NCN@Purdue - Intel Summer School: July 14-25, 2008

Physics of Nanoscale Transistors: Lecture 7:

Connection to the Bottom Up Approach

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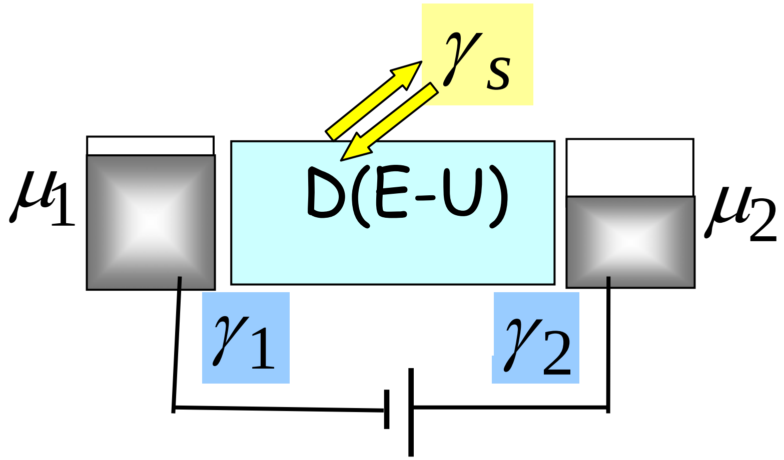
Datta's generic model for a nanodevice



$$N = D(E-U)\left[\frac{\gamma_1 f_1 + \gamma_2 f_2}{\gamma_1 + \gamma_2}\right]$$

$$I = \frac{q}{h}\, D(E-U)\, \frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2}\left[f_1 - f_2\right]$$

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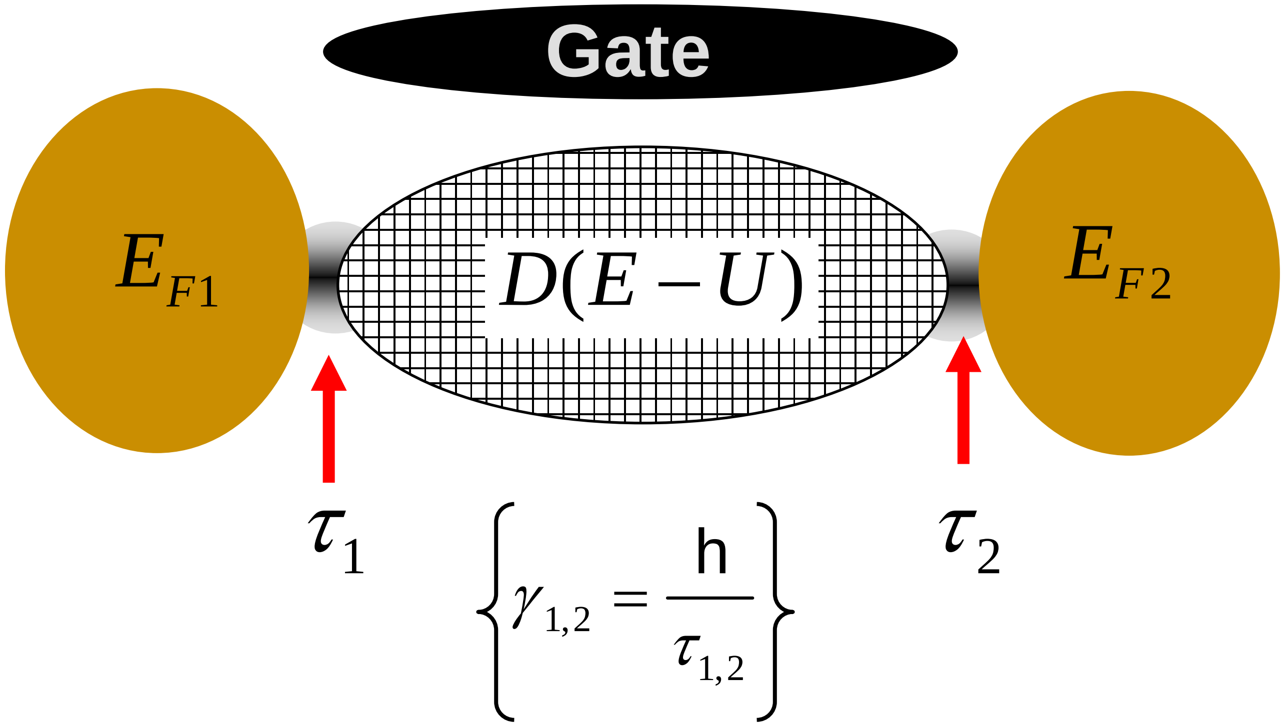
http://www.nanohub.org/courses/cqt

outline

1) Introduction
2) **Bottom-up approach**
3) The ballistic MOSFET
4) Treatment of scattering
5) Summary

