EE-612:
Lecture 11:
Effective Mobility

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outline

1) Review of mobility
2) “Effective” mobility
3) Physics of the effective mobility
4) Measuring effective mobility
5) Discussion
6) Summary

mobility

$$\mu = \frac{q\tau}{m^*}$$

$(1/\tau)\,\Delta t$ = probability per second of scattering

[$(1/\tau)$ is ‘scattering rate’]

τ = average time between scattering events

$$\mu_1 = \frac{q\tau_1}{m^*} \qquad \mu_2 = \frac{q\tau_2}{m^*} \qquad \mu_{12} = ? \qquad \mu_{12} \neq \mu_1 + \mu_2$$

$$\mu_{12} = \left(\frac{1}{\mu_1} + \frac{1}{\mu_2} \right)^{-1}$$

Mathiessen’s Rule

diffusion coefficient

$$D = \frac{\upsilon_T \lambda}{2} = \upsilon_R \lambda$$

υ_T unidirectional thermal velocity

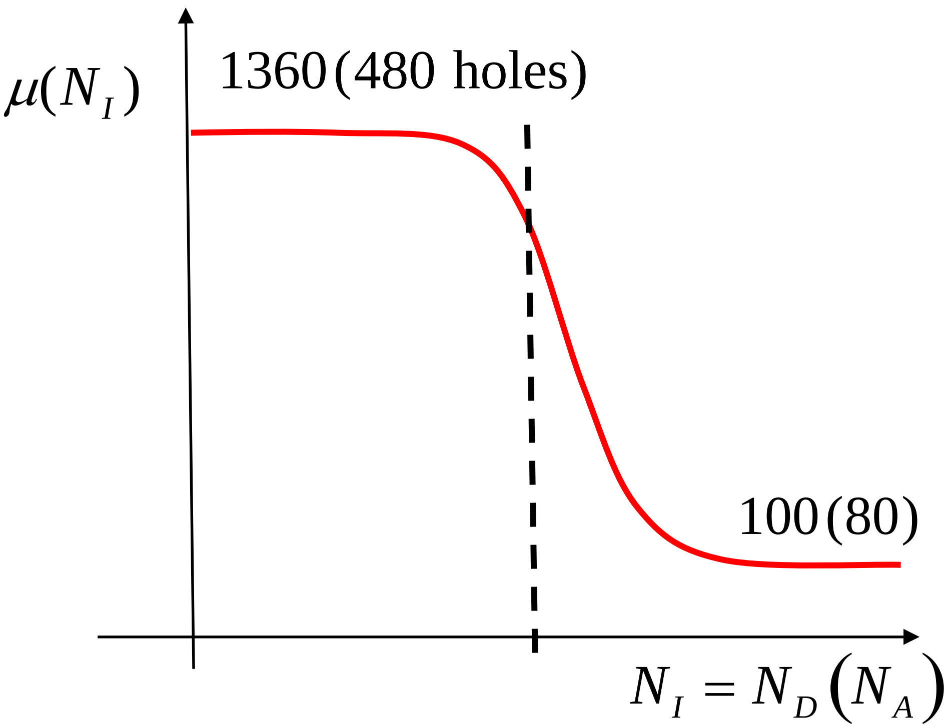
υ_R Richardson velocity

λ mean-free-path for scattering

$$\frac{D}{\mu} = \frac{k_B T}{q}$$

$$D \leftrightarrow \mu$$

mobility vs. doping density (silicon)



$$N_{CR} \approx 10^{17}\ \text{cm}^{-3}$$

$$\mu_L : T^{-3/2}$$

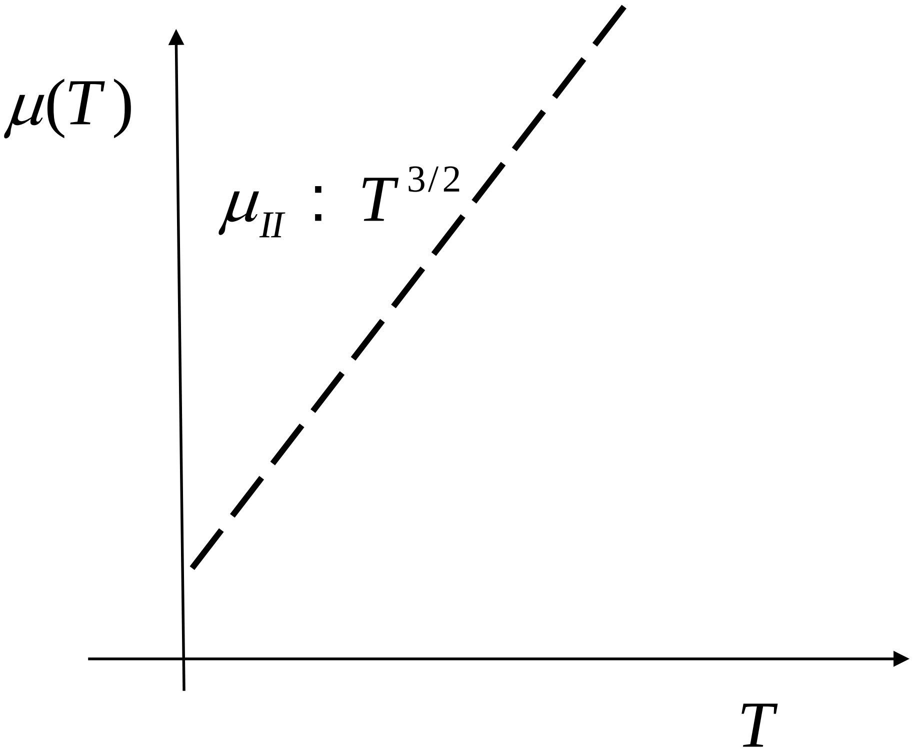
$$\mu_{II} : T^{3/2} / N_I$$

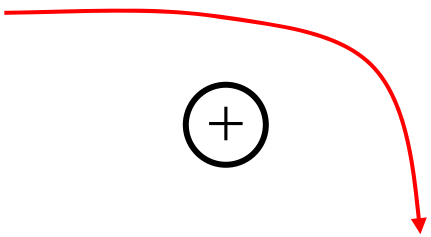
$$\mu = \left(\frac{1}{\mu_L} + \frac{1}{\mu_{II}} \right)^{-1}$$

$$\mu\left(N_I << N_{CR}\right) \approx \mu_L$$

$$\mu\left(N_I >> N_{CR}\right) \approx \mu_I$$

μ_{II} temperature dependence (silicon)



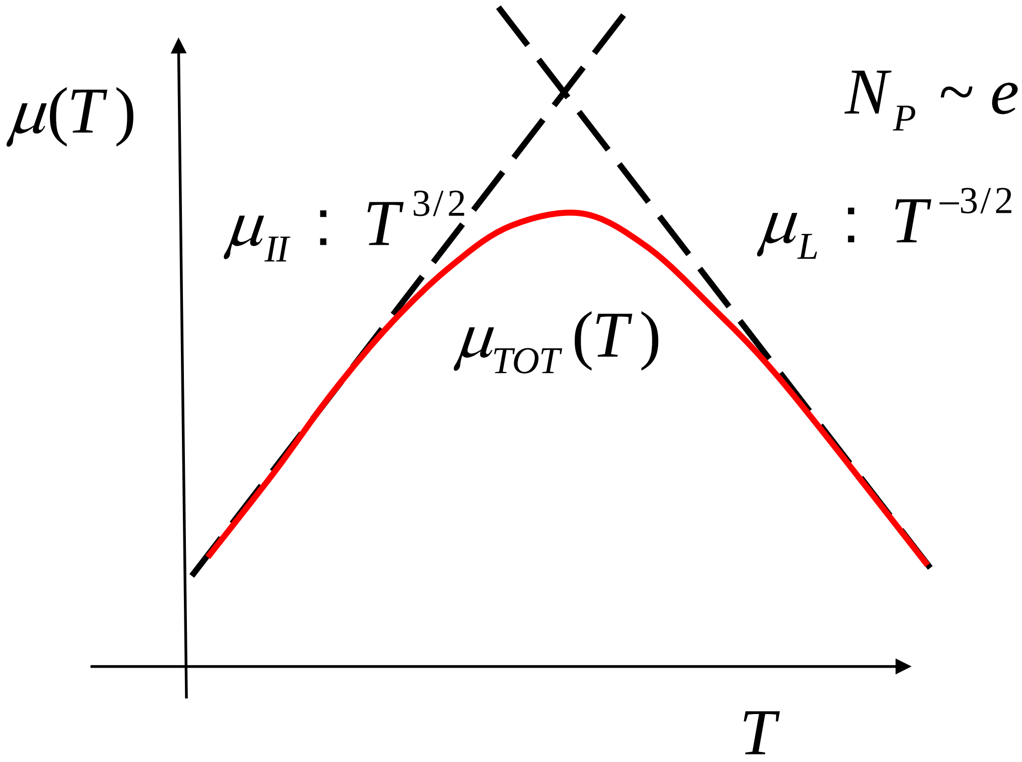


$$\frac{1}{2}m^*\upsilon^2 = \frac{3}{2}k_B T$$

lattice scattering (silicon)

