+R)}$$

$$\langle v(0) \rangle = v_T \frac{(1-R)}{(1+R)} \leq v_T$$



# questions

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- 1) We expected current to be proportional to transmission ( $T = 1 - R$ ), but where does the  $(1 + R)$  in the denominator come from?
- 2) How do we generalize the result for Fermi-Dirac statistics?

approximate answer:

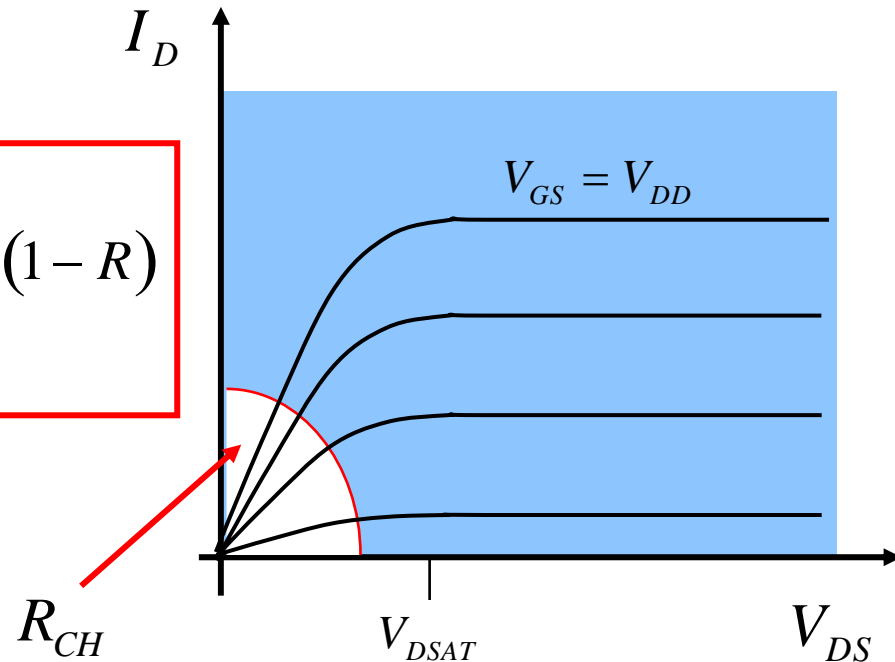
$$I_D = WC_{ox} (V_{GS} - V_T) \frac{\mathcal{E}_F}{T} \frac{(1 - R)}{(1 + R)}$$

# low drain bias

$$I_D = WC_{ox} (V_{GS} - V_T) v_T \left( \frac{(1-R) - (1-R)e^{-qV_{DS}/k_B T}}{(1+R) + (1-R)e^{-qV_{DS}/k_B T}} \right)$$

$$G_{CH} = \frac{I_D}{V_{DS}} = WC_{ox} (V_{GS} - V_T) \left[ \frac{v_T}{2(k_B T/q)} \right] (1-R)$$

$$G_{CH} (\text{scattering}) = TG_{CH} (\text{ballistic})$$





# questions

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- 1) For low  $V_{DS}$ , the drain current is proportional to transmission ( $T = 1 - R$ ). Why in this case but not for high  $V_{DS}$ ?
- 2) How do we generalize the result for Fermi-Dirac statistics?

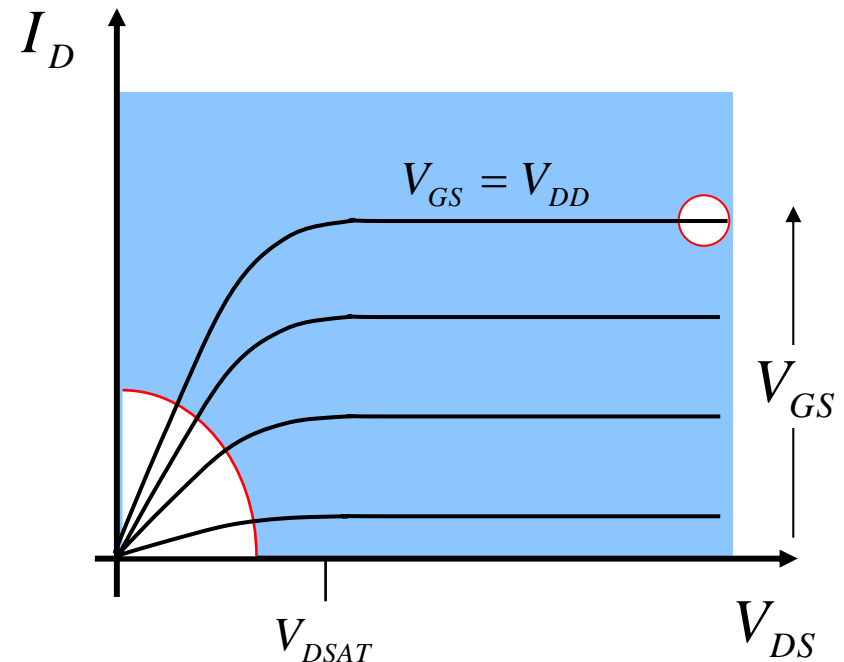
approximate answer:

$$G_{CH} = \left( WC_{ox} (V_{GS} - V_T) \frac{v_T}{(2k_B T / q)} \right) \left[ \frac{\mathcal{F}_{-1/2}(\eta_{F1})}{\mathcal{F}_0(\eta_{F1})} \right] (1 - R)$$

# summary of the scattering model

$$\left\{ \begin{array}{l} I_D \approx WC_{ox} (V_{GS} - V_T) \theta_F \left( \frac{(1-R) - (1-R) \mathcal{F}_{1/2}(\eta_{F2}) / \mathcal{F}_{1/2}(\eta_{F1})}{(1+R) + (1-R) \mathcal{F}_0(\eta_{F2}) / \mathcal{F}_0(\eta_{F1})} \right) \\ G_{CH} \approx \left( WC_{ox} (V_{GS} - V_T) \frac{v_T}{(2k_B T / q)} \right) \left[ \frac{\mathcal{F}_{-1/2}(\eta_{F1})}{\mathcal{F}_0(\eta_{F1})} \right] (1-R) \\ I_{ON} \approx WC_{ox} (V_{GS} - V_T) \theta_F \frac{(1-R)}{(1+R)} \end{array} \right.$$