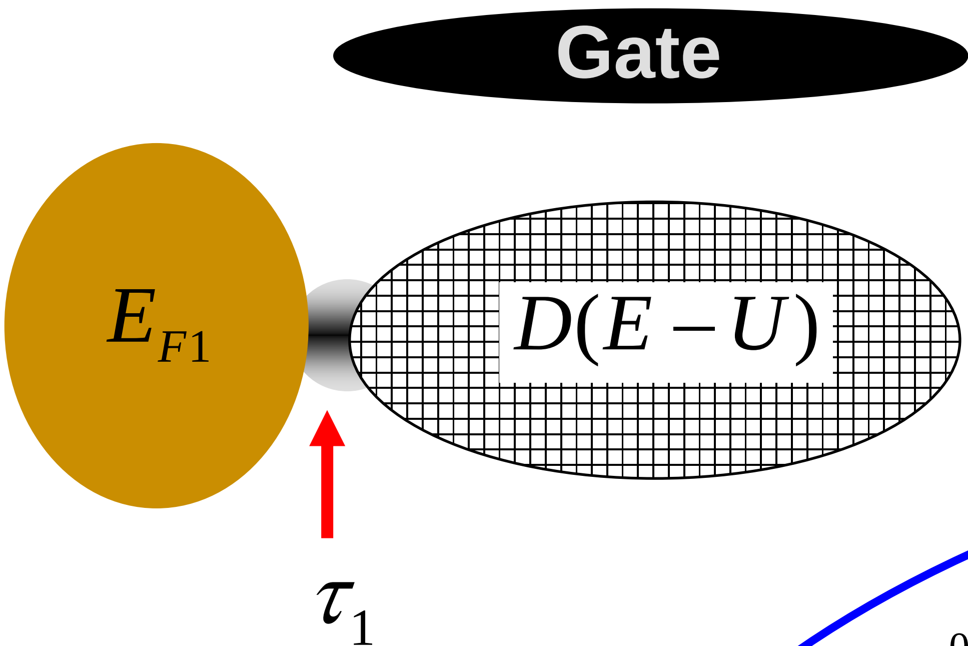
generic model



S. Datta, *Quantum Transport: Atom to Transistor*, Cambridge, 2005
("Concepts of Quantum Transport" **nanohub.org**)

filling states from the left contact

Assumption:
Each energy channel is independent

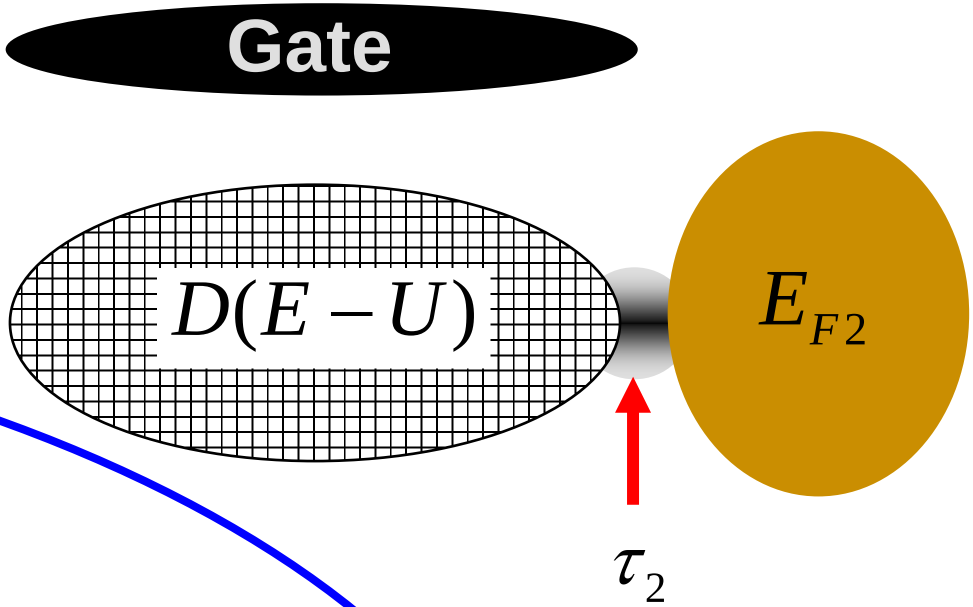


includes spin

$$N_1^0(E) = D(E-U)\,f_1(E)$$

$$\frac{dN(E)}{dt} = \frac{N_1^0(E) - N}{\tau_1}$$

filling states from the right contact



$$N_2^0(E) = D(E-U)\,f_2(E)$$

$$\frac{dN(E)}{dt} = \frac{N_2^0(E) - N}{\tau_2}$$

steady-state

$$\frac{dN(E)}{dt} = \frac{N_1^0 - N}{\tau_1} + \frac{N_2^0 - N}{\tau_2} = 0$$

$$(1/\tau_1)N_1^0 - (1/\tau_1)N + (1/\tau_2)N_2^0 - (1/\tau_2)N = 0$$

$$\boxed{N(E) = \frac{(1/\tau_1)}{(1/\tau_1)+(1/\tau_2)}N_1^0(E) + \frac{(1/\tau_2)}{(1/\tau_1)+(1/\tau_2)}N_2^0(E)}$$

$$\left\{ \begin{array}{ll} N_1^0(E) \equiv D(E-U)\,f_1(E) & \gamma_1 = h/\tau_1 \\ N_2^0(E) \equiv D(E-U)\,f_2(E) & \gamma_1 = h/\tau_2 \end{array} \right\}$$

steady-state electron number, *N*(*E*)

$$N(E)=\frac{\gamma_1}{\gamma_1+\gamma_2}D(E-U)f_1(E)+\frac{\gamma_2}{\gamma_1+\gamma_2}D(E-U)f_2(E)$$

$$\boxed{N(E)=D_1(E-U)f_1(E)+D_2(E-U)f_2(E)}$$

$$D_1(E-U_{SCF})=\frac{\gamma_1}{\gamma_1+\gamma_2}D(E-U_{SCF})$$ DOS that can be filled by contact 1

$$D_2(E-U_{SCF})=\frac{\gamma_2}{\gamma_1+\gamma_2}D(E-U_{SCF})$$ DOS that can be filled by contact 2

