NCN@Purdue-Intel Summer School
Notes on Percolation Theory

Lecture 1
Percolation in Electronic Devices

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PURDUE UNIVERSITY

plan for the lecture series

1) Percolative transport in electronic devices
2) Basic concepts: threshold, island sizes, fractal dimensions
3) Electrical conduction in random media
4) Theory of stick percolation: application to nanonet transistors
5) 2D nets in 3D world: sensors, solar cells, super-capacitors

Acknowledgement:

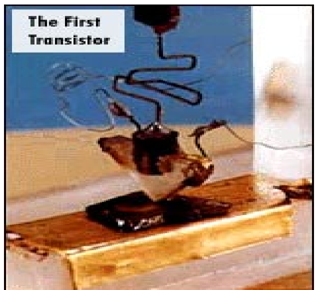
D. Varghese, P. Nair, and E. Islam

outline of lecture 1

1) Order is an anomaly … randomness rules
    1) Randomness in electronics
    2) Randomness in nature
2) Why did we not hear about it
    1) Averaging over large numbers
    2) Quantization and coherence
3) Approximate randomness at your own peril
    1) Weibull distribution vs. Gaussian distribution.
    2) Poisson vs. 'Fish' arrival distributions
4) Conclusions

randomness in electronics ...

1947 ....



