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Physics of Nanoscale Transistors: Lecture 4:

Carrier Scattering in Nanoscale MOSFETs

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outline

1) **Review and introduction**
2) Scattering theory of the MOSFET
3) Transmission under low V_{DS}
4) Transmission under high V_{DS}
5) Discussion
6) Summary

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review: ballistic I-V

$$I_D = \frac{2q}{h}\int_{-\infty}^{\infty} T(E)M(E)\left(f_1 - f_2\right)dE$$

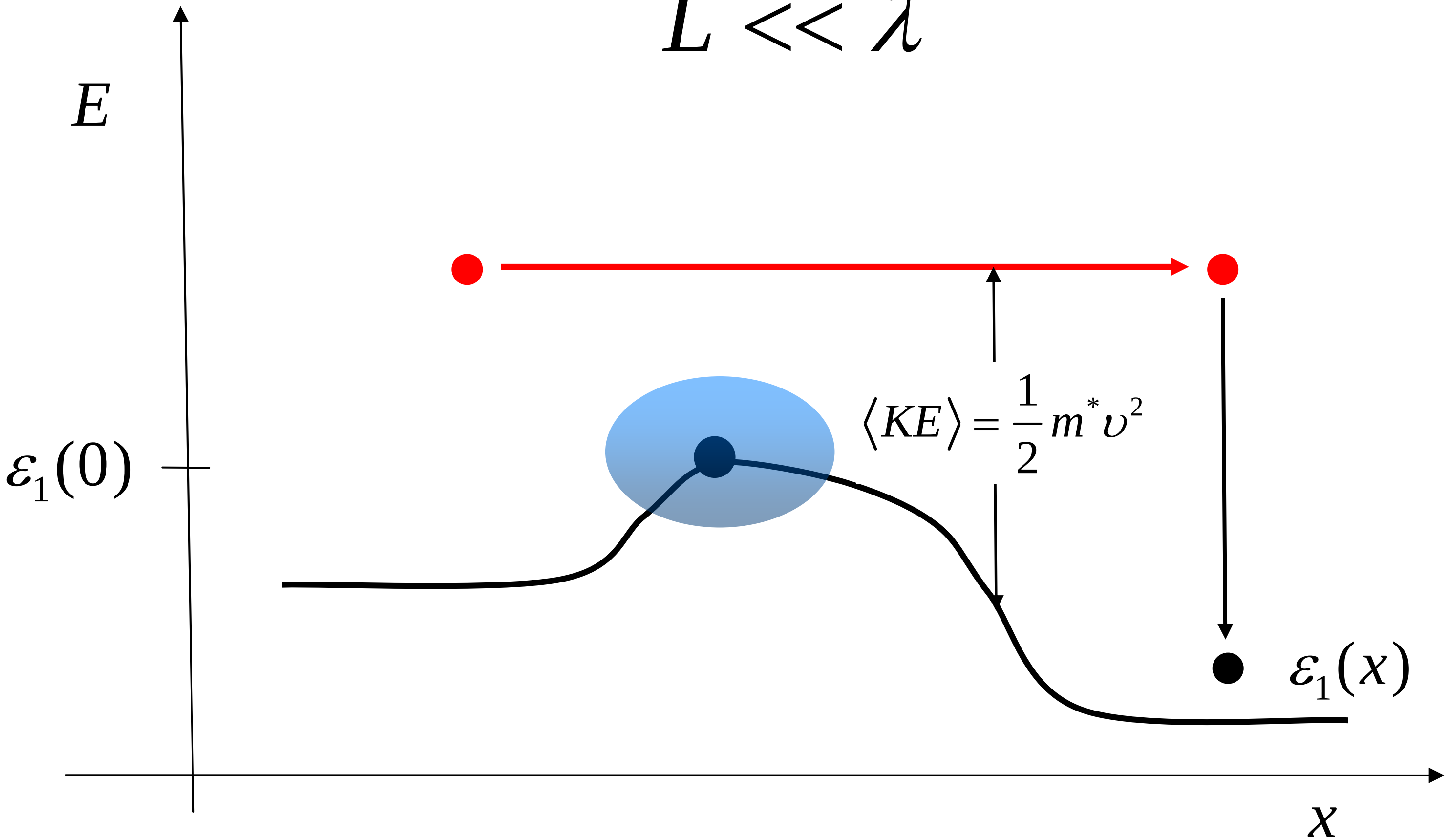
$$N = \int_{-\infty}^{\infty}\left[f_1(E)D_1(E) + f_2(E)D_2(E)\right]dE$$

$$I_D = WC_{ox}\left(V_{GS} - V_T\right)\tilde{\upsilon}_T\left[\frac{1 - \mathcal{F}_{1/2}\left(\eta_{F2}\right)/\mathcal{F}_{1/2}\left(\eta_{F1}\right)}{1 + \mathcal{F}_{0}\left(\eta_{F2}\right)/\mathcal{F}_{0}\left(\eta_{F1}\right)}\right]$$

$$\tilde{\upsilon}_T \equiv \sqrt{\frac{2k_BT}{\pi m^*}}\,\frac{\mathcal{F}_{1/2}\left(\eta_{F1}\right)}{\mathcal{F}_{0}\left(\eta_{F1}\right)} = \upsilon_T\,\frac{\mathcal{F}_{1/2}\left(\eta_{F1}\right)}{\mathcal{F}_{0}\left(\eta_{F1}\right)}$$

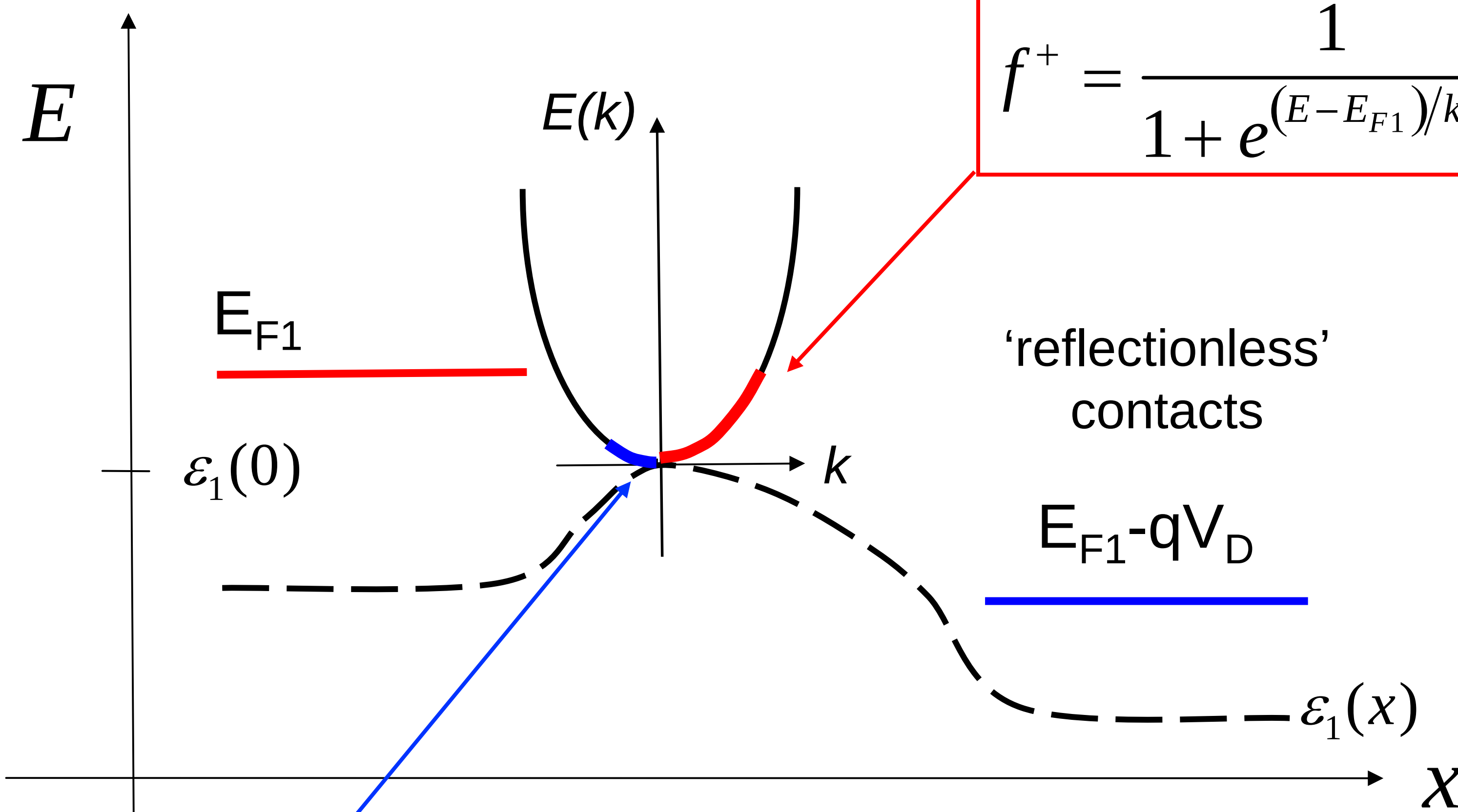
review: ballistic transport in a MOSFET

$$L << \lambda$$



review: filling states in a ballistic MOSFET

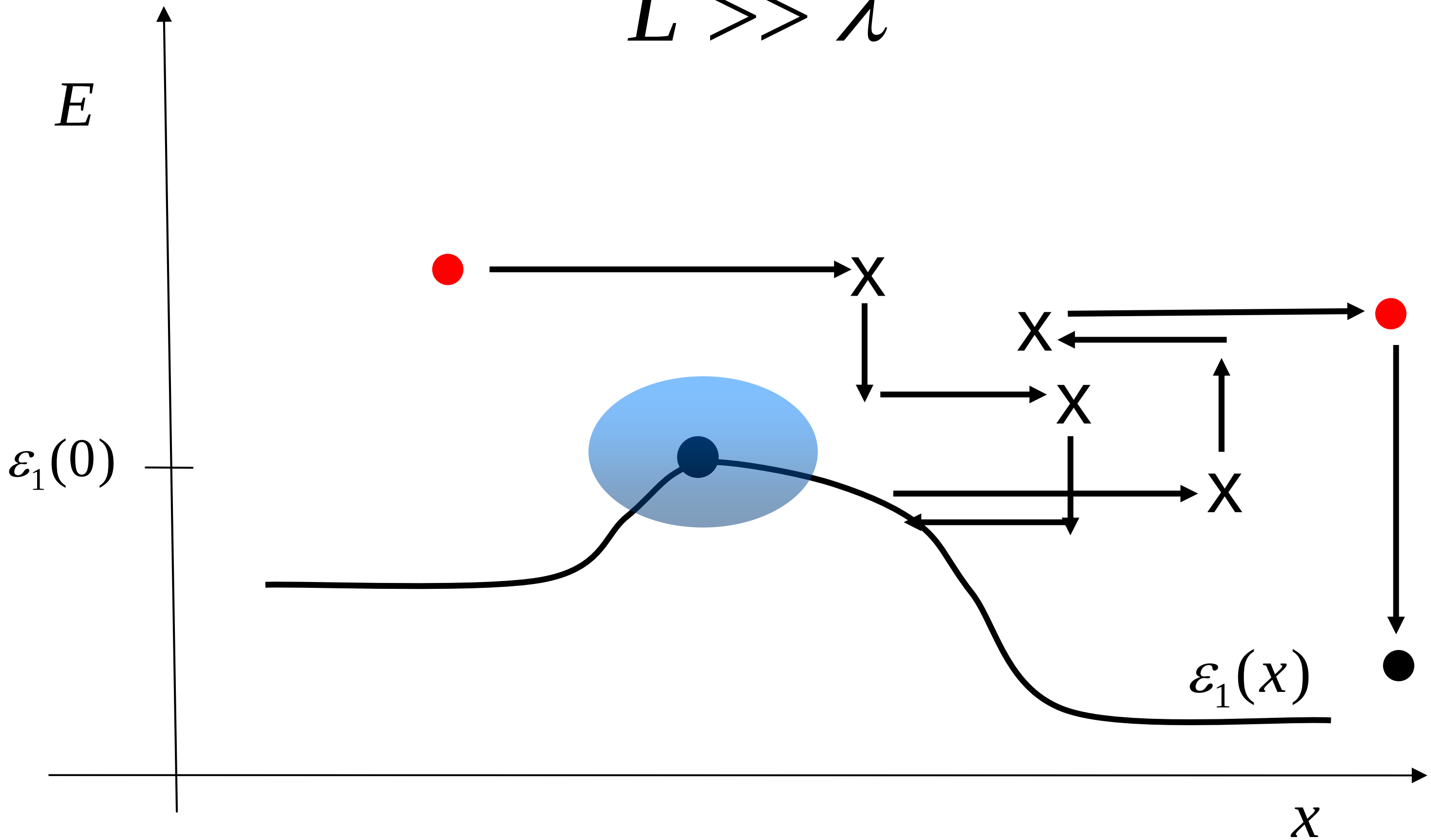
$$f^{+} = \frac{1}{1 + e^{(E - E_{F1})/k_B T}}$$