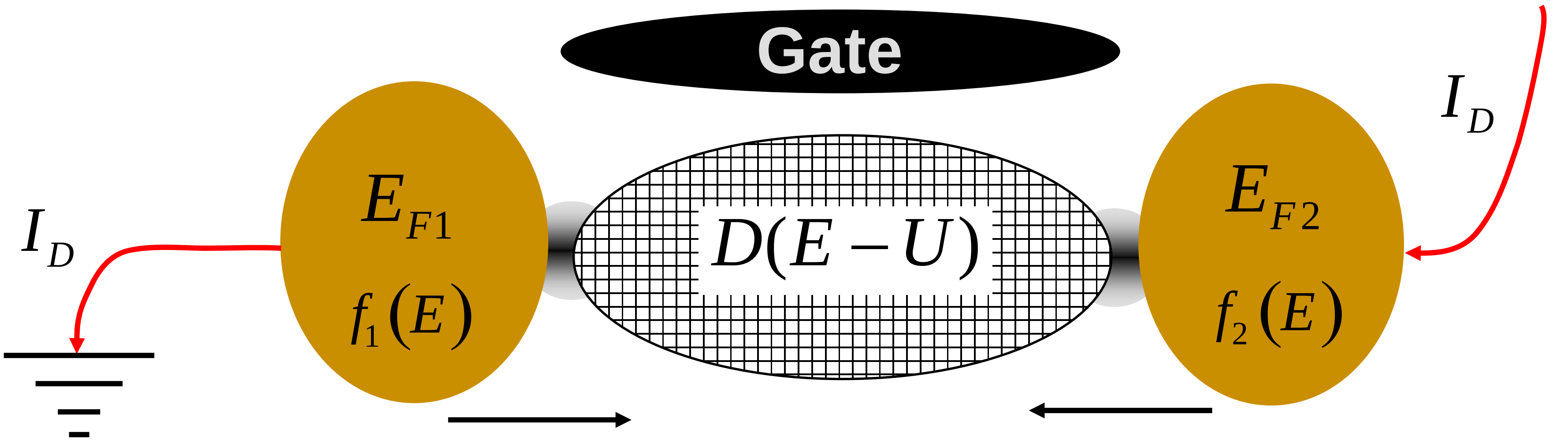
steady-state electron number, *N*

$$\boxed{N = \int \left[D_1(E-U) f_1(E) + D_2(E-U) f_2(E) \right] dE}$$

in equilibrium, we use:

$$\boxed{N = \int D(E-U) f_0(E) dE}$$

steady-state current, *I*



$$\left.\frac{dN(E)}{dt}\right|_1 = \frac{N_1^0(E) - N(E)}{\tau_1}$$

$$\left.\frac{dN(E)}{dt}\right|_2 = \frac{N_2^0(E) - N}{\tau_2}$$

$$\boxed{I_D(E) = +q\left.\frac{dN(E)}{dt}\right|_1 = -q\left.\frac{dN(E)}{dt}\right|_2}$$

results

$$I(E) = \frac{q}{h}\left(\frac{\gamma_1\gamma_2}{\gamma_1+\gamma_2}\right)D(E-U)\left(f_1 - f_2\right)$$

$$\gamma_1 = \gamma_2 = \gamma$$

$$I_D = \int I(E)dE = \frac{2q}{h}\int\left(\frac{\gamma}{2}\right)\pi D(E-U)\left(f_1 - f_2\right)dE$$

$$N = \int N(E)dE = \int\left[\frac{D(E-U)}{2}\left(f_1(E) + f_2(E)\right)\right]dE$$

final results

$$\gamma_1 = \gamma_2 = \gamma$$

$$I_D = \frac{2q}{h}\int \gamma\, \pi D'(E-U)\left(f_1 - f_2\right)dE$$

$$N = \int D'(E-U)\left[f_1(E) + f_2(E)\right]dE$$

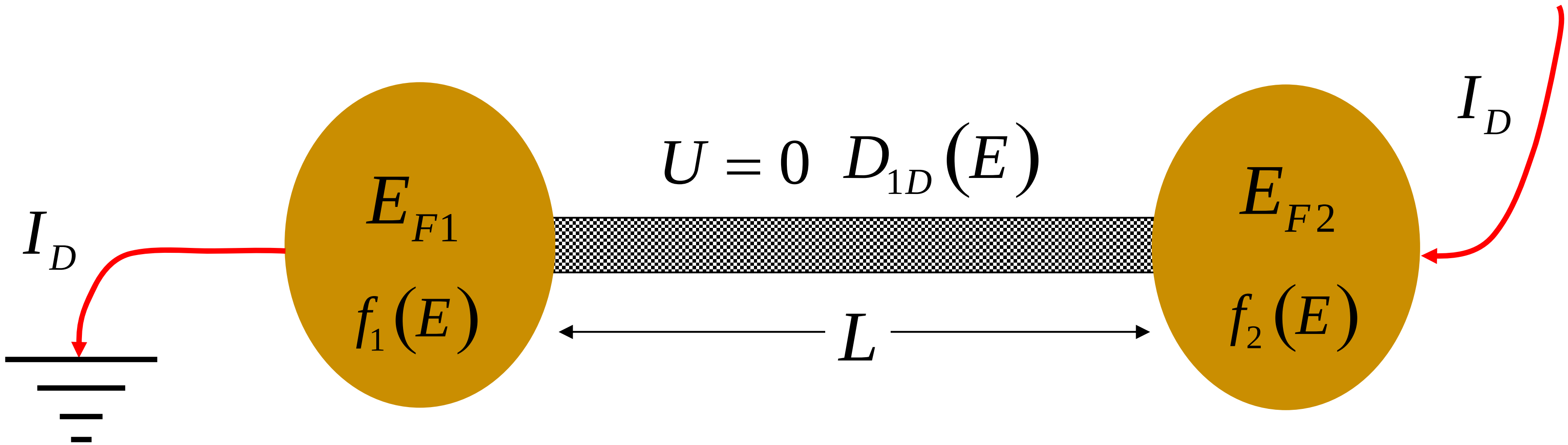
$$D'(E-U) = \frac{D(E-U)}{2}$$

density-of-states per spin

outline

1) Introduction
2) Bottom-up approach
3) **Applications**
4) The ballistic MOSFET
5) Summary

