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"Electronics from the Bottom Up"

Exercises on Carrier Scattering in Nanoscale MOSFETs

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Objective:
The objective of these exercises is to help you gain familiarity with the scattering theory of nanoscale MOSFETs as described in Lecture 5.

1) Consider a modern, Si MOSFET, with $L \approx 100$nm, $I_{ON} \approx 1$ mA/μm, $\mu_n \approx 200$ cm^2/V-s, operating under on-current conditions with $n_S \approx 10^{13}$ cm^{-2}. Estimate the critical length for backscattering, ℓ.

2) Consider a modern, Si MOSFET, with $L \approx 100$nm, $\mu_n \approx 200$ cm^2/V-s, operating in the linear region with $n_S \approx 10^{13}$ cm^{-2}.

   a) Determine whether the device is operating in the ballistic or diffusive regime.

   b) Repeat part a) but now assume that the channel material is InGaAs with $\mu_n \approx$ 10,0000 cm^2/V-s

3) In Lecture 5, we developed an expression for the drain current for a planar MOSFET in the presence of carrier backscattering as described by the transmission, T, or reflection, R = 1-T. We assumed Boltzmann statistics for carriers and obtained:

$$I_D = WC_{ox}\left(V_{GS} - V_T\right)\upsilon_T \left(\frac{(1-R)-(1-R)e^{-qV_{DS}/k_BT}}{(1+R)+(1-R)e^{-qV_{DS}/k_BT}} \right)$$

   Develop the analogous expressions for without assuming Boltzmann statistics for carriers. Carefully state the assumptions you make.

4) In Lecture 5, we showed that the high drain bias current is:

$$I_D = WC_{ox}\left(V_{GS} - V_T\right)\upsilon_T \frac{(1-R)}{(1+R)}$$

when Boltzmann tatistics are assumed. Derive an analogous expression without assuming Boltzmann statistics. Carefully state the assumptions you make.

HINT: You can either begin with the result of problem 3) for the entire I-V characteristic or you can assume high V_{DS} from the outset and derive the result.

5) In Lecture 5, we assumed Boltzmann statistics and showed that the channel conductance in the presence of scattering is given by

$$G_{CH} = \frac{I_D}{V_{DS}} = WC_{ox}\left(V_{GS} - V_T\right)\left[\frac{\upsilon_T}{2\left(k_B T/q\right)}\right](1-R)$$

Develop an analogous expression without assuming Boltzmann carrier statistics.

HINT: You can either begin with the result of problem 3) for the entire I-V characteristic or you can assume low V_{DS} from the outset and derive the result.

6) The mobility can be related to the mean-free-path for back-scattering, For Boltzmann carrier statistics, the relation is

$$D_n = \frac{k_B T}{q}\mu_n = \frac{\upsilon_T}{2}\lambda_0 .$$

Derive an analogous expression without assuming Boltzmann statistics.

HINT: begin with the channel conductance, which does not include a mobility, and equate it to the conventional expression for channel conductance, which does contain a mobility.

7) Develop a "Mathiesson's Rule' for nanotransistors as follows. Begin with the channel conductance,

$$G_{CH} = \left(WC_{ox}\left(V_{GS} - V_T\right)\frac{\upsilon_T}{\left(2k_B T/q\right)}\right)\left[\frac{\mathcal{F}_{-1/2}\left(\eta_{F1}\right)}{\mathcal{F}_0\left(\eta_{F1}\right)}\right]T$$

where

$$T = \frac{\lambda_0}{\lambda_0 + L}.$$

Show that the result can be expressed as

$$G_{CH} = \frac{W}{L}\left(\frac{1}{\mu_n} + \frac{1}{\mu_B}\right)^{-1} C_{ox}\left(V_{GS} - V_T\right)$$

where

$$\mu_n = \frac{\upsilon_T \lambda_0}{2\,k_B T/q}\left[\frac{\mathcal{F}_{-1/2}\left(\eta_{F1}\right)}{\mathcal{F}_0\left(\eta_{F1}\right)}\right]$$

relates the mobility to the mean-free-path for backscattering, and

$$\mu_B = \frac{\upsilon_T L}{2\,k_B T/q}\left[\frac{\mathcal{F}_{-1/2}\left(\eta_{F1}\right)}{\mathcal{F}_0\left(\eta_{F1}\right)}\right]$$

is the so-called ballistic mobility. The ballistic limit occurs when $\mu_B << \mu_n$ and the diffusive limit when $\mu_B >> \mu_n$.