$$N_P \sim \frac{k_B T}{\hbar\omega} \quad \text{(acoustic phonons)}$$

$$N_P \sim e^{-\hbar\omega/k_B T} \quad \text{(optical phonons)}$$

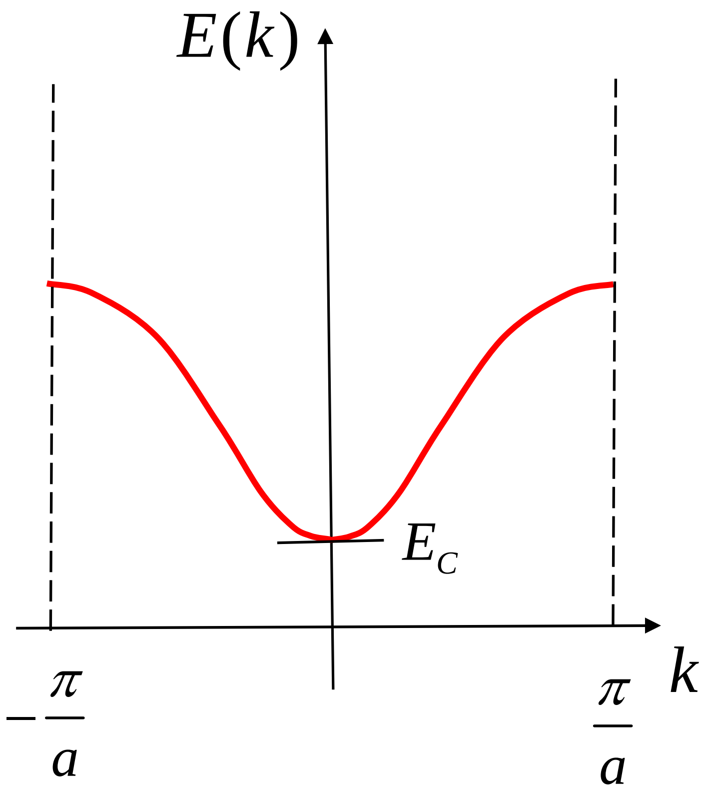


$$\mu_{TOT} = \left(\frac{1}{\mu_L} + \frac{1}{\mu_{II}}\right)^{-1}$$

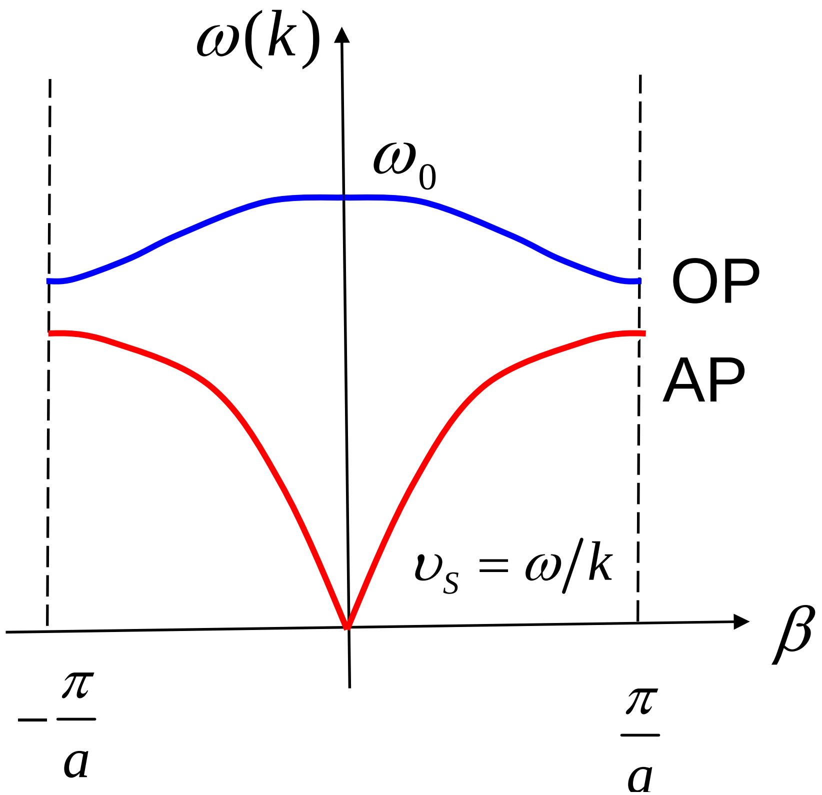
Mathiessen's Rule

comment (added after lecture)

electrons



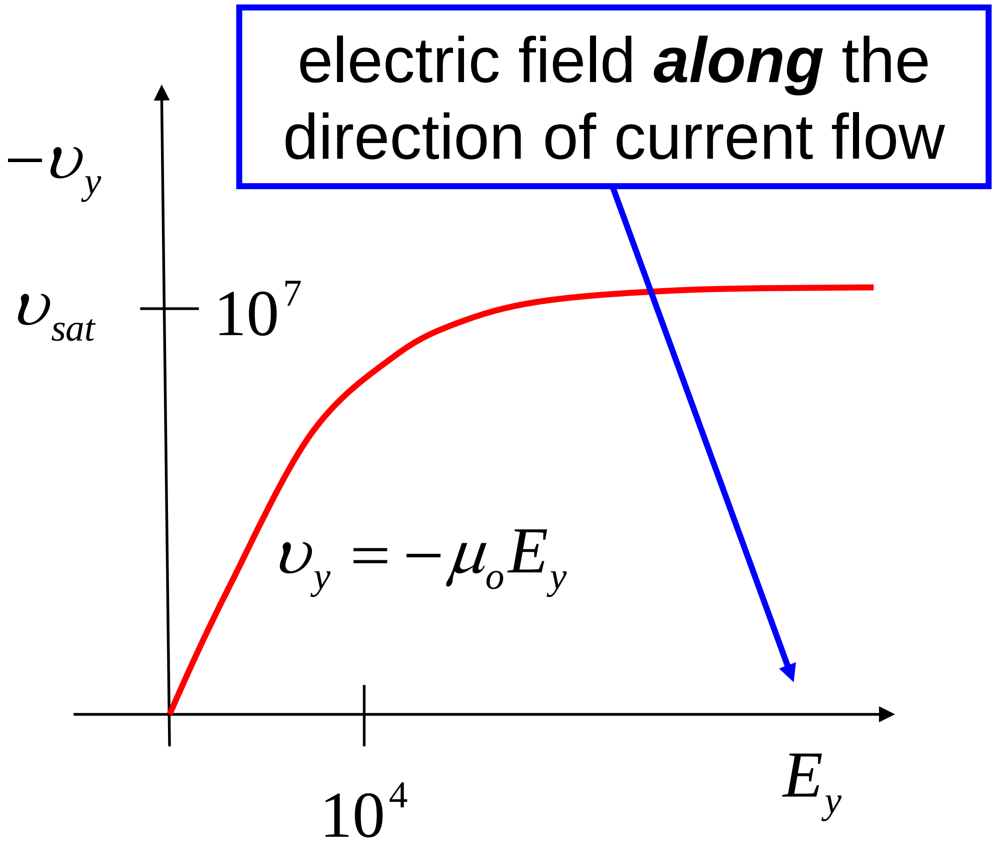
phonons



Electrons in a periodic structure are characterized by "dispersion curve" $E(k)$.

Phonons in a periodic structure are characterized by "dispersion curve" $\omega(\beta)$. Two types of phonons, acoustic (AP) and optical (OP).

high-field transport (silicon)



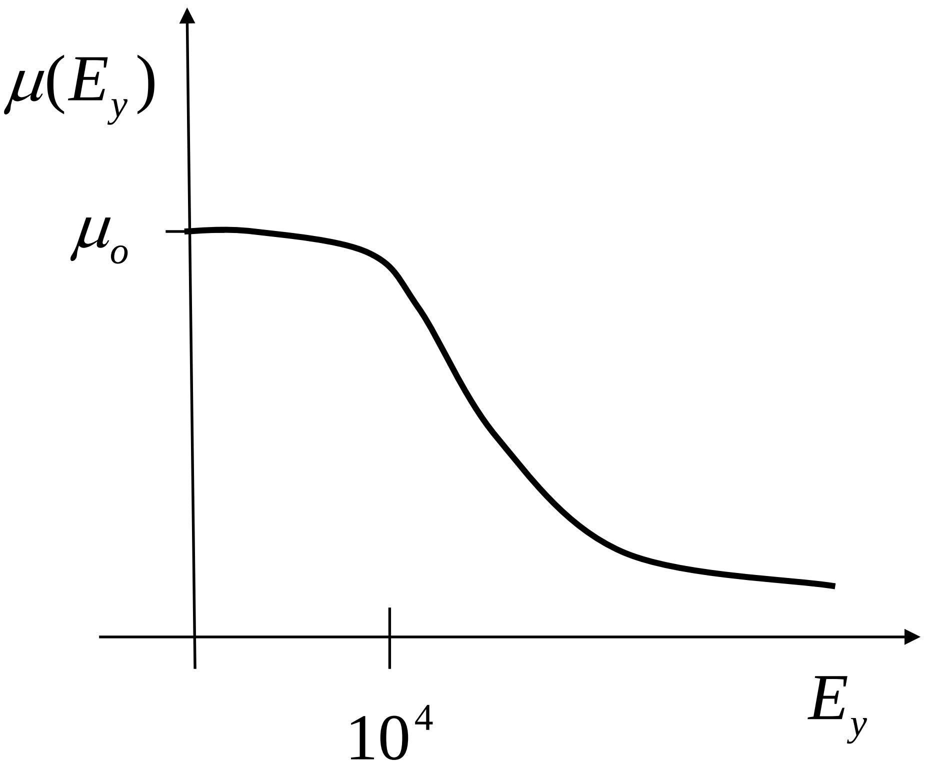
$$\upsilon_y = \frac{-\mu_o E_y}{\sqrt{1+\left(E_y / E_{cr}\right)^2}}$$