To proceed, we need to understand  $R(V_{GS}, V_{DS})$

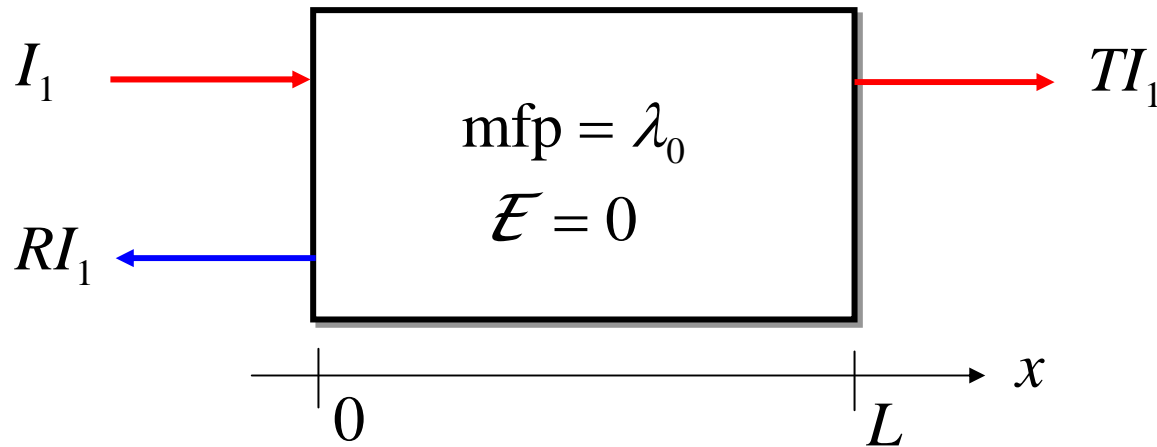


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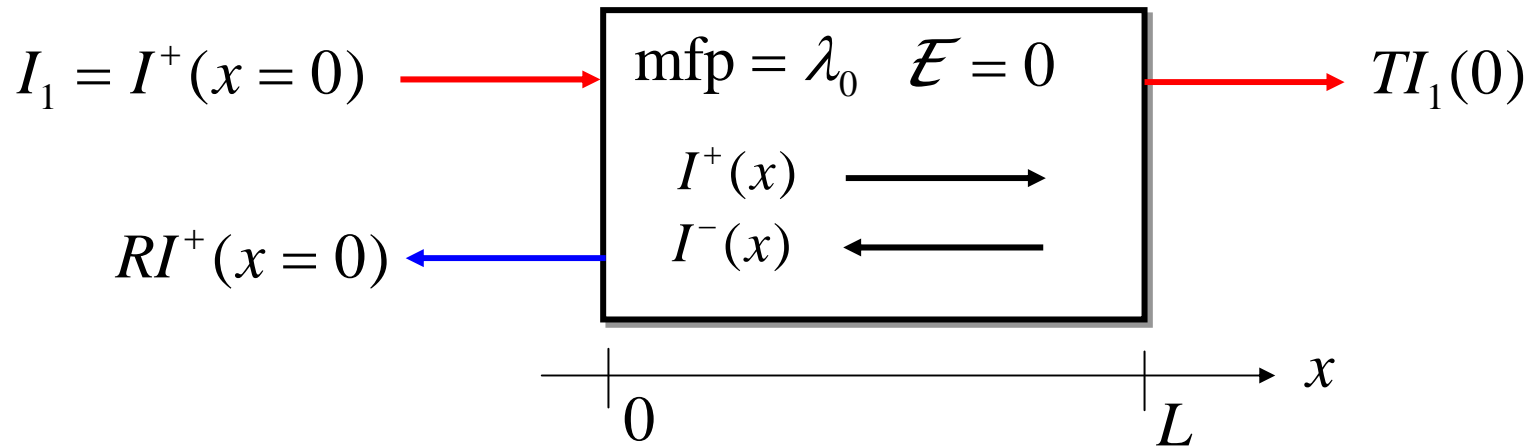
# transmission across a field-free slab



Consider a flux of carriers injected into a field-free slab of length,  $L$ . The flux that emerges at  $x = L$  is  $T$  times the incident flux, where  $0 < T < 1$ . The flux that emerges from  $x = 0$  is  $R$  times the incident flux, where  $T + R = 1$ , assuming no carrier recombination-generation.

How is  $T$  related to the mean-free-path for backscattering within the slab?

# transmission



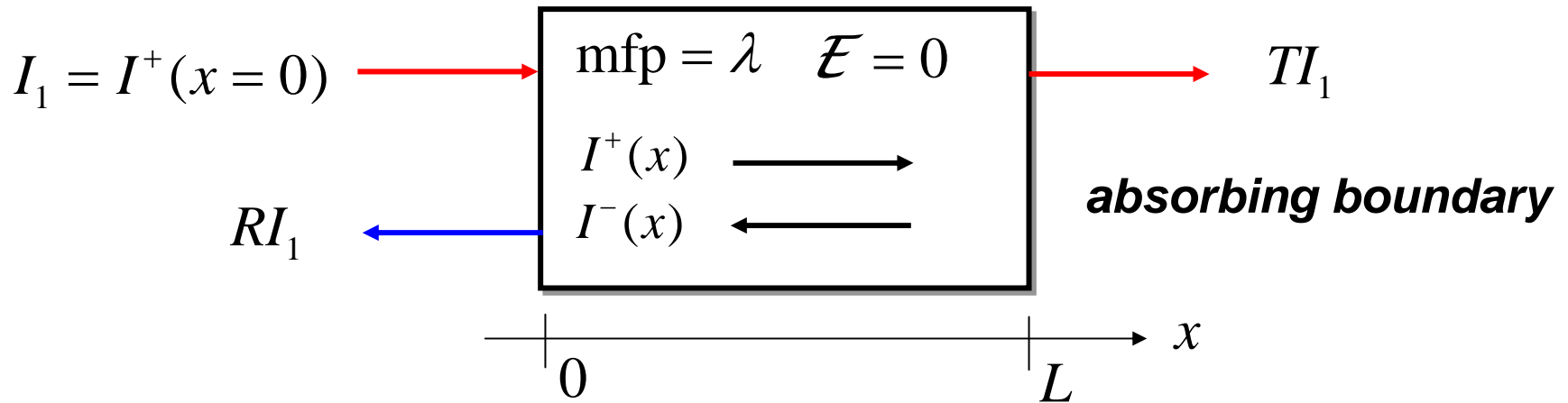
$$\frac{dI^+(x)}{dx} = -\frac{I^+(x)}{\lambda_0} + \frac{I^-(x)}{\lambda_0}$$

$$I^-(x) = I^+(x) - I$$

$$I = I^+(x) - I^-(x) \quad (\text{constant})$$

$$\frac{dI^+(x)}{dx} = -\frac{I}{\lambda_0}$$

## transmission (ii)



$$\frac{dI^+(x)}{dx} = -\frac{I}{\lambda_0}$$

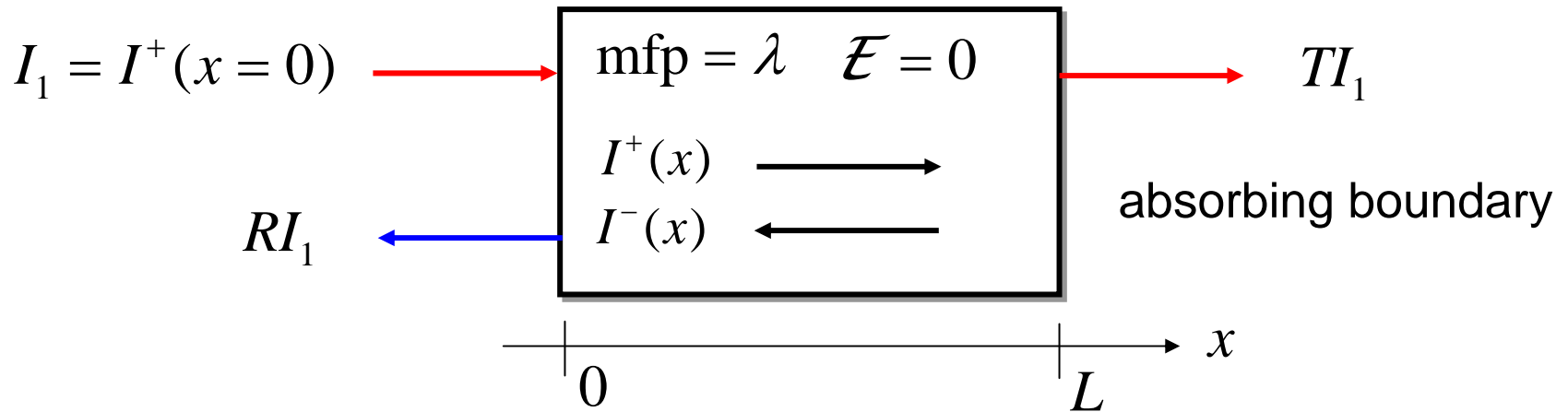
$$I^+(x) = I^+(0) - I \frac{x}{\lambda_0}$$