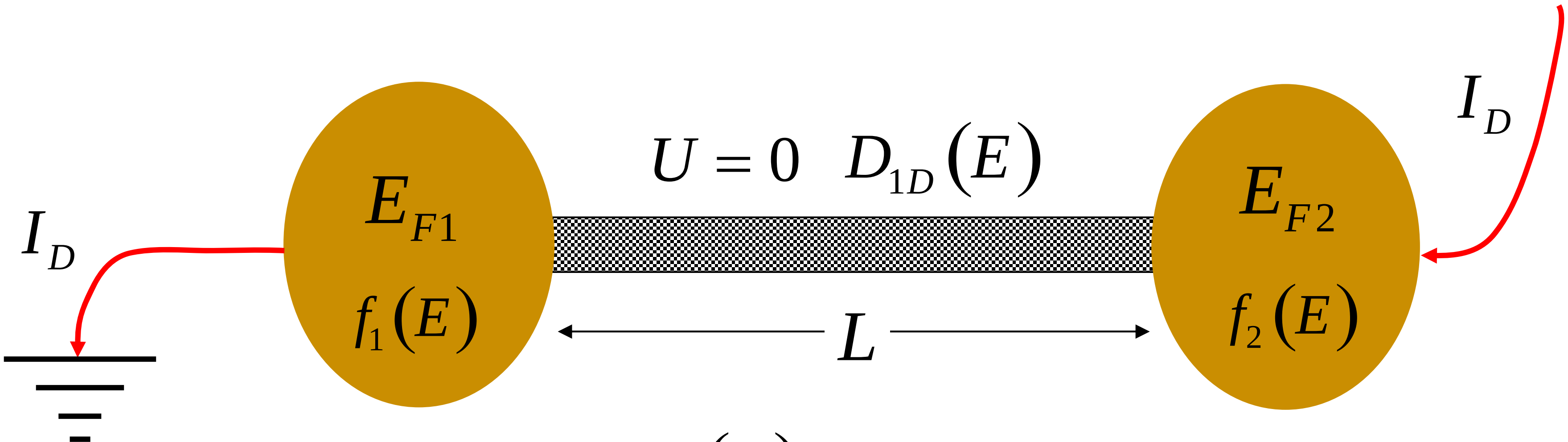
current in 1D



$$I_D = \frac{2q}{h}\int \gamma\,\pi D'(E-U)(f_1 - f_2)dE$$

$$I_D = \frac{2q}{h}\int M(E)(f_1 - f_2)dE \quad \text{(ballistic)}$$

modes in 1D



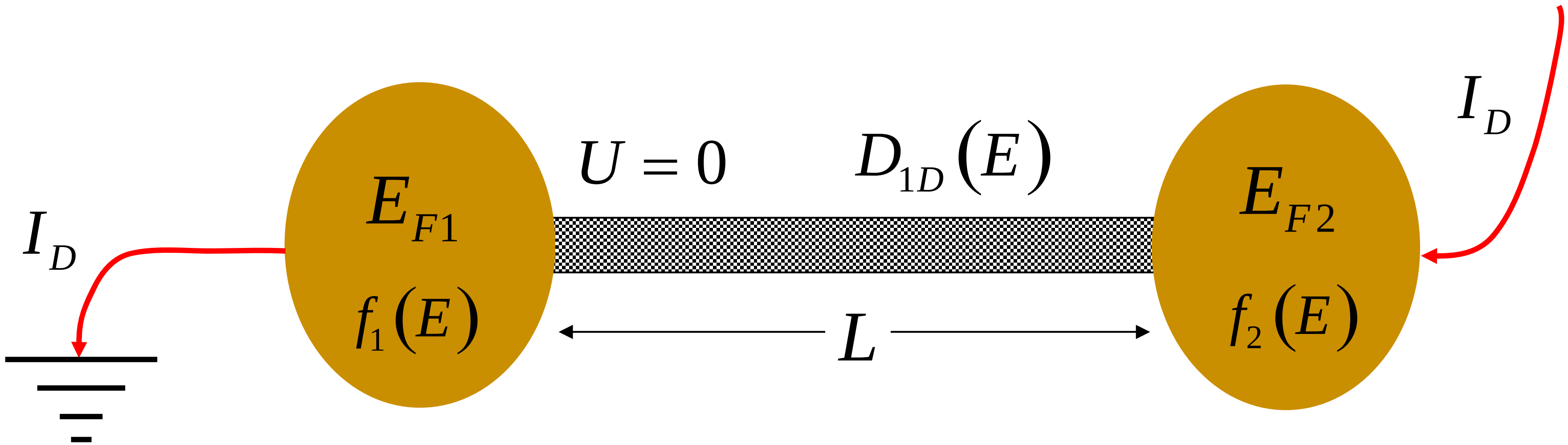
$$\gamma\,\pi D'(E) = M_{1D}(E) = 1?$$

$$\gamma\,\pi D'(E) = \frac{h\upsilon_x}{L}\pi\frac{1}{\pi h\upsilon_x} = 1 \qquad 4$$

$$D'_{1D}(E) = \frac{1}{\pi h\upsilon_x}L \qquad \gamma = \frac{h}{\tau} \qquad \gamma = \frac{h\upsilon_x}{L}$$

$$\left\{ I_D = \frac{qN}{L}\upsilon_x = \frac{qN}{\tau} \right.$$

conductance in 1D



$$I_D = \frac{2q}{h}\int \gamma\,\pi\,D'(E)\left(f_1 - f_2\right)dE$$

$$I_D = \frac{2q}{h}\int \left(f_1 - f_2\right)dE = \frac{2q}{h}V_D \quad \left(T = 0K\right) \qquad 4$$

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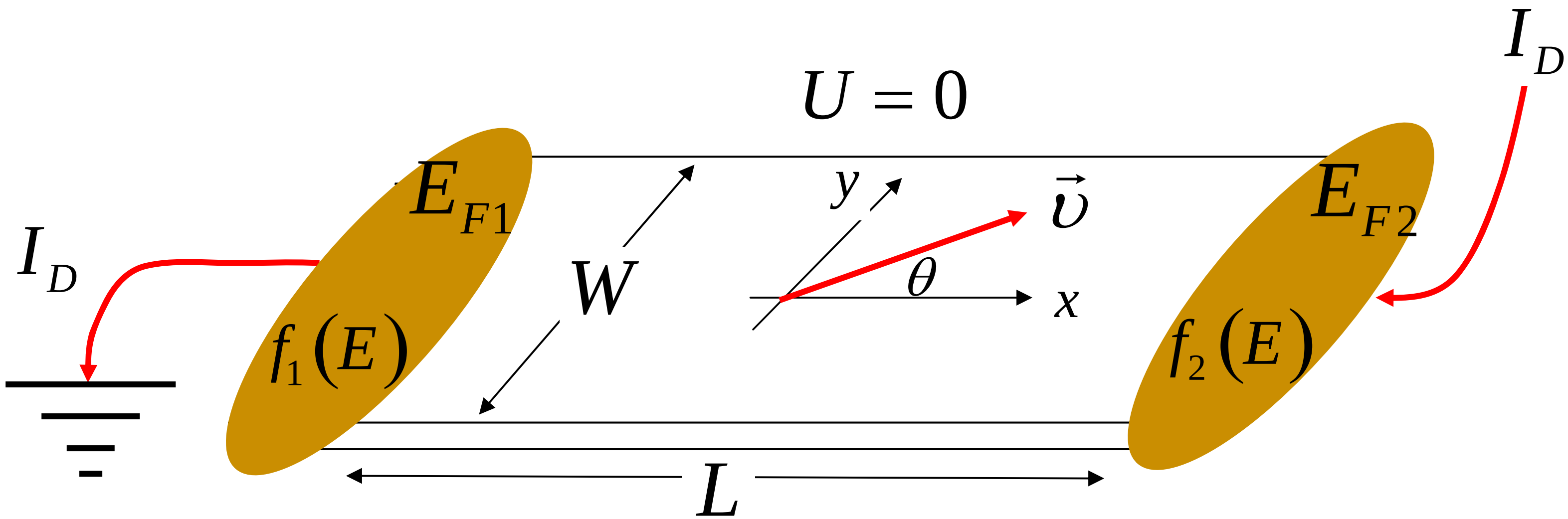
current in 2D