$$E_y >> E_{cr} \rightarrow$$

$$\left|\upsilon_y\right| \rightarrow \mu_o E_{cr} \equiv \upsilon_{sat}$$

$$E_{cr} = \frac{\upsilon_{sat}}{\mu}$$

field-dependent mobility



$$\upsilon_y = \mu(E_y)E_y$$

$$\mu(E_y) \equiv \frac{-\upsilon_y}{E_y}$$

$$= \frac{\mu_o}{\sqrt{1 + (E_y/E_{cr})^2}}$$

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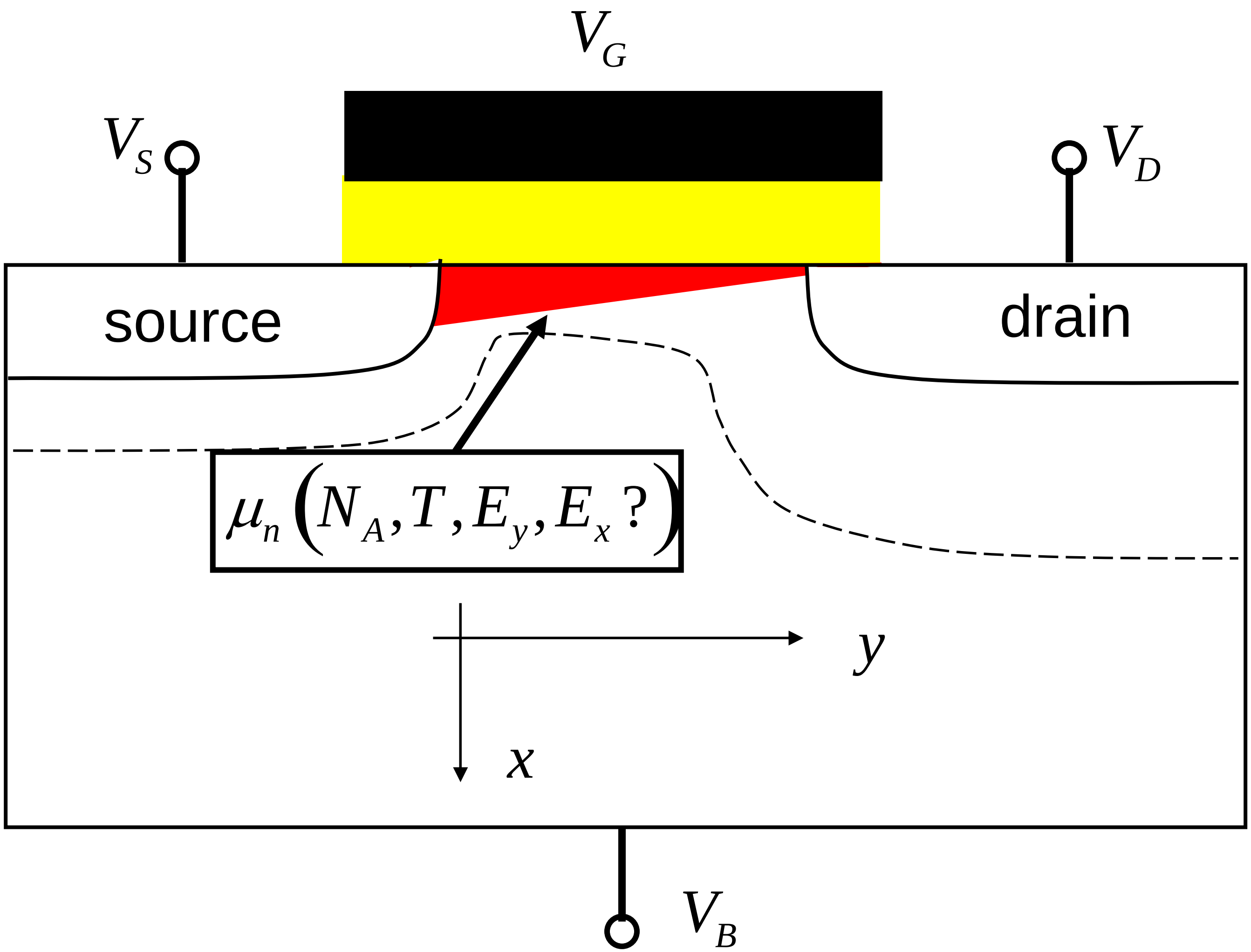
mobility in an inversion layer



effective mobility

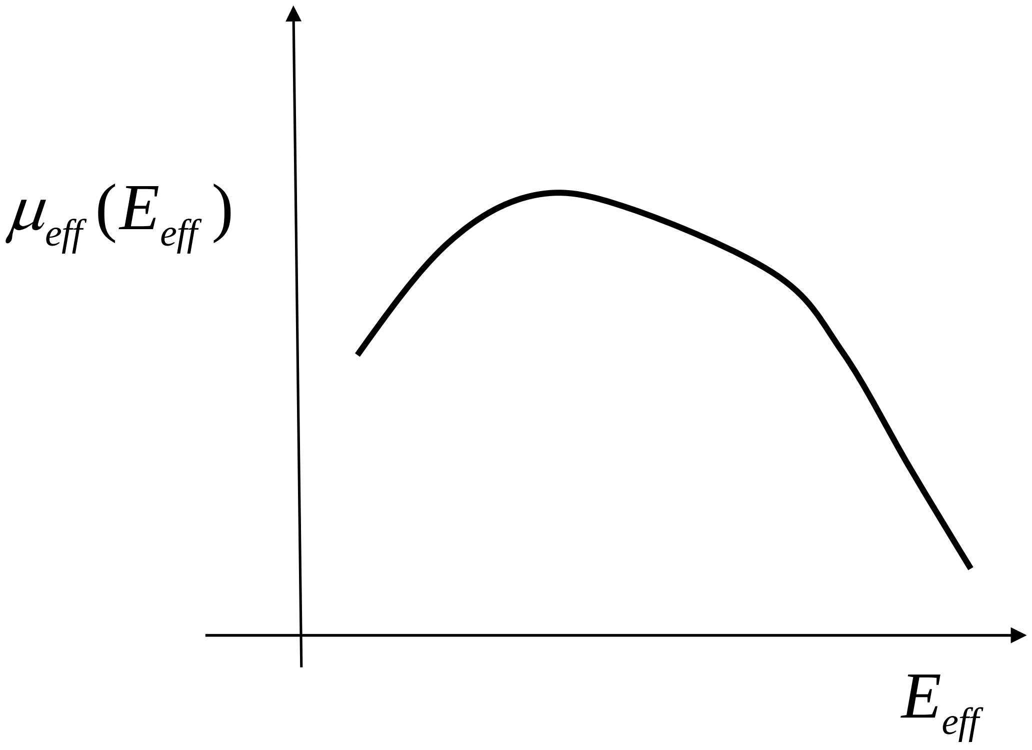
$$I_D = -WQ_i(y)\upsilon_y = WC_G\left[V_G - V_T - mV(y)\right]\underline{\mu_n(x,y)}\frac{dV}{dy}$$

$$I_D\int_0^L dy = WC_G\,\mu_{eff}\int_0^{V_{DS}}\left[V_G - V_T - mV(y)\right]dV$$

$$\mu_{eff} = \frac{\int_0^\infty n(x)\mu_n(x)dx}{\int_0^\infty n(x)dx}$$

(If we only consider the x-dependence, i.e. normal to the surface)

effective normal field



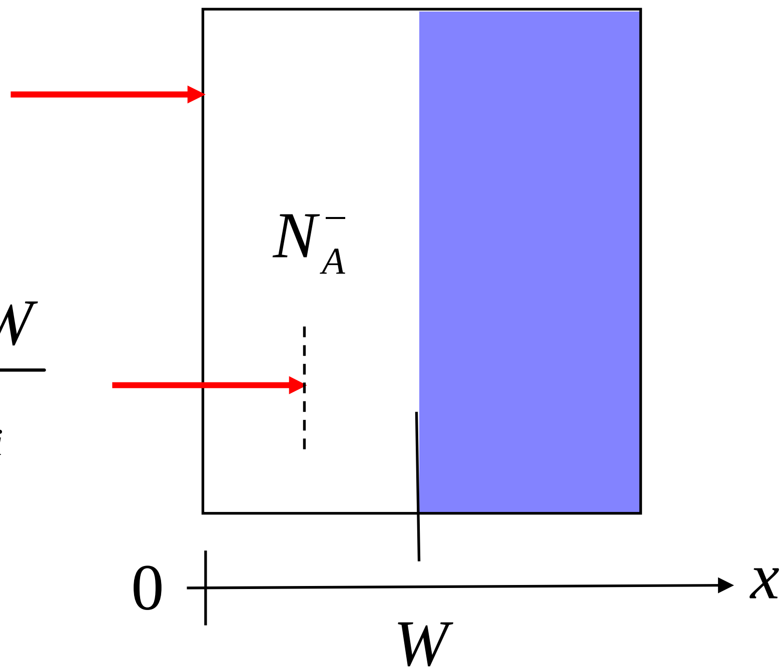
$$E_{eff} \equiv \frac{\int_0^\infty n(x)E_x(x)dx}{\int_0^\infty n(x)dx}$$

effective normal field

below threshold

$$E_S = \frac{qN_A W}{\varepsilon_{Si}}$$

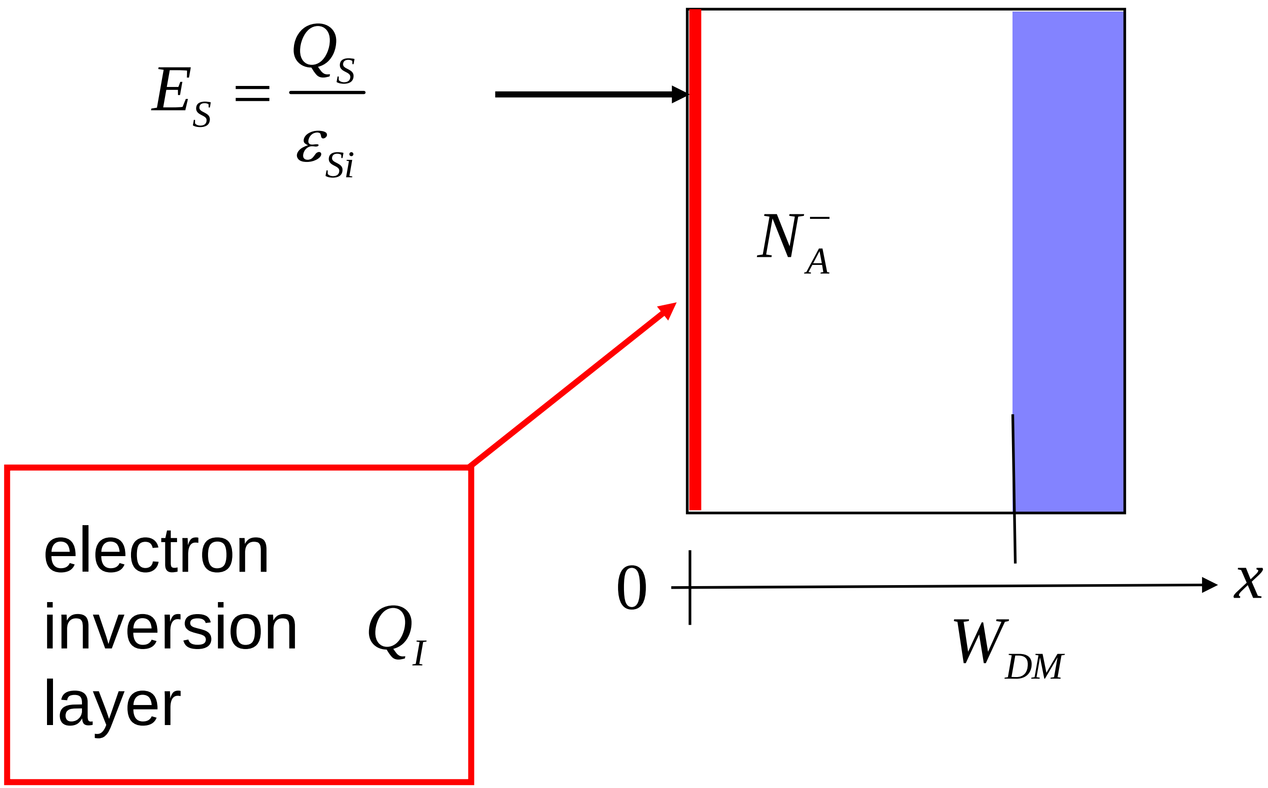
$$E(W/2) = \frac{qN_A W}{2\varepsilon_{Si}}$$



effective normal field (ii)