$$f^{-} = \frac{1}{1 + e^{(E - E_{F2})/k_B T}}$$

review: diffusive transport in a MOSFET

$$L >> \lambda$$



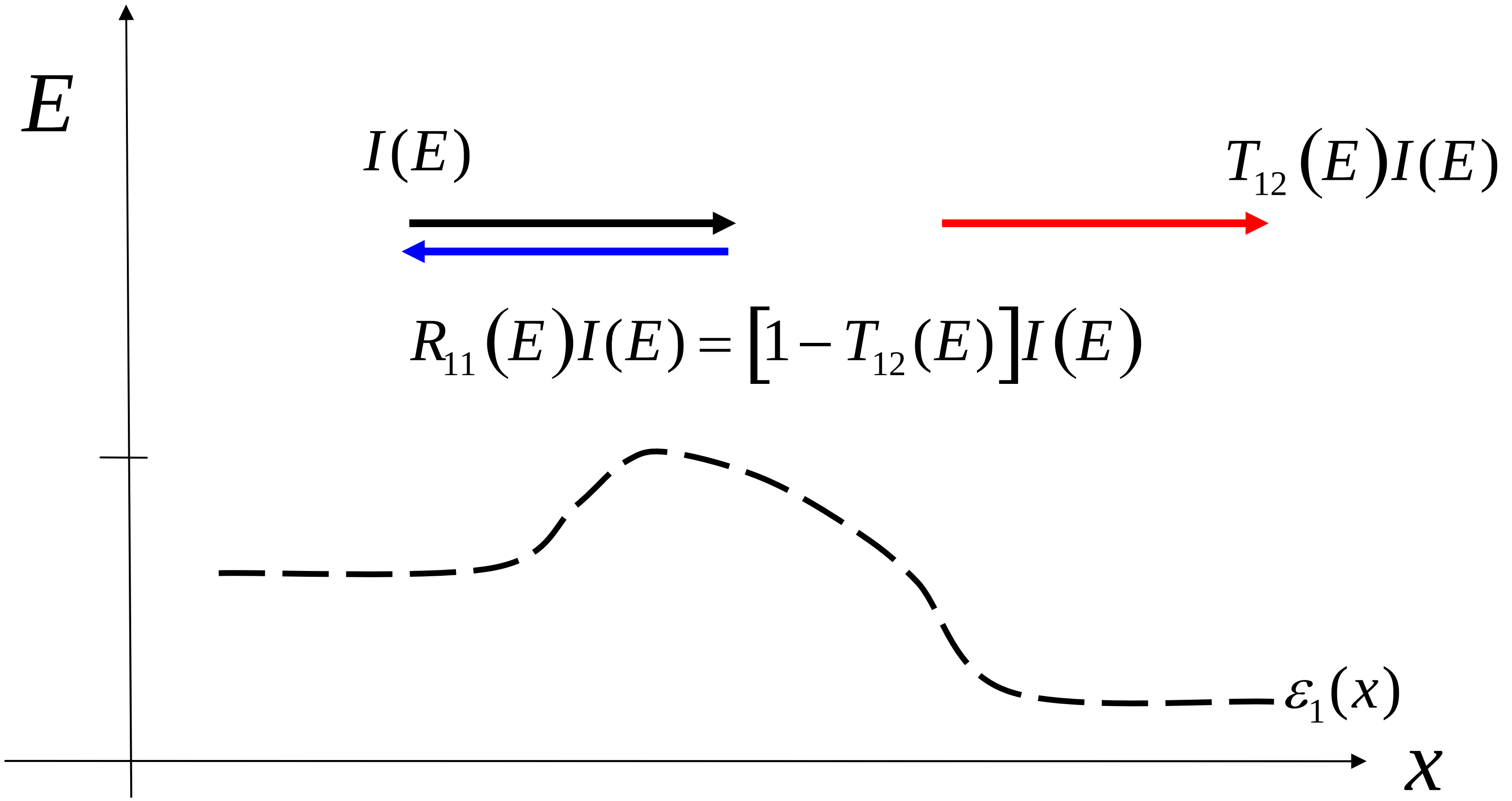
nanoscale MOSFETs

Nanoscale MOSFETs are neither fully ballistic nor fully diffusive; they operate in a ‘quasi-ballistic’ regime.

How do we ***understand*** how carrier scattering affects the performance of a nanoscale MOSFET?

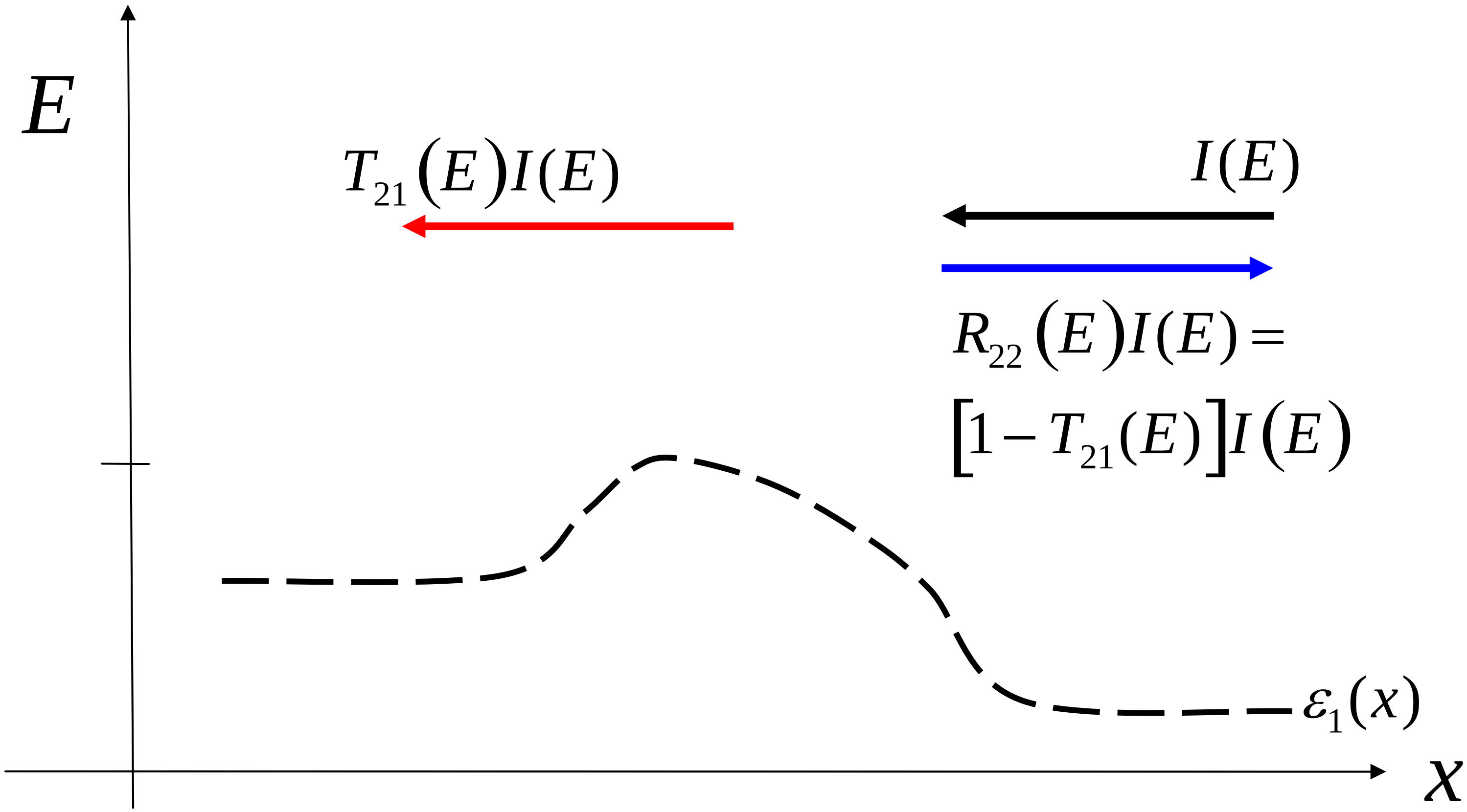
current transmission in a MOSFET

elastic scattering….



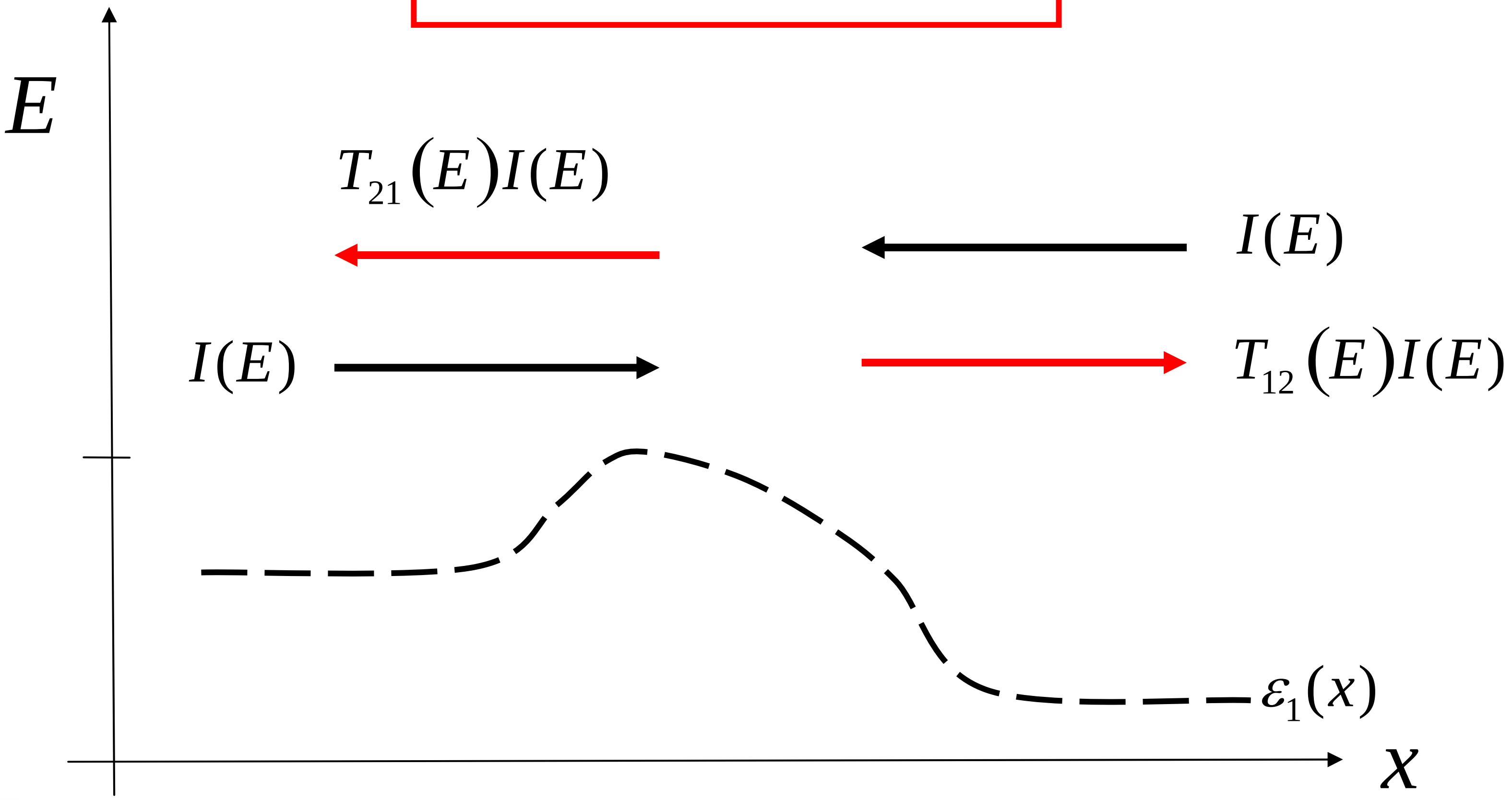
current transmission in a MOSFET

elastic scattering….



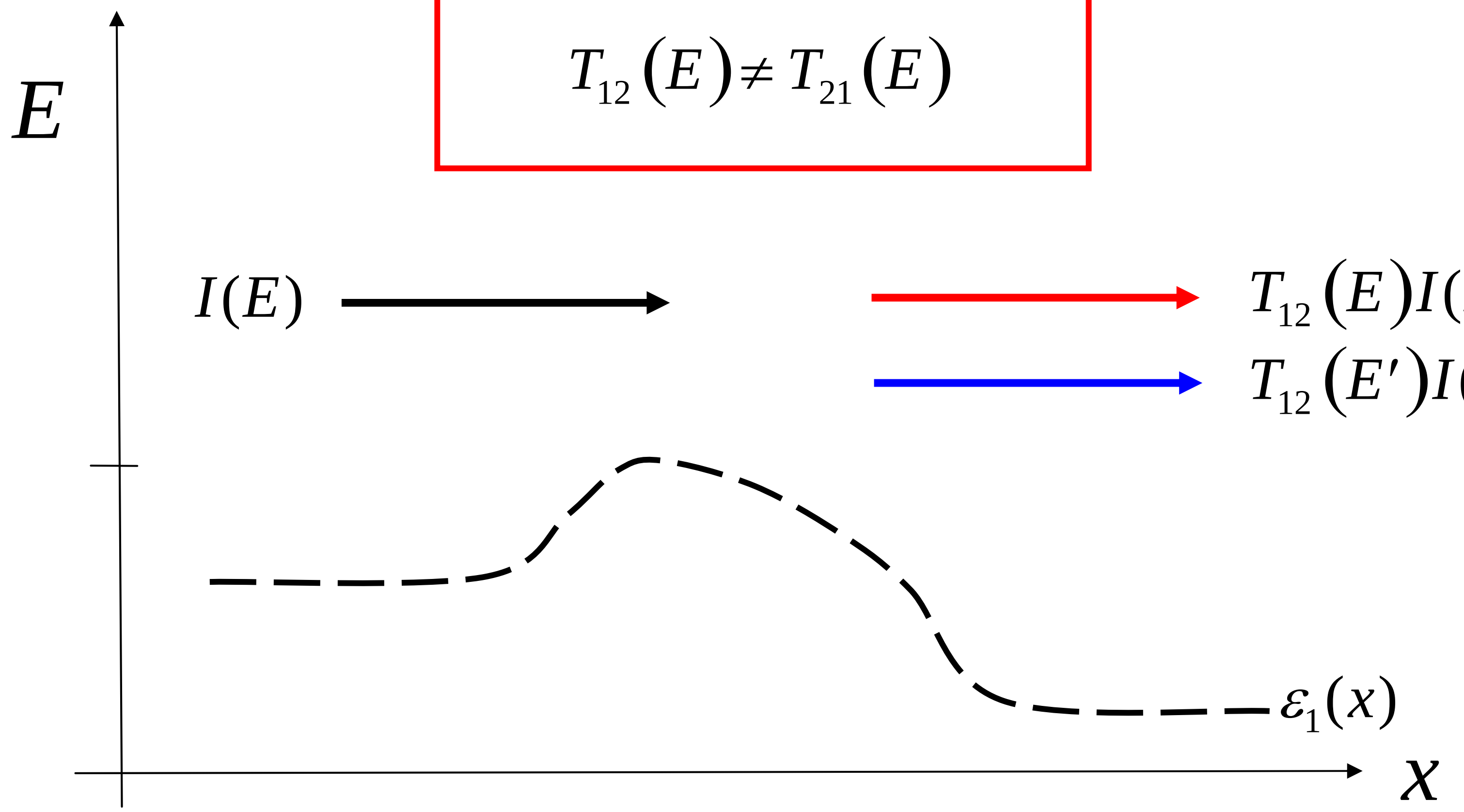
transmisson in the presence of **elastic** scattering

