

Exercises on Theory of the Ballistic MOSFET Theory: Part 2

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Objective:

The objective of these exercises is to help you gain familiarity with the theory of the ballistic MOSFET as presented in “Physics of Nanoscale Transistors: Lectures 3 and 4: Theory of the Ballistic MOSFET.”

- 1) In Lecture 3, we developed an expression for the ballistic drain current for a planar MOSFET (i.e. one with a 2D channel). We obtained:

$$I_{DS} = I^+ - I^-$$

where

$$I^+ = Wq \left(\frac{N_{2D}}{2} v_T \right) \mathcal{F}_{1/2}(\eta_{F1})$$

$$I^- = Wq \left(\frac{N_{2D}}{2} v_T \right) \mathcal{F}_{1/2}(\eta_{F2})$$

Develop the analogous expressions for a 1D channel MOSFET (i.e. for a nanowire MOSFET).

- 2) In Lecture 3, we also showed that the states at the top of the barrier in a 2D MOSFET are filled according to:

$$n_s^+(0) = \frac{N_{2D}}{2} \mathcal{F}_0(\eta_{F1}) \text{ cm}^{-2}$$

$$n_s^-(0) = \frac{N_{2D}}{2} \mathcal{F}_0(\eta_{F2}) \text{ cm}^{-2}$$

where

$$N_{2D} = \frac{m^* k_B T}{\pi \hbar^2} \text{ #/cm}^2$$

$$\eta_{F1} \equiv (E_{F1} - \varepsilon_1(0))/k_B T$$

$$\eta_{F2} \equiv \eta_{F1} - qV_{DS}/k_B T$$

Develop analogous expressions for the electron density per unit length, $n_L^+(\eta_{F1})$ and $n_L^+(\eta_{F2})$.

- 3) In Lecture 3, we developed an expression for the I-V characteristic of a ballistic MOSFET as

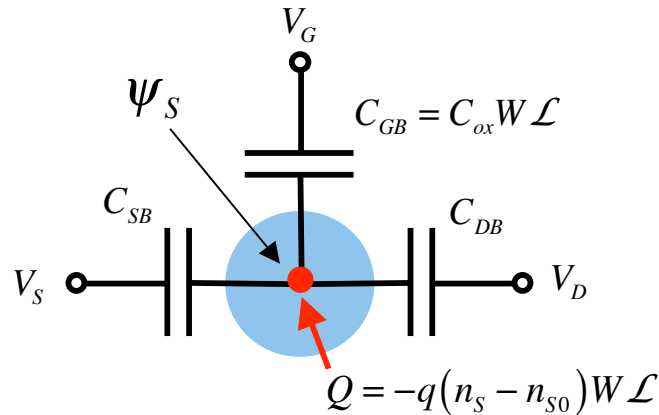
$$I_{DS} = WC_{ox} (V_{GS} - V_T) \tilde{v}_T \left[\frac{1 - \mathcal{F}_{1/2}(\eta_{F2})/\mathcal{F}_{1/2}(\eta_{F1})}{1 + \mathcal{F}_0(\eta_{F2})/\mathcal{F}_0(\eta_{F1})} \right]$$

where

$$\tilde{v}_T \equiv \sqrt{\frac{2k_B T}{\pi m^*} \frac{\mathcal{F}_{1/2}(\eta_{F1})}{\mathcal{F}_0(\eta_{F1})}} = v_T \frac{\mathcal{F}_{1/2}(\eta_{F1})}{\mathcal{F}_0(\eta_{F1})}.$$

Use the results of problems 1) and 2) to develop similar expressions for a 1D (nanowire) MOSFET. Note that for the planar MOSFET, $C_{ox} = \varepsilon_0 \kappa_{ox} / t_{ox}$ F/cm². For a nanowire MOSFET with a cylindrical gate, the analogous expression would be $C_{ox} = 2\pi\varepsilon_0\kappa_{ox} / \ln[(2t_{ox} + t_{wire})/t_{wire}]$, where t_{wire} is the diameter of the nanowire.

- 4) For a planar MOSFET, we know the saturated drain current varies with V_{GS} according to $I_D \propto (V_{GS} - V_T)^\alpha$ where $\alpha = 1$ in the non-degenerate case and $\alpha = 1.5$ in the degenerate case. What happens for a nanowire transistor? Use the results of problem 3) to determine the range of α for a ballistic nanowire MOSFET.
- 5) In Lecture 3, we showed how to derive the density-of-states and conductivity effective masses for the unprimed subbands of (100) Si. Repeat the derivation for the primed subbands. Do this for a transport direction of [100] and for the industry standard direction of [110].
- 6) Use the three capacitor model below to relate the surface potential, ψ_s , to the applied voltages and charge at the top of the barrier.



Prove that
$$\psi_S = V_G \left(\frac{C_{GB}}{C_\Sigma} \right) + V_D \left(\frac{C_{DB}}{C_\Sigma} \right) + V_S \left(\frac{C_{SB}}{C_\Sigma} \right) - \frac{q[n_S(\psi_S) - n_{S0}]W\mathcal{L}}{C_\Sigma}.$$

HINT: Use superposition. Begin by assuming that the charge, Q , is zero, and then use superposition again to relate V_G to ψ_S assuming V_S and $V_D = 0$. Repeat for V_S and then for V_D . Next, assume $V_G = V_S = V_D = 0$, and relate ψ_S to the charge, Q . Add the solution for $Q = 0$ to that for Q nonzero.

7) Use the capacitor model,

$$\psi_S = V_G \left(\frac{C_{GB}}{C_\Sigma} \right) + V_D \left(\frac{C_{DB}}{C_\Sigma} \right) + V_S \left(\frac{C_{SB}}{C_\Sigma} \right) - \frac{q[n_S(\psi_S) - n_{S0}]W\mathcal{L}}{C_\Sigma}$$

and assume that $C_{GB} = C_{ox}W\mathcal{L}$ is known, then answer the following questions.

7a) Show that the subthreshold swing, S , is given by

$$S = 2.3 \frac{k_B T}{q} \left(\frac{C_{GB}}{C_\Sigma} \right)$$

7b) Show that the drain-induced barrier lowering, DIBL, is given by

$$DIBL = \frac{C_{DB}}{C_{GB}}.$$

HINT: Recall that $I_D = Wq(N_{2D}v_T/2)[\mathcal{F}_{1/2}(\eta_{F1}) - \mathcal{F}_{1/2}(\eta_{F2})]$, so for non-degenerate carrier statistics, $I_D \approx e^{q\psi_s/k_B T}$. Because S and DIBL are defined below threshold non-degenerate statistics can safely be used.

7c) Show also that above threshold,

$$R_o = \frac{1}{2.3I_D} \frac{S}{DIBL}$$

Explain why the expression for output resistance may not be as accurate as the expressions for S and DIBL.

- 8) For $T = 0\text{K}$, the ballistic channel conductance is $G_{CH} = M(2e^2/h)$ where M is the number of conducting channels. Show that the ballistic channel conductance of a planar MOSFET depends only on the inversion layer density, n_s , and not on the effective mass.