$$I_D = \frac{2q}{h}\int \gamma\, \pi D'(E)\left(f_1 - f_2\right)dE$$

$$\gamma\, \pi D'(E) = M_{2D}(E) = \frac{\sqrt{2m^* E}}{\pi \hbar} \quad ?$$

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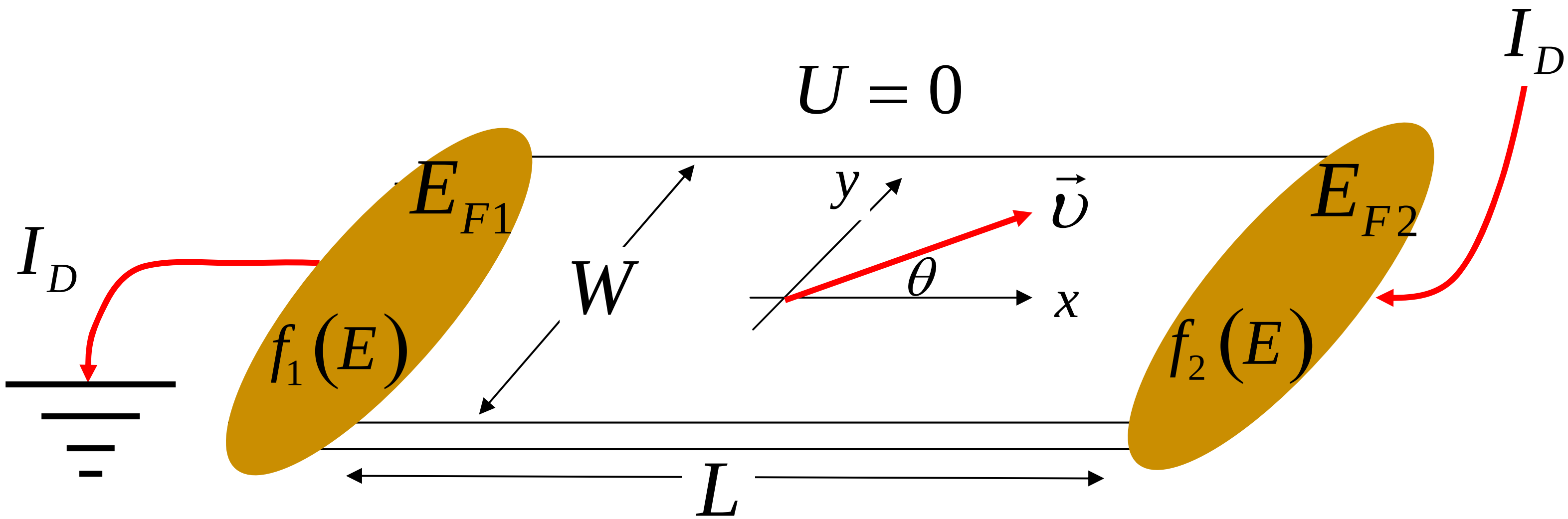
current in 2D



$$D'_{2D}(E) = \frac{m^*}{2\pi\hbar^2} WL$$

$$\gamma = \frac{\hbar}{\langle \tau \rangle} \quad \gamma = \frac{\hbar \langle \upsilon_x \rangle}{L} = \frac{\hbar \langle \upsilon \cos\theta \rangle}{L} = \frac{\hbar \upsilon}{L}\left(\frac{2}{\pi}\right) \qquad \upsilon = \sqrt{2E/m^*}$$

current in 2D

$$I_D = \frac{2q}{h}\int \gamma\, \pi D'_{2D}(E)\left(f_1 - f_2\right)dE$$

$$\begin{cases} \gamma = \dfrac{\hbar\upsilon}{L}\left(\dfrac{2}{\pi}\right) = \dfrac{\hbar\sqrt{2E/m^*}}{L}\left(\dfrac{2}{\pi}\right) \\ D'_{2D}(E) = \dfrac{m^*}{2\pi\hbar^2}WL \end{cases}$$

$$\gamma\, \pi D'_{2D} = \frac{\hbar}{L}\sqrt{\frac{2E}{m^*}}\left(\frac{2}{\pi}\right)\pi\frac{m^*}{2\pi\hbar^2}WL = W\frac{\sqrt{2m^*E}}{\pi\hbar} = M_{2D}(E) \quad 4$$