$$\boxed{T_{12}(E) = T_{21}(E) = T(E)}$$



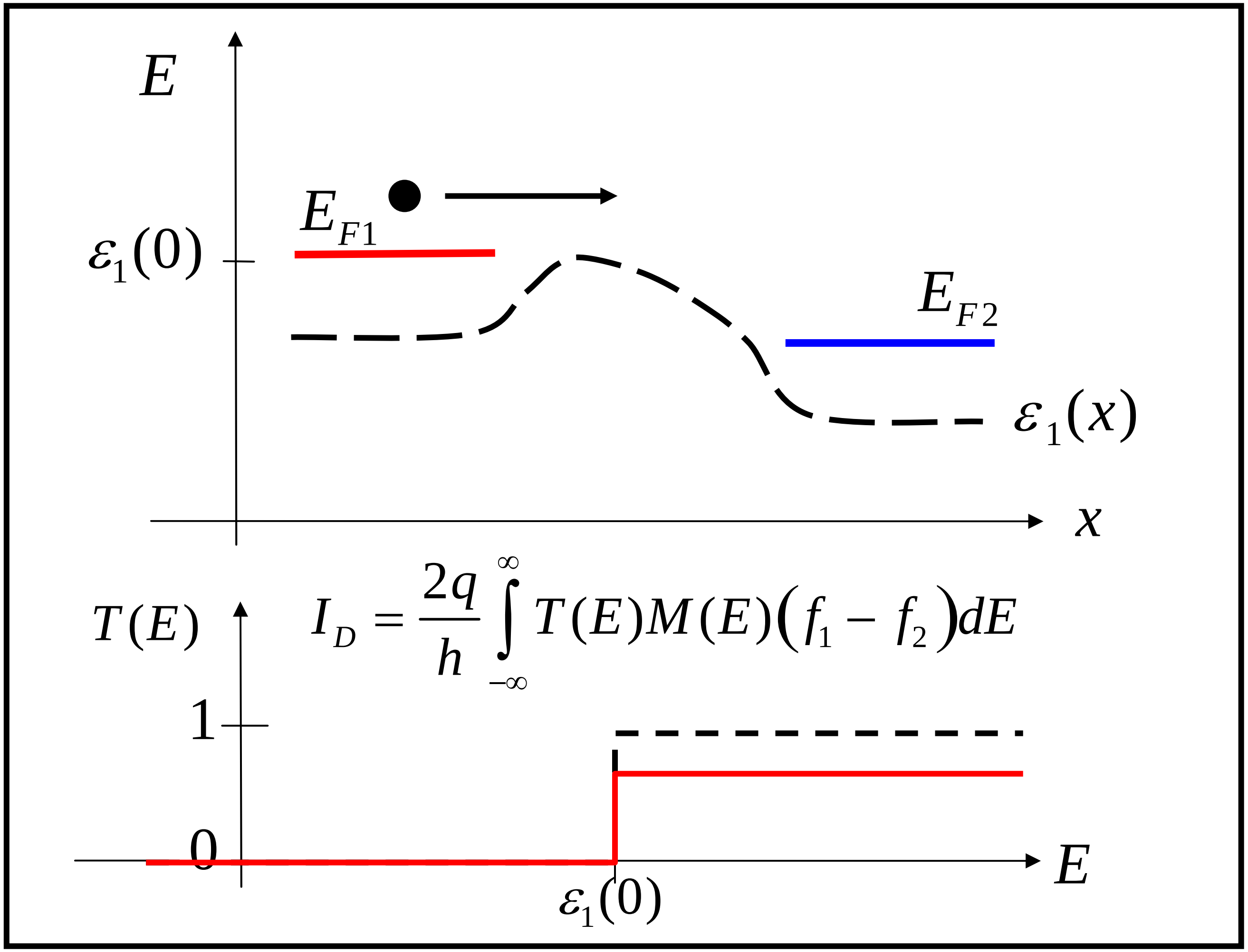
inelastic scattering

$$\boxed{T_{12}(E) \neq T_{21}(E)}$$

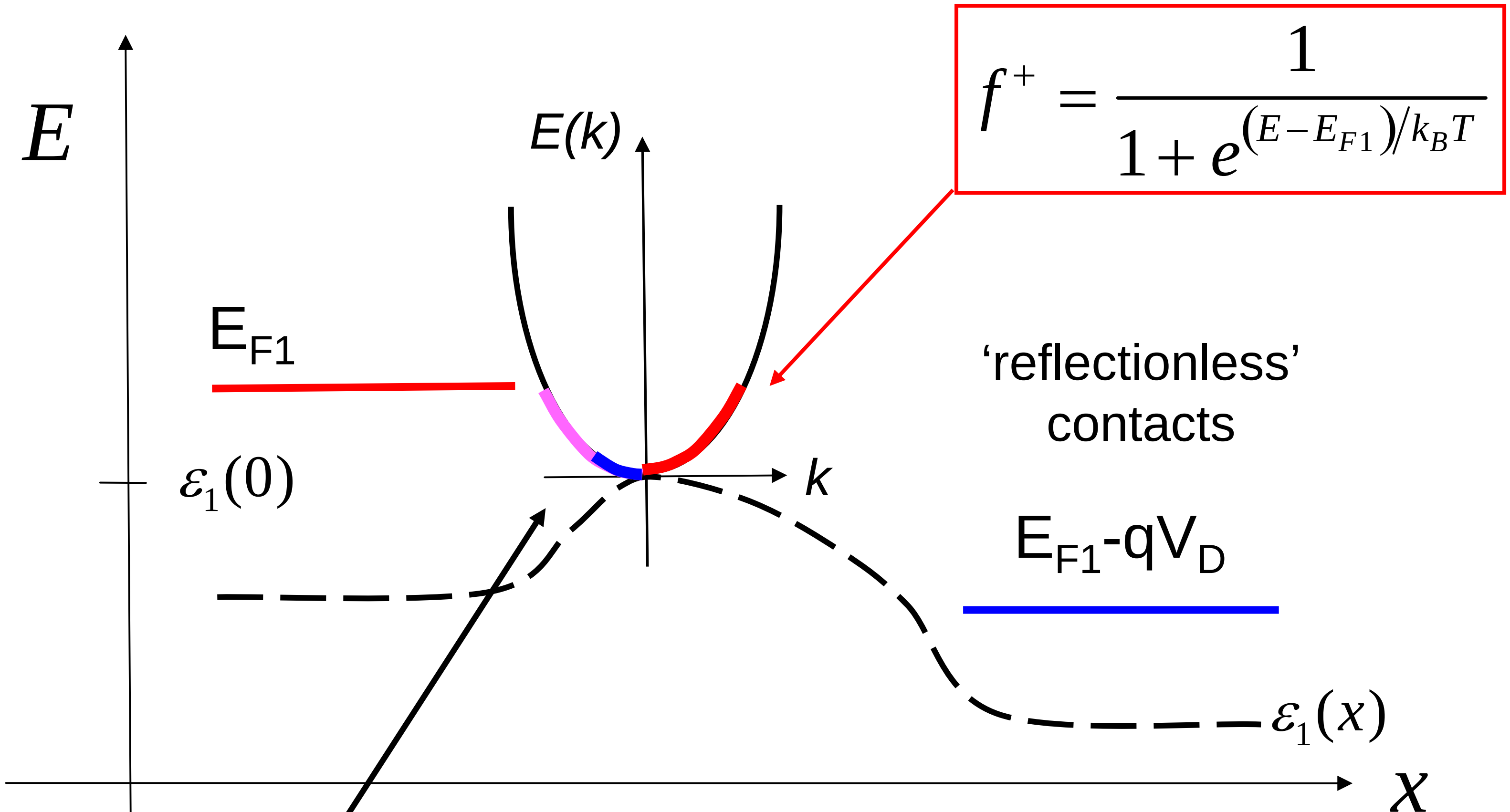


S. Datta, *Electronic Transport in Mesoscopic Systems*, Cambridge, 1995.

transmision and the IV characteristic



filling states in a quasi-ballistic MOSFET



some states are still filled from the drain, but some are now filled by backscattering from source-injected flux.

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scattering theory of the MOSFET

Goal:

To illustrate the influence on scattering on the I-V characteristic of a MOSFET by developing a very simple theory.

Assumptions:

1) Average quantities, not energy-resolved.

2) Boltzmann statistics for carriers

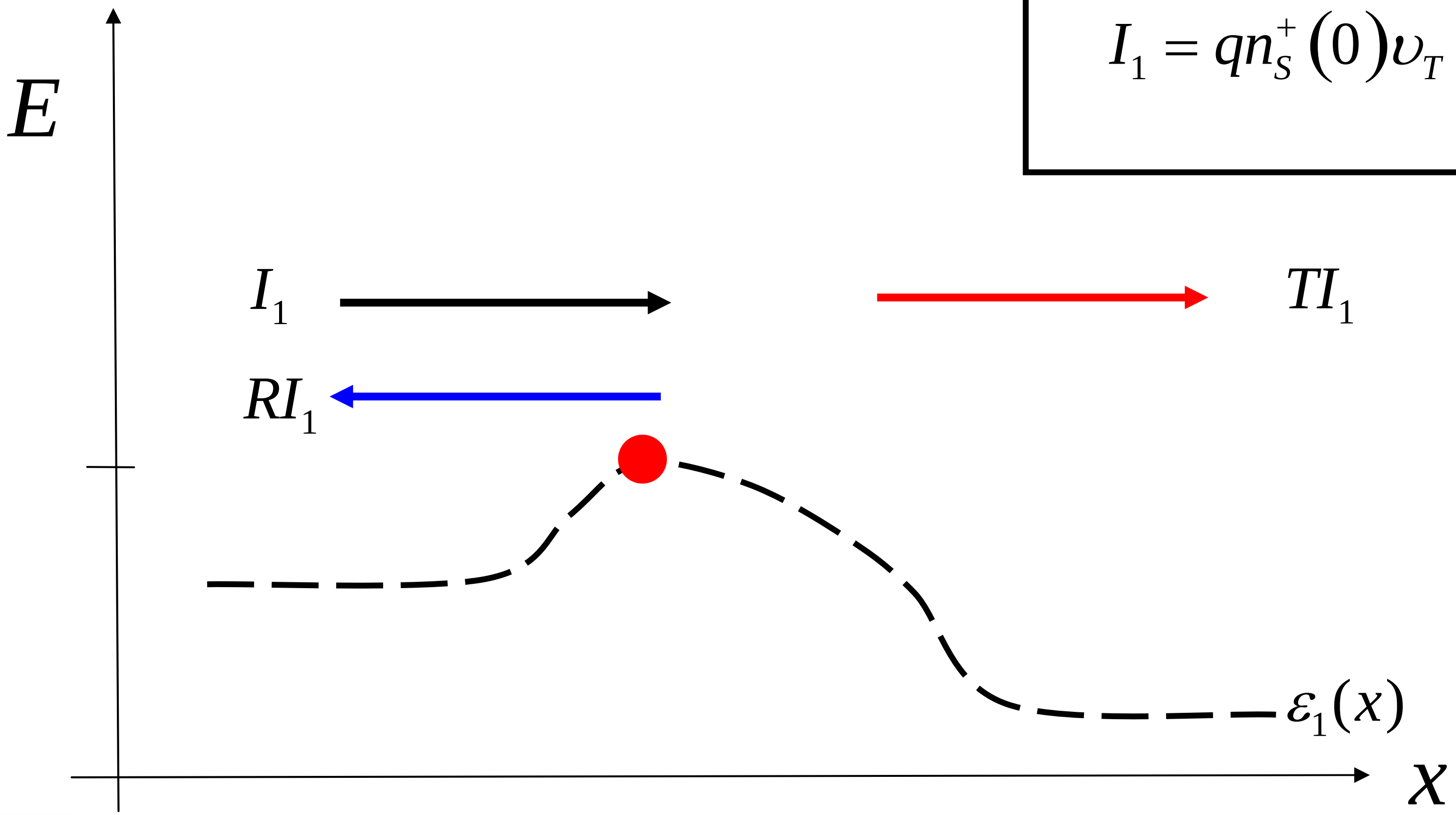
3) $T_{12} = T_{21} = T$

4) *Average velocity of backscattered carriers equals that of the injected carriers.*

