

## Exercises on Carrier Scattering in Nanoscale MOSFETs

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### Objective:

The objective of these exercises is to help you gain familiarity with the scattering theory of nanoscale MOSFETs as described in Lecture 5.

- 1) Consider a modern, Si MOSFET, with  $L \approx 100\text{nm}$ ,  $I_{ON} \approx 1 \text{ mA}/\mu\text{m}$ ,  $\mu_n \approx 200 \text{ cm}^2/\text{V-s}$ , operating under on-current conditions with  $n_S \approx 10^{13} \text{ cm}^{-2}$ . Estimate the critical length for backscattering,  $\ell$ .
- 2) Consider a modern, Si MOSFET, with  $L \approx 100\text{nm}$ ,  $\mu_n \approx 200 \text{ cm}^2/\text{V-s}$ , operating in the linear region with  $n_S \approx 10^{13} \text{ cm}^{-2}$ .
  - a) Determine whether the device is operating in the ballistic or diffusive regime.
  - b) Repeat part a) but now assume that the channel material is InGaAs with  $\mu_n \approx 10,000 \text{ cm}^2/\text{V-s}$
- 3) In Lecture 5, we developed an expression for the drain current for a planar MOSFET in the presence of carrier backscattering as described by the transmission,  $T$ , or reflection,  $R = 1 - T$ . We assumed Boltzmann statistics for carriers and obtained:

$$I_D = WC_{ox}(V_{GS} - V_T)v_T \left( \frac{(1 - R) - (1 - R)e^{-qV_{DS}/k_B T}}{(1 + R) + (1 - R)e^{-qV_{DS}/k_B T}} \right)$$

Develop the analogous expressions for without assuming Boltzmann statistics for carriers. Carefully state the assumptions you make.

- 4) In Lecture 5, we showed that the high drain bias current is:

$$I_D = WC_{ox}(V_{GS} - V_T)v_T \frac{(1-R)}{(1+R)}$$

when Boltzmann statistics are assumed. Derive an analogous expression without assuming Boltzmann statistics. Carefully state the assumptions you make.

HINT: You can either begin with the result of problem 3) for the entire I-V characteristic or you can assume high  $V_{DS}$  from the outset and derive the result.

- 5) In Lecture 5, we assumed Boltzmann statistics and showed that the channel conductance in the presence of scattering is given by

$$G_{CH} = \frac{I_D}{V_{DS}} = WC_{ox}(V_{GS} - V_T) \left[ \frac{v_T}{2(k_B T/q)} \right] (1-R)$$

Develop an analogous expression without assuming Boltzmann carrier statistics.

HINT: You can either begin with the result of problem 3) for the entire I-V characteristic or you can assume low  $V_{DS}$  from the outset and derive the result.

- 6) The mobility can be related to the mean-free-path for back-scattering, For Boltzmann carrier statistics, the relation is

$$D_n = \frac{k_B T}{q} \mu_n = \frac{v_T}{2} \lambda_0.$$

Derive an analogous expression without assuming Boltzmann statistics.

HINT: begin with the channel conductance, which does not include a mobility, and equate it to the conventional expression for channel conductance, which does contain a mobility.

- 7) Develop a ‘Mathiessen’s Rule’ for nanotransistors as follows. Begin with the channel conductance,

$$G_{CH} = \left( WC_{ox}(V_{GS} - V_T) \frac{v_T}{(2k_B T/q)} \right) \left[ \frac{\mathcal{F}_{-1/2}(\eta_{F1})}{\mathcal{F}_0(\eta_{F1})} \right] T$$

where

$$T = \frac{\lambda_0}{\lambda_0 + L}.$$

Show that the result can be expressed as

$$G_{CH} = \frac{W}{L} \left( \frac{1}{\mu_n} + \frac{1}{\mu_B} \right)^{-1} C_{ox} (V_{GS} - V_T)$$

where

$$\mu_n = \frac{v_T \lambda_0}{2 k_B T / q} \left[ \frac{\mathcal{F}_{-1/2}(\eta_{F1})}{\mathcal{F}_0(\eta_{F1})} \right]$$

relates the mobility to the mean-free-path for backscattering, and

$$\mu_B = \frac{v_T L}{2 k_B T / q} \left[ \frac{\mathcal{F}_{-1/2}(\eta_{F1})}{\mathcal{F}_0(\eta_{F1})} \right]$$

is the so-called ballistic mobility. The ballistic limit occurs when  $\mu_B \ll \mu_n$  and the diffusive limit when  $\mu_B \gg \mu_n$ .