$$I_D = \frac{2q}{h}\int M_{2D}(E)\left(f_1 - f_2\right)dE$$

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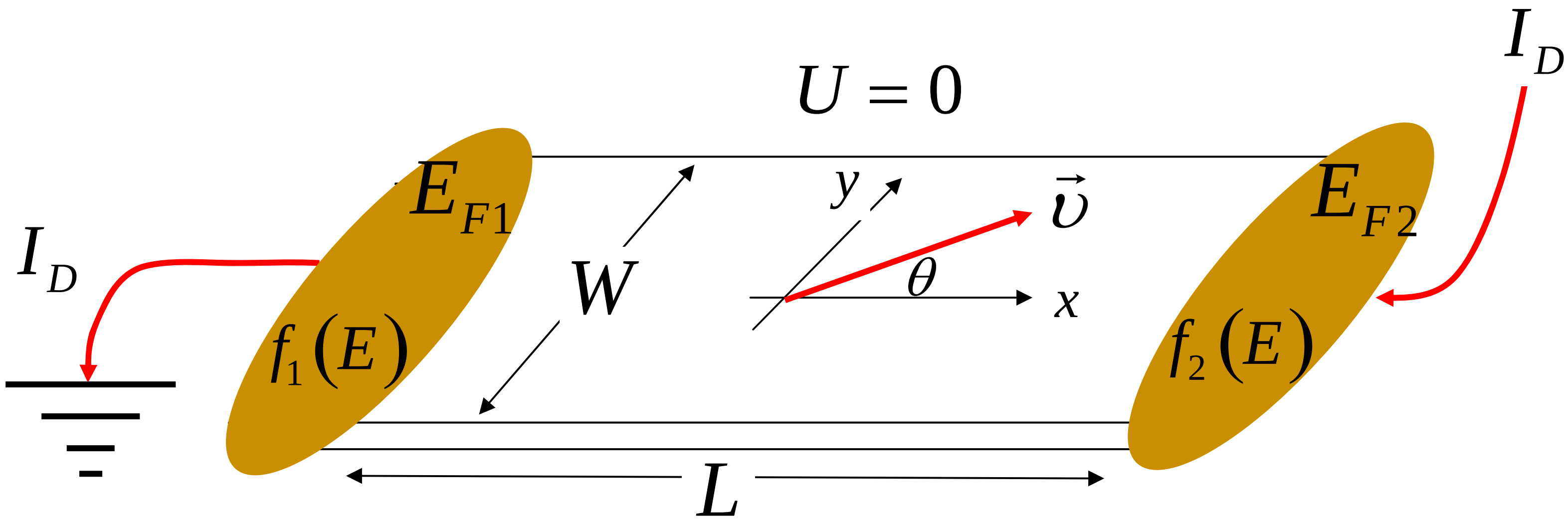
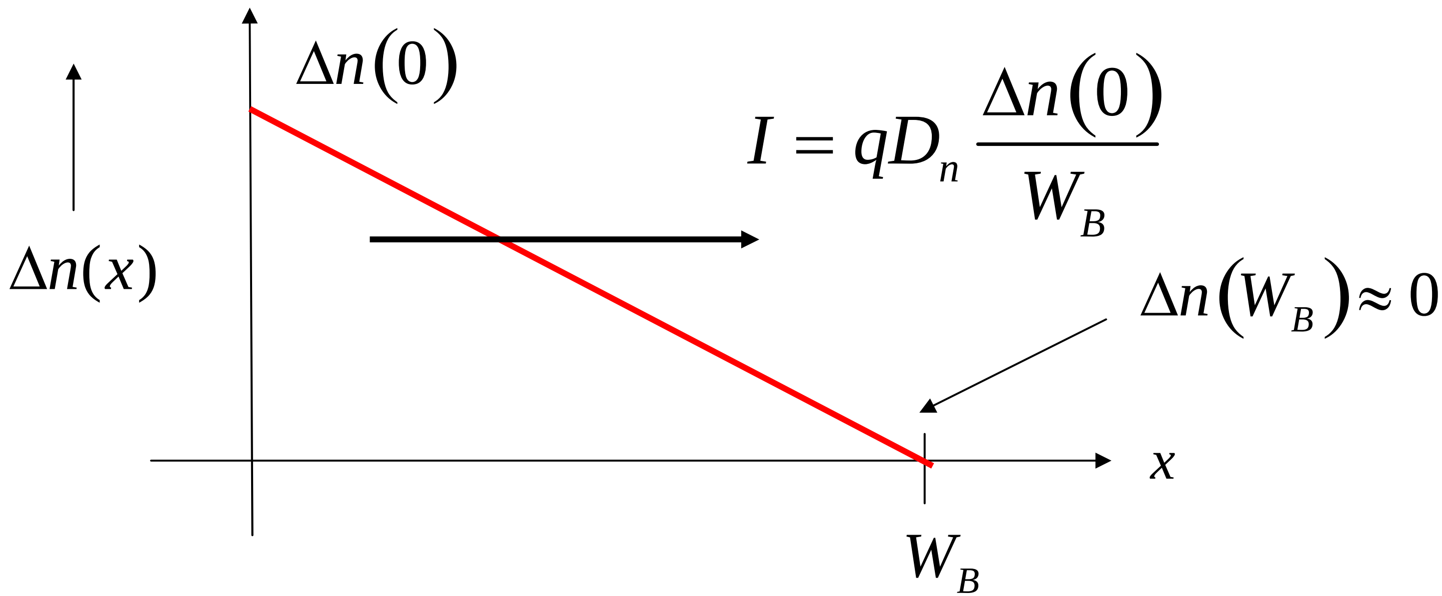
diffusive current in 2D