$$I^+(x) = I^+(0) - \left( I^+(x) - I^-(x) \right) \frac{x}{\lambda_0}$$

$$I^+(L) = I^+(0) - \left( I^+(L) - I^-(L) \right) \frac{L}{\lambda_0}$$

$$I^-(L) = 0$$

# transmission (iii)



$$I^+(L) = I^+(0) - I^+(L) \frac{L}{\lambda_0}$$

$$I^+(L) = \frac{I^+(0)}{1 + L/\lambda_0}$$

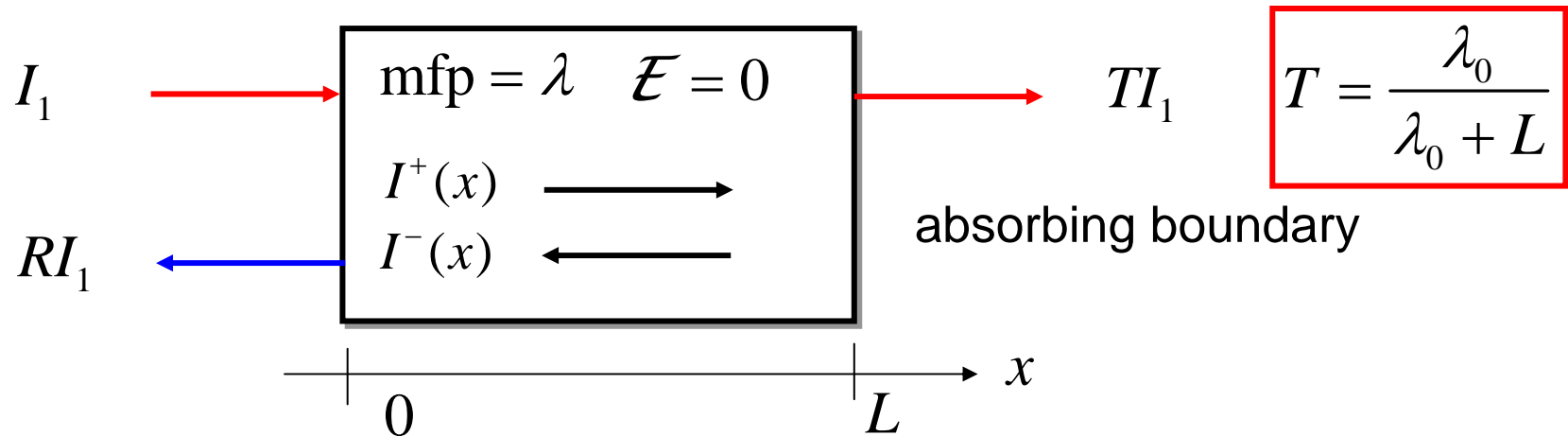
$$\frac{I^+(L)}{I^+(0)} = T = \frac{\lambda_0}{\lambda_0 + L}$$

$$T = \frac{\lambda_0}{\lambda_0 + L} \quad R = \frac{L}{\lambda_0 + L}$$

$$T \rightarrow 0 \quad L \gg \lambda_0$$

$$T \rightarrow 1 \quad L \ll \lambda_0$$

# mean-free-path



How do we relate  $\lambda_0$  to known parameters?

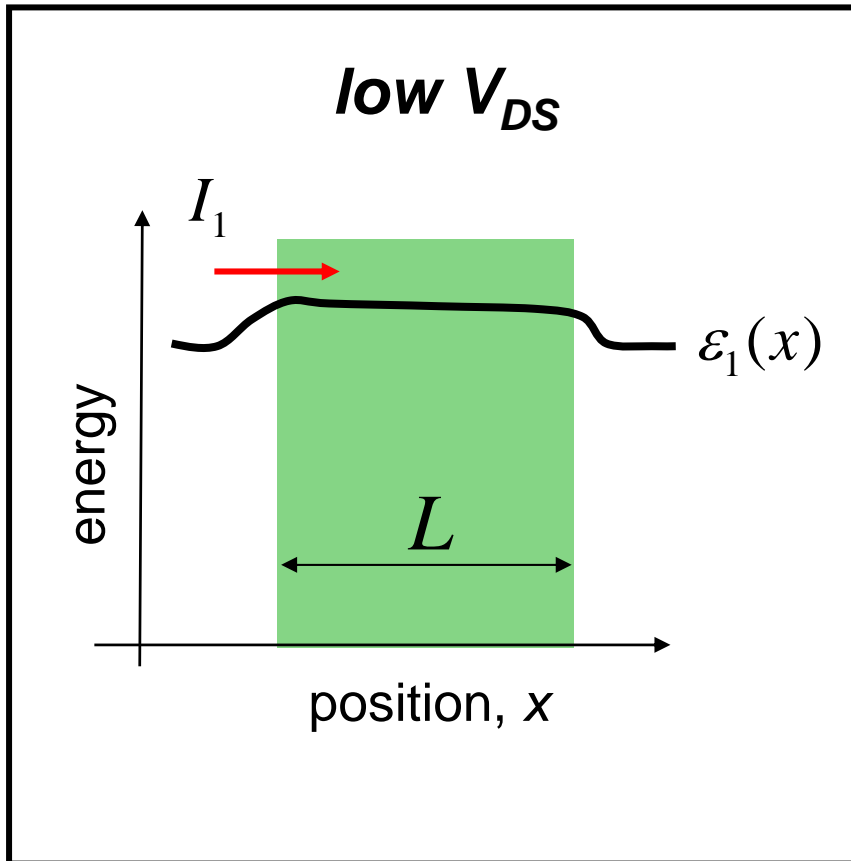
If  $I_1$  is a thermal equilibrium injected flux,  $I_1 = n^+(0)v_T$   
then, it can be shown that:

$$D_n = \frac{k_B T}{q} \mu_n = \frac{v_T}{2} \lambda_0$$

(non-degenerate  
carrier statistics)



# example



$$\mu_n \approx 200 \text{ cm}^2/\text{V-s}$$

$$\mu_n = \frac{v_T}{2(k_B T/q)} \lambda_0$$

$$\lambda_0 \approx 9 \text{ nm}$$

$$L \approx 50 \text{ nm}$$

$$T \approx \frac{\lambda_0}{L + \lambda_0} \approx 0.15$$

## relation to conventional theory

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$$G_{CH} = \left( WC_{ox} (V_{GS} - V_T) \frac{v_T}{(2k_B T / q)} \right) (1 - R)$$

$$1 - R = T = \frac{\lambda_0}{\lambda_0 + L} \approx \frac{\lambda_0}{L} \quad (\text{diffusive limit})$$

$$\lambda_0 = \frac{2k_B T / q}{v_T} \mu_n$$

$$G_{CH} = \frac{W}{L} \mu_n C_{ox} (V_{GS} - V_T)$$

(non-degenerate carrier statistics)

The scattering model works in the diffusive limit, as well as the ballistic limit, and in the quasi-ballistic regime in between.