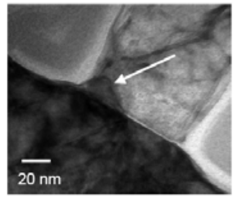
ballistic

$$G \propto \frac{\tau}{m^*}\frac{1}{L}$$

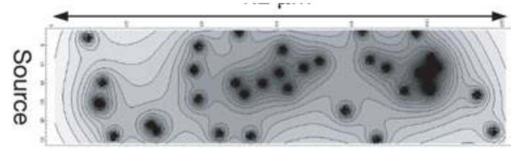
Quantum transport

percolation

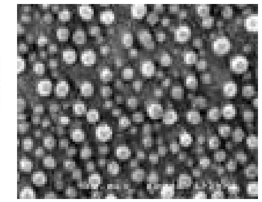
Dielectric BD

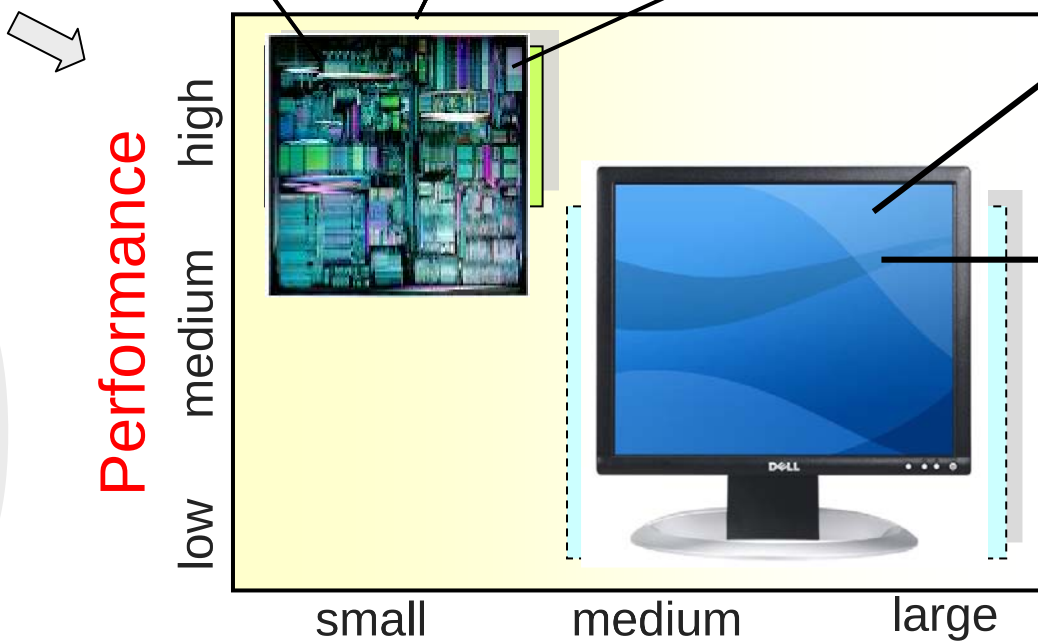


Random Dopants

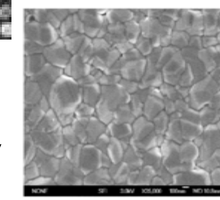


NC Flash

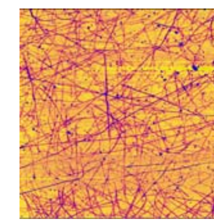




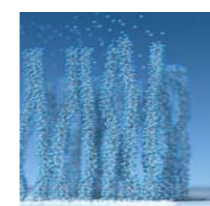
Area



Poly-Si



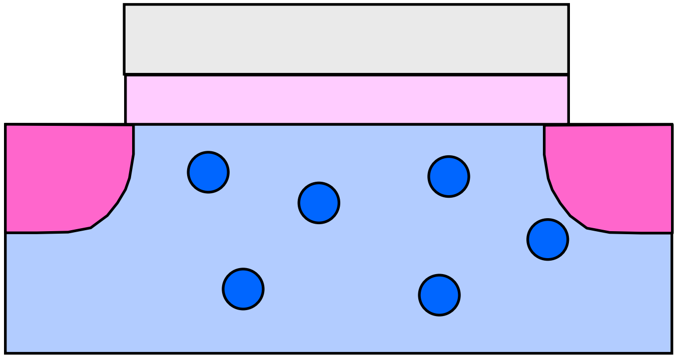
NanoNet Biosensors



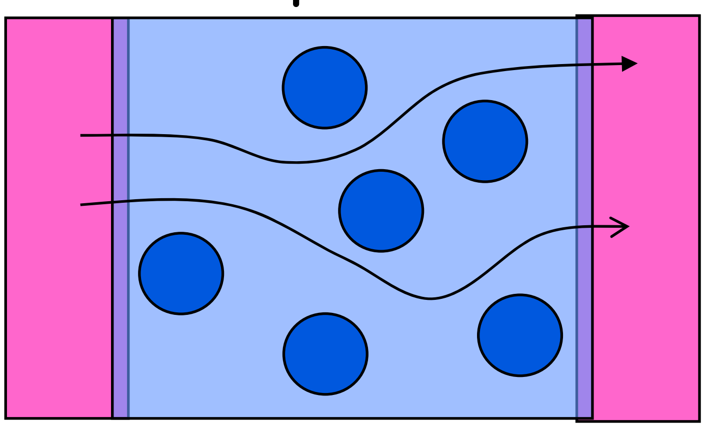
super-capacitors

random dopant fluctuation

side view



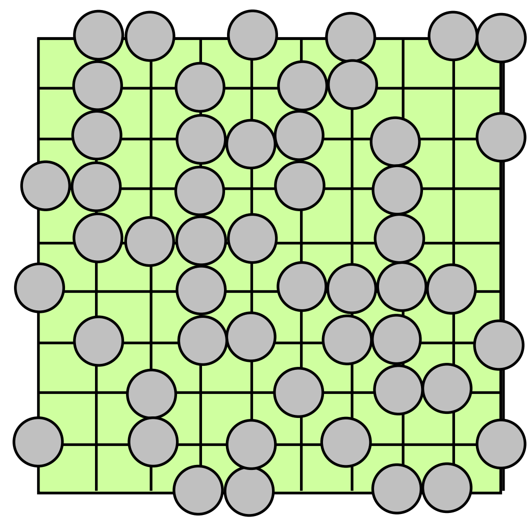
top view



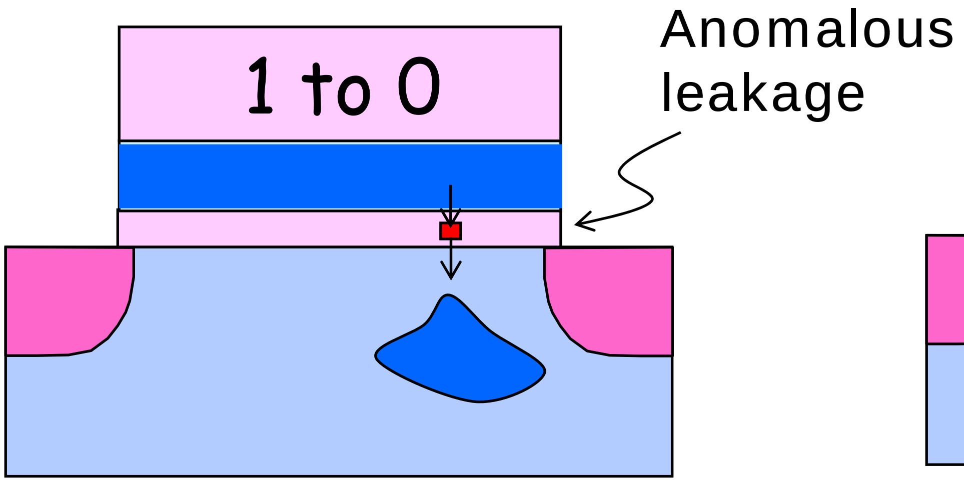
@1e18/cm^3

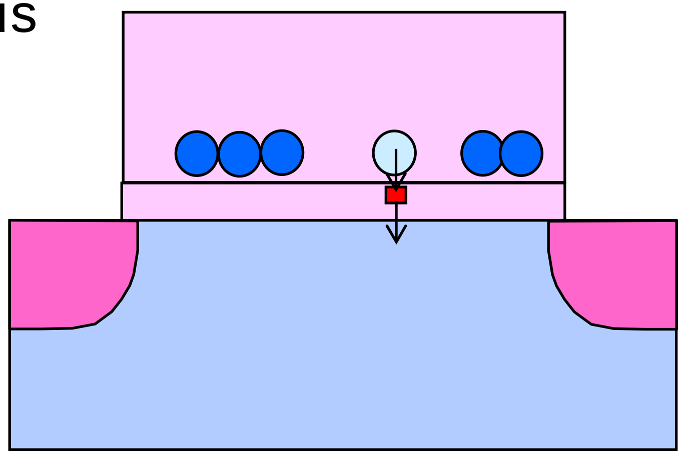
100 nm --- 1000 dopants
20 nm --- 40 dopants
10 nm --- 10 dopants