$$I_D = \frac{2q}{h}\int \gamma\, \pi D'(E)\left(f_1 - f_2\right)dE$$

$$\gamma = \frac{\hbar}{\langle \tau \rangle} \qquad \langle \tau \rangle = ?$$

base transit time



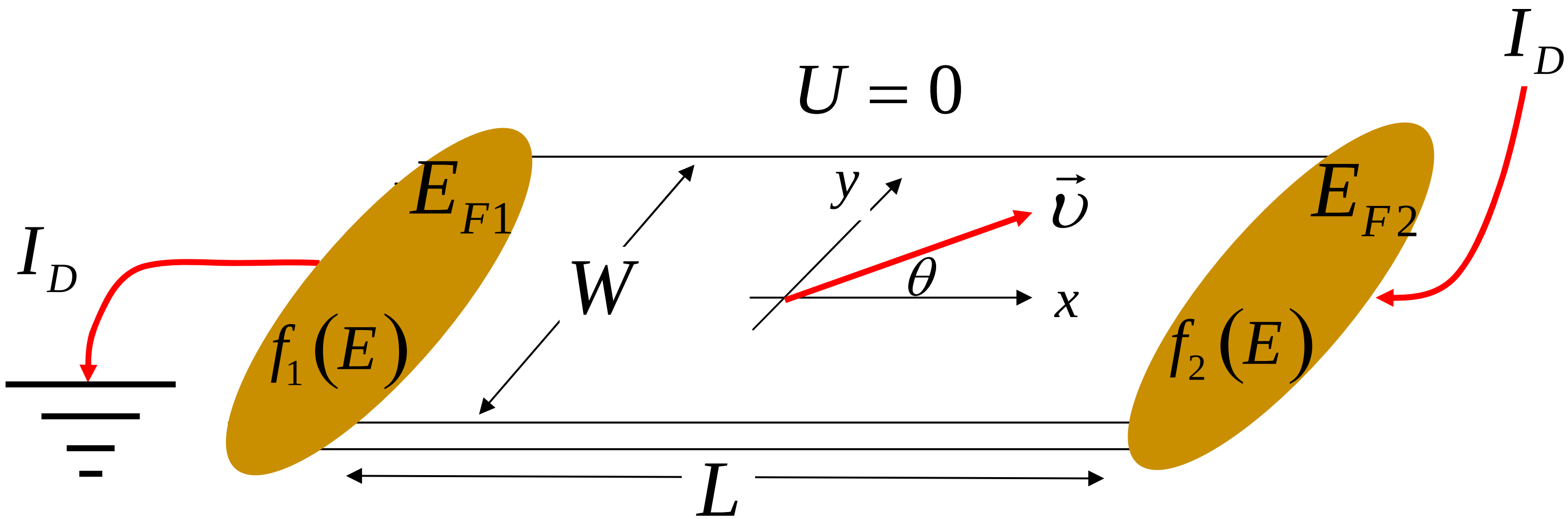
$$I = \frac{qN}{\tau}$$

$$\frac{qN}{\tau} = qD_n \frac{\Delta n(0)}{W_B}$$

$$N = \frac{1}{2} \Delta n(0) W_B$$

$$\tau = \frac{W_B^2}{2D_n}$$

diffusive current in 2D



$$I_D = \frac{2q}{h} \int \gamma \, \pi D'(E)(f_1 - f_2) dE$$

$$\gamma = \frac{\hbar}{\langle \tau \rangle} \qquad \langle \tau \rangle = \frac{L^2}{2D_n}$$

diffusive current in 2D

$$I_D = \frac{2q}{h}\int \gamma\, \pi D'(E)\left(f_1 - f_2\right)dE$$

$$\gamma = \frac{\mathrm{h}}{\langle \tau \rangle}$$

$$\langle \tau \rangle = \frac{L^2}{2D_n}$$

$$D_n = \frac{\upsilon(2/\pi)}{2}\lambda$$

$$\gamma = \frac{2D_n \mathrm{h}}{L^2} \quad \rightarrow \quad \gamma = \frac{\upsilon(2/\pi)\lambda\, \mathrm{h}}{L^2} = \overset{T}{\frac{\lambda}{L}}\, \underbrace{\frac{\mathrm{h}\upsilon}{L}\left(\frac{2}{\pi}\right)}_{\gamma_B}$$

$$I_D = \frac{2q}{h}\int T M_{2D}(E)\left(f_1 - f_2\right)dE \qquad 4$$

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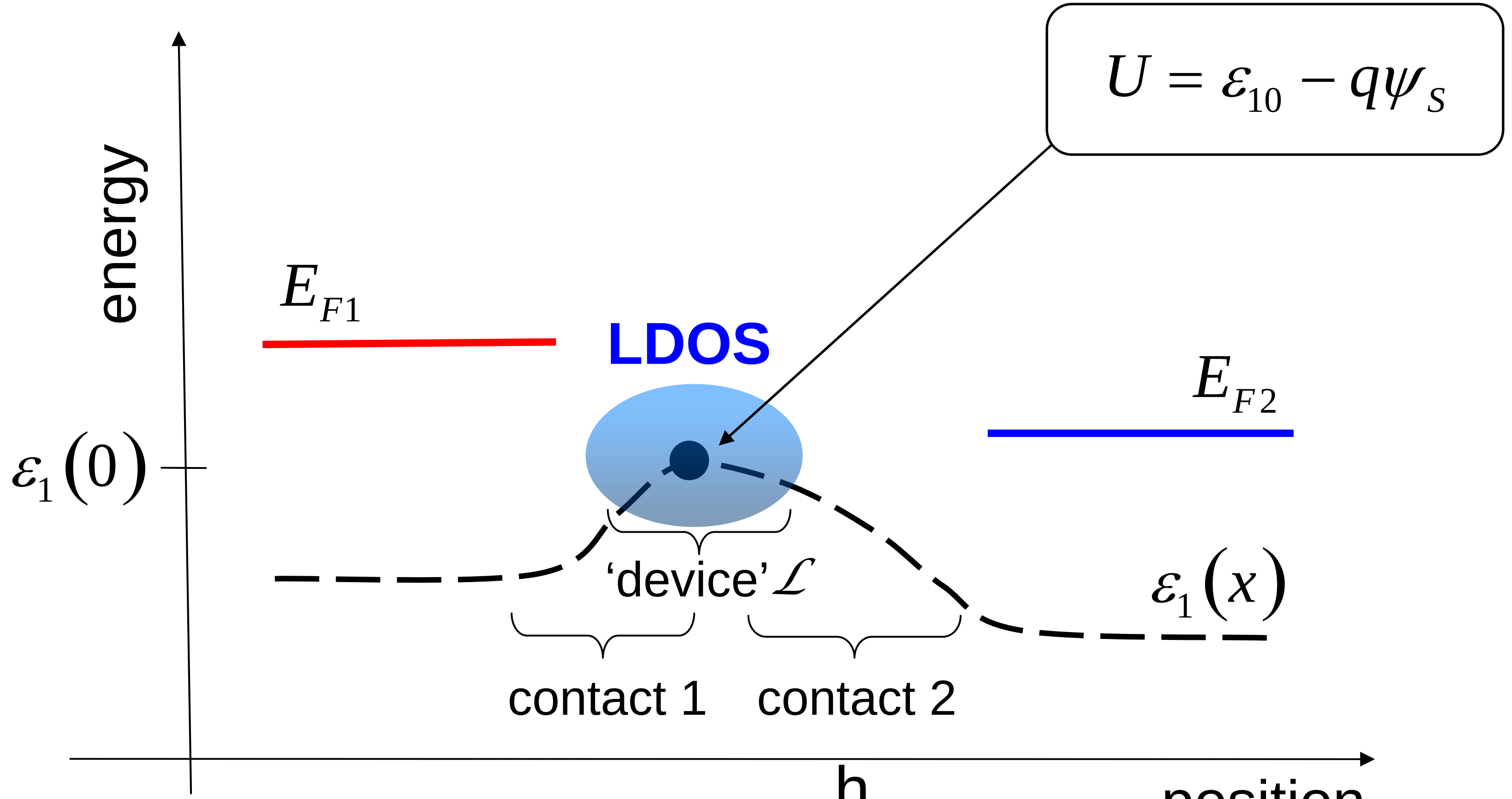
"top of the barrier model"