above threshold

$$E_S = \frac{Q_S}{\varepsilon_{Si}}$$



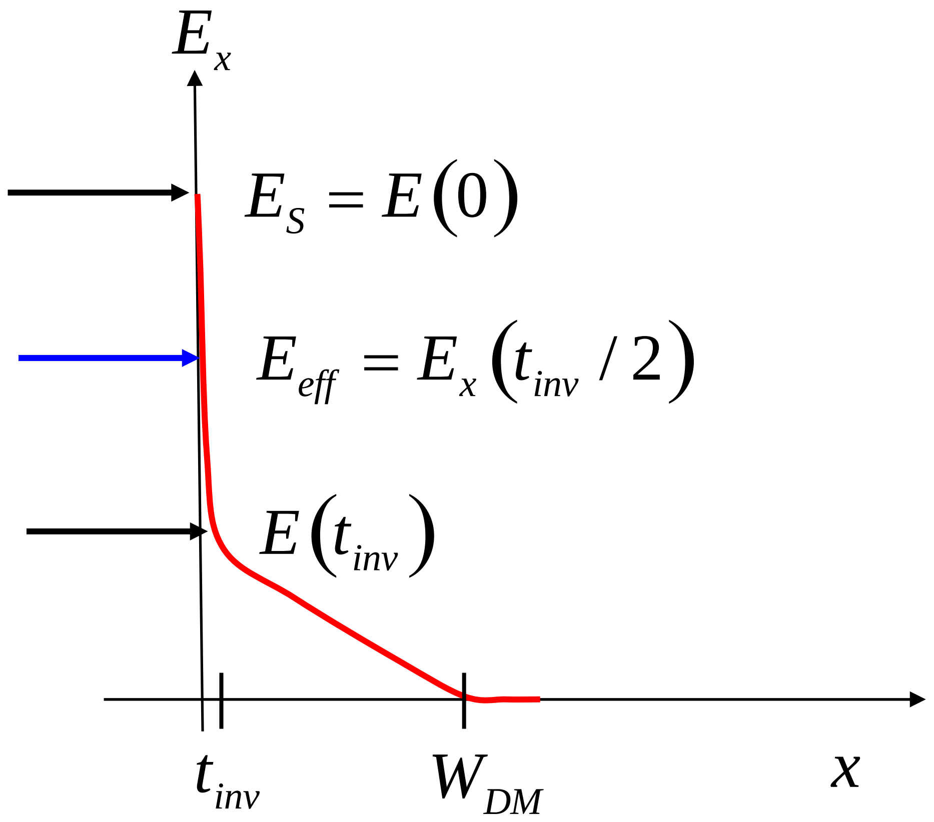
effective normal field (iii)

$$E_S = \frac{|Q_S|}{\varepsilon_{Si}}$$

$$E_S = \frac{1}{\varepsilon_{Si}}\left(|Q_{DM}| + |Q_I|\right)$$

$$E(t_{inv}) = \frac{|Q_{DM}|}{\varepsilon_{Si}}$$

$$E_{eff} = \frac{1}{\varepsilon_{Si}}\left(|Q_{DM}| + \frac{|Q_I|}{2}\right)$$



effective normal field (iv)

$$E_{eff} = \frac{1}{\varepsilon_{Si}}\left(\left|Q_{DM}\right| + \frac{\left|Q_I\right|}{2}\right) \qquad (1)$$

$$V_T = V_{FB} + 2\psi_B + \frac{\left|Q_{DM}\right|}{C_{ox}}$$

$$\left|Q_I\right| = C_G\left(V_{GS} - V_T\right)$$

$$\begin{cases} \left|Q_{DM}\right| = C_{ox}\left(V_T - V_{FB} - 2\psi_B\right) \\ \left|Q_I\right| \approx C_{ox}\left(V_{GS} - V_T\right) \end{cases}$$

$$E_{eff} = \frac{C_{ox}}{\varepsilon_{Si}}\left(V_T - V_{FB} - 2\psi_B + \frac{V_G}{2} - \frac{V_T}{2}\right)$$

effective normal field (v)

$$E_{eff} = \frac{C_{OX}}{\varepsilon_{Si}}\left(V_T - V_{FB} - 2\psi_B + \frac{V_G}{2} - \frac{V_T}{2}\right)$$

$$V_{FB} = -\frac{E_G}{2q} - \psi_B$$

$$E_{eff} = \frac{\varepsilon_{OX}}{\varepsilon_{Si}\, t_{OX}}\left[\left(\frac{V_G + V_T}{2}\right) + \frac{E_G}{2q} - \psi_B\right]$$

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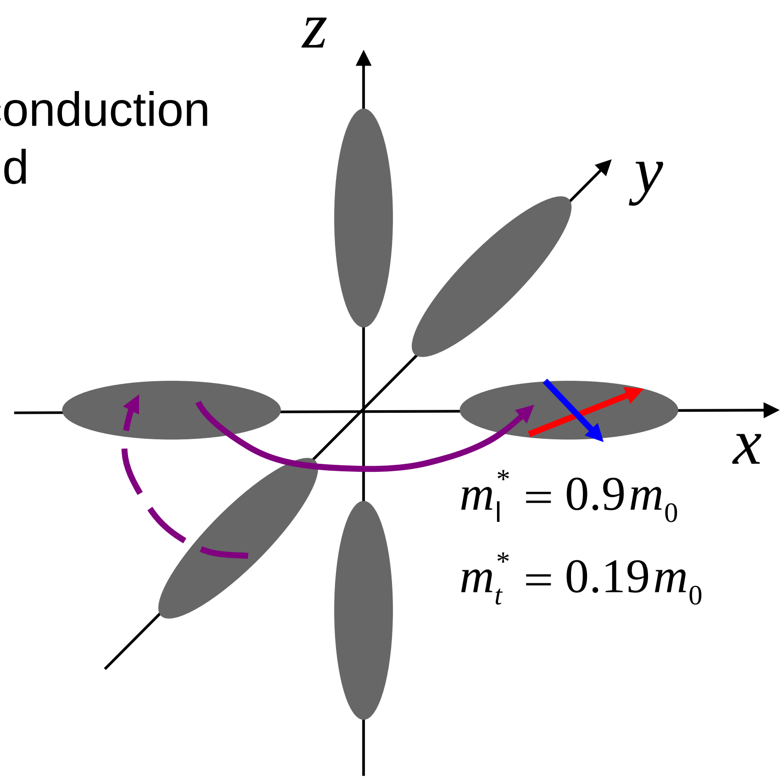
transport in bulk Si

Si conduction band



under low (and modest) fields:

-6 equivalent ellipsoids

-*n*/6 electrons in each one

$$m_C^* = \left(\frac{2}{6m_l^*} + \frac{4}{6m_t^*} \right)^{-1} = 0.26m_0$$

dominant scattering processes:

(low-field, room temperature)

-acoustic phonons (ADP)

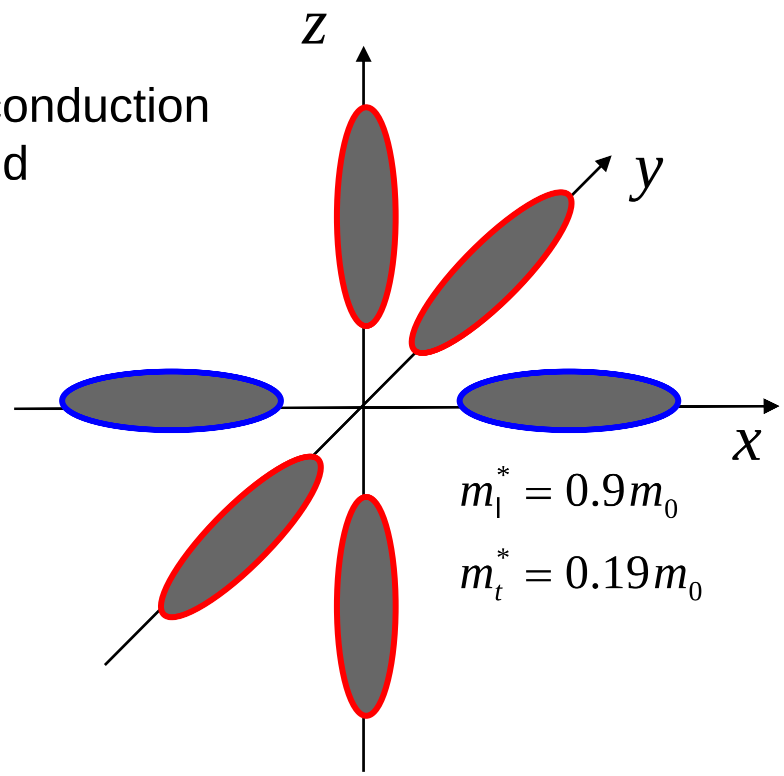
-ionized impurities (II)

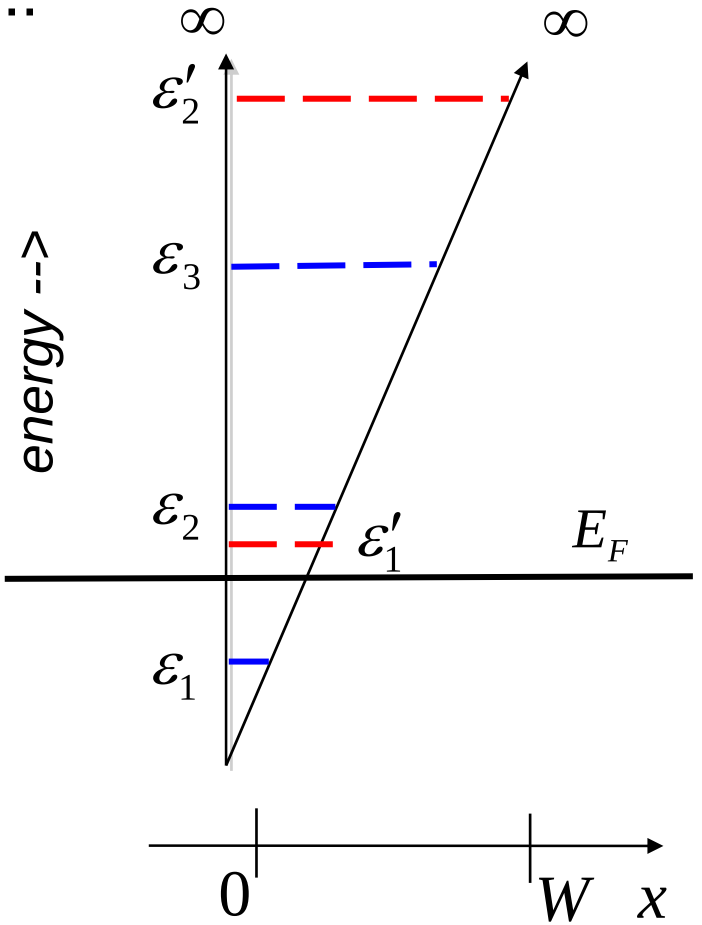
-intervalley phonons (IV)

transport in Si inversion layers

is different from transport in bulk Si…..