# channel conductance

$$G_{CH} = \left( WC_{ox} (V_{GS} - V_T) \frac{v_T}{(2k_B T/q)} \right) \left[ \frac{\mathcal{F}_{-1/2}(\eta_{F1})}{\mathcal{F}_0(\eta_{F1})} \right] T$$

$$T = \frac{\lambda_0}{\lambda_0 + L}$$

one can show that:

$$G_{CH} = \frac{W}{L} \left( \frac{1}{\mu_n} + \frac{1}{\mu_B} \right)^{-1} C_{ox} (V_{GS} - V_T)$$

$$\left\{ \begin{array}{l} \mu_n = \frac{v_T \lambda_0}{2k_B T/q} \left[ \frac{\mathcal{F}_{-1/2}(\eta_{F1})}{\mathcal{F}_0(\eta_{F1})} \right] \\ \mu_B = \frac{v_T L}{2k_B T/q} \left[ \frac{\mathcal{F}_{-1/2}(\eta_{F1})}{\mathcal{F}_0(\eta_{F1})} \right] \end{array} \right.$$

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# transmission under high drain bias

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## ***scattering model:***

$$I_{ON} = WC_{ox} (V_{GS} - V_T) \vartheta_F \frac{(1-R)}{(1+R)} = WC_{ox} (V_{GS} - V_T) \vartheta_F \frac{T}{(2-T)}$$

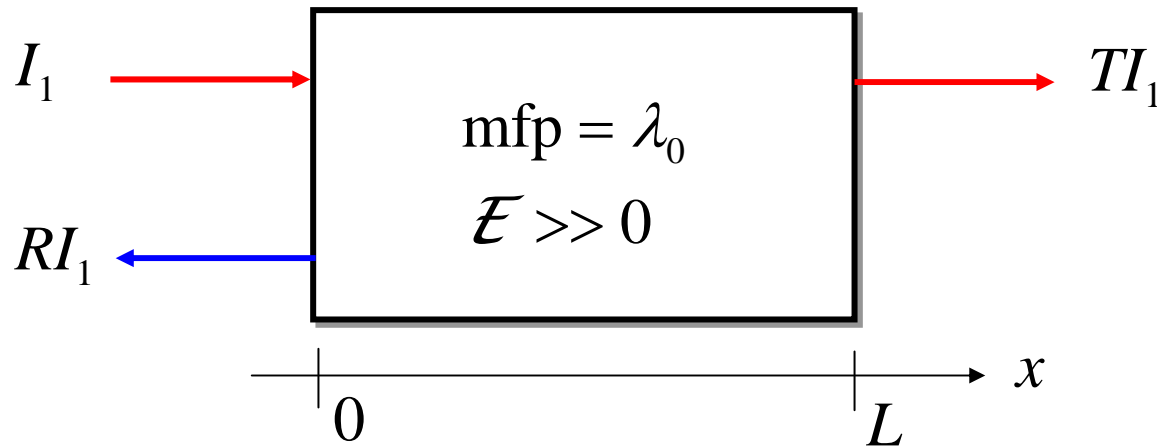
## ***in practice:***

$$B \equiv \frac{I_{ON}(\text{measured})}{I_{ON}(\text{ballistic})} \approx 0.50$$

$$B = \frac{T}{(2-T)} \rightarrow T \approx 0.67 \gg 0.15 \quad \textbf{Why?}$$

# transmission across a slab with an electric field

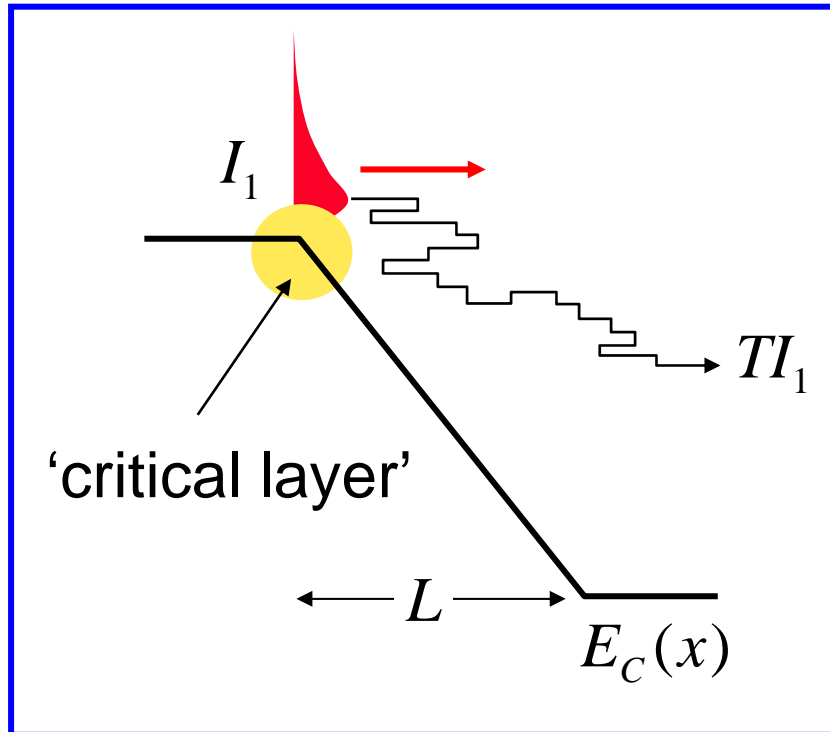
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When the electric field is strong and position-dependent and several scattering mechanisms operate, this turns out to be a difficult problem.

How can we understand the essential physics?