$$\gamma_1 = \gamma_2 = \frac{\mathrm{h}}{\langle \tau \rangle}$$

MOSFET current

$$I_D = \frac{2q}{h}\int \gamma\,\pi D'(E)\left(f_1 - f_2\right)dE$$

$$\gamma_B \pi D'_{2D}(E) = M_{2D}(E)$$

$$\boxed{I_D = \frac{2q}{h}\int T(E)M_{2D}(E)\left(f_1 - f_2\right)dE}$$

$$\gamma = \frac{\gamma_D \gamma_B}{\gamma_D + \gamma_B}$$

$$\frac{\gamma_D}{\gamma_D + \gamma_B}\gamma_B \pi D'_{2D}(E) = T(E)M_{2D}(E)$$

$$T(E) = \frac{\gamma_D}{\gamma_D + \gamma_B} = \frac{\lambda(E)}{\lambda(E) + L}$$

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summary

$$\gamma_1 = \gamma_2 = \gamma$$

$$I_D = \frac{2q}{h}\int \gamma\,\pi D'(E-U)\left(f_1 - f_2\right)dE$$

$$N = \int \frac{D(E-U)}{2}\left[f_1(E) + f_2(E)\right]dE$$

$$\gamma = \frac{\mathrm{h}}{\langle \tau \rangle} = \frac{\gamma_D \gamma_B}{\gamma_D + \gamma_B}$$

$$\gamma\,\pi D'(E-U) = T(E)M(E)$$

$$T(E) = \frac{\gamma_D}{\gamma_D + \gamma_B} \qquad M(E) = \gamma_B \pi D'(U)$$

summary

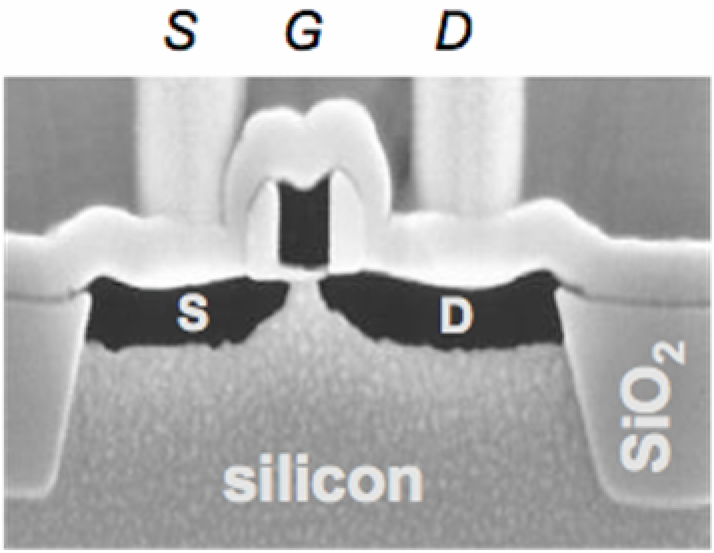
$$I_D = \frac{2q}{h}\int \gamma\, \pi D'(E-U)\left(f_1 - f_2\right)dE$$

or

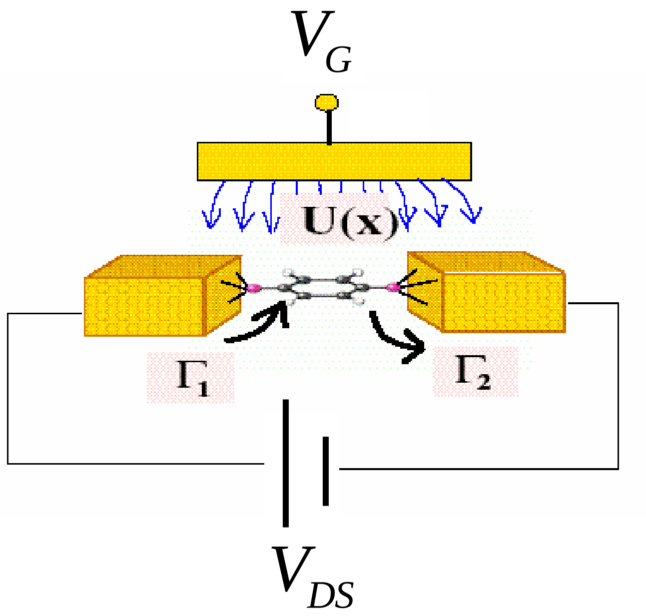
$$I_D = \frac{2q}{h}\int T(E)M(E)\left(f_1 - f_2\right)dE$$

?

molecular-scale MOSFETs



(Texas Instruments, 1997)



Avik W. Ghosh, Titash Rakshit, and Supriyo Datta, “Gating of a Molecular Transistor: Electrostatic and Conformational,” *Nano Lett*., **4**, 565-568, (2004).

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