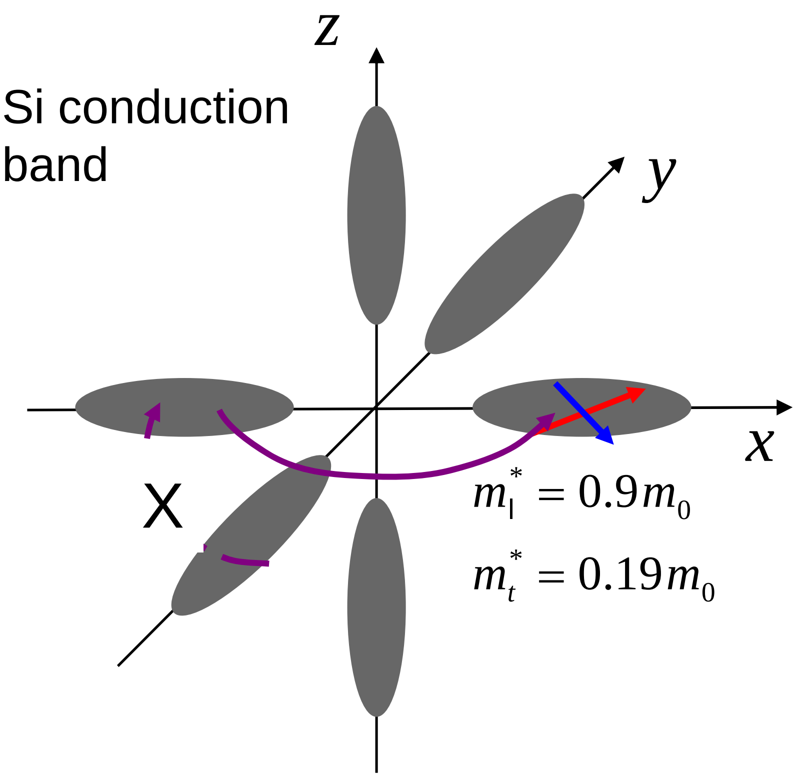
Si conduction band





transport in Si inversion layers

Si conduction band



expectations:

most carriers in unprimed subbands

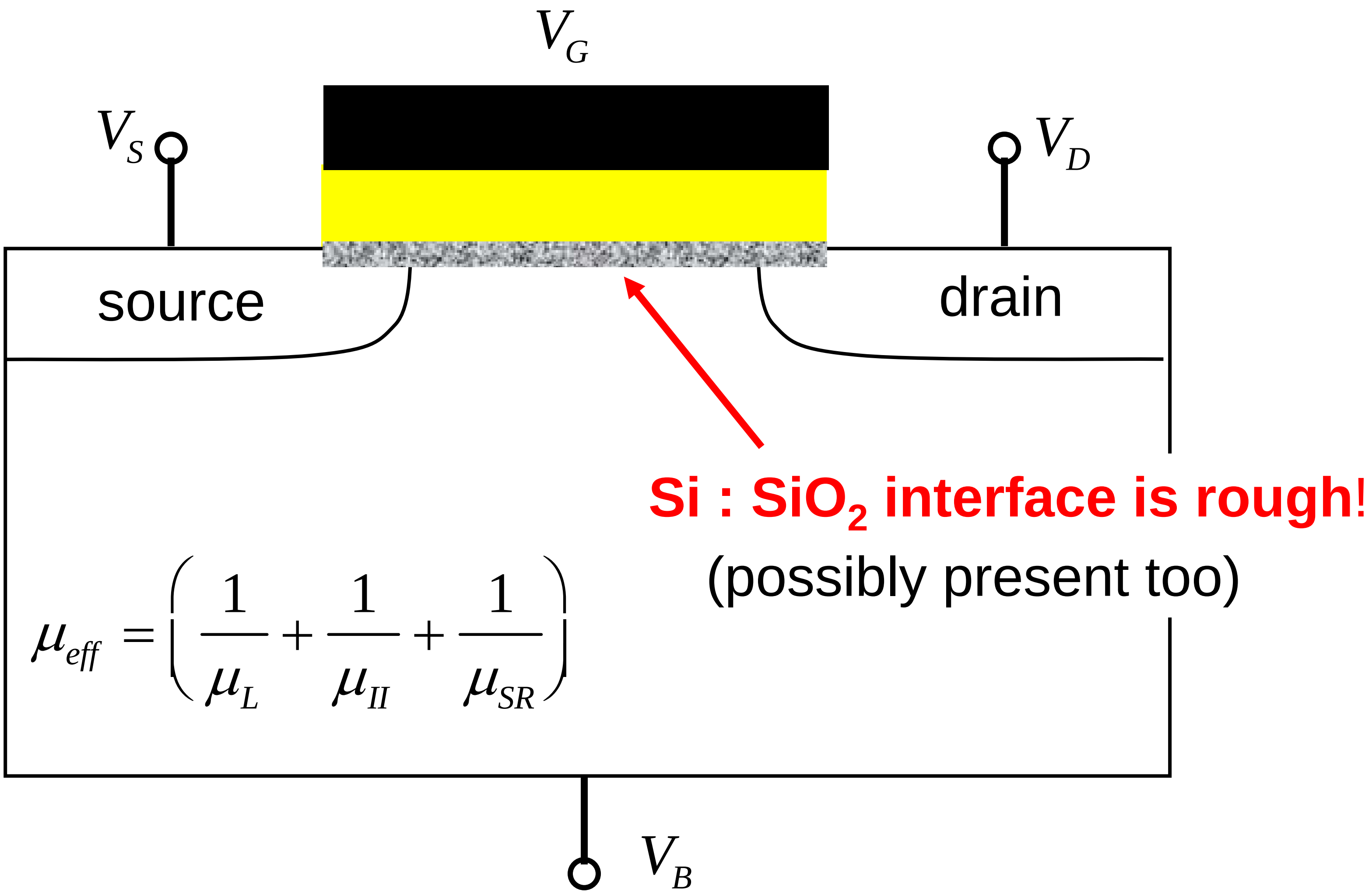
-lighter conductivity *m**

$$m_C^* \approx 0.19 m_0$$

-suppressed intersubband scattering

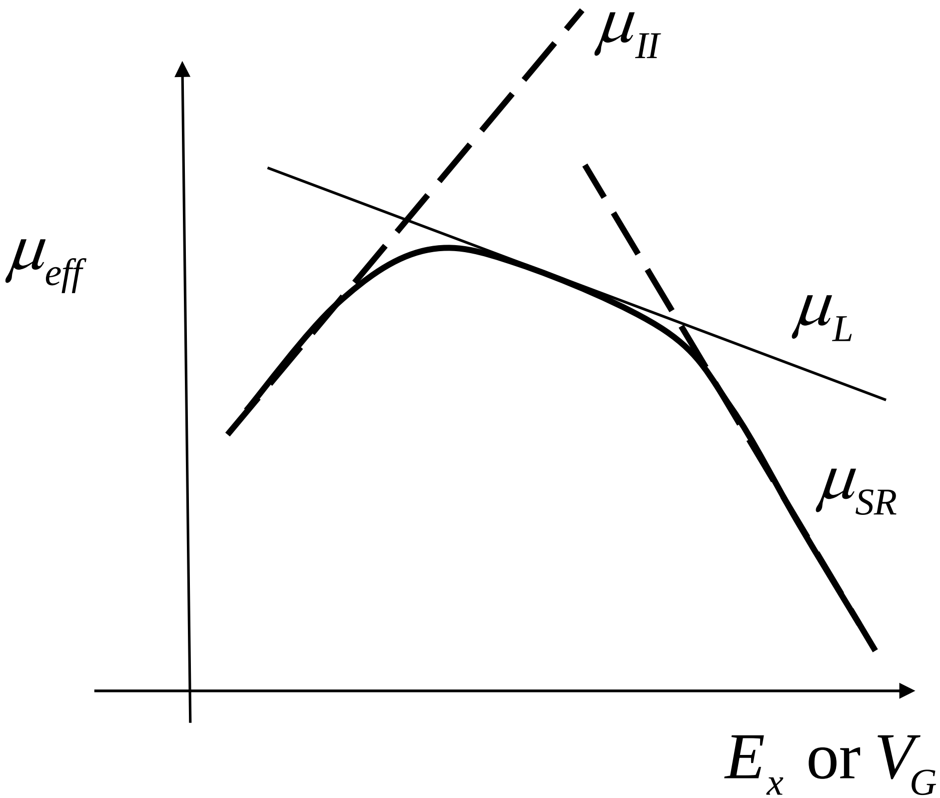
-enhanced intra subband phonon scattering

surface roughness



$$\mu_{eff} = \left(\frac{1}{\mu_L} + \frac{1}{\mu_{II}} + \frac{1}{\mu_{SR}} \right)$$

