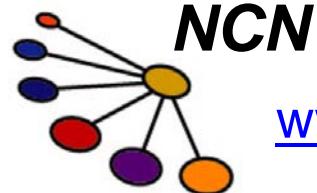


EE-612:

Lecture 5

MOSFET IV: Part 1

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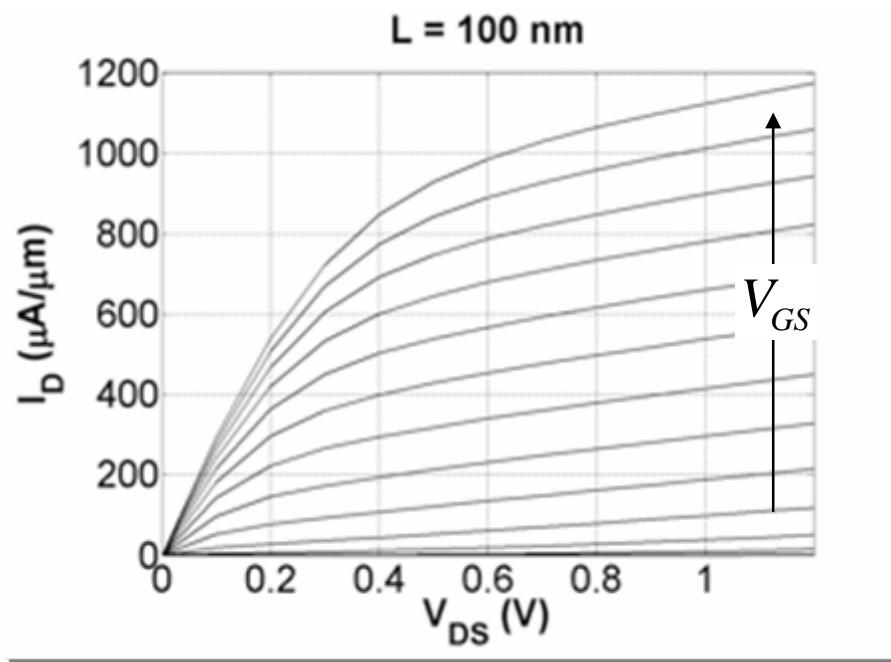
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outline

- 1) Introduction
- 2) Square law theory
- 3) PN junction effects on MOSFETs
- 4) Bulk charge theory (exact)
- 5) Summary

typical Si NMOS characteristics



(Courtesy, Shuji Ikeda, ATDF, Dec. 2007)

MOSFET IV: summary

linear region: $V_{DS} \ll V_{DSAT}$

$$I_D = \frac{W}{L} \mu_{eff} C_{ox} (V_{GS} - V_T) V_{DS}$$

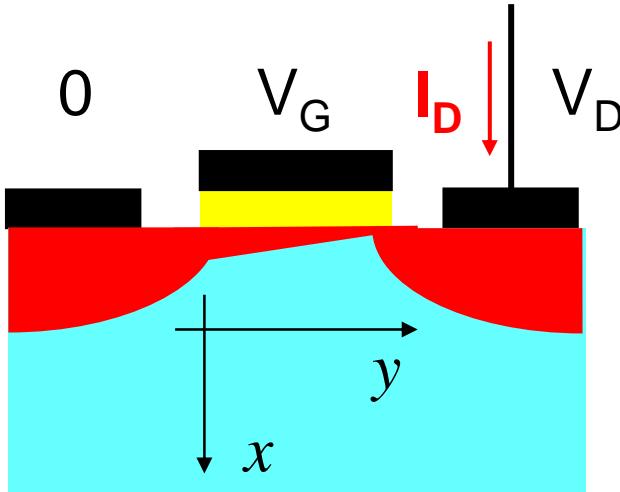
saturated region: $V_{DS} > V_{DSAT}$

$$I_D = \frac{W}{2L} \mu_{eff} C_{ox} (V_{GS} - V_T)^2 \quad V_{DS} = V_{GS} - V_T$$

$$I_D = W C_{ox} v_{sat} (V_{GS} - V_T) \quad V_{DS} > V_{DSAT}$$

See: "A Review of MOSFET Fundamentals," M. Lundstrom,
<http://nanohub.edu/resources/5307>

I-V formulation



$$I_D = W Q_I(y) v_y(y) \text{ Amperes}$$

$$I_D = -W Q_I(y) \mu_{eff} \frac{dV}{dy}$$

$$I_D dy = -W Q_I(V) \mu_{eff} dV$$

to include diffusion:

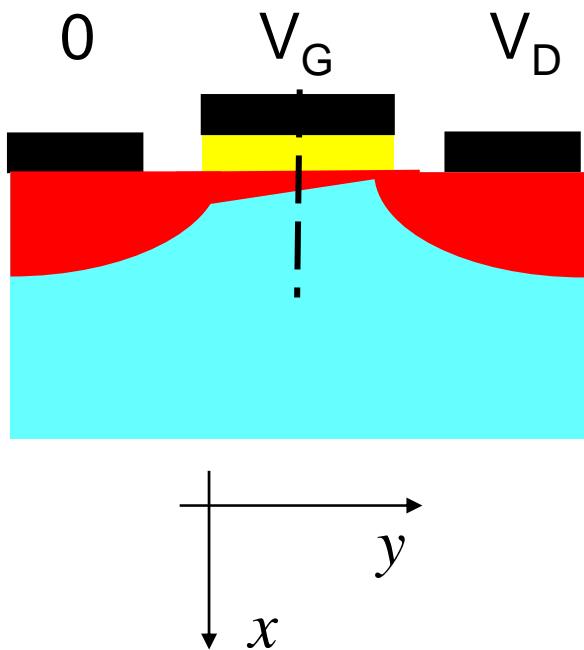
$$\frac{dV}{dy} \rightarrow \frac{dF_n}{dy}$$

$$I_D = -\frac{W}{L} \mu_{eff} \int_0^{V_{DS}} Q_I(V) dV$$

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gradual channel approximation