model

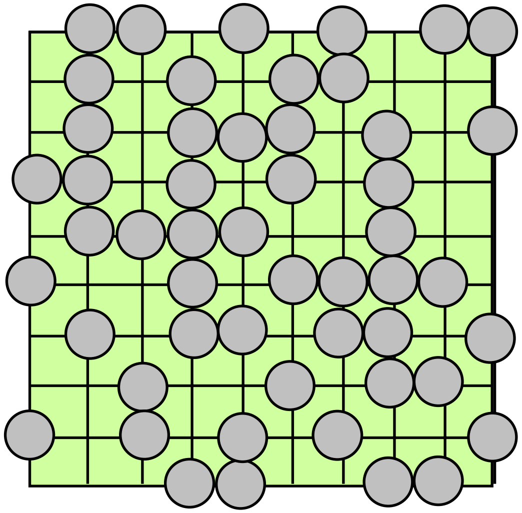


Flash vs. Nanocrystal Flash

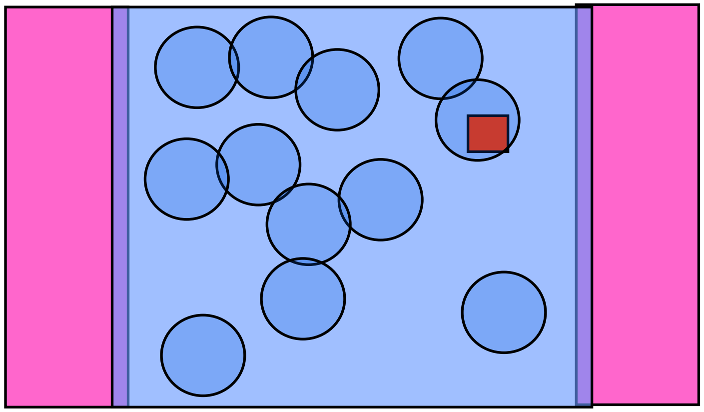




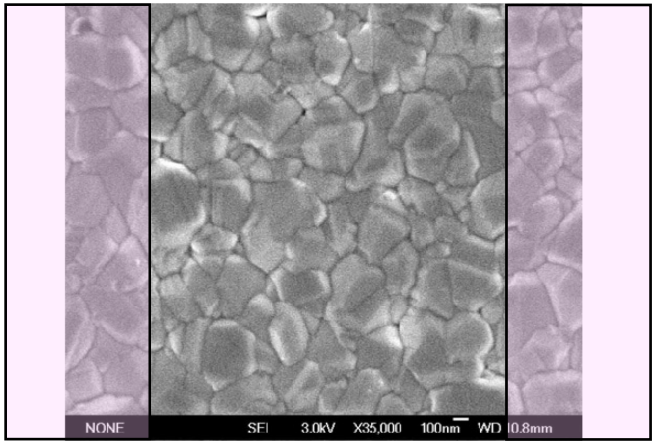
model



Top view



solar cells and display electronics

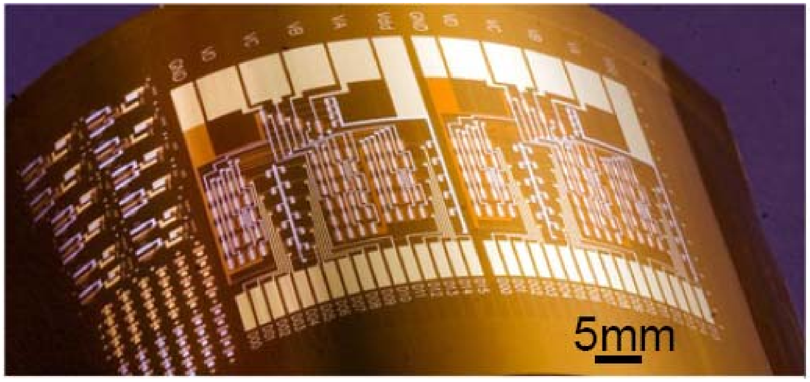


Key issues:

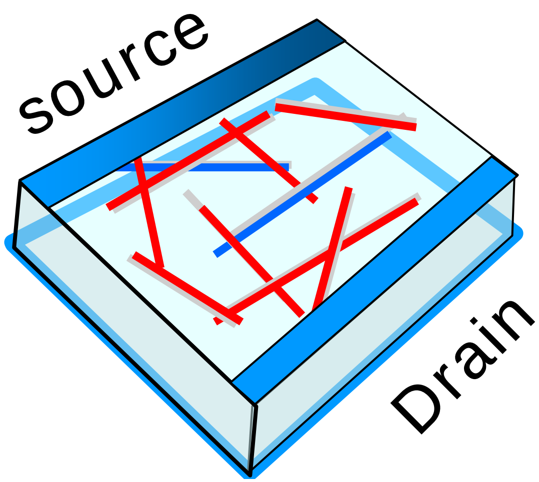
Transport through barriers created by grain boundaries