scattering in a nano-MOSFET

$$T + R = 1$$

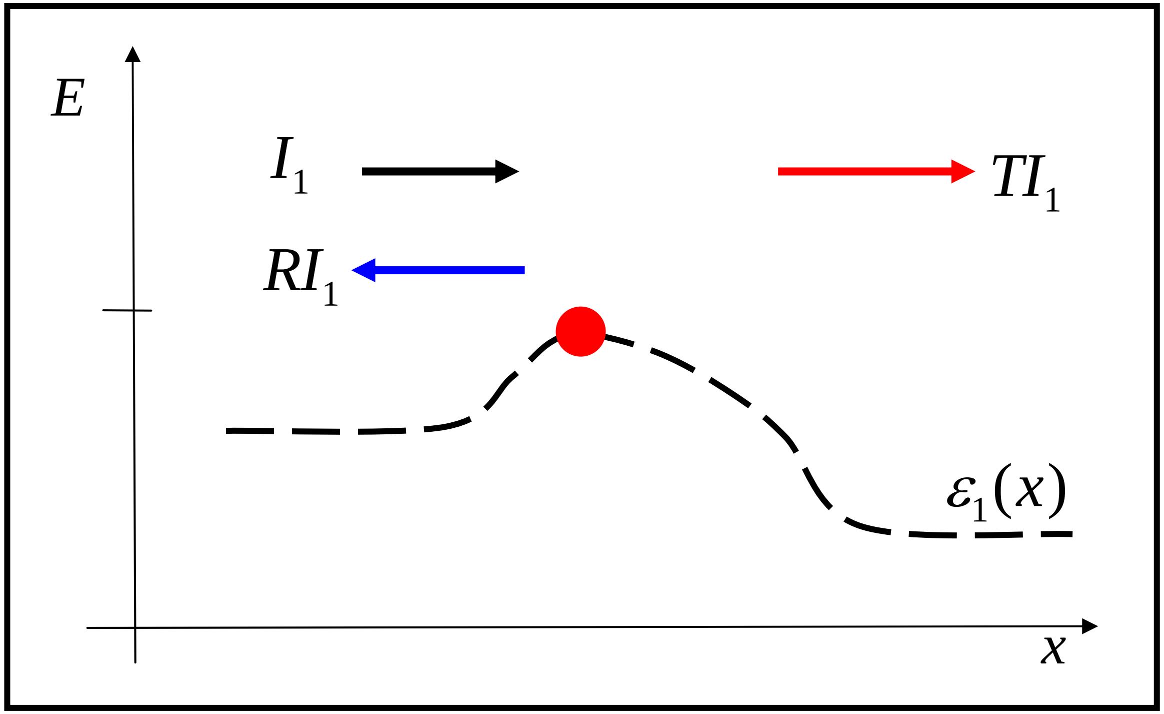
$$I_1 = q n_S^+(0) \upsilon_T$$



current

$$I_D = W\left(qn_S^+(0)\upsilon_T - qn_S^-(0)\upsilon_T\right) = Wqn_S^+\left(0\right)\upsilon_T\left[1 - n_S^-(0)/n_S^+(0)\right] \qquad (1)$$

$$n_S\left(0\right) = n_S^+\left(0\right) + n_S^-\left(0\right) = n_S^+\left(0\right)\left[1 + n_S^-\left(0\right)/n_S^+\left(0\right)\right] \qquad (2)$$



current

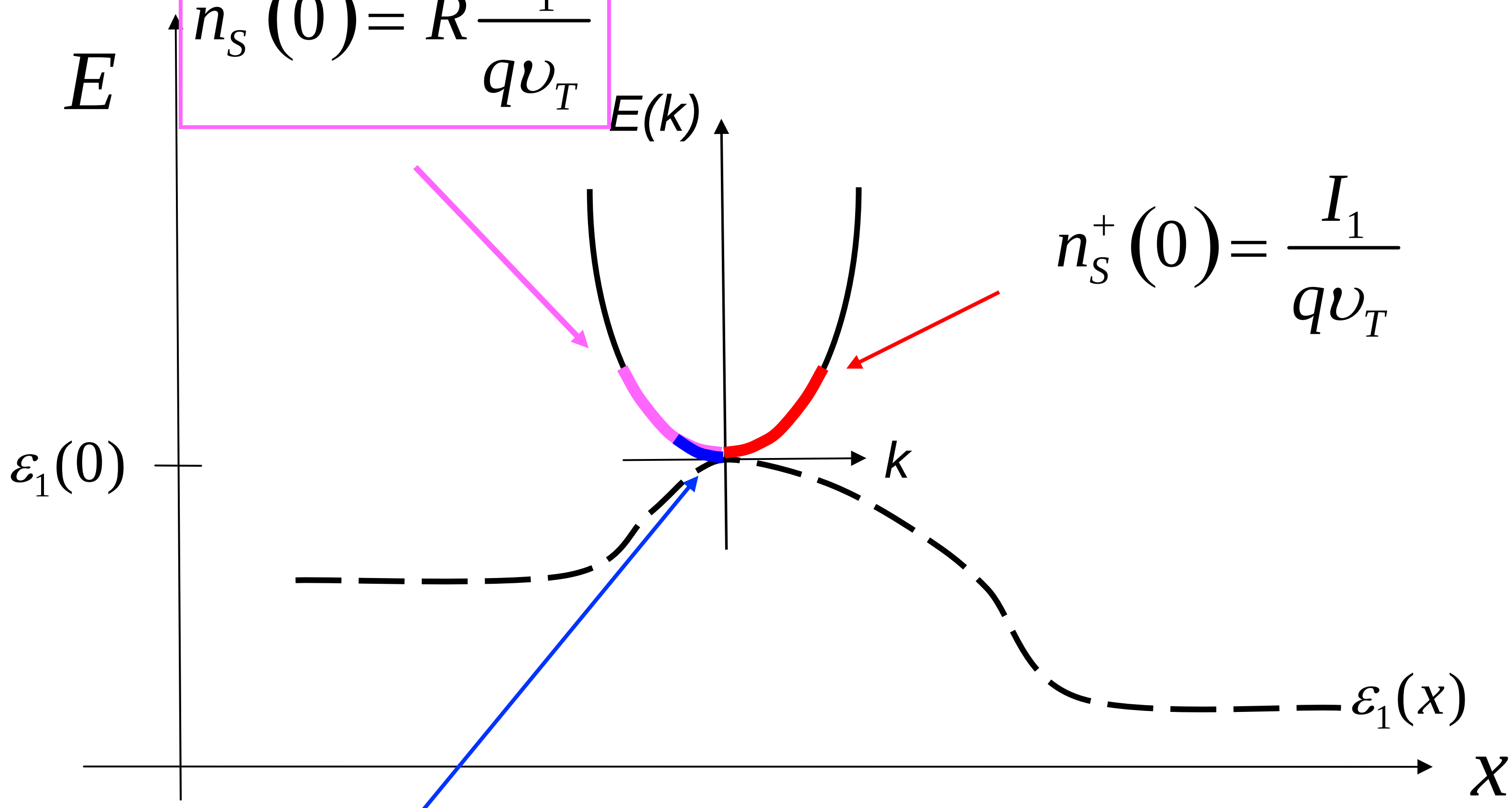
$$I_D = Wqn_S(0)\upsilon_T\left(\frac{1 - n_S^-(0)/n_S^+(0)}{1 + n_S^-(0)/n_S^+(0)}\right)$$

$$\boxed{I_D = WQ_I(0)\upsilon_T\left(\frac{1 - n_S^-(0)/n_S^+(0)}{1 + n_S^-(0)/n_S^+(0)}\right)}$$

Exactly the same result we had for the ballistic case, but the (- velocity) carrier density at the top of the barrier is altered by scattering.

carrier densities at the top of the barrier

$$n_S^-(0) = R\frac{I_1}{q\upsilon_T}$$



$$n_S^+(0) = \frac{I_1}{q\upsilon_T}$$

$$n_S^-(0) = Tn_S^+(0)e^{-qV_{DS}/k_BT}$$

from carrier densities to drain current

$$n_S^-(0) = Rn_S^+(0) + Tn_S^+(0)e^{-qV_{DS}/k_BT} = n_S^+(0)\left[R + (1-R)e^{-qV_{DS}/k_BT}\right]$$

$$\frac{n_S^-(0)}{n_S^+(0)} = R + (1-R)e^{-qV_{DS}/k_BT}$$

$$I_D = WQ_I(0)\upsilon_T\left(\frac{1 - n_S^-(0)/n_S^+(0)}{1 + n_S^-(0)/n_S^+(0)}\right)$$

$$I_D = WQ_I(0)\upsilon_T\left(\frac{(1-R) - (1-R)e^{-qV_{DS}/k_BT}}{(1+R) + (1-R)e^{-qV_{DS}/k_BT}}\right)$$

the MOSFET I-V with scattering

$$I_{DS} = WC_{ox}\left(V_{GS} - V_T\right)\upsilon_T \left(\frac{1 - e^{qV_{DS}/k_BT}}{1 + e^{qV_{DS}/k_BT}}\right)$$ (ballistic, Boltzmann statistics)

$$\boxed{I_D = WC_{ox}\left(V_{GS} - V_T\right)\upsilon_T \left(\frac{(1-R) - (1-R)e^{-qV_{DS}/k_BT}}{(1+R) + (1-R)e^{-qV_{DS}/k_BT}}\right)}$$

$$T = (1 - R)$$

$$I_D = WC_{ox}\left(V_{GS} - V_T\right)\upsilon_T T \left(\frac{1 - e^{-qV_{DS}/k_BT}}{(2-T) + Te^{-qV_{DS}/k_BT}}\right)$$

$$I_D(\text{scattering}) \neq TI_D(\text{ballistic})$$

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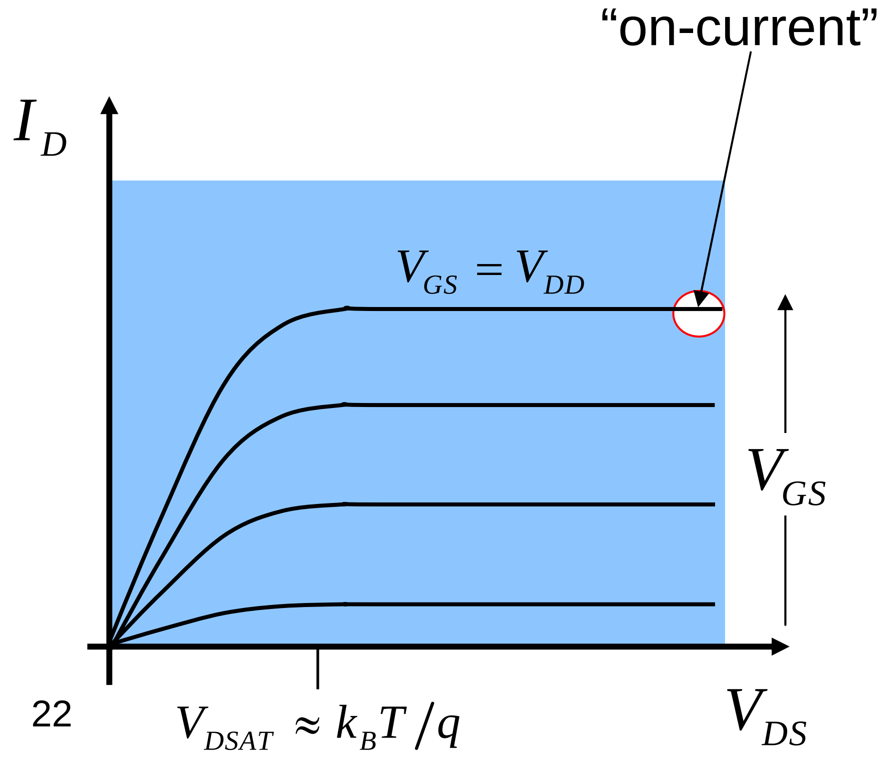
high drain bias

$$I_D = WC_{ox}\left(V_{GS} - V_T\right)\upsilon_T \left(\frac{\left(1-R\right) - \left(1-R\right)e^{-qV_{DS}/k_BT}}{\left(1+R\right) + \left(1-R\right)e^{-qV_{DS}/k_BT}} \right)$$

$$I_D = WC_{ox}\left(V_{GS} - V_T\right)\upsilon_T \frac{\left(1-R\right)}{\left(1+R\right)}$$

$$\left\langle \upsilon(0) \right\rangle = \upsilon_T \frac{\left(1-R\right)}{\left(1+R\right)} \leq \upsilon_T$$



questions

1) We expected current to be proportional to transmission ($T = 1 - R$), but where does the $(1 + R)$ in the denominator come from?

2) How do we generalize the result for Fermi-Dirac statistics?

approximate answer:

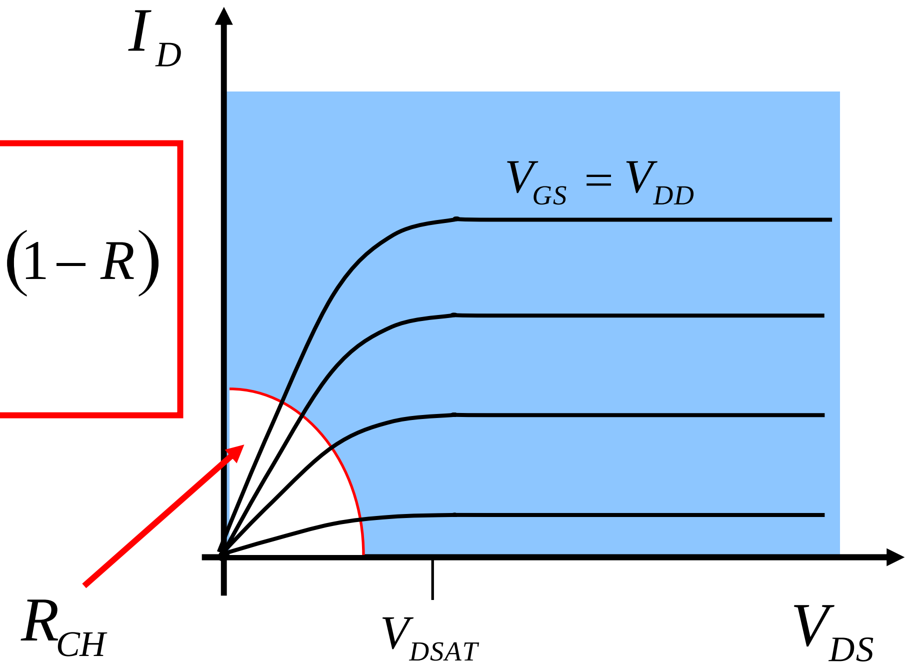
$$I_D = W C_{ox} \left(V_{GS} - V_T \right) \upsilon_T \frac{(1 - R)}{(1 + R)}$$

low drain bias

$$I_D = WC_{ox}\left(V_{GS} - V_T\right)\upsilon_T\left(\frac{\left(1-R\right)-\left(1-R\right)e^{-qV_{DS}/k_BT}}{\left(1+R\right)+\left(1-R\right)e^{-qV_{DS}/k_BT}}\right)$$

$$\boxed{G_{CH} = \frac{I_D}{V_{DS}} = WC_{ox}\left(V_{GS} - V_T\right)\left[\frac{\upsilon_T}{2\left(k_BT/q\right)}\right]\left(1-R\right)}$$

$$G_{CH}\left(\text{scattering}\right) = TG_{CH}\left(\text{ballistic}\right)$$



questions

1) For low V_{DS}, the drain current is proportional to transmission ($T = 1 - R$). Why in this case but not for high V_{DS}?