for $0 \leq y \leq L$

$$V = V(y)$$

$$V(0) = 0 \quad V(L) = V_D$$

$$Q_I = Q_I(y)$$

$$1\text{D MOS-C: } Q_I = -C_{ox} (V_G - V_T)$$

$$GCA: E_y \ll E_x$$

$$V_T \rightarrow V_T(y) = V_T + V(y)$$

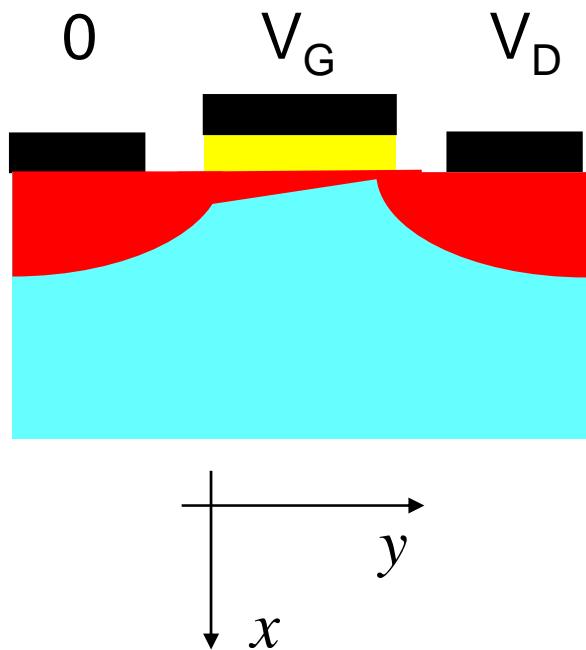
$$V_G - V_T > 0$$



(subthreshold current will
require a separate treatment)

$$Q_I(y) = -C_{ox} [V_G - V_T - V(y)]$$

IV relation

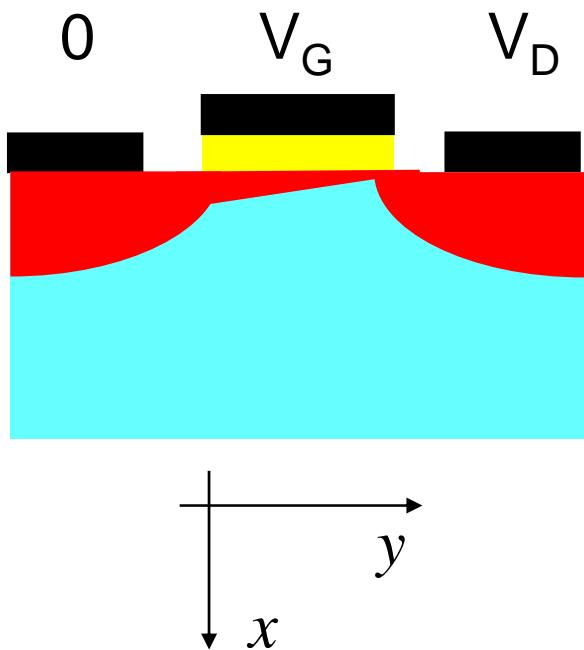


$$I_D = -\frac{W}{L} \mu_{eff} \int_0^{V_{DS}} Q_I(V) dV$$

$$I_D = +\mu_{eff} C_{ox} \frac{W}{L} \int_0^{V_D} [V_G - V_T - V] dV$$

$$I_D = +\mu_{eff} C_{ox} \frac{W}{L} \left[(V_{GS} - V_T) V_{DS} - \frac{V_{DS}^2}{2} \right]$$

pinch-off



$$Q_I(L) = -C_{ox} [V_G - V_T - V_D]$$

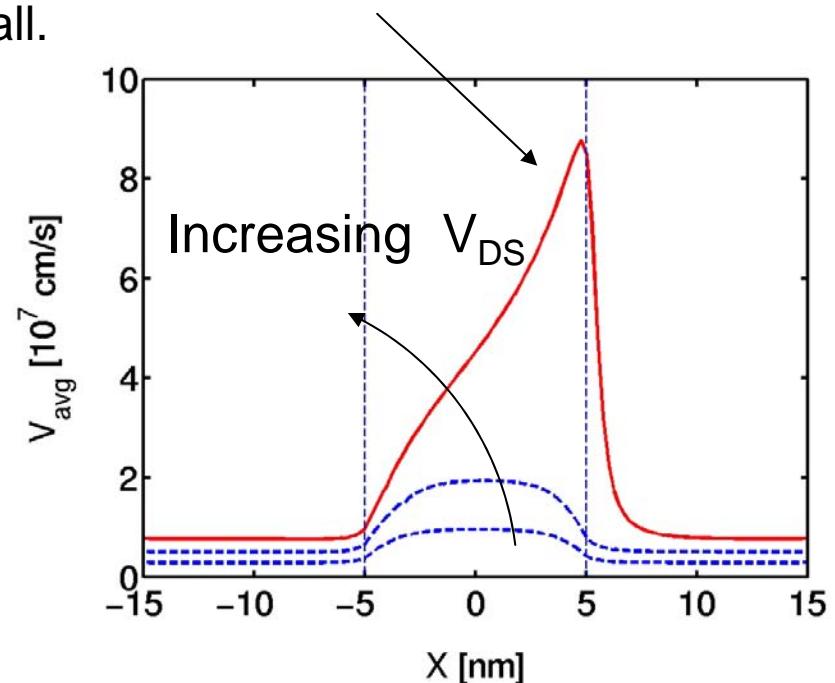
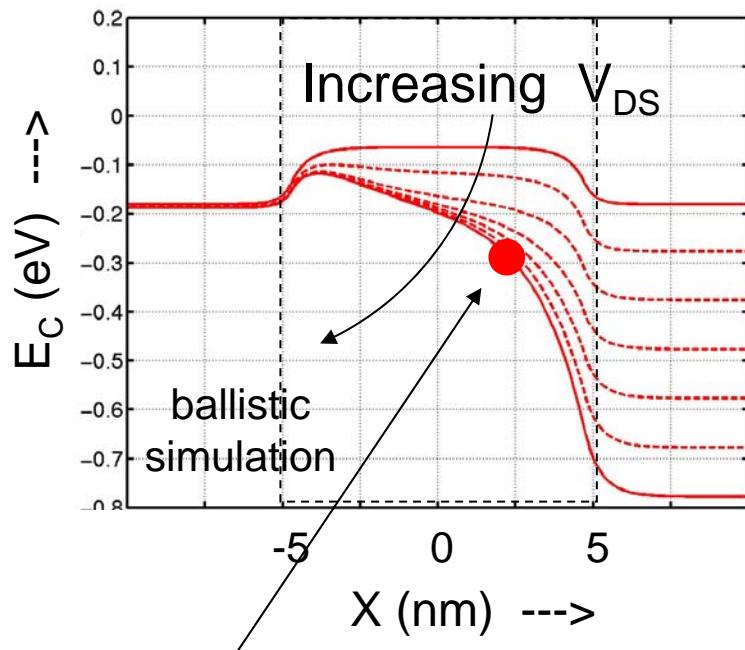
when $V_D = V_G - V_T$,
then $Q_i(L) = 0$

$E_y \gg E_x$ GCA fails!

but current still flows!