Device/device fluctuation

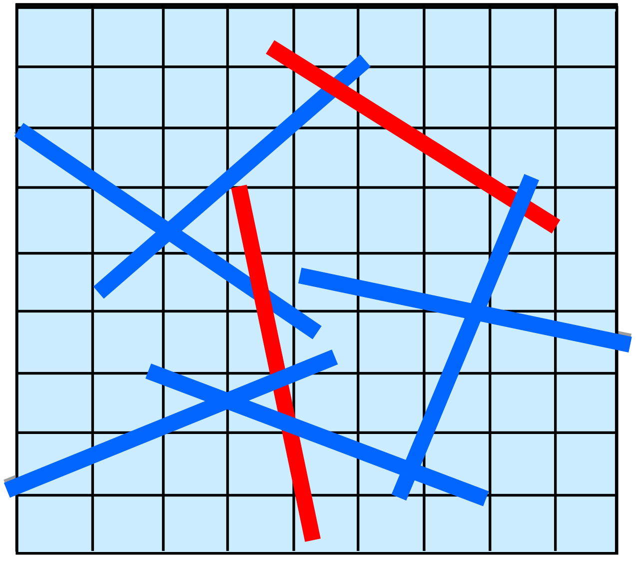
Flexible nanonet electronics



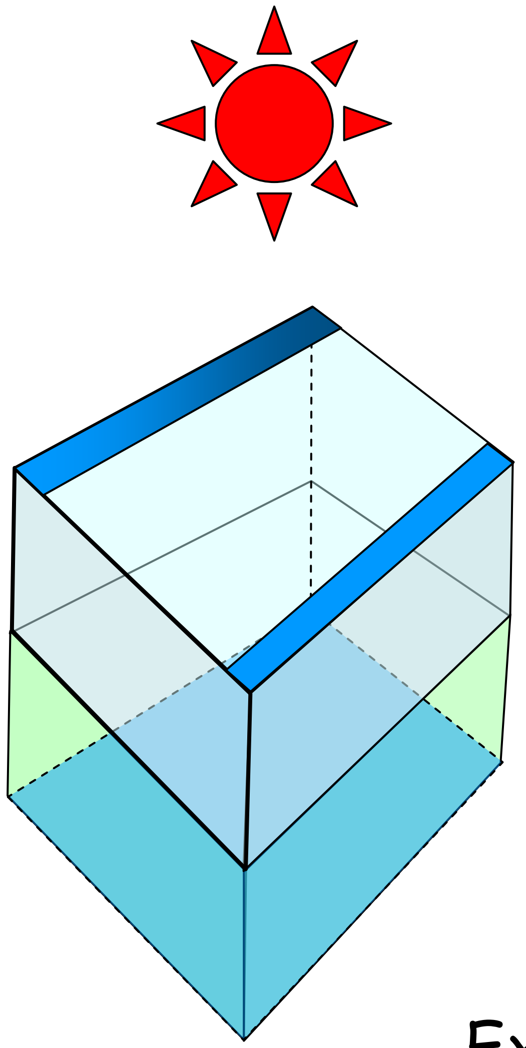
Cao, Nature, 2008



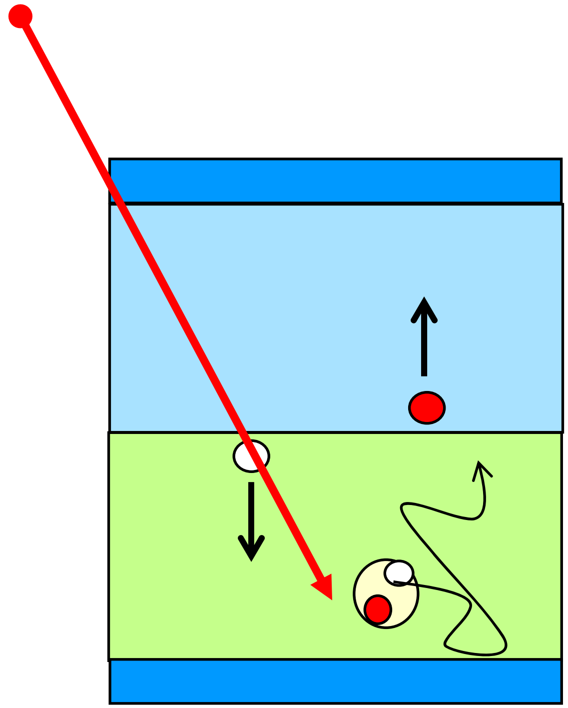
Heterogeneous percolation



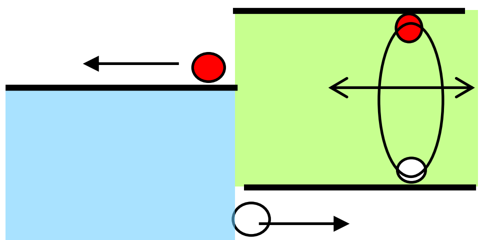
solar cells



side view

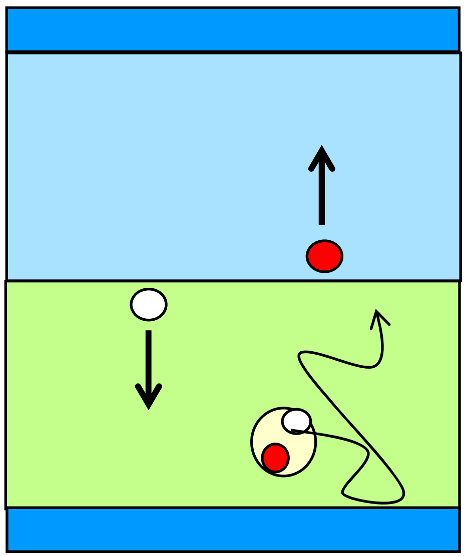


Band-diagram

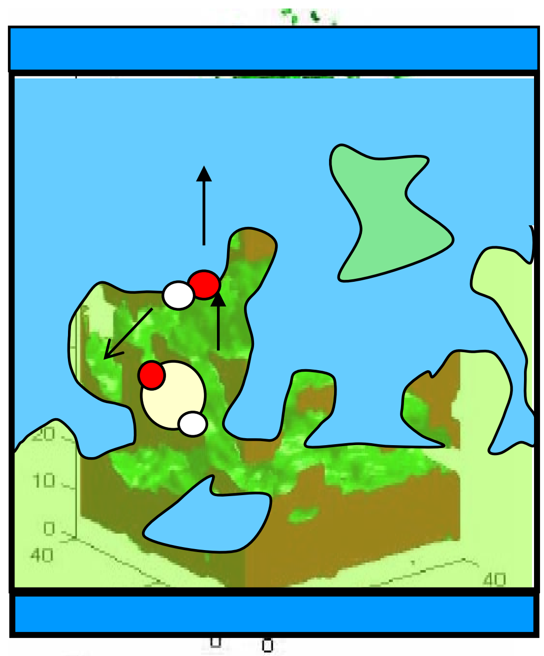


Exciton recombination before dissociation at the junction makes it a poor cell …

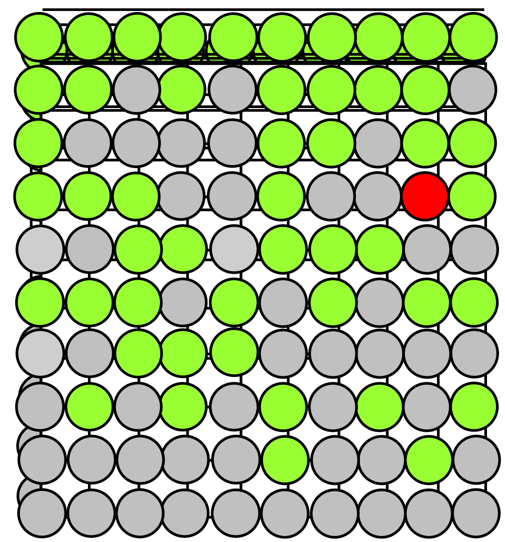