effective mobility vs. effective field

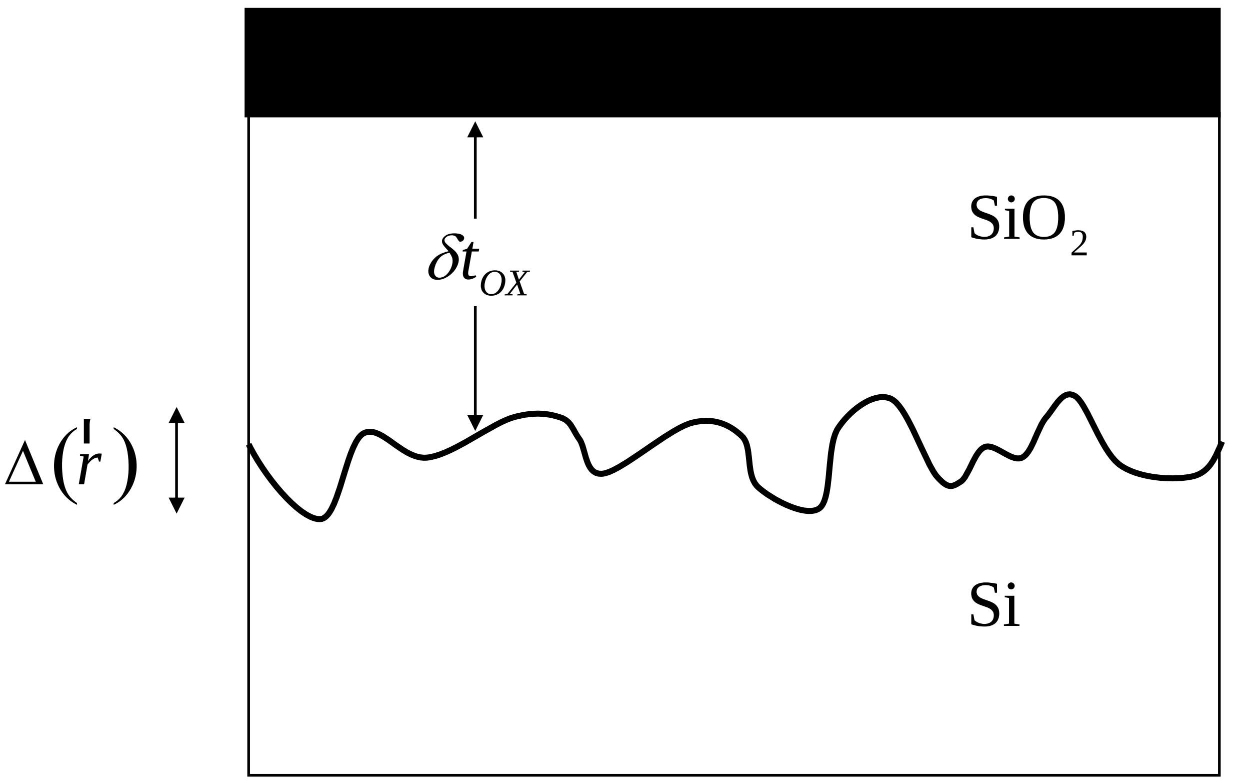


1) screening

2) electron confinement

3) proximity to the surface

surface roughness scattering

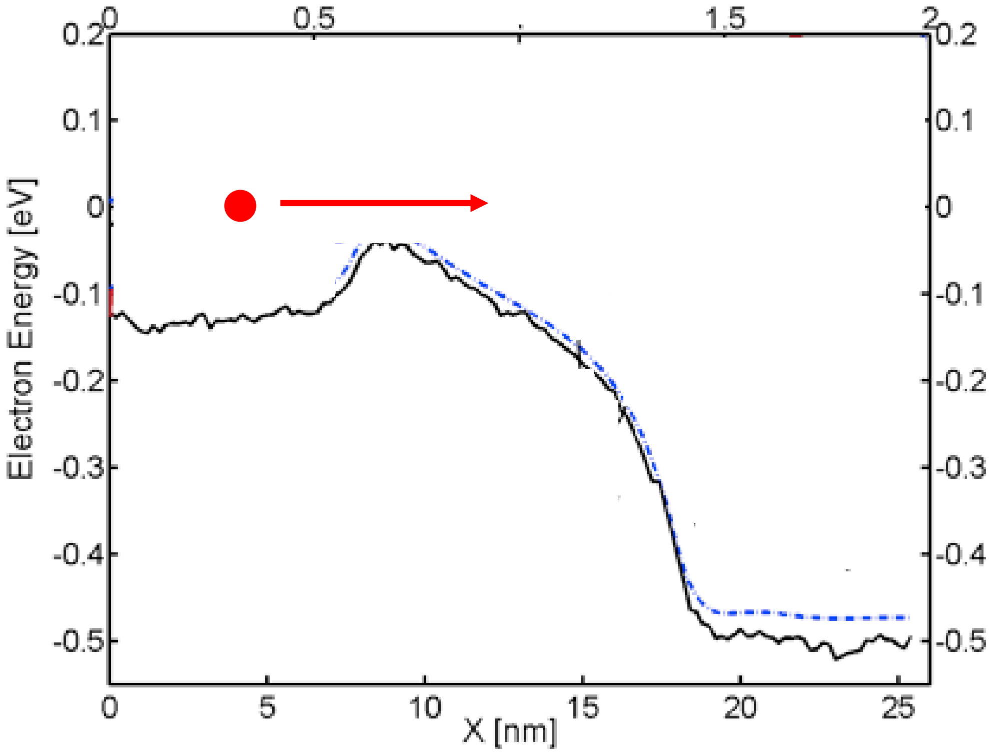


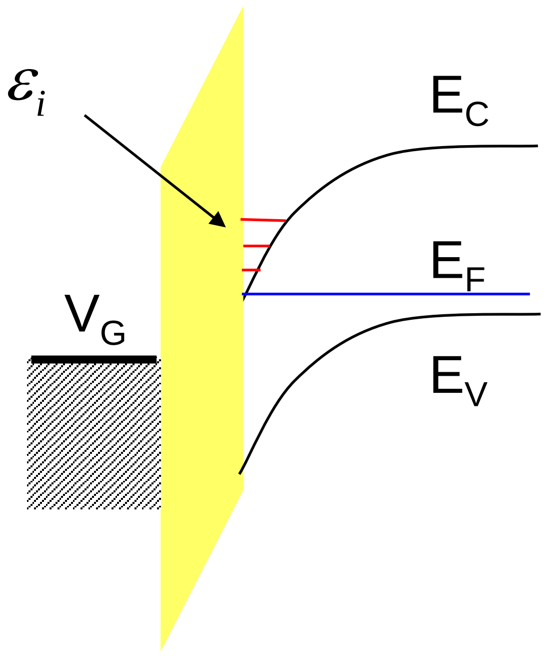
$$V_G = V_{FB} + \psi_S - Q_S / C_{ox}$$

$$\delta t_{OX} \Rightarrow \delta \psi_S$$

surface roughness scattering (ii)

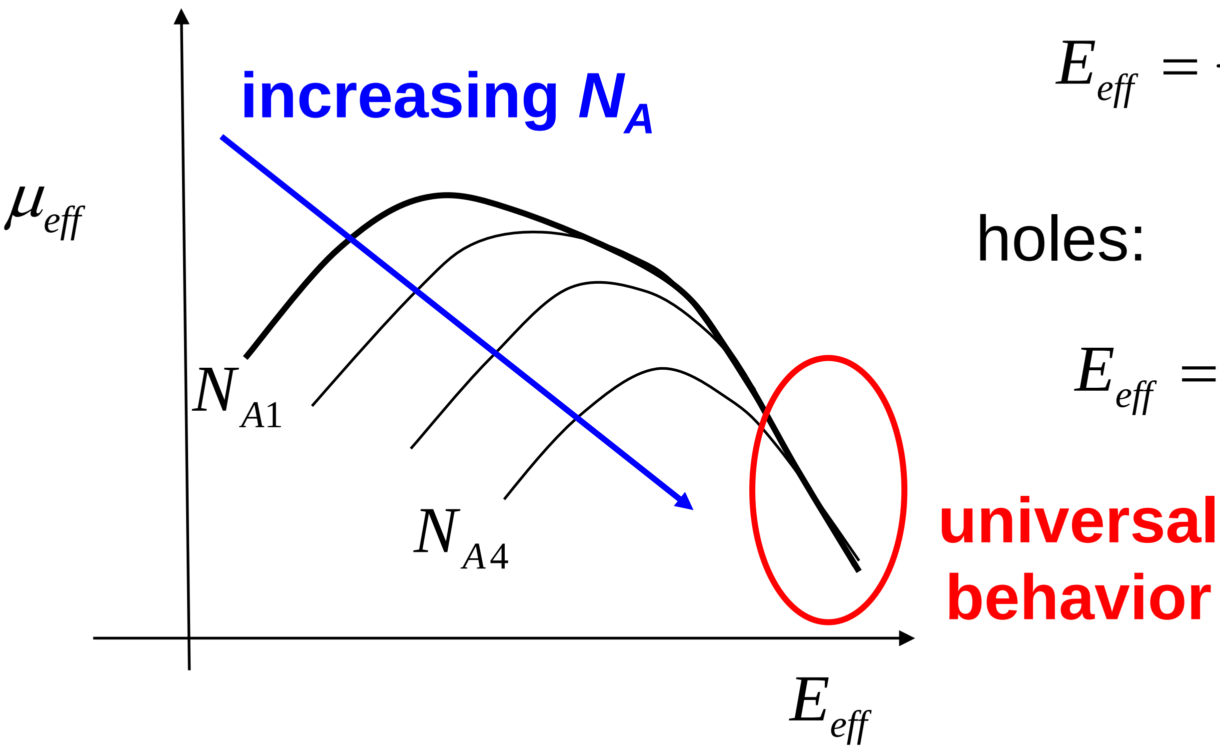
$$\delta t_{OX} \Rightarrow \delta \psi_S$$





from Jing Wang, et al., *Appl. Phys. Lett.*, Aug. 2005

'universal' mobility



electrons:

$$E_{eff} = \frac{1}{\varepsilon_{Si}}\left(\left|Q_{DM}\right| + \frac{\left|Q_I\right|}{2} \right)$$

holes:

$$E_{eff} = \frac{1}{\varepsilon_{Si}}\left(\left|Q_{DM}\right| + \frac{\left|Q_I\right|}{3} \right)$$

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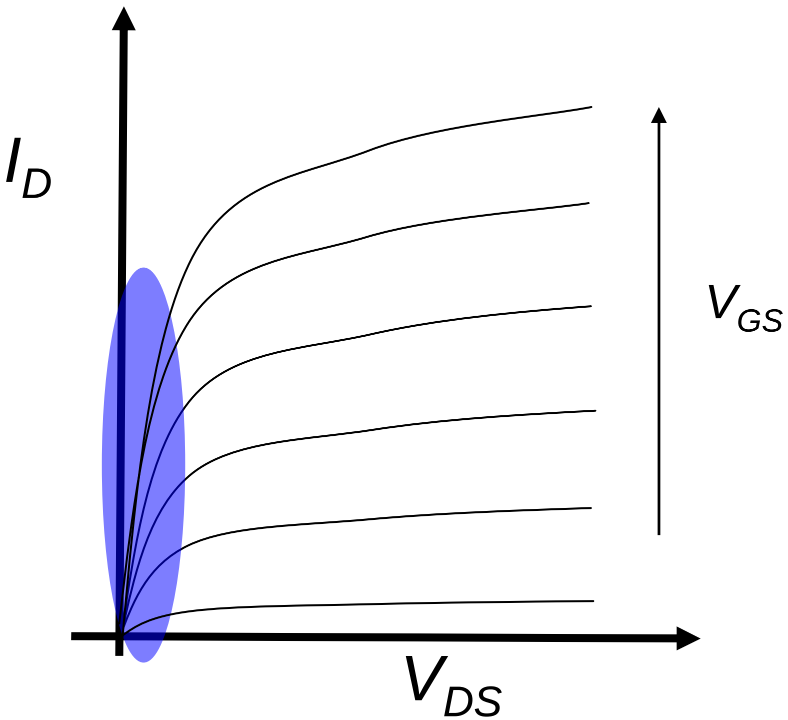
measuring μ_{eff}