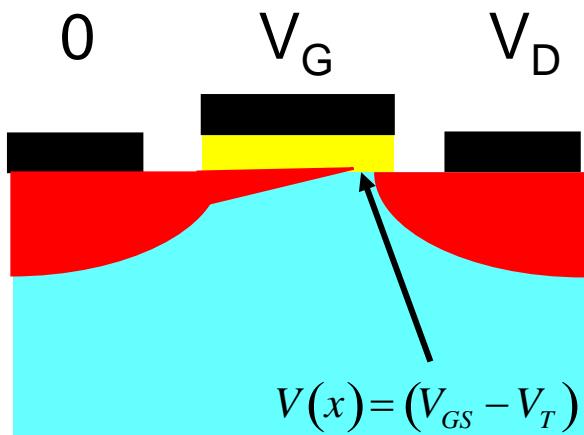
pinch off in a MOSFET

The electron velocity is very high in the pinch-off region. High velocity implies low inversion layer density (because I_D is constant). In the textbook model, we say $Q_i \approx 0$, but it is not really zero - just very small.



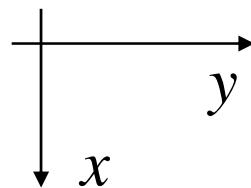
pinch-off point: where the electric field along the channel becomes very large. Note that electrons are simply swept across the high-field (pinched-off) portion at very high velocity.

beyond pinch-off



$$I_D = +\mu_{eff} C_{ox} \frac{W}{L} \left[(V_{GS} - V_T) V_{DS} - \frac{V_{DS}^2}{2} \right]$$

$$I_D \approx I_D (V_{DS} = V_{GS} - V_T)$$



$L' \approx L$

$$I_D = +\mu_{eff} C_{ox} \frac{W}{2L'} (V_{GS} - V_T)^2$$

$$V_{GS} > V_T$$

$$V_{DS} > V_{GS} - V_T$$

the electric field

small V_{DS}

$$I_D = \mu_{eff} C_{ox} \frac{W}{L} (V_{GS} - V_T) V_{DS}$$

$$I_D = W C_{ox} (V_{GS} - V_T) \mu_{eff} E_y(0)$$

$$E_y(0) = \frac{V_{DS}}{L}$$

large V_{DS}

$$I_D = \mu_{eff} C_{ox} \frac{W}{2L} (V_{GS} - V_T)^2$$

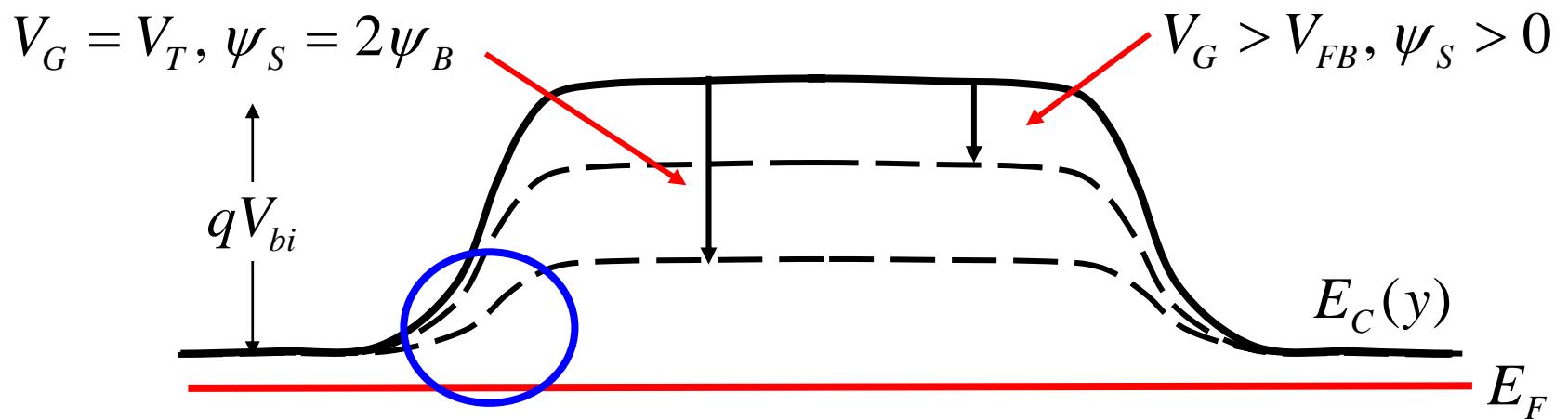
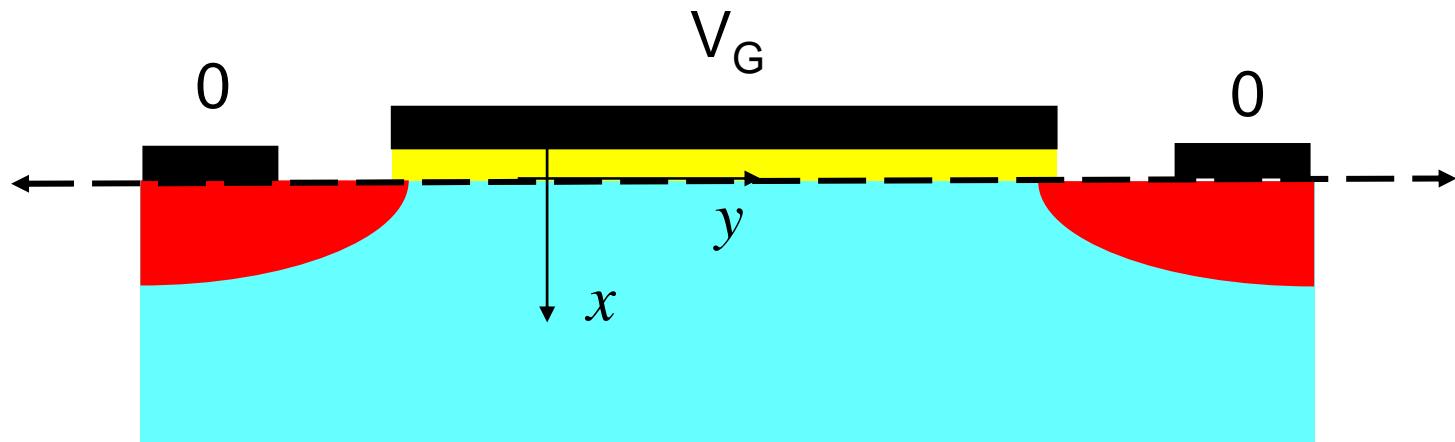
$$I_D = W C_{ox} (V_{GS} - V_T) \mu_{eff} E_y(0)$$