nano-structured solar cells



mixed layers

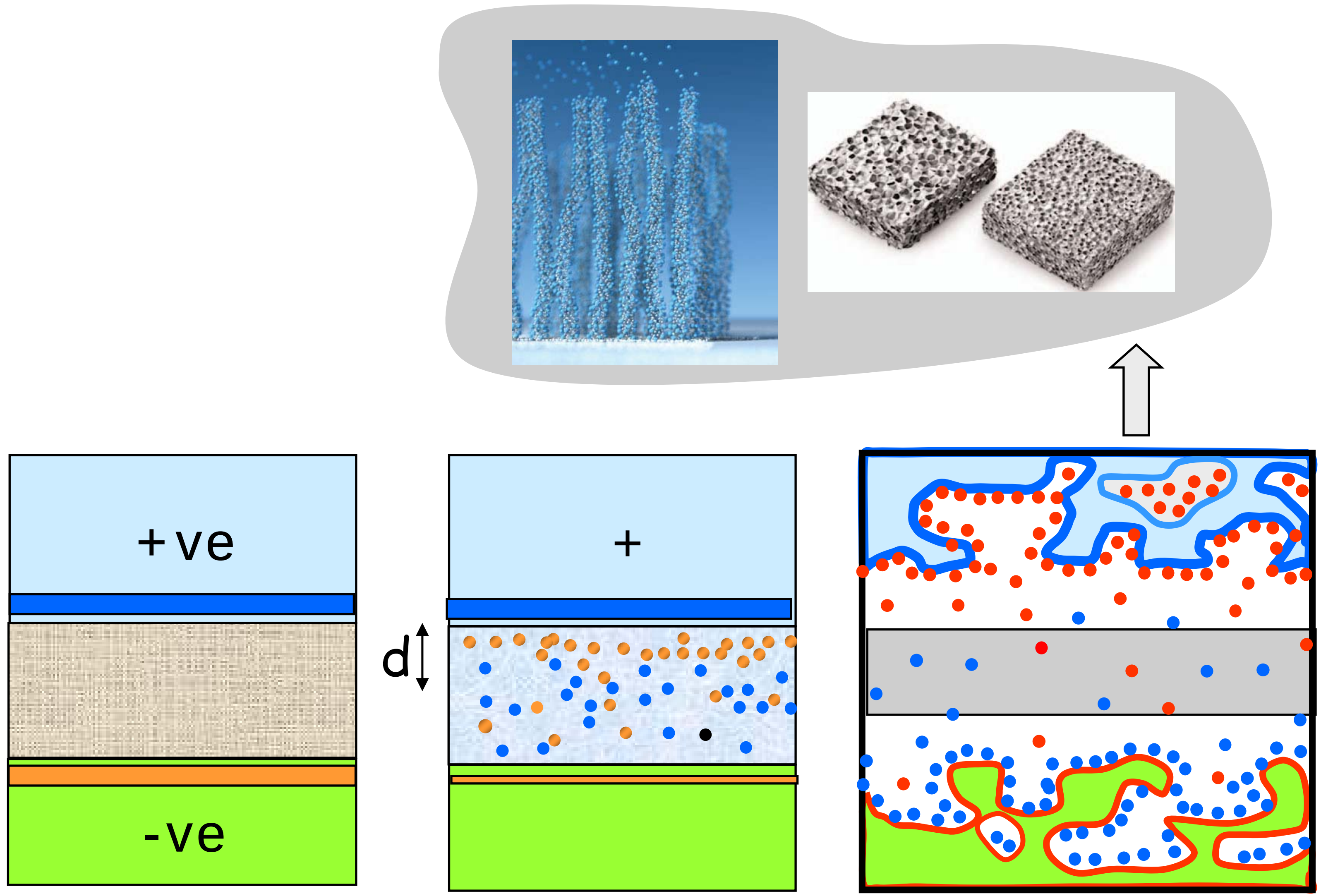


heterogeneous percolation



How do we describe exciton dissociation/charge collection

super-capacitors

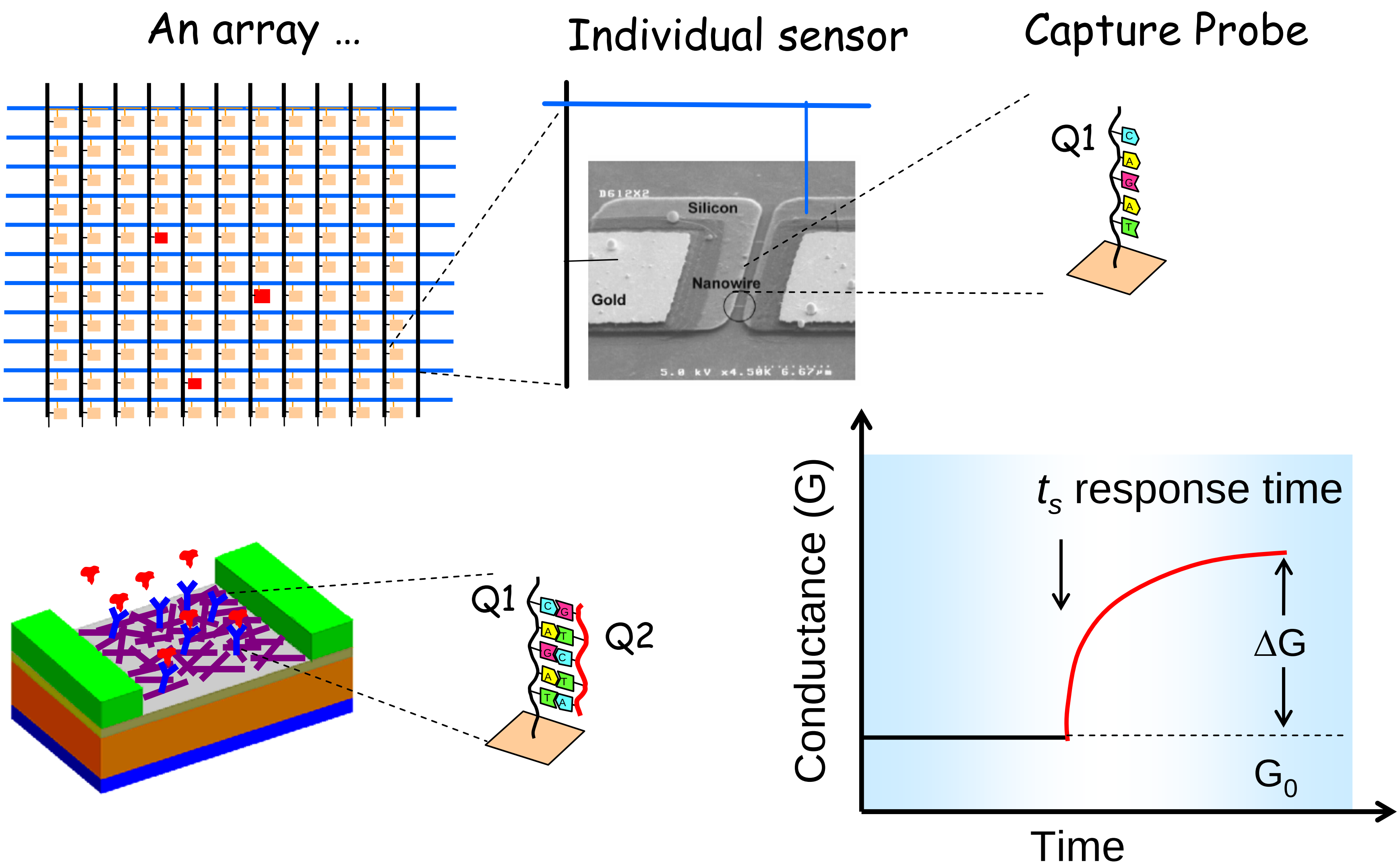


Parallel plate capacitor

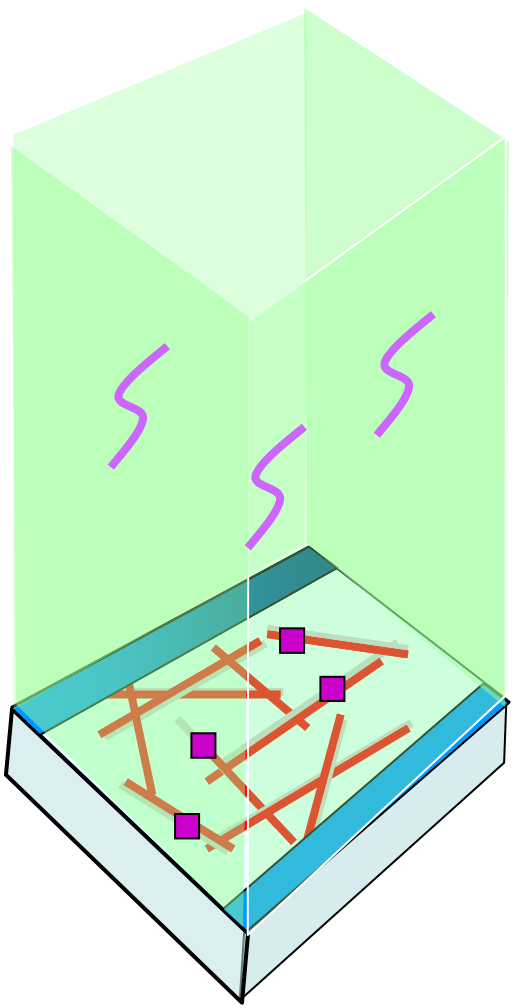
Electrolyte capacitor

Super-capacitor

biosensors …

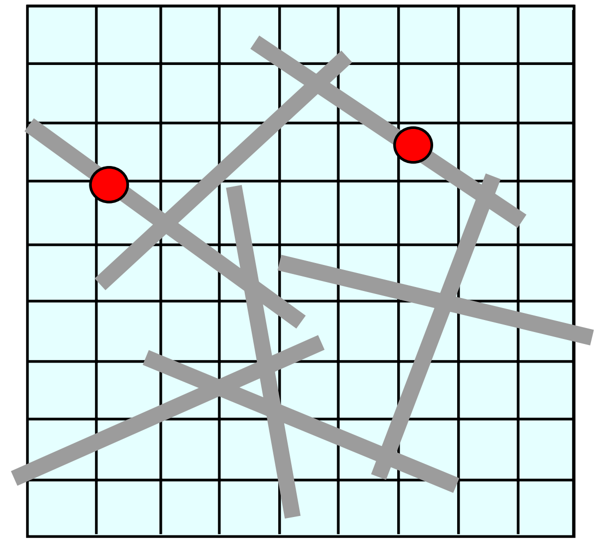

An array …
Individual sensor
Capture Probe
Silicon
Nanowire
Gold
Q1
C
A
G
A
T
Q2
t_s response time
ΔG
G_0
Conductance (G)
Time