2) How do we generalize the result for Fermi-Dirac statistics?

approximate answer:

$$G_{CH} = \left(W C_{ox} \left(V_{GS} - V_T \right) \frac{\upsilon_T}{\left(2 k_B T / q \right)} \right) \left[\frac{\mathcal{F}_{-1/2}\left(\eta_{F1}\right)}{\mathcal{F}_0\left(\eta_{F1}\right)} \right] \left(1 - R\right)$$

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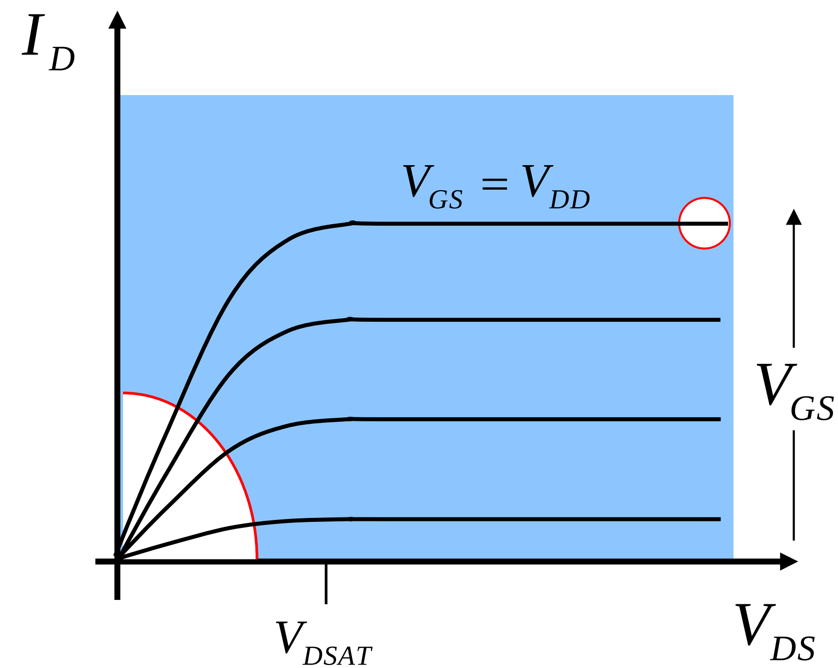
summary of the scattering model

$$
\begin{cases}
I_D \approx W C_{ox} \left(V_{GS} - V_T\right) \upsilon_T \left(\dfrac{(1-R) - (1-R)\mathcal{F}_{1/2}(\eta_{F2}) / \mathcal{F}_{1/2}(\eta_{F1})}{(1+R) + (1-R)\mathcal{F}_{0}(\eta_{F2}) / \mathcal{F}_{0}(\eta_{F1})} \right) \\[2ex]
G_{CH} \approx \left(W C_{ox} \left(V_{GS} - V_T\right) \dfrac{\upsilon_T}{(2k_B T/q)} \right) \left[\dfrac{\mathcal{F}_{-1/2}(\eta_{F1})}{\mathcal{F}_{0}(\eta_{F1})} \right] (1-R) \\[2ex]
I_{ON} \approx W C_{ox} \left(V_{GS} - V_T\right) \upsilon_T \dfrac{(1-R)}{(1+R)}
\end{cases}
$$



To proceed, we need to understand $R\left(V_{GS}, V_{DS}\right)$

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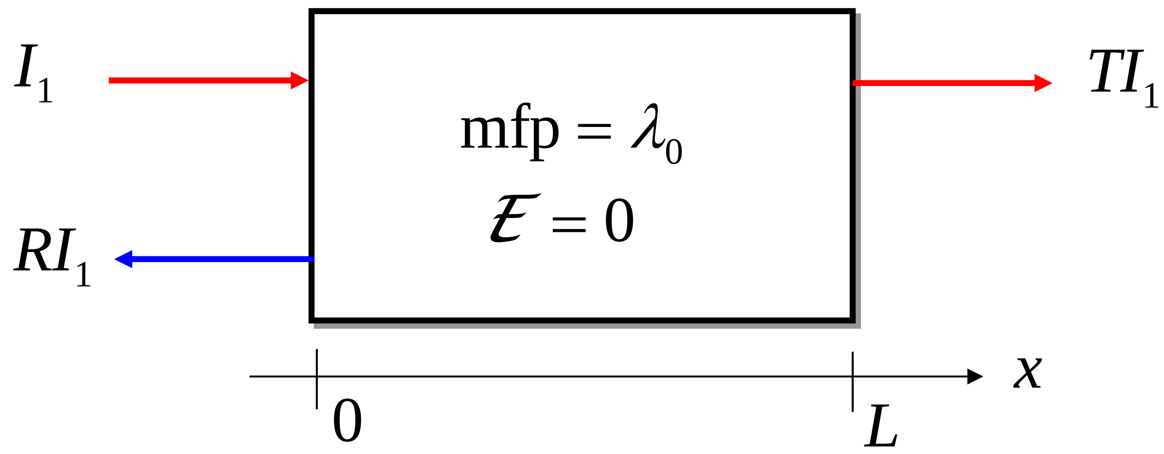
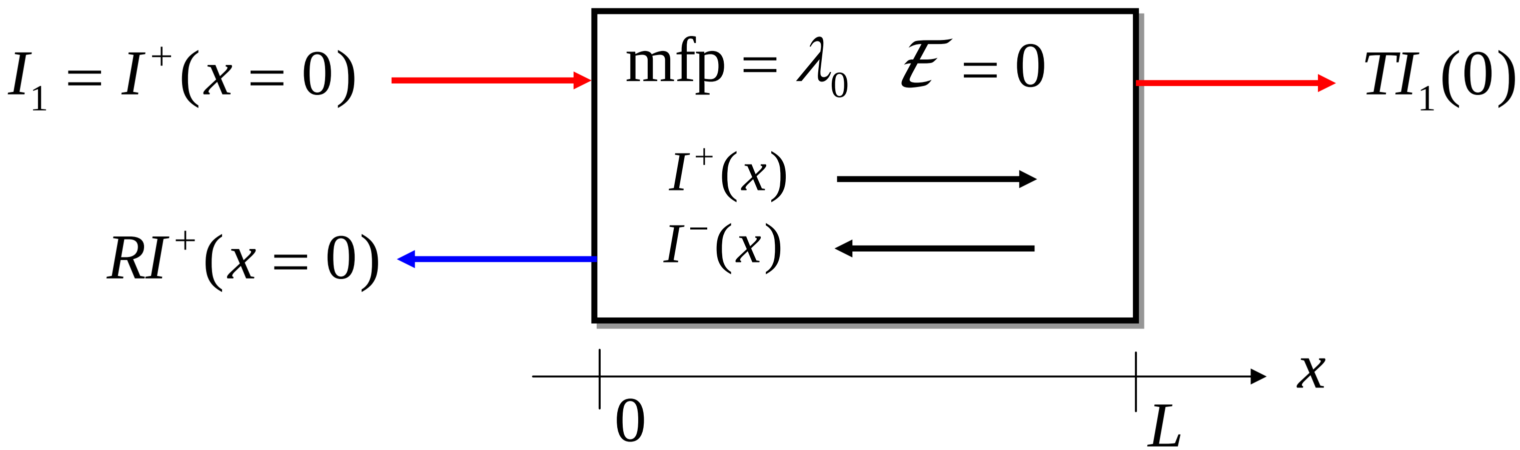
transmission across a field-free slab



Consider a flux of carriers injected into a field-free slab of length, *L*. The flux that emerges at $x = L$ is T times the incident flux, where $0 < T < 1$. The flux that emerges from $x = 0$ is R times the incident flux, where $T + R = 1$, assuming no carrier recombination-generation.

How is T related to the mean-free-path for backscattering within the slab?

transmission



$$\frac{dI^+(x)}{dx} = -\frac{I^+(x)}{\lambda_0} + \frac{I^-(x)}{\lambda_0}$$

$$I^-(x) = I^+(x) - I$$

$$I = I^+(x) - I^-(x) \quad \text{(constant)}$$

$$\boxed{\frac{dI^+(x)}{dx} = -\frac{I}{\lambda_0}}$$

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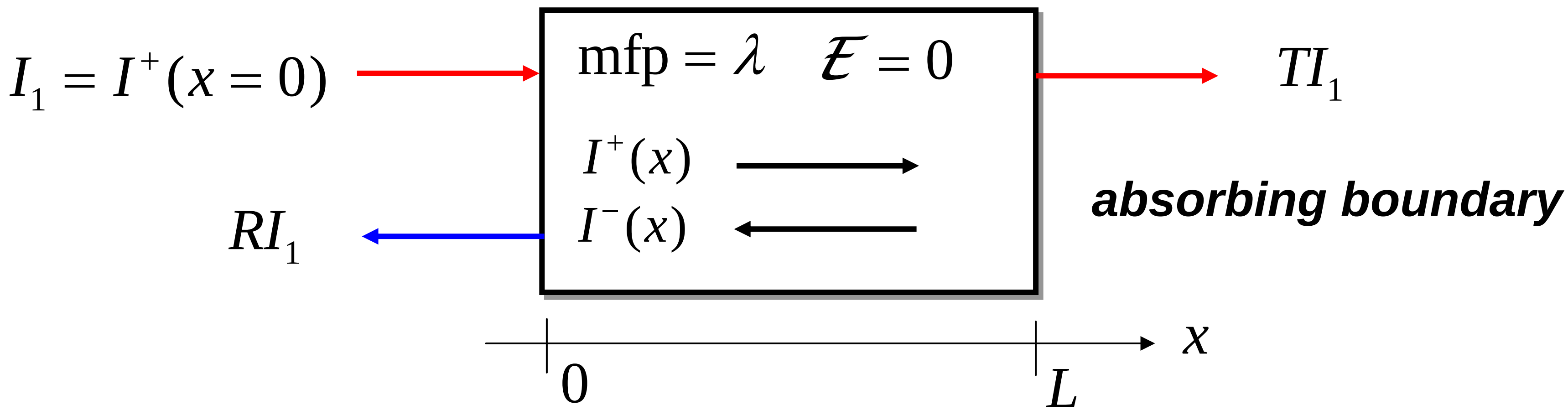
transmission (ii)



$$\boxed{\frac{dI^+(x)}{dx} = -\frac{I}{\lambda_0}}$$

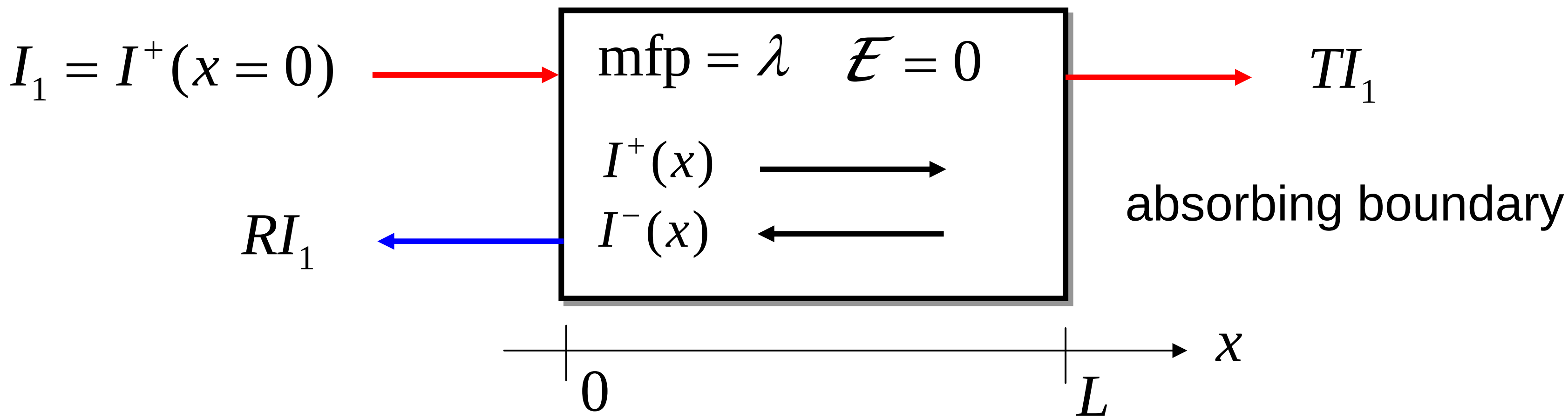
$$I^+(x) = I^+(0) - \left(I^+(x) - I^-(x)\right)\frac{x}{\lambda_0}$$

$$I^+(x) = I^+(0) - I\,\frac{x}{\lambda_0}$$

$$I^+(L) = I^+(0) - \left(I^+(L) - I^-(L)\right)\frac{L}{\lambda_0}$$

$$I^-(L) = 0$$

transmission (iii)



$$I^+(L) = I^+(0) - I^+(L)\frac{L}{\lambda_0}$$

$$I^+(L) = \frac{I^+(0)}{1 + L/\lambda_0}$$

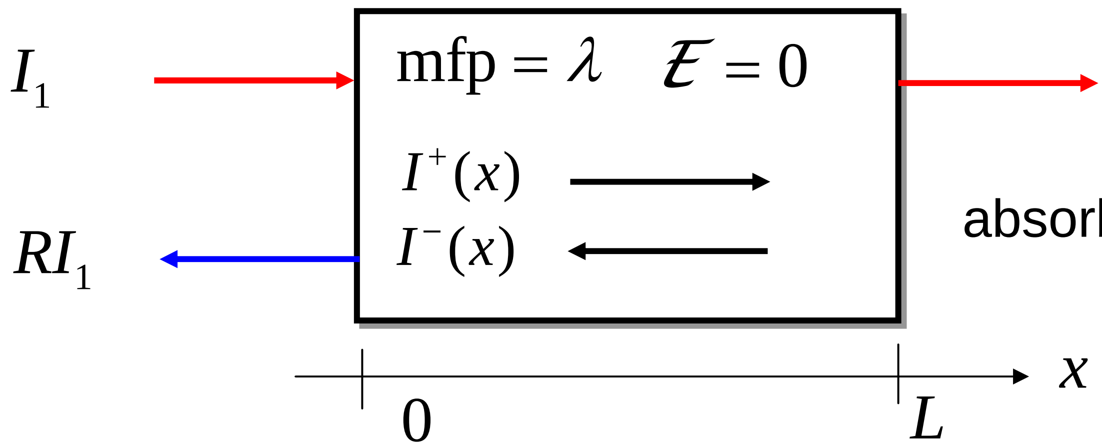
$$\frac{I^+(L)}{I^+(0)} = T = \frac{\lambda_0}{\lambda_0 + L}$$

$$T = \frac{\lambda_0}{\lambda_0 + L} \qquad R = \frac{L}{\lambda_0 + L}$$

$$T \to 0 \quad L >> \lambda_0$$

$$T \to 1 \quad L << \lambda_0$$

mean-free-path



$$\boxed{T = \frac{\lambda_0}{\lambda_0 + L}}$$

absorbing boundary

How do we relate λ_0 to known parameters?