$$E_y(0) = \frac{(V_{GS} - V_T)}{2L}$$

outline

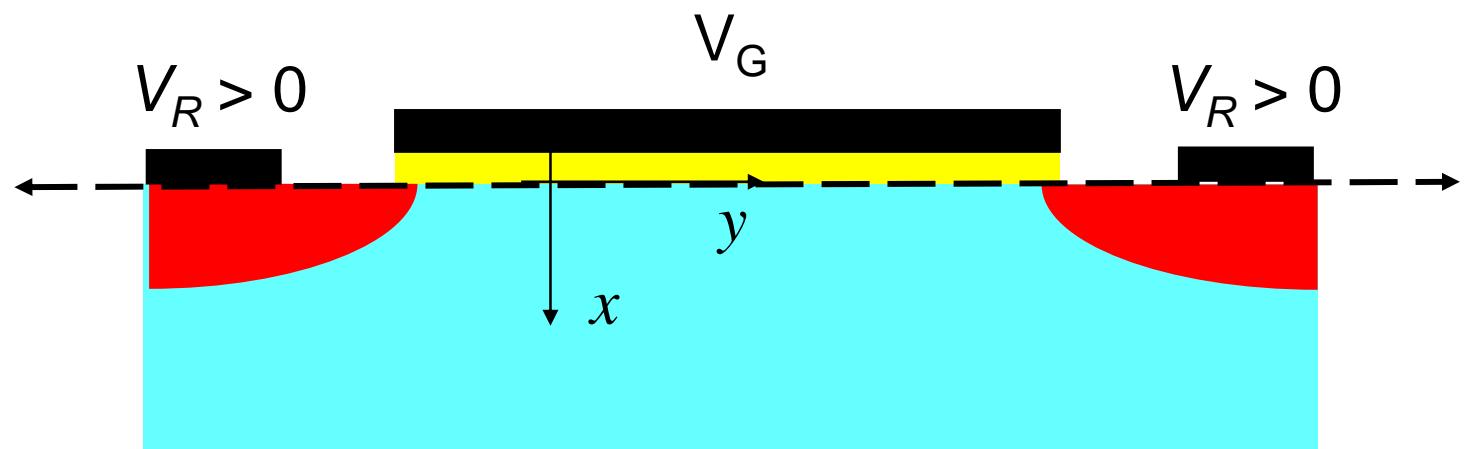
- 1) Introduction
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energy band diagram along the channel