diffusion towards disordered biosensors…

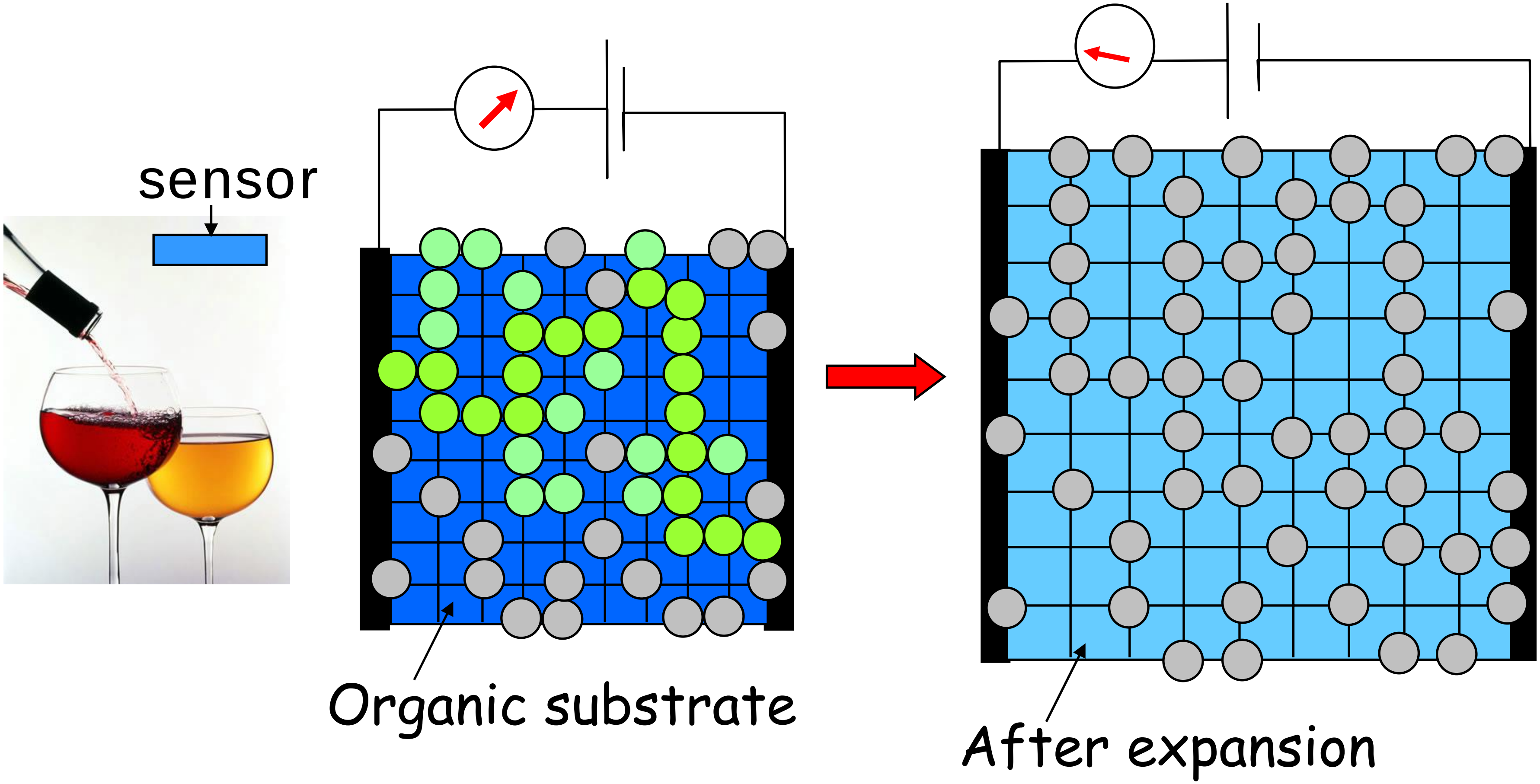


Key issues:

- ❑ density dependent response time
- ❑ conductivity, transfer resistance or substrate dependence
- ❑ Channel length scaling



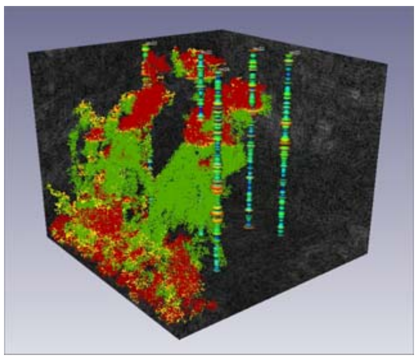
chemical sensors and e-nose

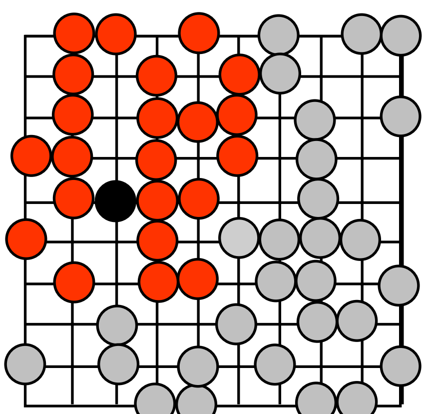


randomness is the rule, not the exception ....