# transport “downhill”



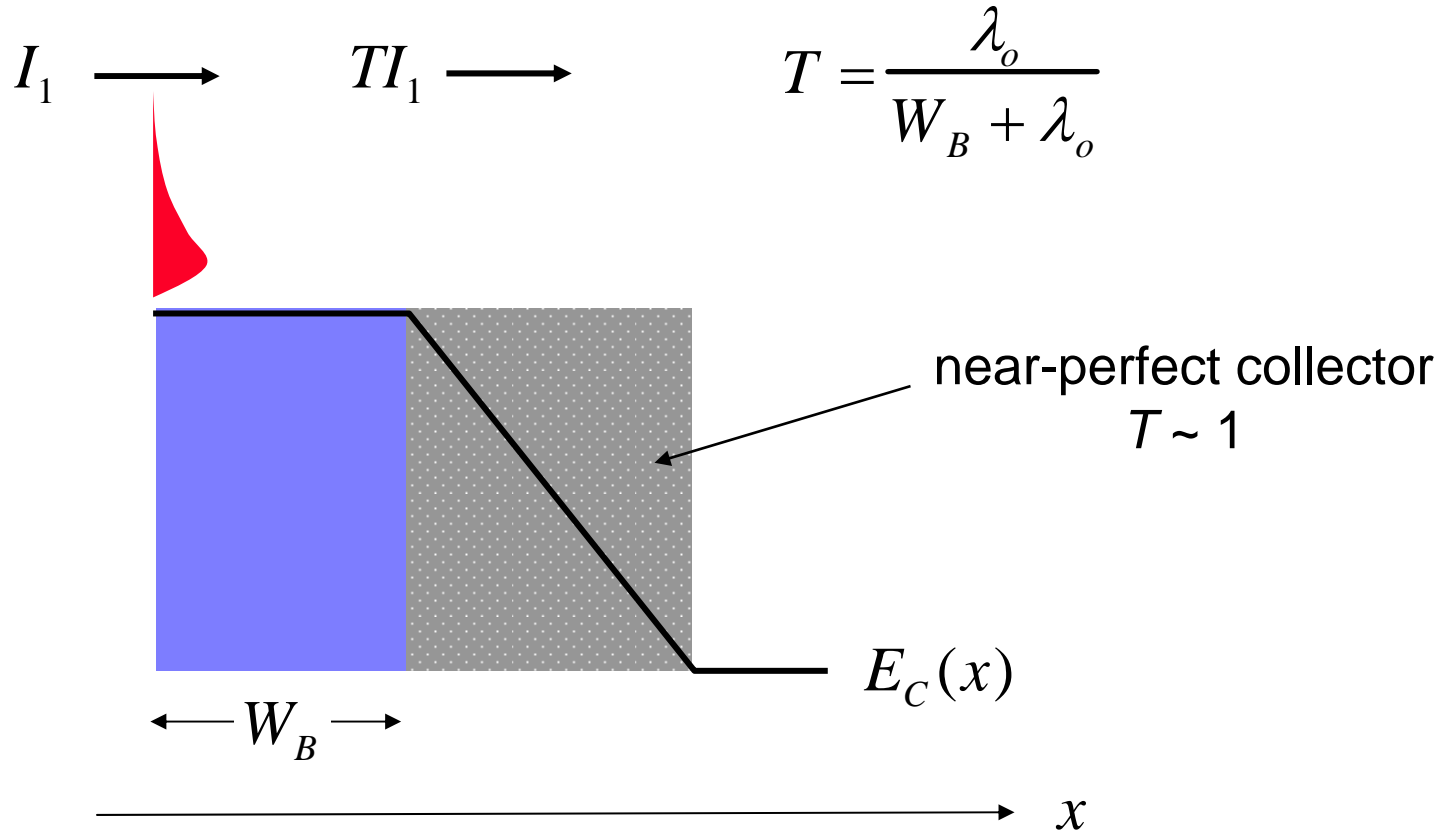
$$T = \frac{\lambda_o}{1 + \lambda_o} \quad \ell \ll L$$

$T \approx 1$ :

High field regions are good carrier collectors.

Peter J, Price, “Monte Carlo calculation of electron transport in solids,”  
*Semiconductors and Semimetals*, **14**, pp. 249-334, 1979

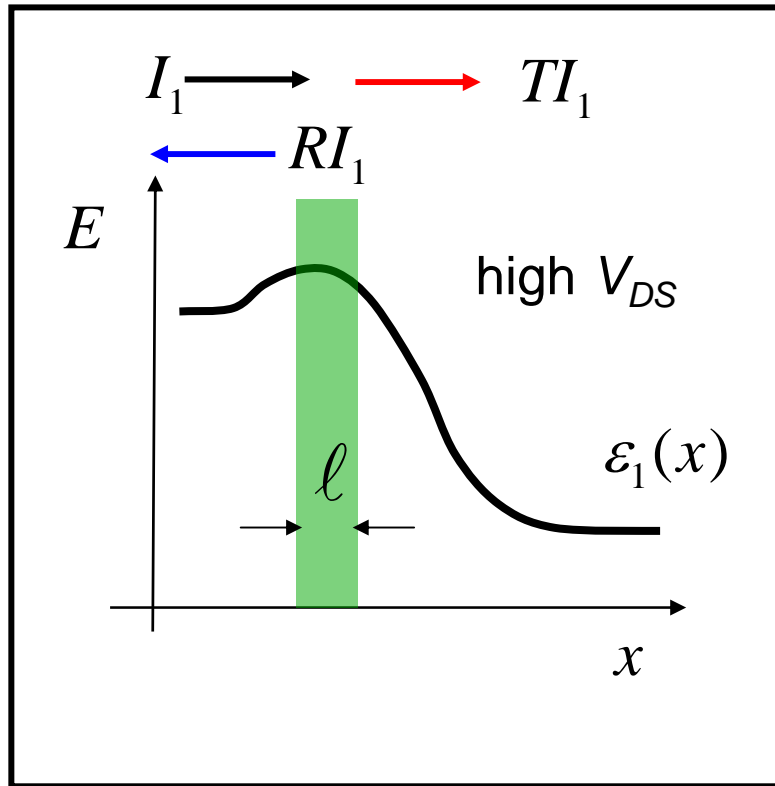
# field-free region followed by high-field region



The base-collector of a bipolar transistor is a low-field region followed by a high-field region. **Transmission is controlled by the low-field region.**