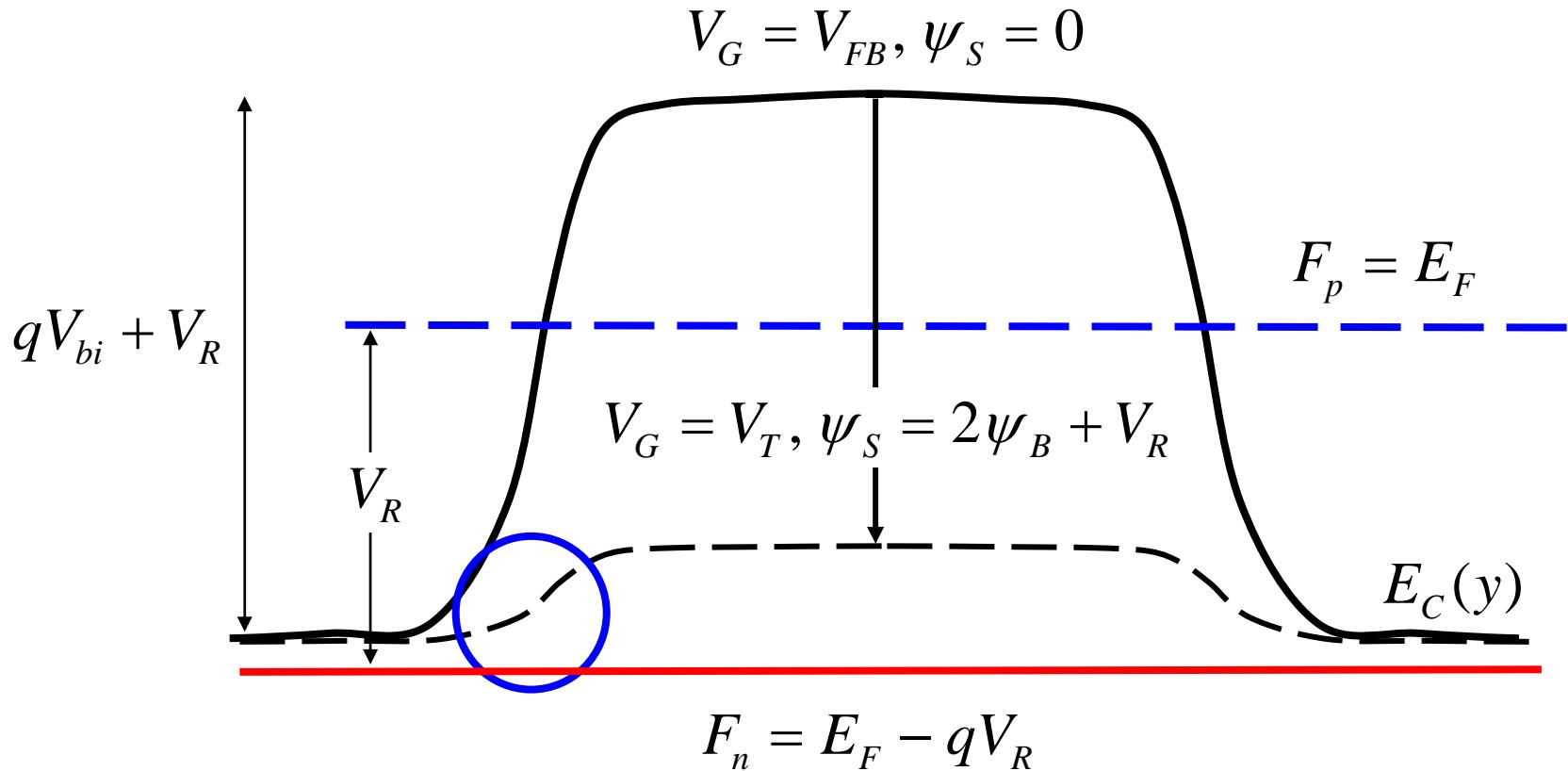


$$\Delta E = q(V_{bi} - 2\psi_B) = k_B T \ln(N_D/N_A) \approx 0.1 \text{ eV}$$

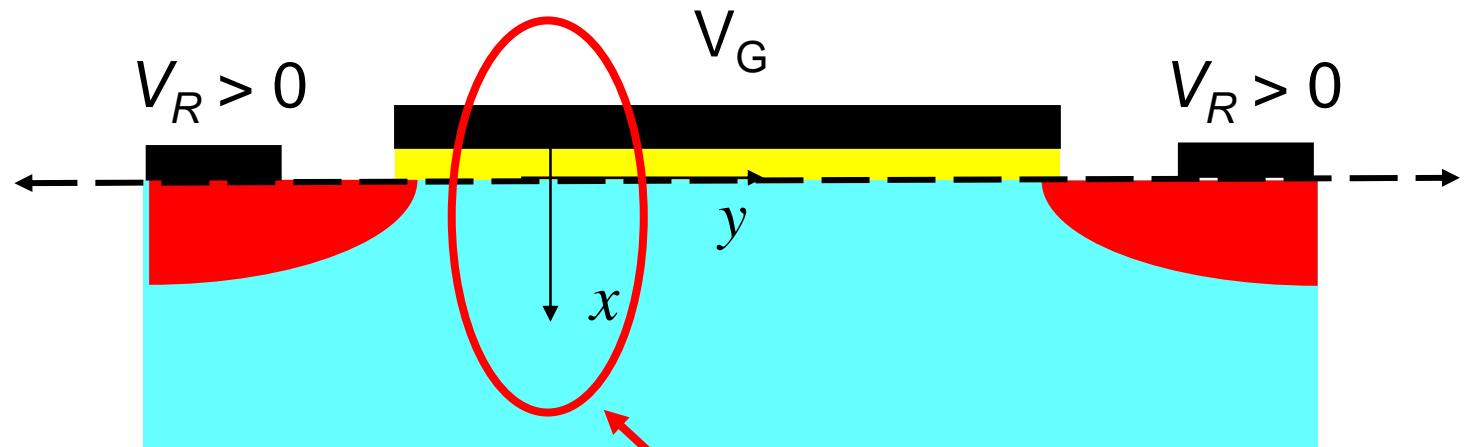
effect of a reverse bias



effect of a reverse bias

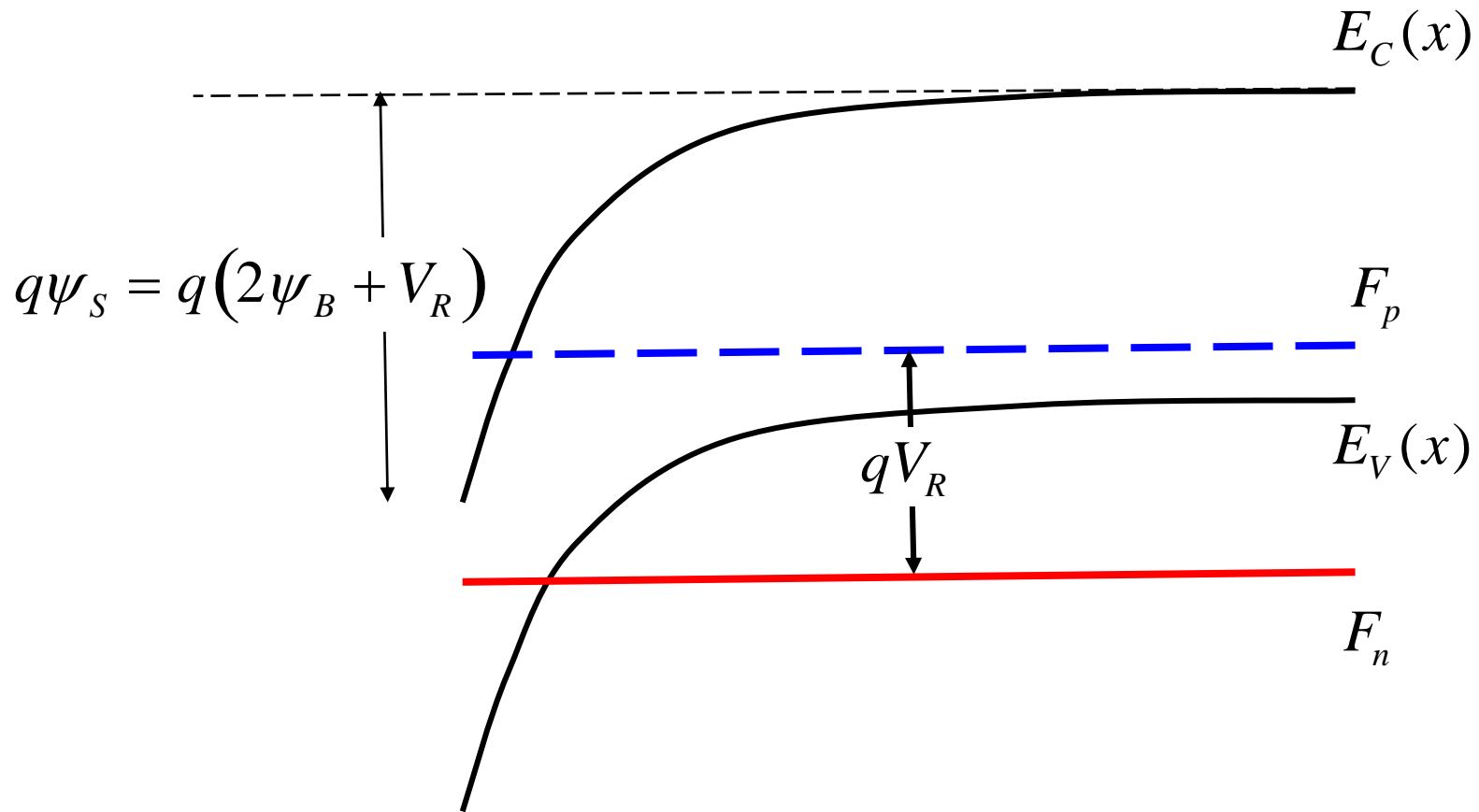


effect of a reverse bias



now look at E_C vs. x

effect of a reverse bias



effect of a reverse bias

no reverse bias:

$$V_T = V_{FB} + 2\psi_B + \sqrt{2q\varepsilon_{Si}N_A(2\psi_B)} / C_{ox}$$

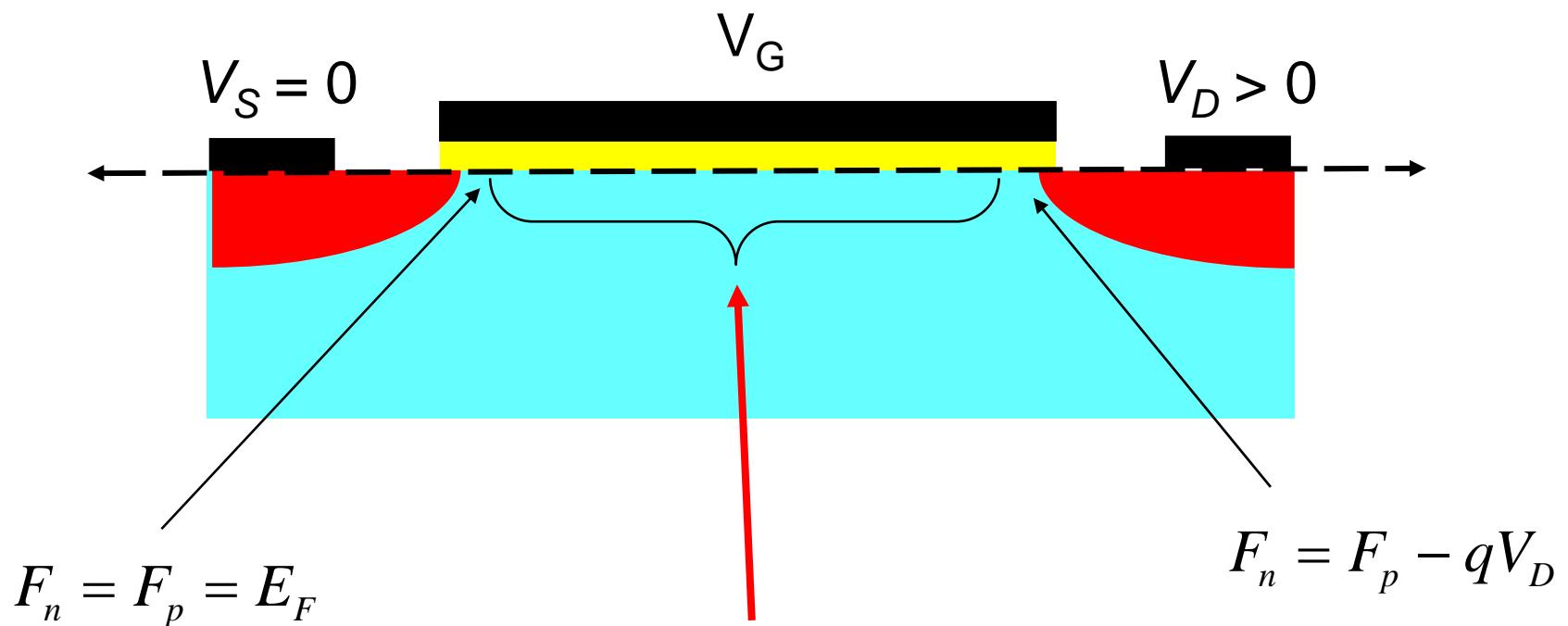
with reverse bias (V_G at onset of inversion):

$$V_G = V_{FB} + 2\psi_B + V_R + \underline{\sqrt{2q\varepsilon_{Si}N_A(2\psi_B + V_R)}} / C_{ox}$$

V_T of the MOSFET is defined as V_{GS} at the onset of inversion

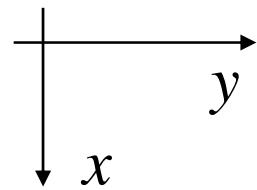
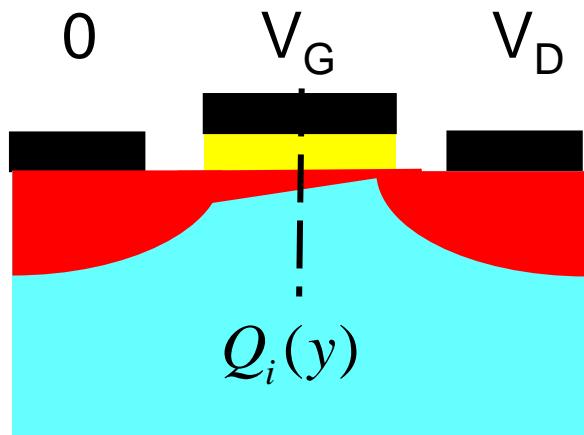
$$V_T = V_G - V_R = V_{FB} + 2\psi_B + \sqrt{2q\varepsilon_{Si}N_A(2\psi_B + V_R)} / C_{ox}$$

back to the MOSFET



F_n increasingly negative from source to drain
(reverse bias increases from source to drain)

back to MOSFET I-V

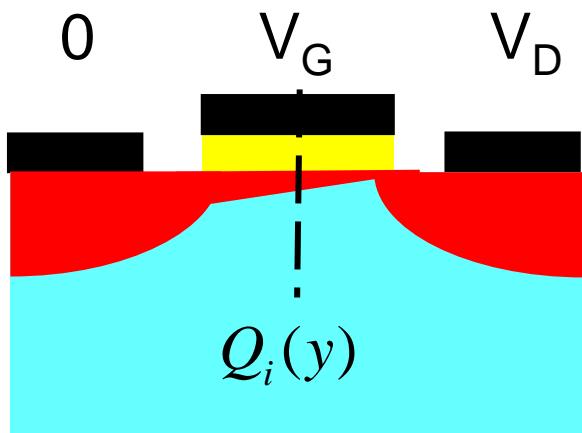


$$Q_I(y) = -C_{ox} [V_G - V_T(y)]$$

bulk charge

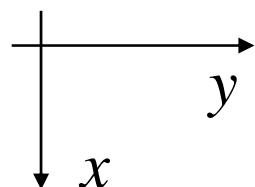
$$V_T(y) = V_{FB} + 2\psi_B + V(y) + \sqrt{2q\epsilon_{Si}N_A [2\psi_B + V(y)]}/C_{ox}$$