Cluster sizes

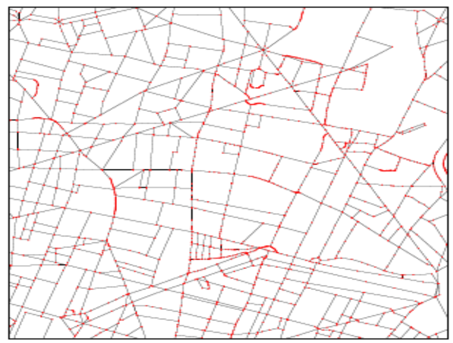
Oil fields, NC Flash

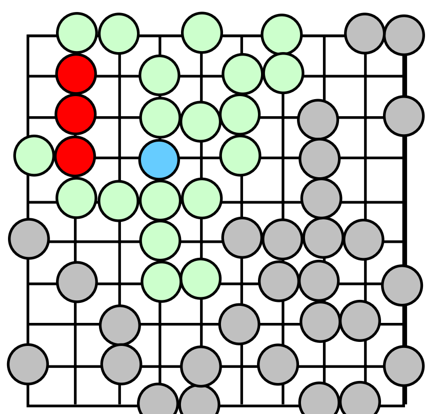




In plane transport

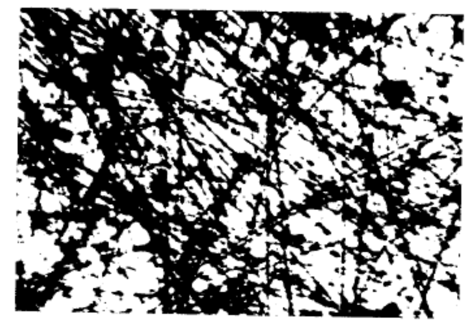
epidemics, forest fire, telecom grid, www, Nanonets, RDF

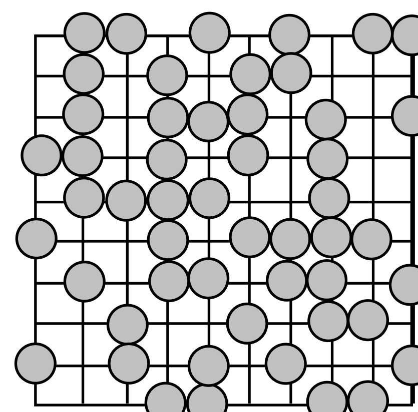




Out of plane transport:

Aerosol, paper, sensors



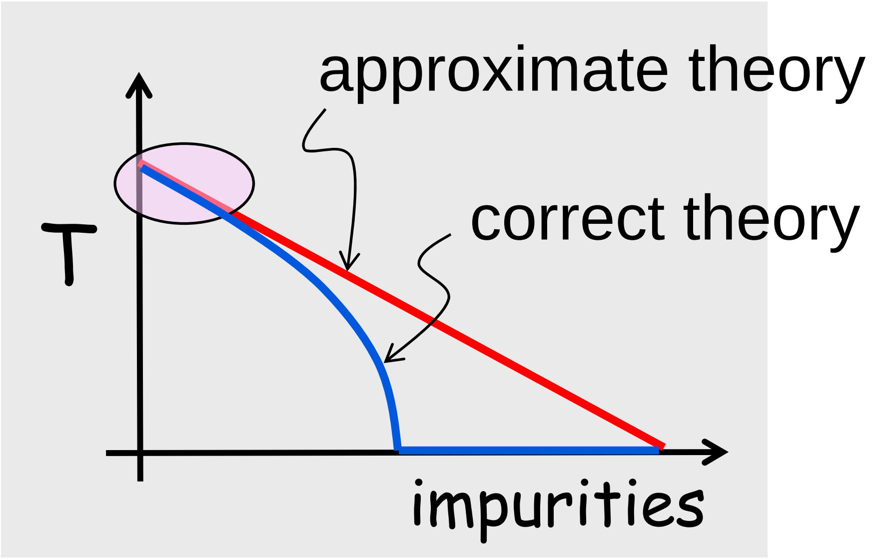


outline of lecture 1

1) Order is an anomaly … randomness is the rule
2) Why did we not hear about it
3) Approximate randomness at your own peril
4) Conclusions

Why did I not hear about it (1) ?

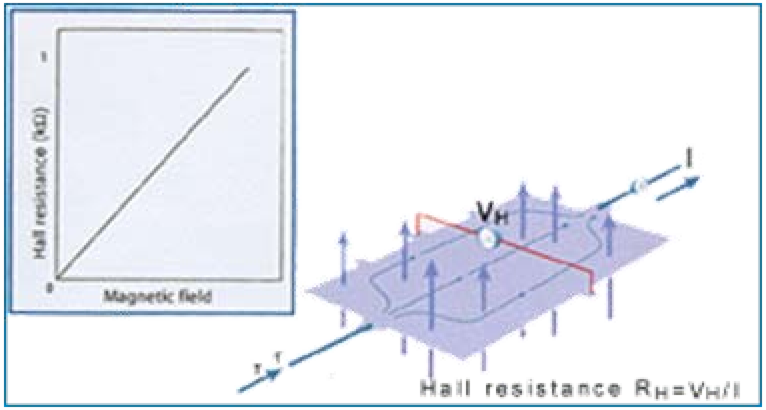
1 mm^3 ~ 10^{17} molecules

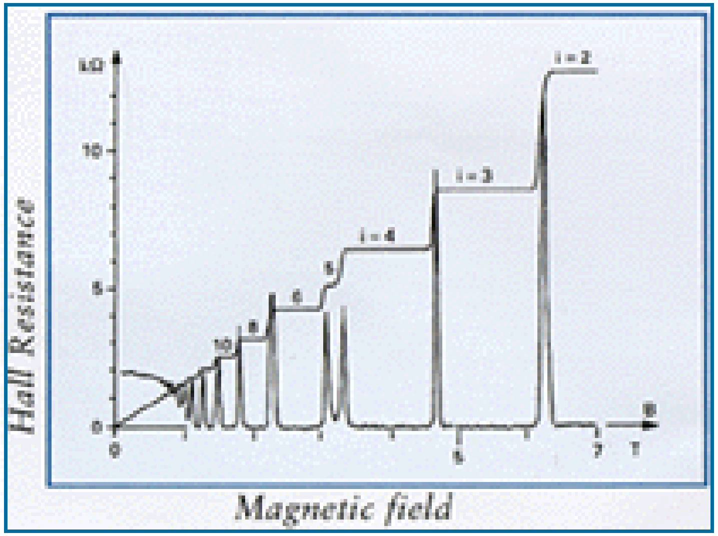


Given traces of impurity, change in property (e.g. transmission) is easily predicted.

Fluctuation in properties of large system is small.

why did I not hear about it (2) ?