If I_1 is a thermal equilibrium injected flux, $I_1 = n^+(0)\upsilon_T$
then, it can be shown that:

$$\boxed{D_n = \frac{k_B T}{q}\mu_n = \frac{\upsilon_T}{2}\lambda_0}$$

(non-degenerate carrier statistics)

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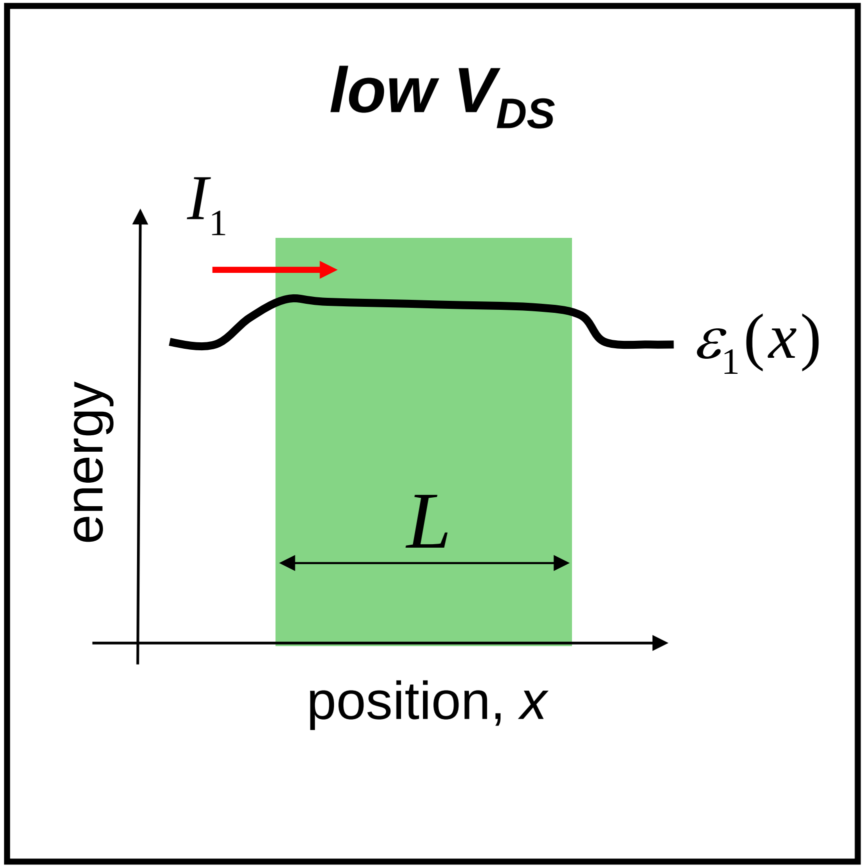
example



$$\mu_n \approx 200 \text{ cm}^2/\text{V-s}$$

$$\mu_n = \frac{\upsilon_T}{2(k_B T/q)}\lambda_0$$

$$\lambda_0 \approx 9 \text{ nm}$$

$$L \approx 50 \text{ nm}$$

$$T \approx \frac{\lambda_o}{L+\lambda_o} \approx 0.15$$

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relation to conventional theory

$$G_{CH} = \left(WC_{ox}\left(V_{GS} - V_T\right)\frac{\upsilon_T}{\left(2k_BT/q\right)} \right)\left(1 - R\right)$$

$$1 - R = T = \frac{\lambda_0}{\lambda_0 + L} \approx \frac{\lambda_0}{L}$$ (diffusive limit)

$$\lambda_0 = \frac{2k_BT/q}{\upsilon_T}\mu_n$$

$$\boxed{G_{CH} = \frac{W}{L}\mu_n C_{ox}\left(V_{GS} - V_T\right)}$$

(non-degenerate carrier statistics)

The scattering model works in the diffusive limit, as well as the ballistic limit, and in the quasi-ballistic regime in between.

channel conductance

$$G_{CH} = \left(WC_{ox}\left(V_{GS} - V_T\right)\frac{\upsilon_T}{\left(2k_B T/q\right)}\right)\left[\frac{\mathcal{F}_{-1/2}\left(\eta_{F1}\right)}{\mathcal{F}_0\left(\eta_{F1}\right)}\right]T$$

$$T = \frac{\lambda_0}{\lambda_0 + L}$$

one can show that:

$$\boxed{G_{CH} = \frac{W}{L}\left(\frac{1}{\mu_n} + \frac{1}{\mu_B}\right)^{-1} C_{ox}\left(V_{GS} - V_T\right)}$$

$$\begin{cases} \mu_n = \dfrac{\upsilon_T \lambda_0}{2k_B T/q}\left[\dfrac{\mathcal{F}_{-1/2}\left(\eta_{F1}\right)}{\mathcal{F}_0\left(\eta_{F1}\right)}\right] \\[2ex] \mu_B = \dfrac{\upsilon_T L}{2k_B T/q}\left[\dfrac{\mathcal{F}_{-1/2}\left(\eta_{F1}\right)}{\mathcal{F}_0\left(\eta_{F1}\right)}\right] \end{cases}$$

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transmission under high drain bias

***scattering model*:**

$$I_{ON} = WC_{ox}\left(V_{GS} - V_T\right)\upsilon_T \frac{\left(1 - R\right)}{\left(1 + R\right)} = WC_{ox}\left(V_{GS} - V_T\right)\upsilon_T \frac{T}{\left(2 - T\right)}$$

in practice:

$$B \equiv \frac{I_{ON}\left(\text{measured}\right)}{I_{ON}\left(\text{ballistic}\right)} \approx 0.50$$

$$B = \frac{T}{\left(2 - T\right)} \quad \rightarrow \quad T \approx 0.67 >> 0.15$$

Why?

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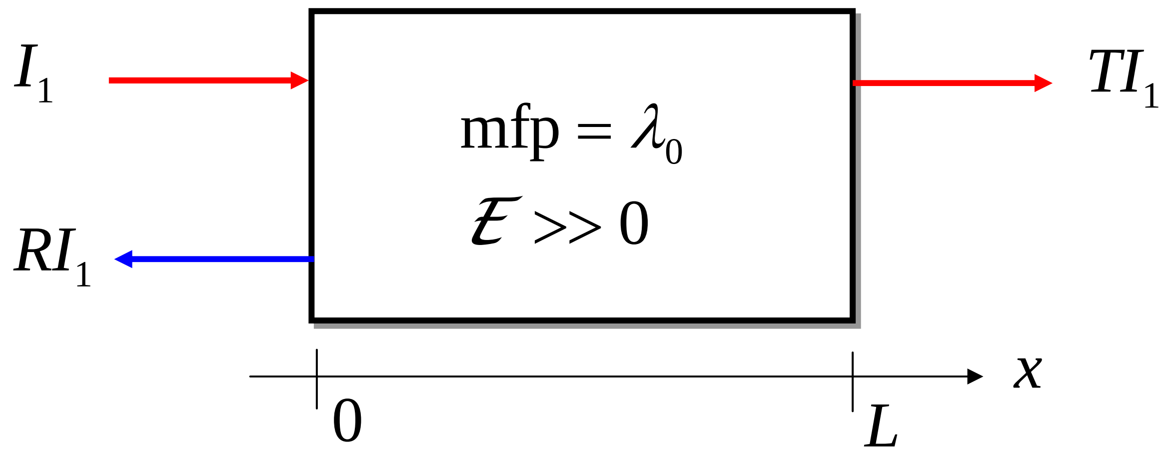
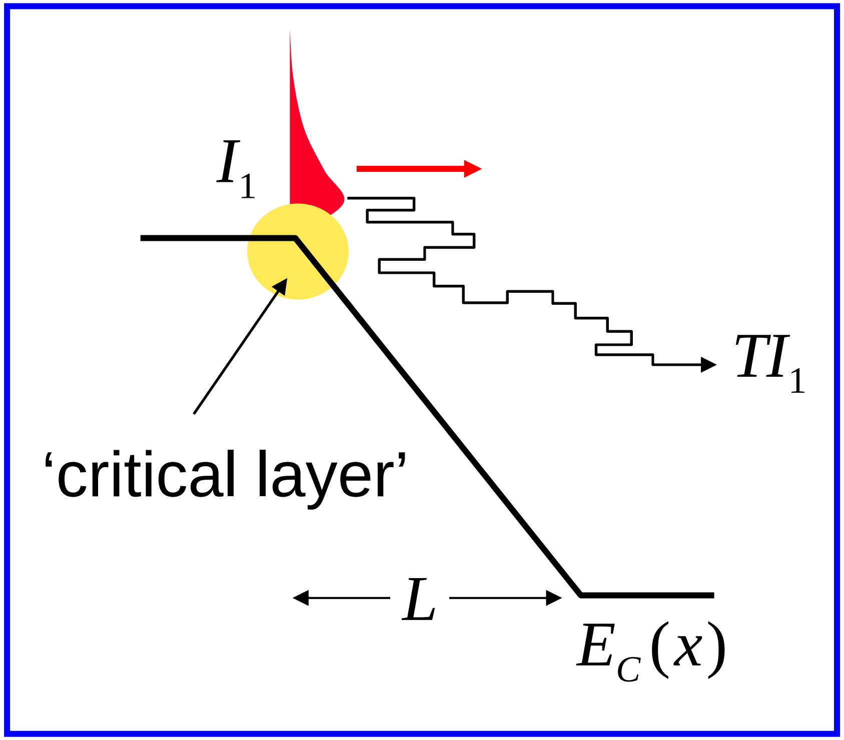
transmission across a slab with an electric field



When the electric field is strong and position-dependent and several scattering mechanisms operate, this turns out to be a difficult problem.

How can we understand the essential physics?

transport “downhill”

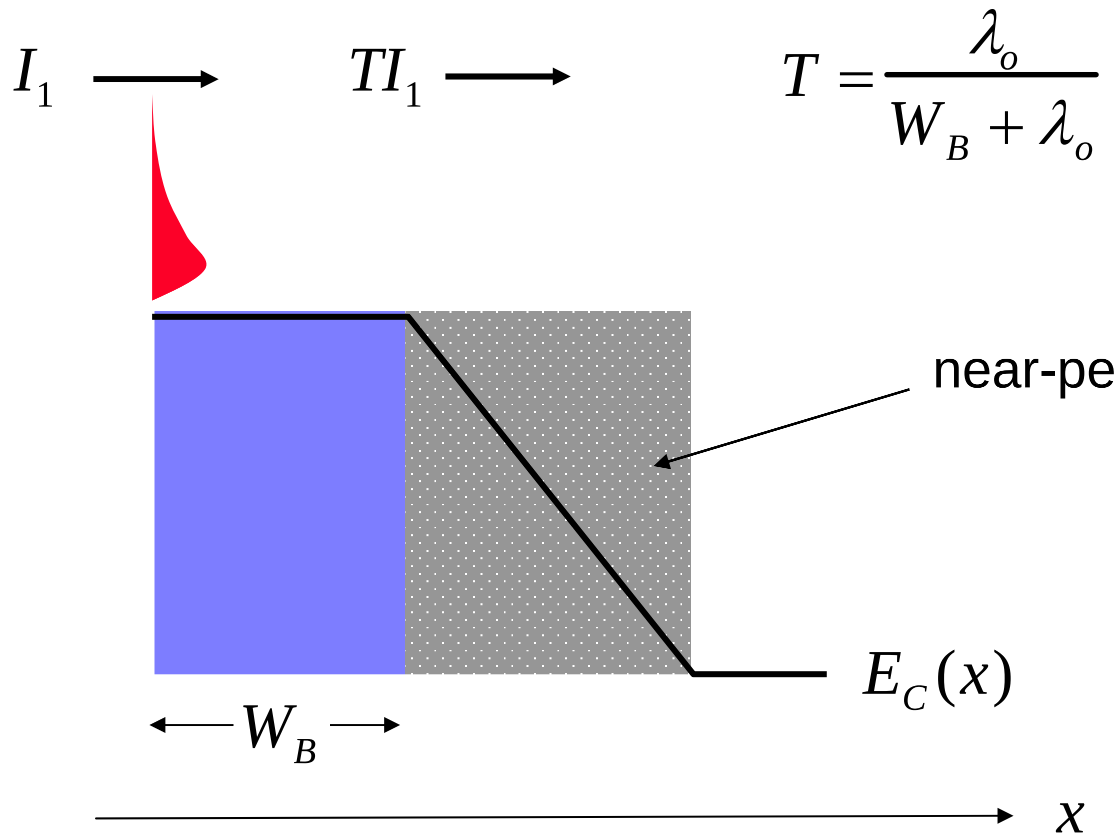


$$T = \frac{\lambda_o}{l + \lambda_o} \qquad \ell << L$$

$T \approx 1$:
High field regions are good carrier collectors.