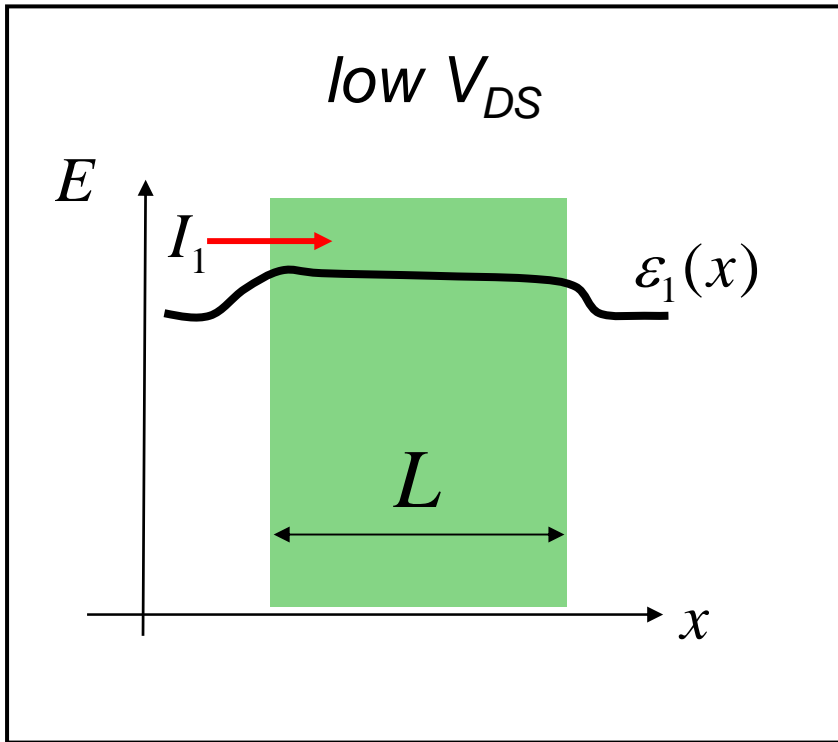
# transport in a MOS transistor



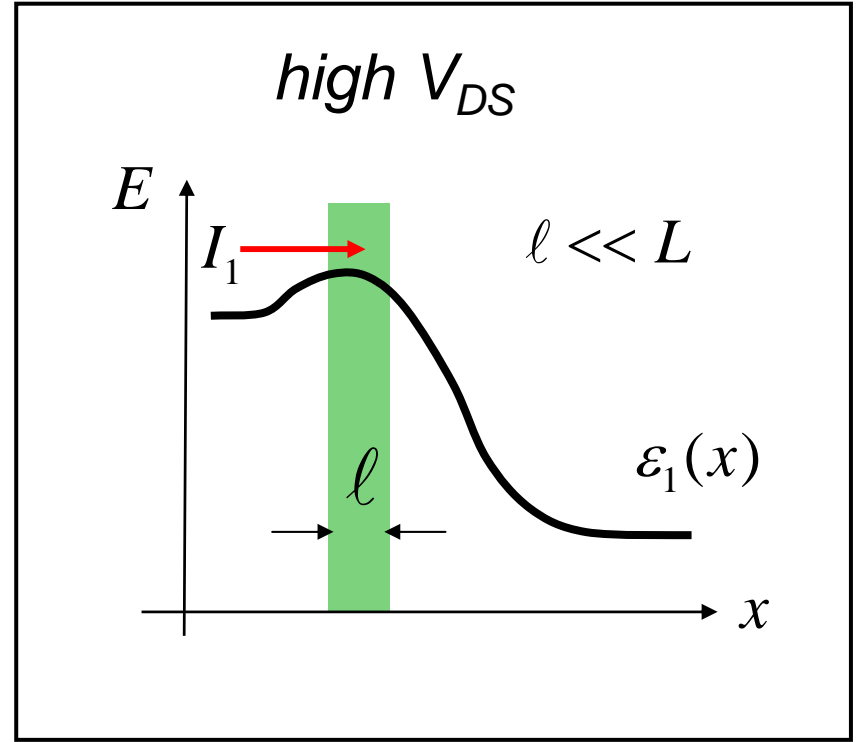
$$T \approx \frac{\lambda_o}{1 + \lambda_o}$$

- 1) A MOSFET consists of a low-field region near the source that is strongly controlled by the gate voltage, and a high-field region near the drain that is strongly controlled by the drain voltage.
- 2) Transmission is controlled by the low-field region near the source.
- 3) Scattering near the drain has a smaller effect on backscattering to the source.
- 4) In contrast to a bipolar transistor, the division between the low and high field regions is not sharp.

# bias-dependent transmission



$$T \approx \frac{\lambda_o}{L + \lambda_o} \approx 0.15$$



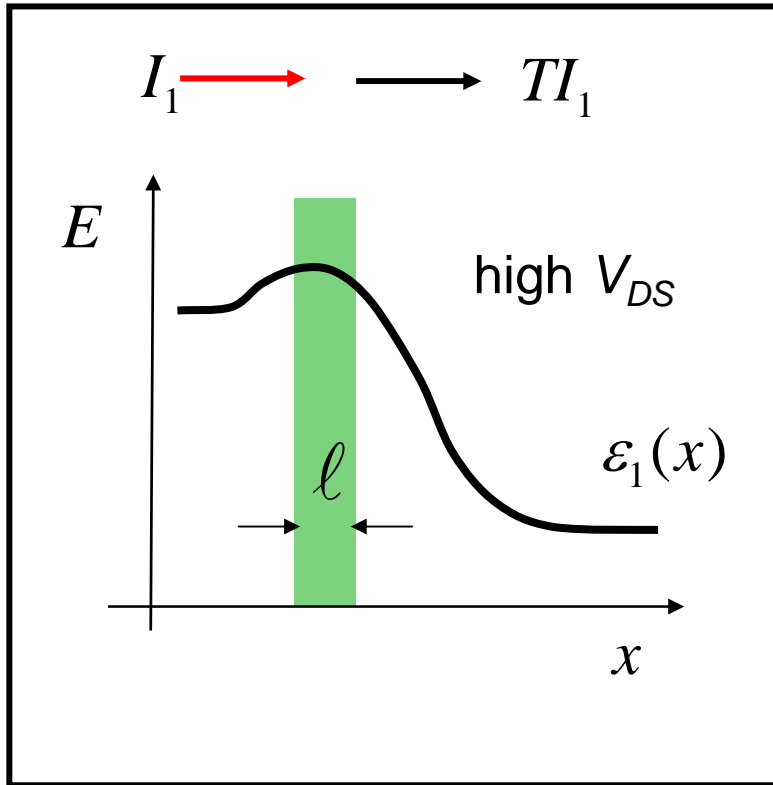
$$T \approx \frac{\lambda_o}{1 + \lambda_o} \approx 0.67$$

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# computing the critical length



$$T \approx \frac{\lambda_o}{1 + \lambda_o}$$

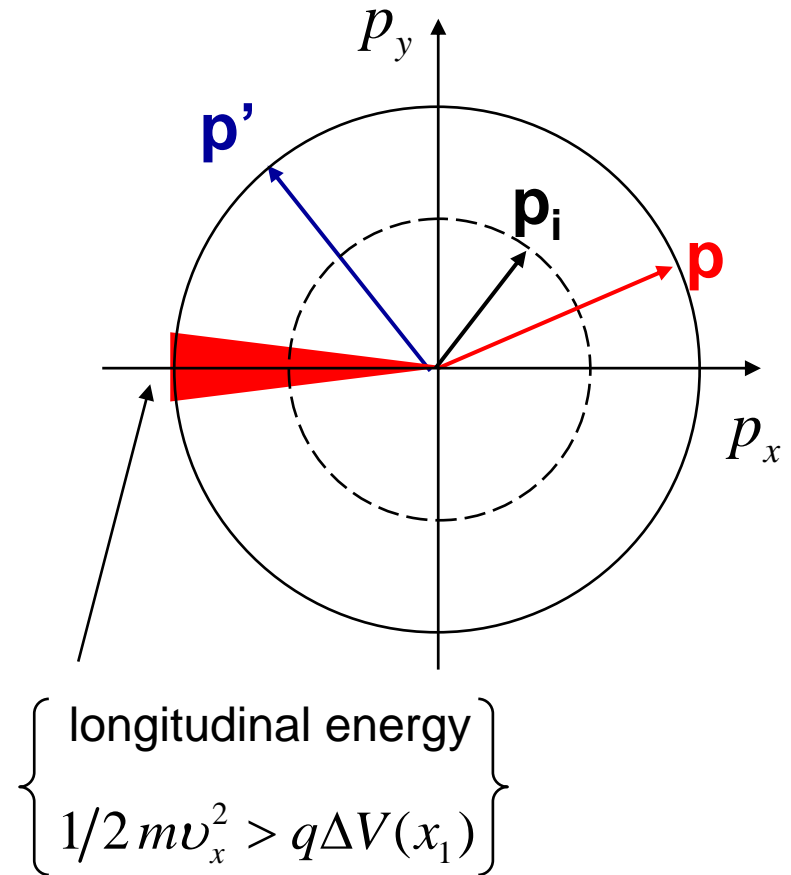
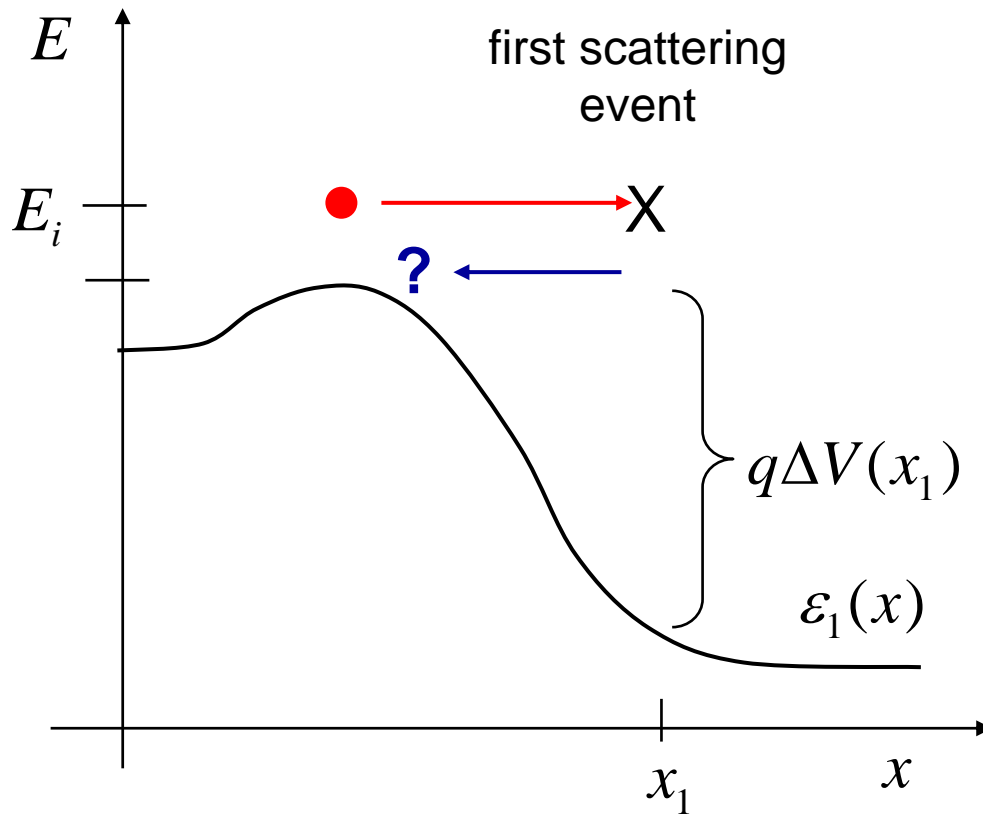
Assuming near-equilibrium transport in the low-field region (i.e. DD), one can show that the critical length is the distance over which the channel potential drops by  $k_B T/q$