$$I_D = \frac{W}{L}\mu_{eff} Q_i V_{DS}$$

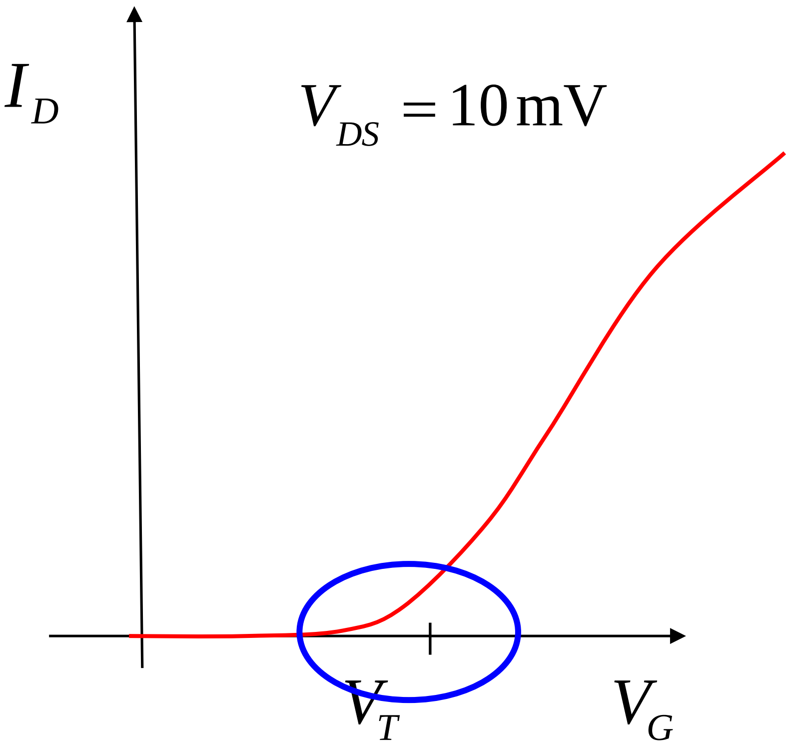
$$R_{CH} = \frac{V_{DS}}{I_D} = \frac{L}{W \mu_{eff} Q_i}$$

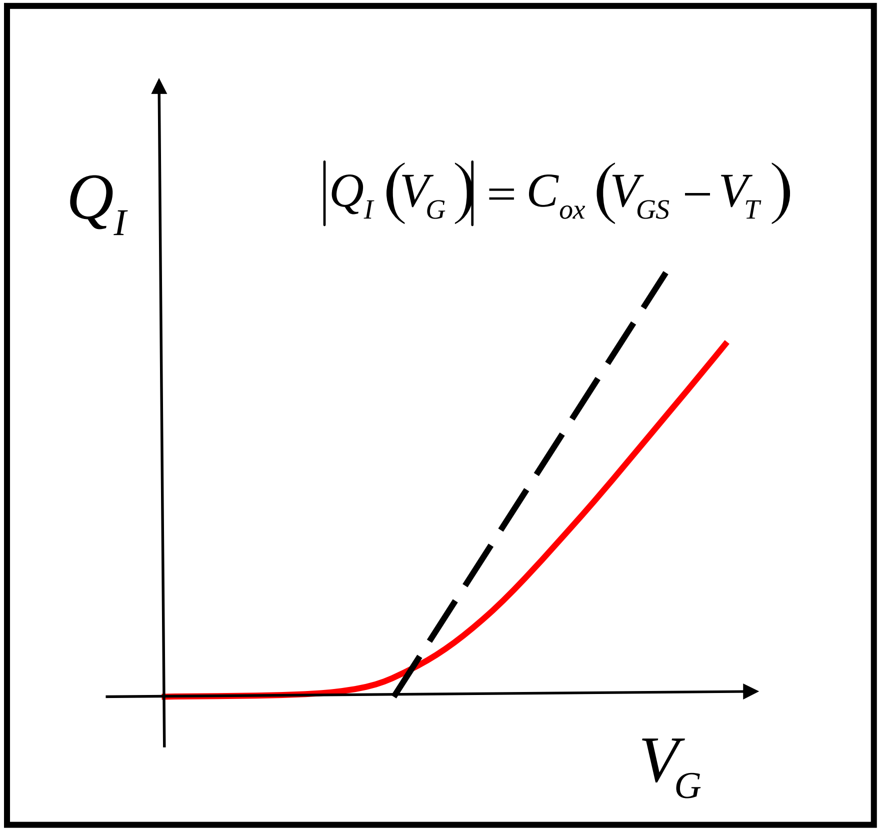
$$\mu_{eff}(V_G) = \frac{L}{W\, R_{CH}(V_G)\, Q_i(V_G)}$$

$$Q_i(V_G) \approx C_G (V_{GS} - V_T)$$

$$R_{CH} >> R_{SD}$$

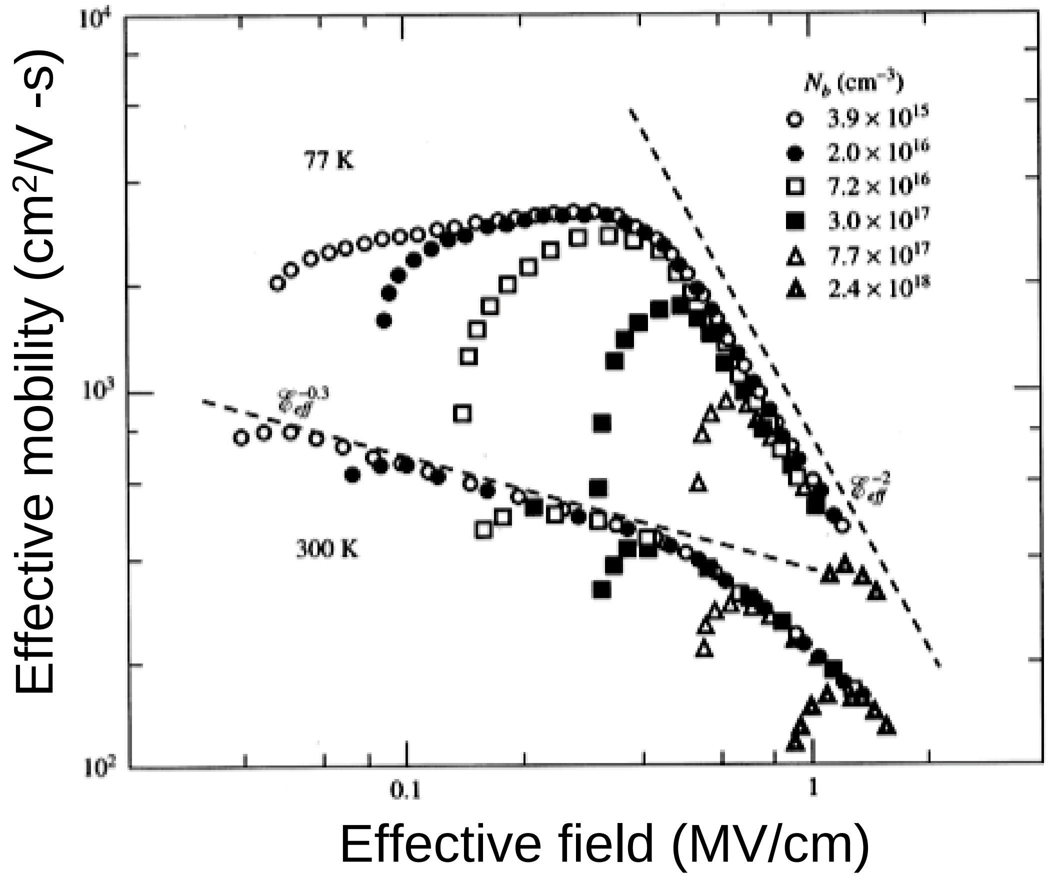
measuring μ_{eff} (ii)





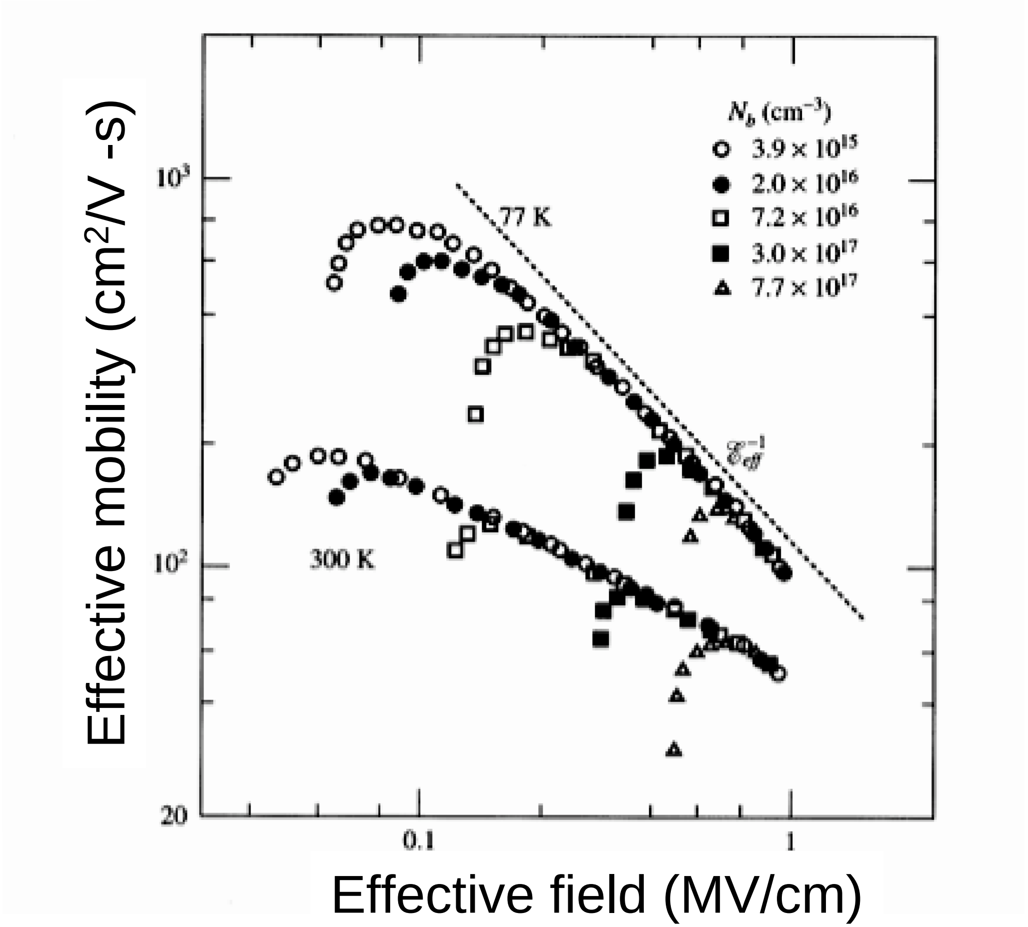
$$Q_I(V_G) = \int_0^{V_G} C_{gs}(V_G)\,dV_G$$

universal mobility for electrons



S. Takagi, A. Toriumi, M. Iwase, and H. Tango, *IEEE Trans. Electron Dev.*, **41**, pp. 2357-2362, 1994

universal mobility for holes



S. Takagi, A. Toriumi, M. Iwase, and H. Tango, *IEEE Trans. Electron Dev*., **41**, pp. 2357-2362, 1994

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“field-effect” mobility

$$I_D = \frac{W}{L}\mu_{eff} Q_I(V_{GS})V_{DS} \approx \frac{W}{L}\mu_{eff} C_G(V_{GS} - V_T)V_{DS}$$

$$g_m = \left.\frac{\partial I_D}{\partial V_{GS}}\right|_{V_{DS}} = \frac{W}{L}\mu\, C_G V_{DS}$$