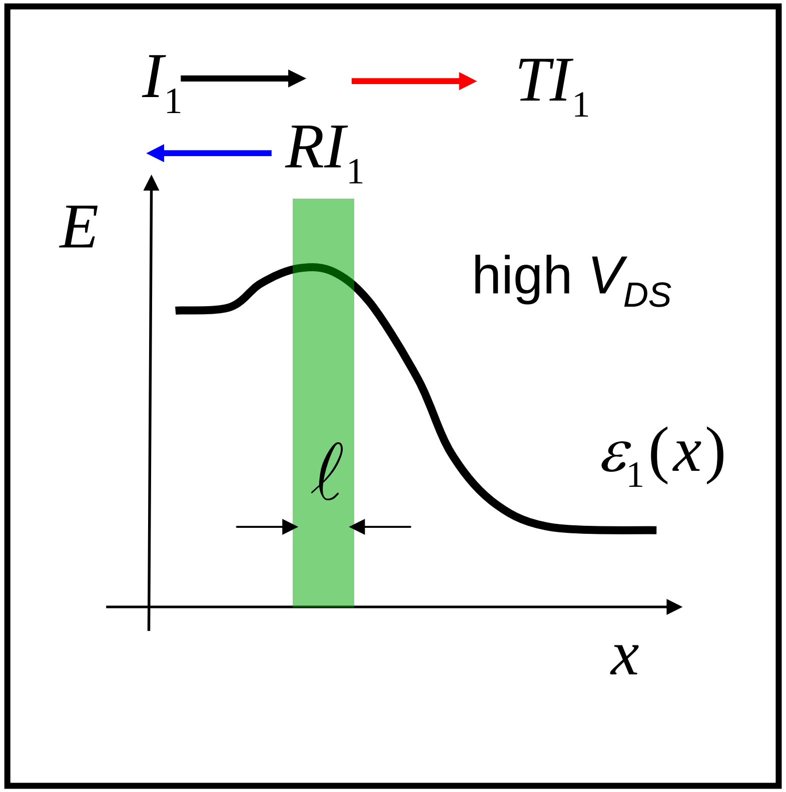
Peter J, Price, “Monte Carlo calculation of electron transport in solids,” *Semiconductors and Semimetals*, **14**, pp. 249-334, 1979

field-free region followed by high-field region



The base-collector of a bipolar transistor is a low-field region followed by a high-field region. **Transmission is controlled by the low-field region.**

transport in a MOS transistor



$$T \approx \frac{\lambda_o}{\ell + \lambda_o}$$

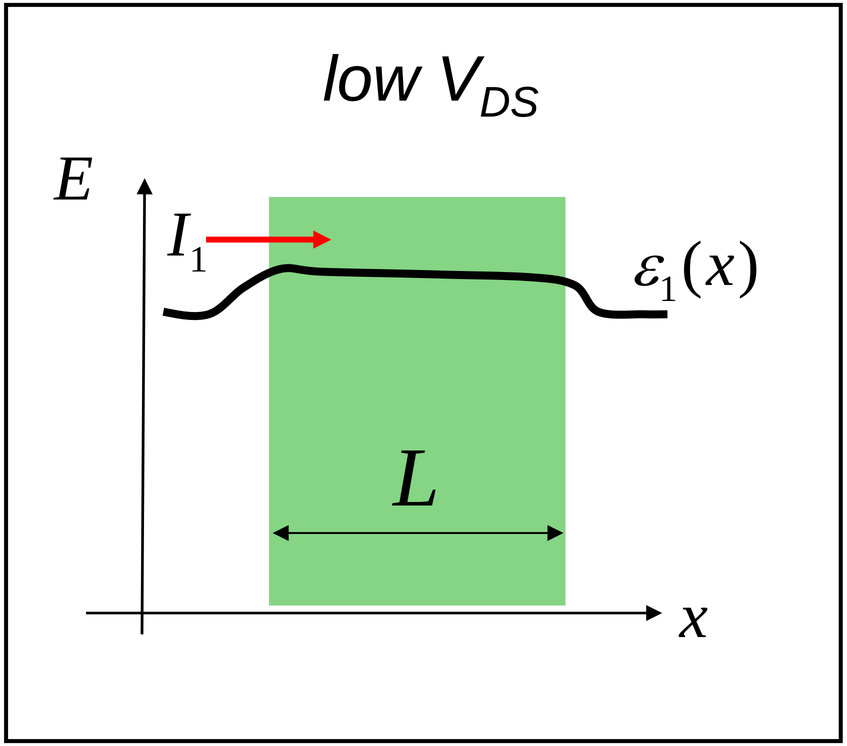
1) A MOSFET consists of a low-field region near the source that is strongly controlled by the gate voltage, and a high-field region near the drain that is strongly controlled by the drain voltage.

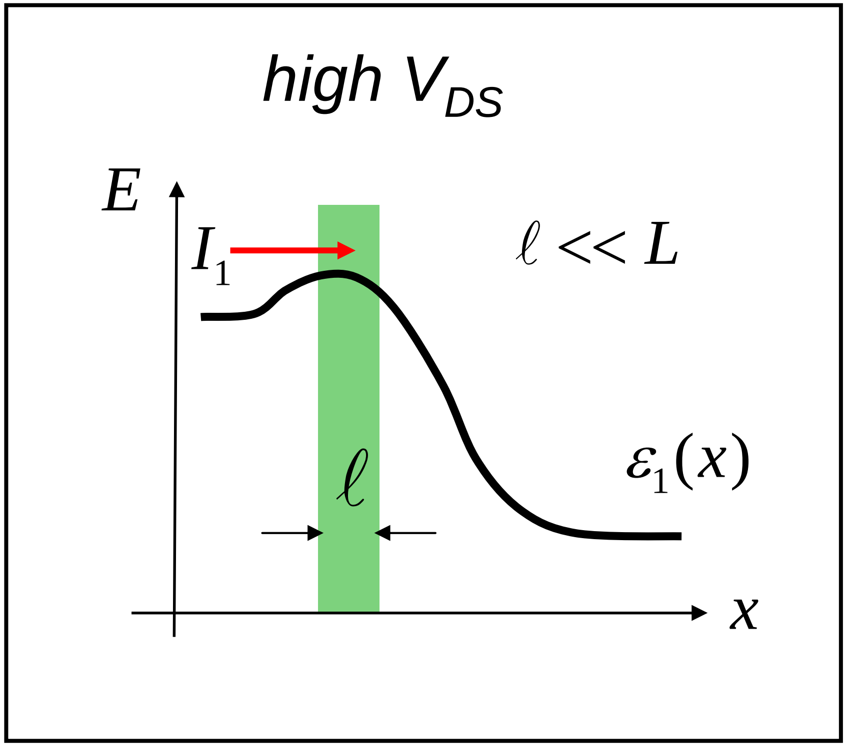
2) Transmission is controlled by the low-field region near the source.

3) Scattering near the drain has a smaller effect on backscattering to the source.

4) In contrast to a bipolar transistor, the division between the low and high field regions is not sharp.

bias-dependent transmission





$$T \approx \frac{\lambda_o}{L + \lambda_o} \approx 0.15$$

$$T \approx \frac{\lambda_o}{\ell + \lambda_o} \approx 0.67$$

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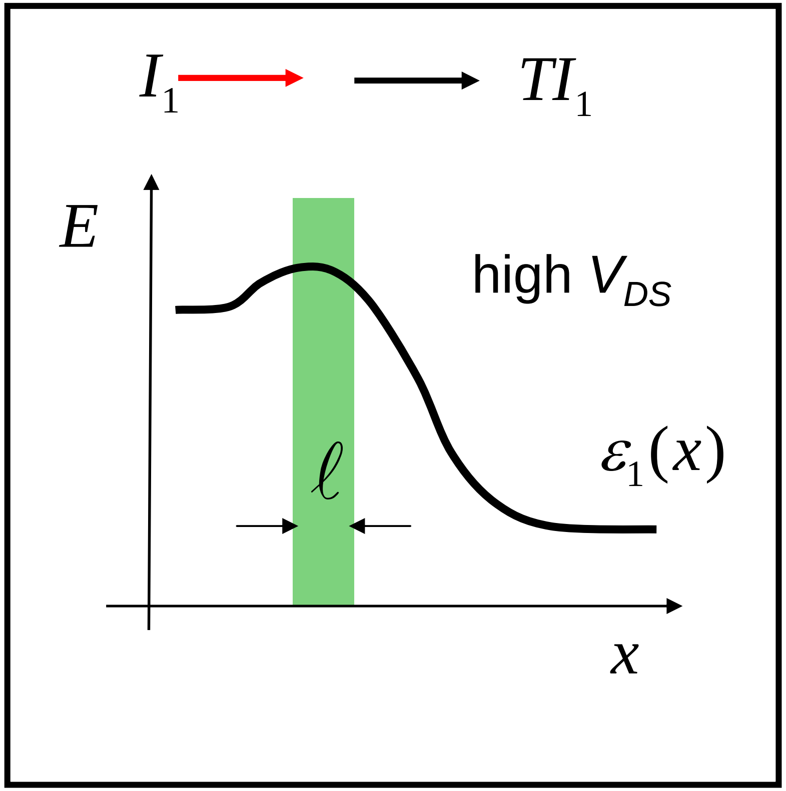
computing the critical length



$$T \approx \frac{\lambda_o}{l + \lambda_o}$$

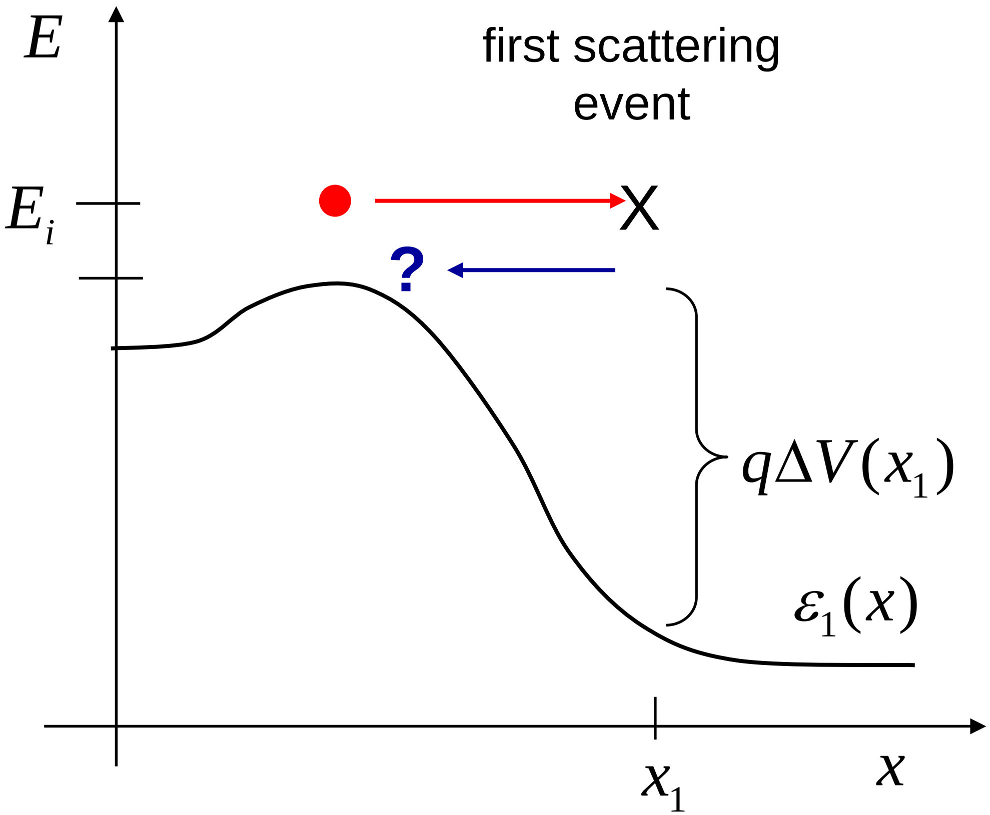
Assuming near-equilibrium transport in the low-field region (i.e. DD), one can show that the critical length is the distance over which the channel potential drops by $k_B T/q$

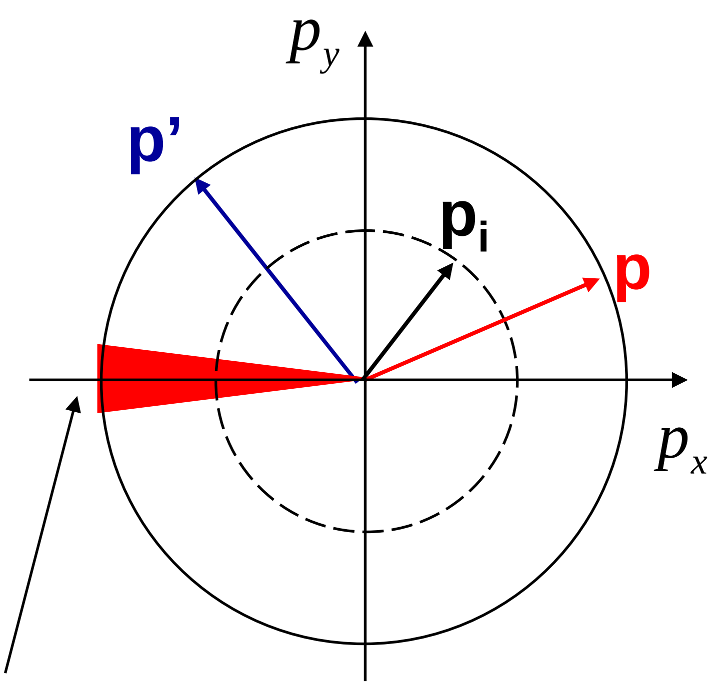
"*kT* layer" $\qquad \ell \approx L_{k_b T}$