“ $kT$  layer”  $\ell \approx L_{k_b T}$

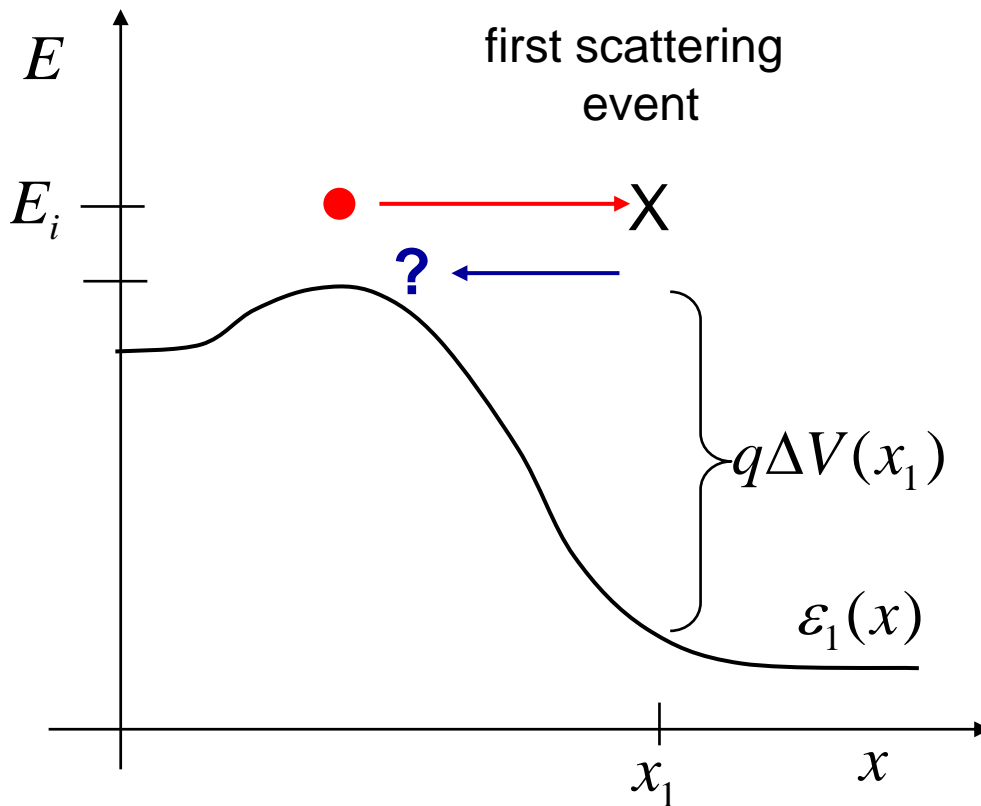
**Two key assumptions:**

- 1) near-equilibrium
- 2) Boltzmann statistics

# physics of elastic back-scattering



# physics of elastic back-scattering



$$x_1 < l$$

probability of returning to the source is high

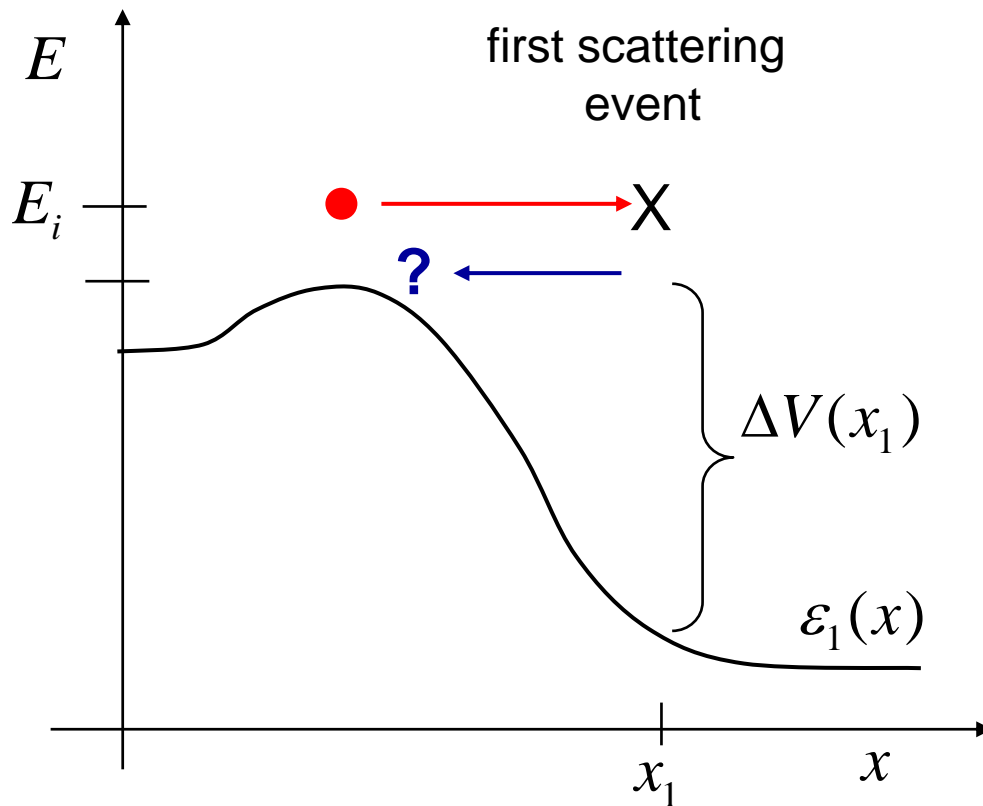
$$x_1 > l$$

probability of returning to the source is low

$$q\Delta V(l) \approx E_i - \varepsilon_1(0)$$

See: Lundstrom and Ren, *IEEE Trans. Electron Dev*, **49**, pp. 133-141, 2002.