(assumes μ is independent of V_G)

$$\mu_{FE} \equiv \frac{L}{W C_G V_{DS}} g_m$$

“field-effect mobility”

For a discussion of mobility measurement techniques, see:
Narain Arora, *MOSFET Models for VLSI Circuit Simulation, Theory and Practice*, Springer-Verlag, New York, 1993.

mobility and on-current

Linear region current is proportional to the effective mobility:

$$I_D = \frac{W}{L}\mu_{eff} C_G \left(V_{GS} - V_T\right) V_{DS}$$

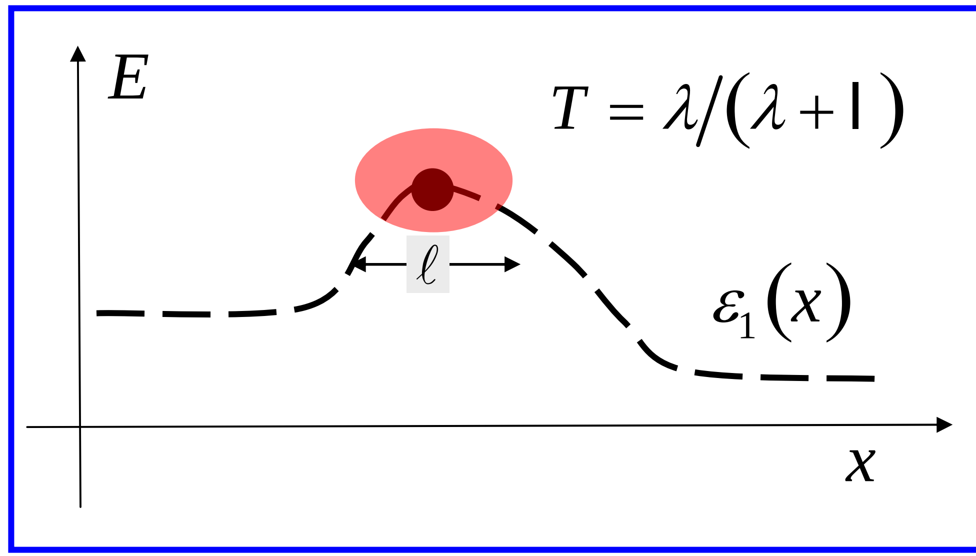
What about the on-current?

$$I_D = W C_G \upsilon_{sat} \left(V_{GS} - V_T\right)$$

mobility and on-current (ii)

$$I_D = W C_G \frac{T}{2-T} \upsilon_T \left(V_{GS} - V_T\right)$$

$$D_n = \upsilon_T \lambda / 2 = \left(k_B T / q\right) \mu_n$$



also:

$$\mu_n = \frac{q\tau}{m^*}$$

smaller m* implies larger ballistic velocity: $\upsilon_T = \sqrt{2 k_B T / \pi m^*}$

effective mobility for the 45 nm technology node

$$E_{eff} = \frac{\varepsilon_{OX}}{\varepsilon_{Si}\, t_{OX}}\left[(V_G + V_T)/2 + E_G/2q - \psi_B\right]$$

$$t_{OX} = 1.1\ \text{nm}$$

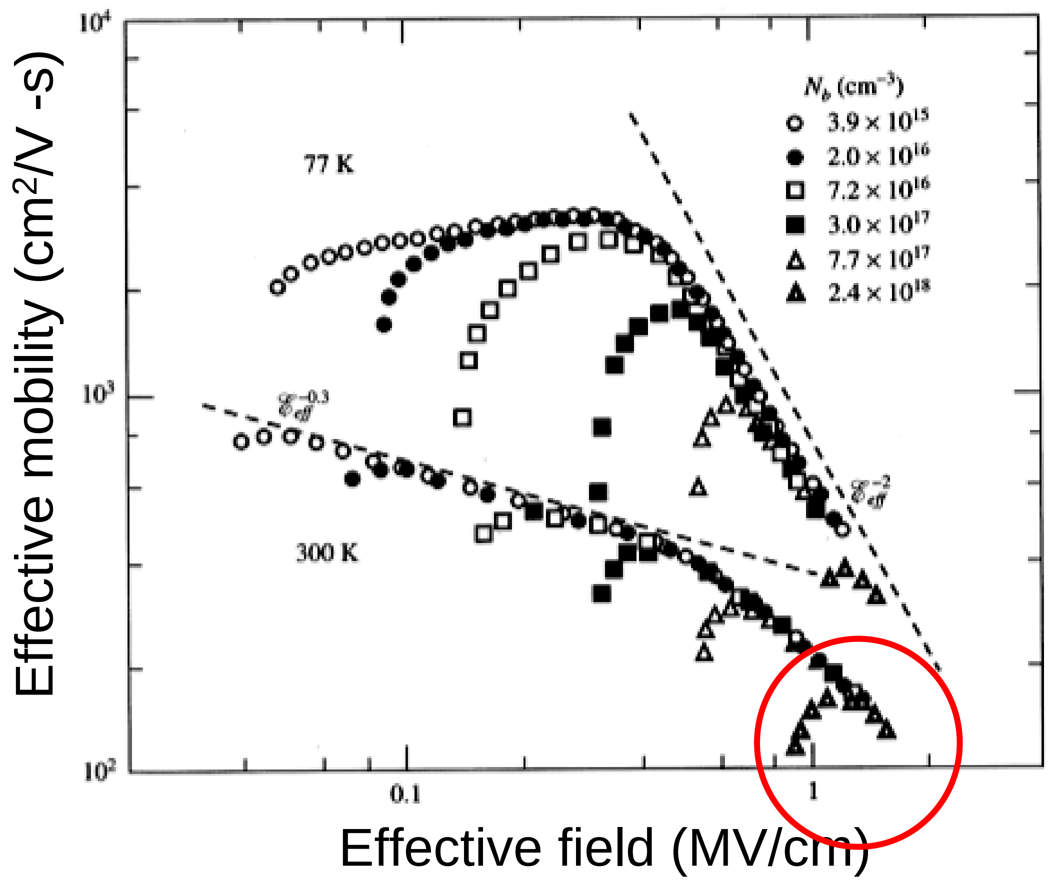
$$V_T \approx 0.25\ \text{V}$$

$$V_G = V_{DD} = 1.0\ \text{V}$$

$$N_A \approx 2.7 \times 10^{18}\ \text{cm}^{-3} \rightarrow \psi_B = 0.48$$

$$E_{eff}\left(V_G = 1\text{V}\right) \approx 2\ \ \text{MV/cm}$$

universal mobility for electrons



S. Takagi, A. Toriumi, M. Iwase, and H. Tango, *IEEE Trans. Electron Dev.*, **41**, pp. 2357-2362, 1994

the tyranny of the effective mobility*

$$E_{eff} = \frac{\varepsilon_{OX}}{\varepsilon_{Si} t_{OX}} \left[(V_G + V_T)/2 + E_G/2q - \psi_B \right]$$

each technology generation, device scaling increases N_A, decreases t_{ox} --> E_{eff} *increases and mobility decreases*

mobility decreases each technology generation, unless we can find 'new' materials with higher mobilities (e.g. strained silicon).

(* Dimitri Antoniadis at MIT describes the problem this way.)

strain engineering

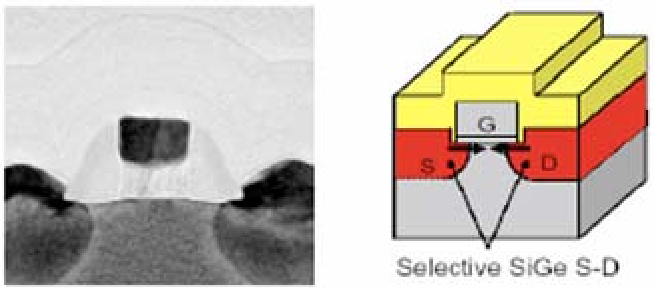


Fig. 1 XTEM of PMOS uni-axial strained silicon transistors developed for low power process.

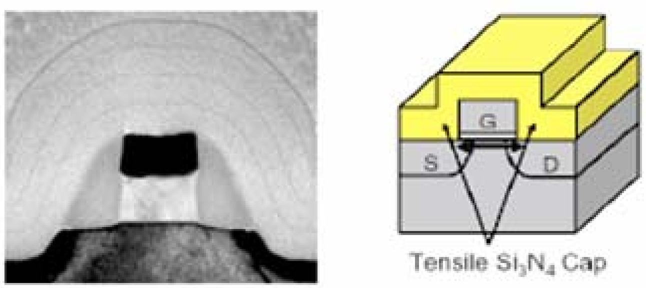


Fig. 2 XTEM of NMOS uni-axial strained silicon transistors developed for low power logic process.

PMOS

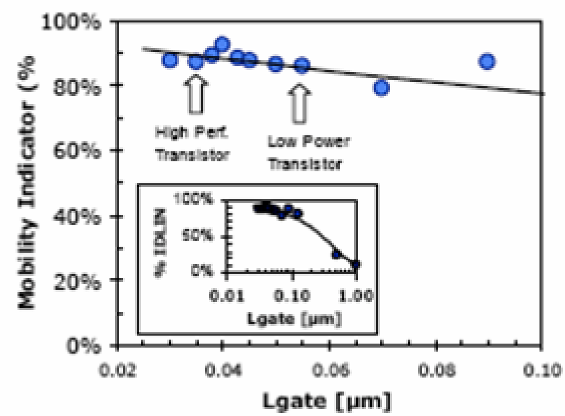


Fig. 3 PMOS mobility improvement vs. unstrained transistor as a function of transistor gate length.

C.-H. Jan, et al., “A 65nm Ultra Low Power Logic Platform Technology using Uni-axial Strained Silicon Transistors,” Intel Corporation, IEDM 2005,

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summary

1) Effective mobility is an important device parameter for both the linear and saturated region currents
2) Effective mobility is strongly reduced by surface roughness scattering
3) Mobility enhancement through *strain engineering* has been an important performance booster in recent years.