Two key assumptions:

1) near-equilibrium
2) Boltzmann statistics

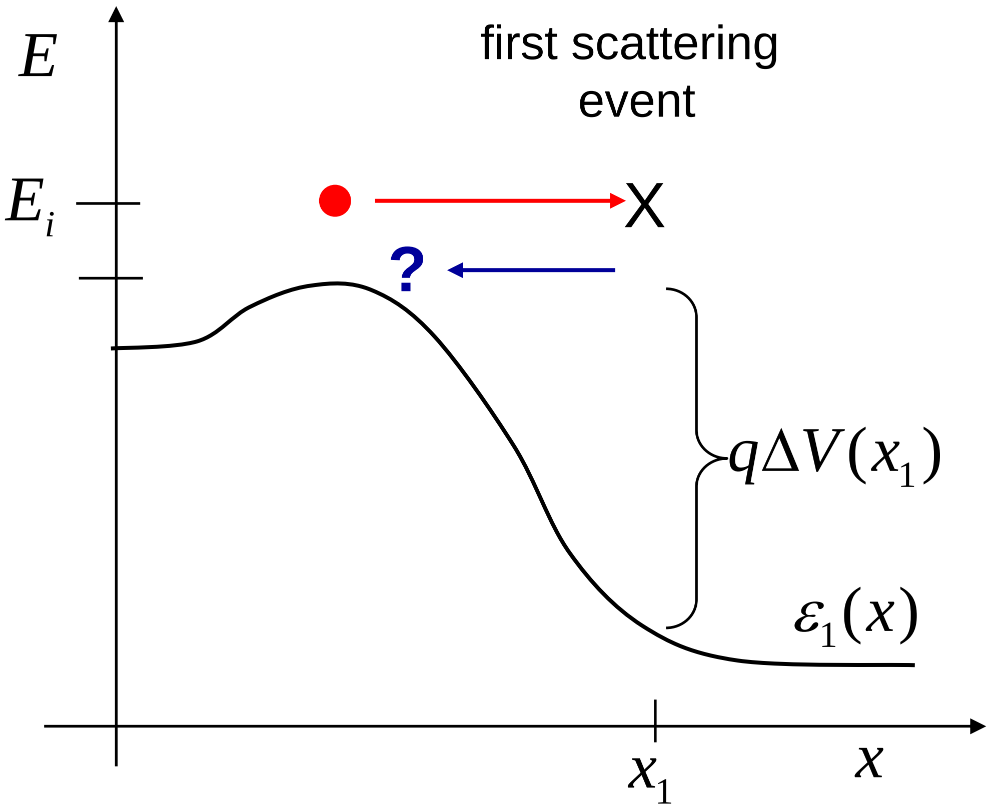
physics of elastic back-scattering





$$\left\{ \begin{array}{c} \text{longitudinal energy} \\ 1/2\, m\upsilon_x^2 > q\Delta V(x_1) \end{array} \right\}$$

physics of elastic back-scattering



$x_1 < \text{l}$

probability of returning to the source is high

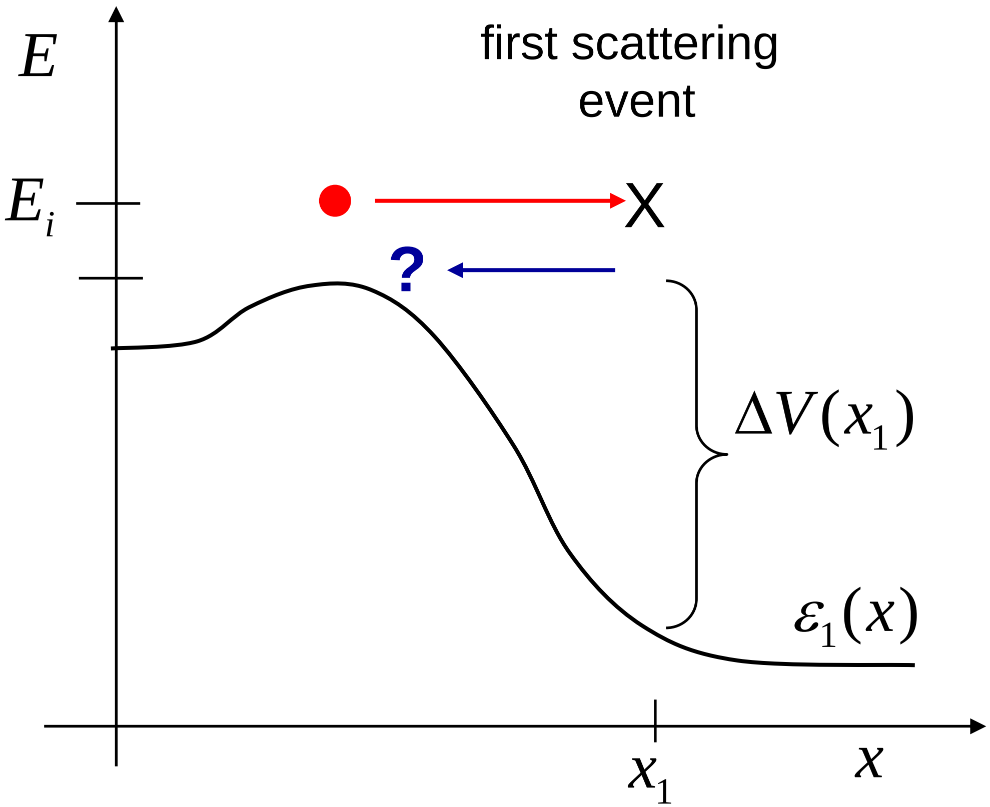
$x_1 > \text{l}$

probability of returning to the source is low

$$q\Delta V(\text{l}) \approx E_i - \varepsilon_1(0)$$

See: Lundstrom and Ren, *IEEE Trans. Electron Dev*, **49**, pp. 133-141, 2002.

physics of elastic back-scattering



For non-degenerate carriers,

$$\langle E_i - \varepsilon_1(0) \rangle = k_B T$$

$$q \Delta V(l) \approx k_B T$$

Above threshold, however,

$$\langle E_i - \varepsilon_1(0) \rangle > k_B T$$

See: Lundstrom and Ren, *IEEE Trans. Electron Dev*, **49**, pp. 133-141, 2002.

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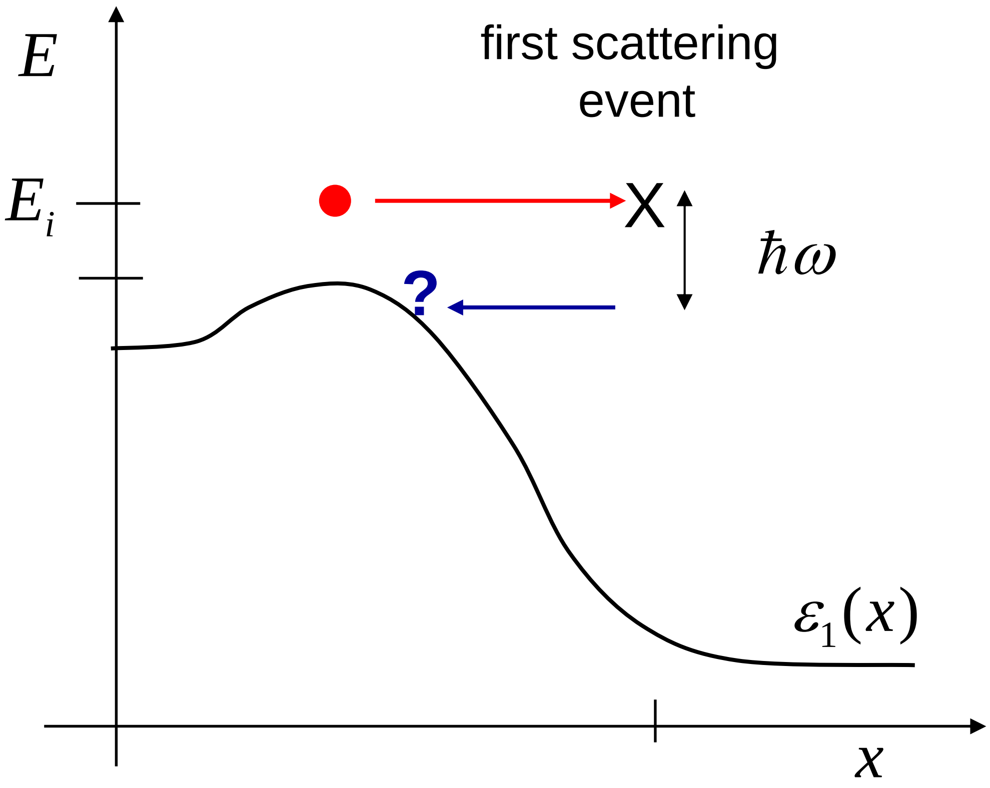
role of inelastic scattering



If an electron backscatters by emitting a phonon, it is less likely to return to the source - even if it is pointed in the right direction.

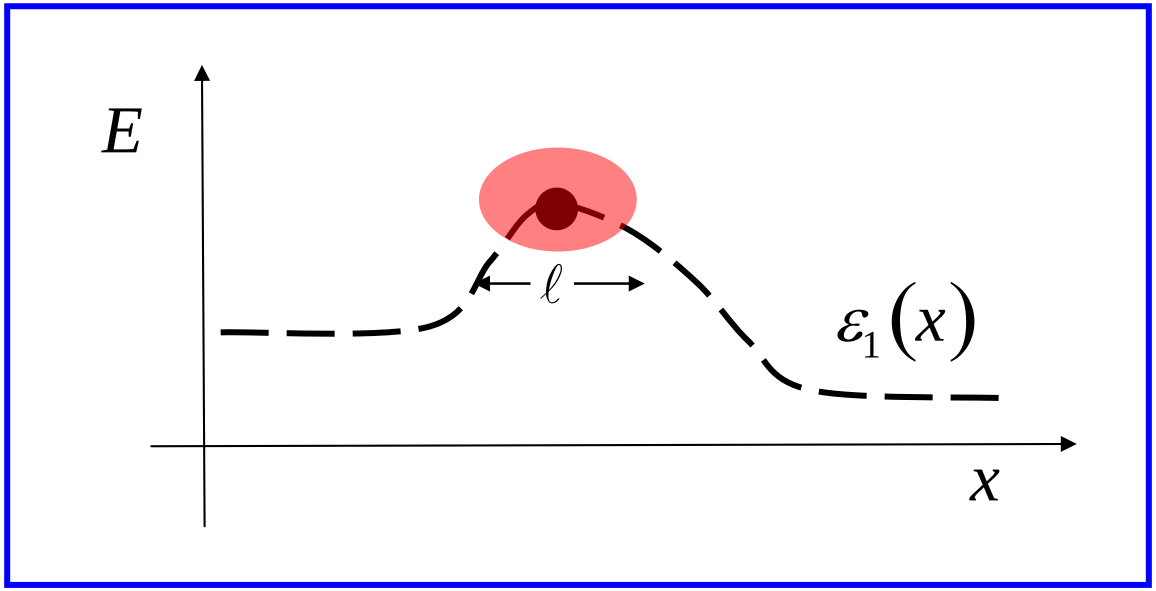
$$L_{k_B T} \rightarrow \text{"h}\omega\text{ length"}$$

K. Natori, *IEEE Electron Dev. Lett*., **23**, pp. 655-657, 2002.

drift-diffusion picture

$$I_D = WC_{ox}\langle \upsilon(0)\rangle \left(V_{GS} - V_T\right)$$

$$\frac{1}{\langle \upsilon(0)\rangle} = \frac{1}{\upsilon_T} + \frac{1}{D_n/\mathrm{l}} \qquad D_n = \left(k_B T/q\right)\mu_n$$



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drift-diffusion vs. scattering model

$$I_D = WC_{ox}\left[\frac{1}{\upsilon_T} + \frac{1}{\left(D_n/\ell\right)}\right]^{-1}\left(V_{GS} - V_T\right)$$

drift-diffusion

$$\boxed{D_n = \upsilon_T \lambda_0/2 \qquad\qquad T = \frac{\lambda_0}{\lambda_0 + \ell}}$$

$$I_D = WC_{ox}\frac{T}{2-T}\upsilon_T\left(V_{GS} - V_T\right)$$

scattering

Monte Carlo simulations

Detailed, numerical simulations confirm the basic physical picture that we have presented, but they show that the critical layer is somewhat longer than L_{kT} and that it depends on the shape of the potential profile. Under some conditions, inelastic scattering can even increase the d.c. current.

See, for example:

P. Palestri, R. Clerc, D. Esseni, L. Lucci, and L. Selmi, “Multi-subband Monte Carlo investigation of the mean free path and of the kT layer in degenerated quasi-ballistic nanoMOSFETs, IEDM Tech. Dig.,pp. 945-948, Dec. 2006.

Raseong Kim and Mark Lundstrom, “Physics of carrier backscattering in one- and two-dimensional nanotransistors,” submitted for publication, June, 2008.

outline

1) Review and introduction
2) Scattering theory of the MOSFET
3) Transmission under low V_{DS}
4) Transmission under high V_{DS}
5) Discussion
6) **Summary**

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summary

1) Modern MOSFETs operate between the ballistic and diffusive limits, so we need to understand transport in the quasi-ballistic regime.

2) Transmission (or scattering) theory provides a simple, physical description of quasi-ballistic transport.

3) The same physics can also be understood at the drift-diffusion level.

4) Quantitative treatments require detailed numerical simulation.