# physics of elastic back-scattering



For non-degenerate carriers,

$$\langle E_i - \varepsilon_1(0) \rangle = k_B T$$

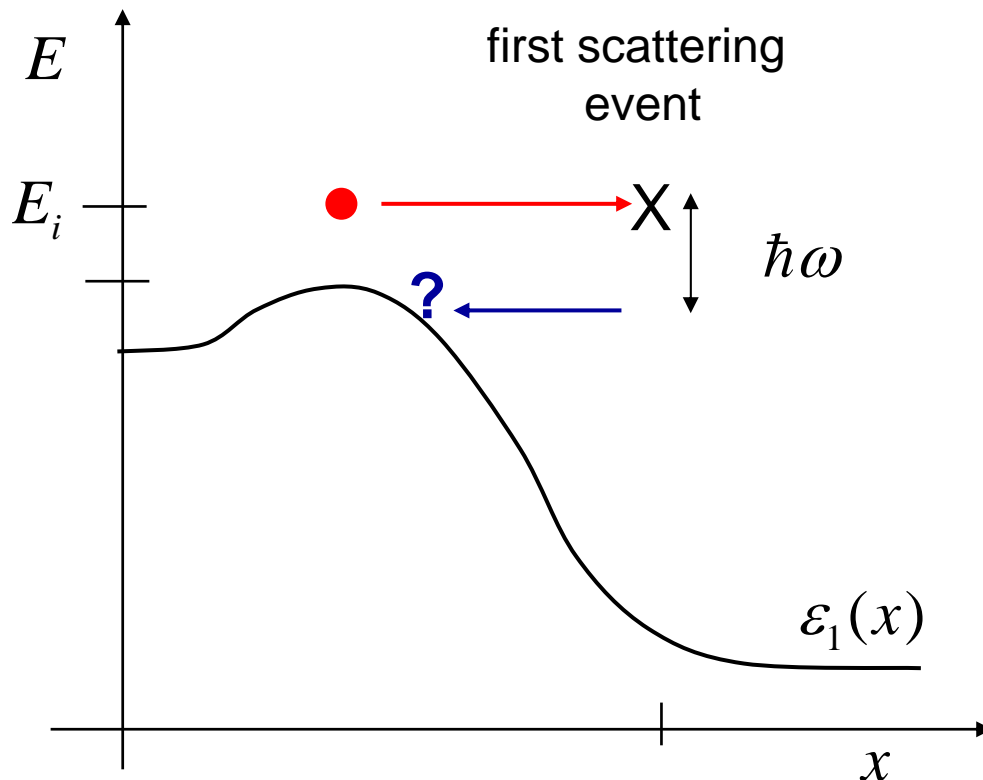
$$q\Delta V(l) \approx k_B T$$

Above threshold, however,

$$\langle E_i - \varepsilon_1(0) \rangle > k_B T$$

See: Lundstrom and Ren,  
*IEEE Trans. Electron Dev*, **49**, pp.  
133-141, 2002.

# role of inelastic scattering



If an electron backscatters by emitting a phonon, it is less likely to return to the source - even if it is pointed in the right direction.

$$L_{k_B T} \rightarrow \text{"}\hbar\omega \text{ length"}$$

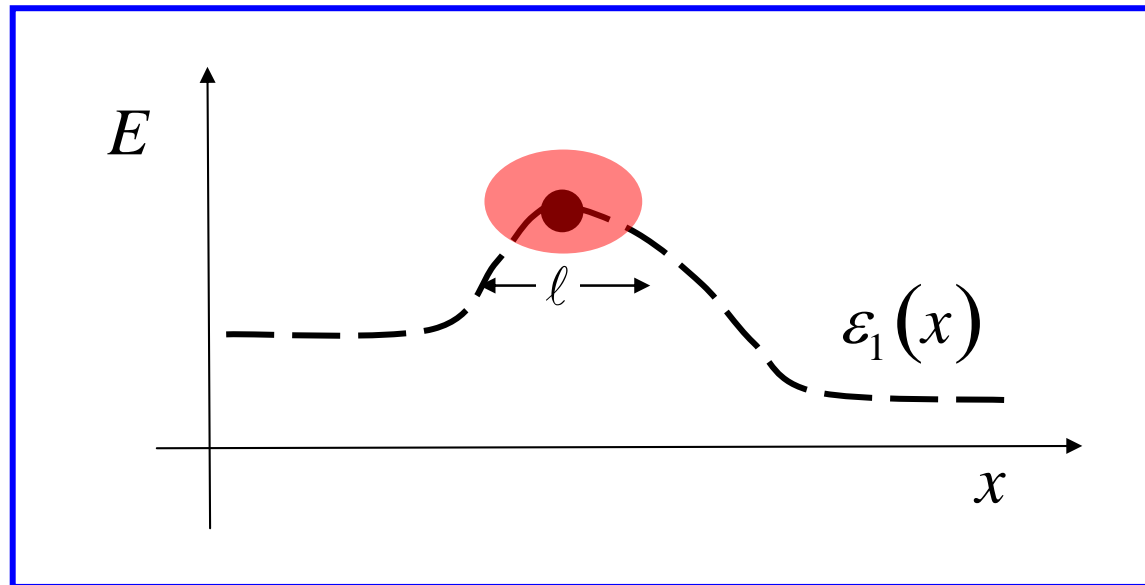
K. Natori, *IEEE Electron Dev. Lett.*, **23**, pp. 655-657, 2002.



# drift-diffusion picture

$$I_D = WC_{ox} \langle v(0) \rangle (V_{GS} - V_T)$$

$$\frac{1}{\langle v(0) \rangle} = \frac{1}{v_T} + \frac{1}{D_n/l} \quad D_n = (k_B T / q) \mu_n$$



# drift-diffusion vs. scattering model

$$I_D = WC_{ox} \left[ \frac{1}{\nu_T} + \frac{1}{(D_n/l)} \right]^{-1} (V_{GS} - V_T)$$

drift-diffusion

$$D_n = \nu_T \lambda_0 / 2 \quad T = \frac{\lambda_0}{\lambda_0 + l}$$

$$I_D = WC_{ox} \frac{T}{2 - T} \nu_T (V_{GS} - V_T)$$

scattering



# Monte Carlo simulations

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Detailed, numerical simulations confirm the basic physical picture that we have presented, but they show that the critical layer is somewhat longer than  $L_{kT}$  and that it depends on the shape of the potential profile. Under some conditions, inelastic scattering can even increase the d.c. current.

See, for example:

P. Palestri, R. Clerc, D. Esseni, L. Lucci, and L. Selmi, “Multi-subband Monte Carlo investigation of the mean free path and of the  $kT$  layer in degenerated quasi-ballistic nanoMOSFETs, IEDM Tech. Dig., pp. 945-948, Dec. 2006.

Raseong Kim and Mark Lundstrom, “Physics of carrier backscattering in one- and two-dimensional nanotransistors,” submitted for publication, June, 2008.

# outline

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- 1) Review and introduction
- 2) Scattering theory of the MOSFET
- 3) Transmission under low  $V_{DS}$
- 4) Transmission under high  $V_{DS}$
- 5) Discussion
- 6) **Summary**

# summary

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- 1) Modern MOSFETs operate between the ballistic and diffusive limits, so we need to understand transport in the quasi-ballistic regime.
- 2) Transmission (or scattering) theory provides a simple, physical description of quasi-ballistic transport.
- 3) The same physics can also be understood at the drift-diffusion level.
- 4) Quantitative treatments require detailed numerical simulation.