relation to square law



$$Q_I(y) = -C_{ox} [V_G - V_T(y)]$$

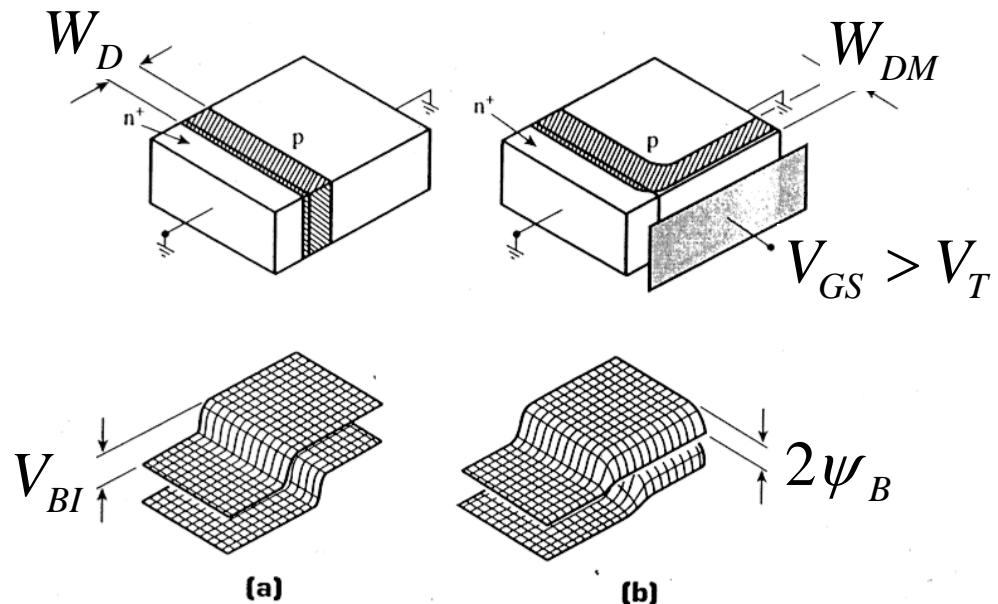
$$Q_I(y) = -C_{ox} [V_G - V_T - V(y)] \quad (\text{square law})$$



$$V_T(y) = V_{FB} + 2\psi_B + V(y) + \sqrt{2q\epsilon_{Si}N_A [2\psi_B + V(y)]}/C_{ox}$$

$$V_T(y) \approx V_{FB} + 2\psi_B + V(y) + \sqrt{2q\epsilon_{Si}N_A (2\psi_B)}/C_{ox} = V_T(0) + V(y)$$

effect of a reverse bias

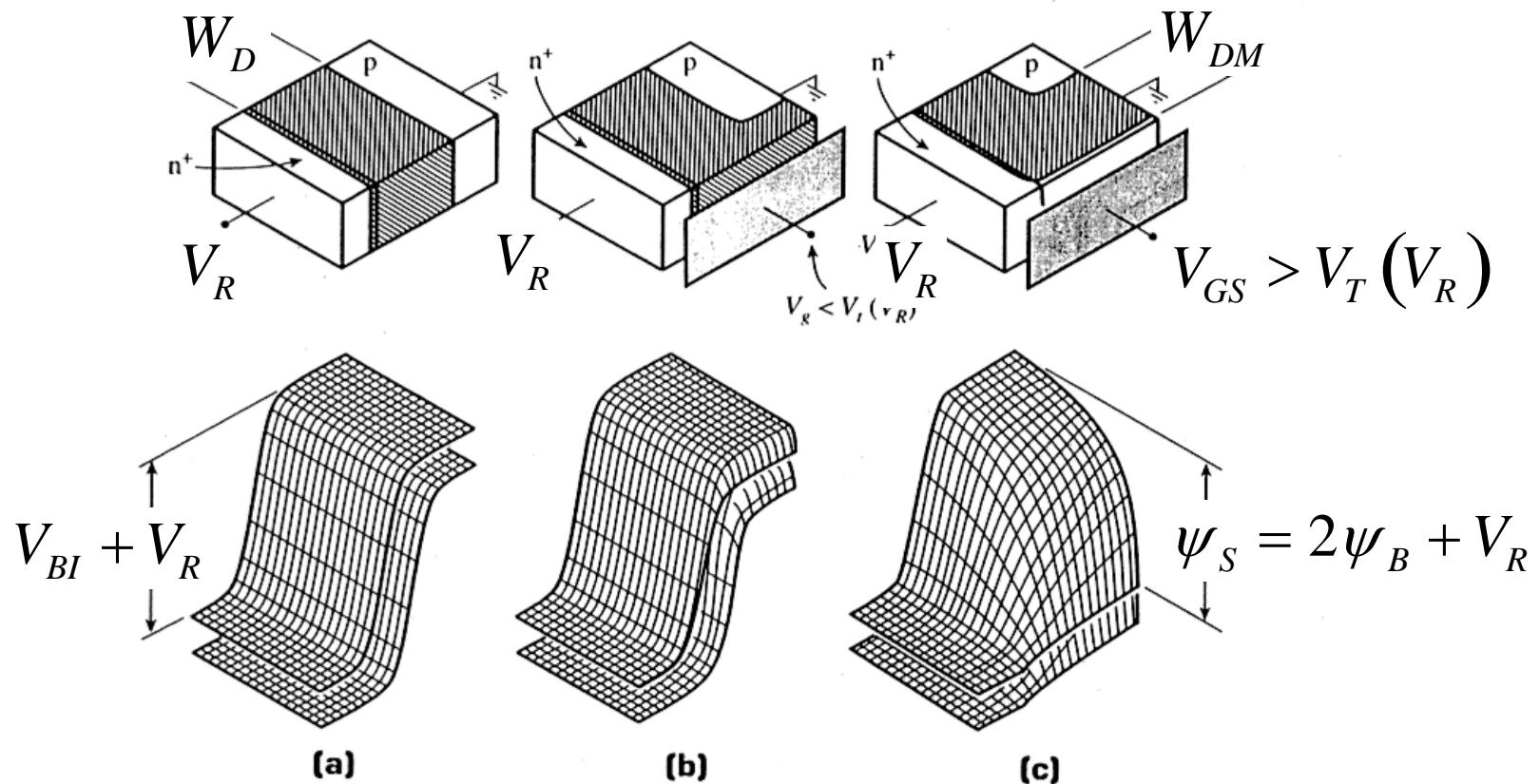


Gated doped or p-MOS with adjacent n⁺ region

- a) gate biased at flat-band
- b) gate biased in inversion

A. Grove, *Physics of Semiconductor Devices*, 1967.

effect of a reverse bias



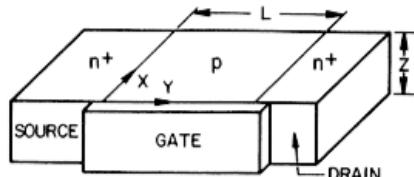
Gated doped or p-MOS with adjacent, reverse-biased n⁺ region

- a) gate biased at flat-band
- b) gate biased in depletion
- b) gate biased in inversion

A. Grove, *Physics of Semiconductor Devices*, 1967.

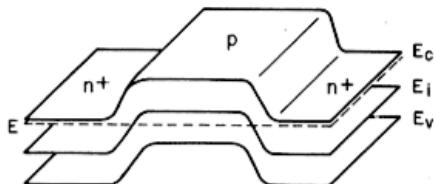
the MOSFET

(a)

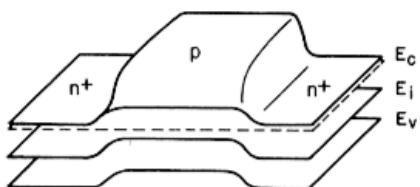


2D e-band diagram for an n-MOSFET

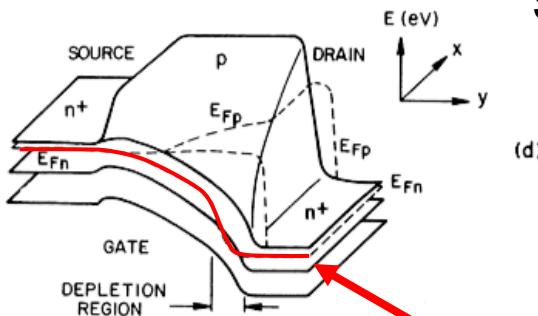
(b)



(c)



(d)



a) device

b) equilibrium (flat band)

c) equilibrium ($\psi_s > 0$)

d) non-equilibrium with V_G and $V_D > 0$ applied

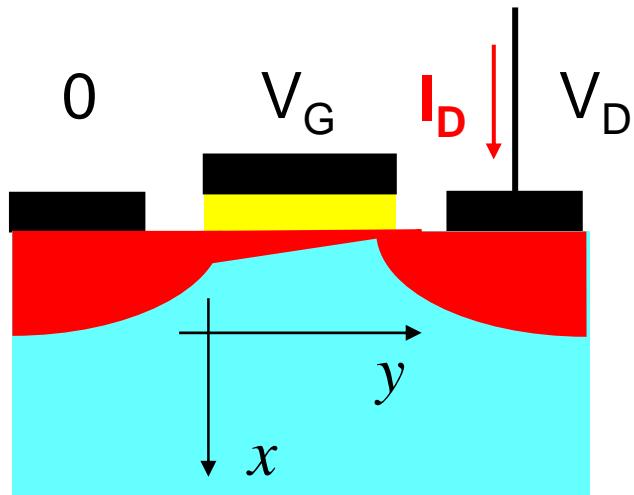
SM. Sze, *Physics of Semiconductor Devices*, 1981
and Pao and Sah.

F_N

outline

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I-V formulation



$$I_D = W Q_I(y) v_y(y)$$

$$I_D = -\frac{W}{L} \mu_{eff} \int_0^{V_{DS}} Q_I(V) dV$$

$$Q_I(y) = -C_{ox} [V_G - V_T(y)]$$

$$V_T(y) = V_{FB} + 2\psi_B + V(y) - Q_D(V)/C_{ox}$$

$$V_T(y) = V_{FB} + 2\psi_B + V(y) + \sqrt{2q\epsilon_{Si}N_A(2\psi_B + V(y))}/C_{ox}$$

IV relation

$$I_D = -\frac{W}{L} \mu_{eff} \int_0^{V_{DS}} Q_I(V) dV \quad (1)$$

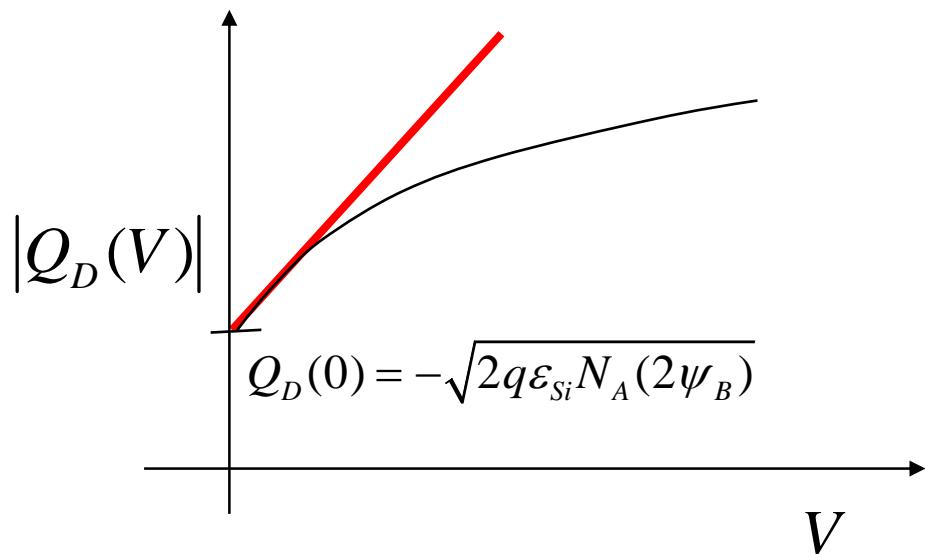
$$Q_I(y) = -C_{ox} \left(V_G - V_{FB} - 2\psi_B - V(y) - \frac{\sqrt{2q\epsilon_{Si}N_A(2\psi_B + V(y))}}{C_{ox}} \right) \quad (2)$$

Insert (2) in (1) and integrate, find eqn. (3.18) of Taur and Ning, but 3/2 powers are inconvenient.

(See Pierret, for a more extended discussion of the bulk charge theory.)

can we approximate Q_D ?

$$Q_D(V) = -\sqrt{2q\varepsilon_{Si}N_A(2\psi_B + V)}$$



can we use a linear
approximation for Q_D ?

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MOSFET IV approaches

$$I_D = -\frac{W}{L} \mu_{eff} \int_0^{V_{DS}} Q_I(V) dV$$

- 1) “exact” (Pao-Sah or Pierret-Shields)
see p. 117 Taur and Ning

- 2) Square Law

$$Q_I(V) = -C_{ox} [V_G - V_T - V]$$

- 3) Bulk Charge

$$Q_I(V) = -C_{ox} \left(V_G - V_{FB} - 2\psi_B - V - \frac{\sqrt{2q\epsilon_{Si}N_A(2\psi_B + V)}}{C_{ox}} \right)$$

MOSFET IV approaches

In practice, a simplified bulk charge theory (to be discussed in Part II) is typically used.

But none of these theories describe modern short channel MOSFETs, for this $I_D \sim (V_{GS} - V_T)$ in the beyond pinch-off region. For that, we need to consider ‘velocity saturation,’ which is also discussed in Part II of this lecture.

suggested reference

For a thorough treatment of MOSFET theory, see:

Yannis Tsividis, *Operation and Modeling of the MOS Transistor*, 2nd Edition, WCB McGraw-Hill, Boston, 1999.

especially Chapters 3 and 4