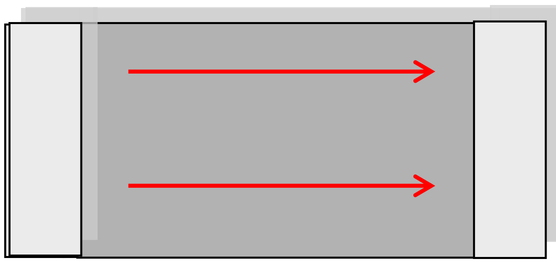


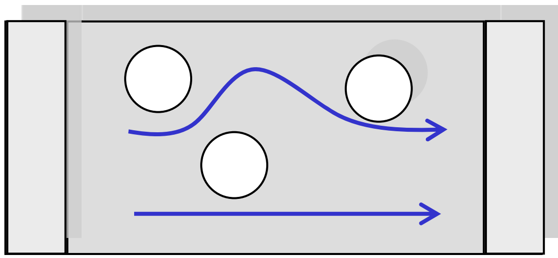


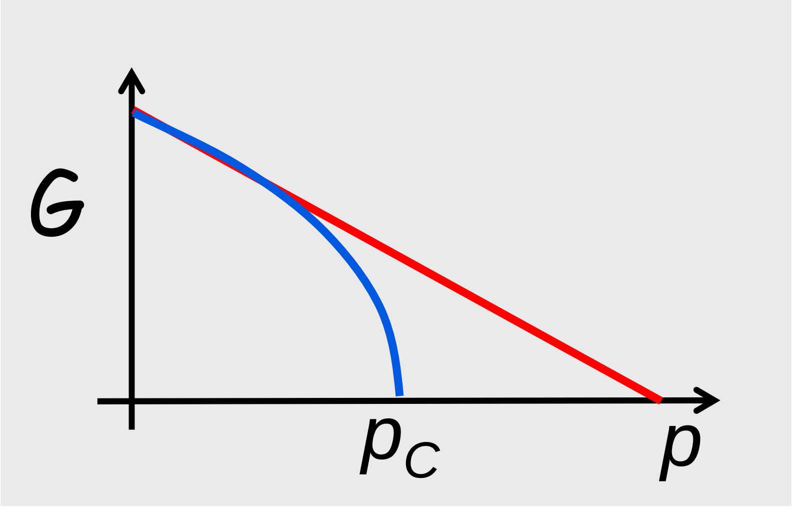
Some small systems have unusually robust properties, (e.g., quantum Hall effect) and physicists often focus on those extra-ordinary aspects of small systems …

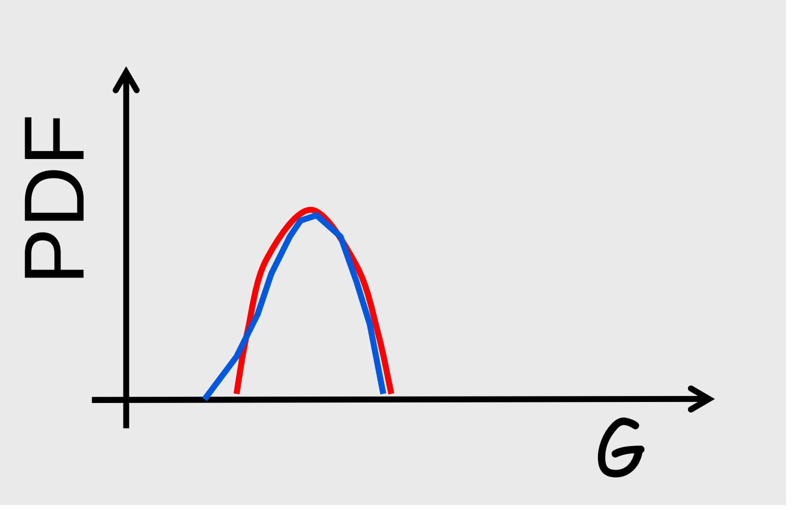
- Small regular systems
- Bio-mimetic materials

mean and deviation



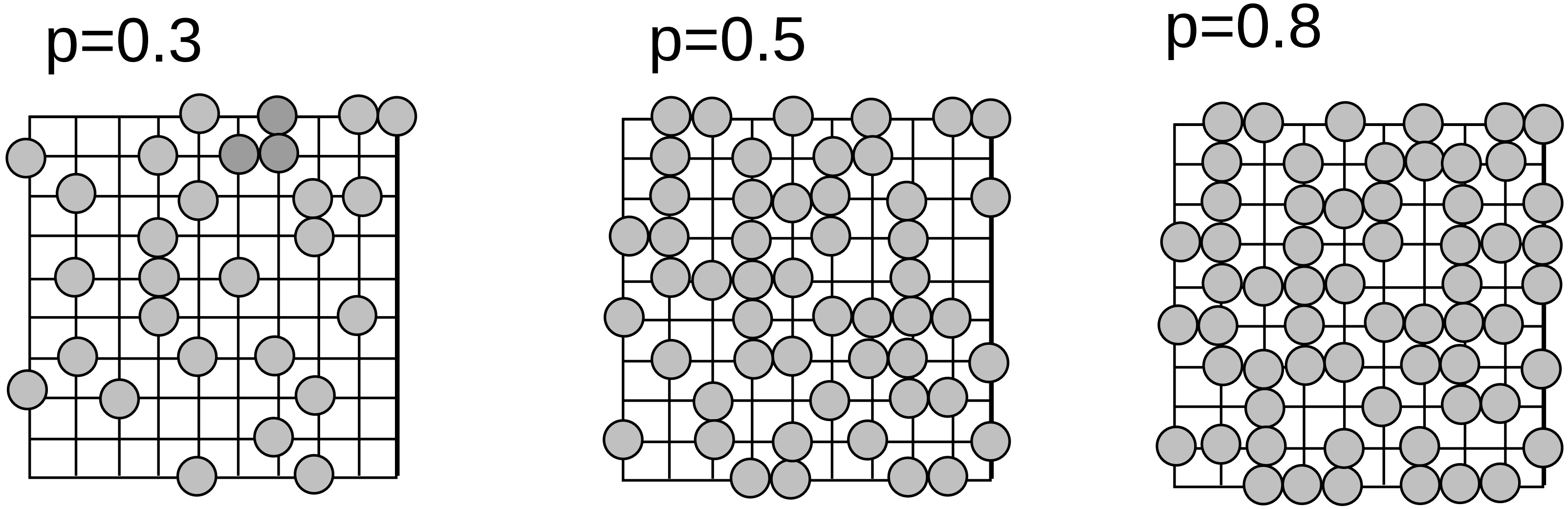


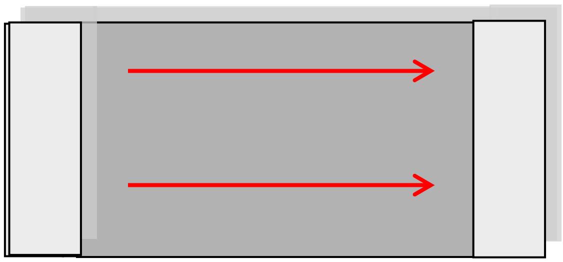


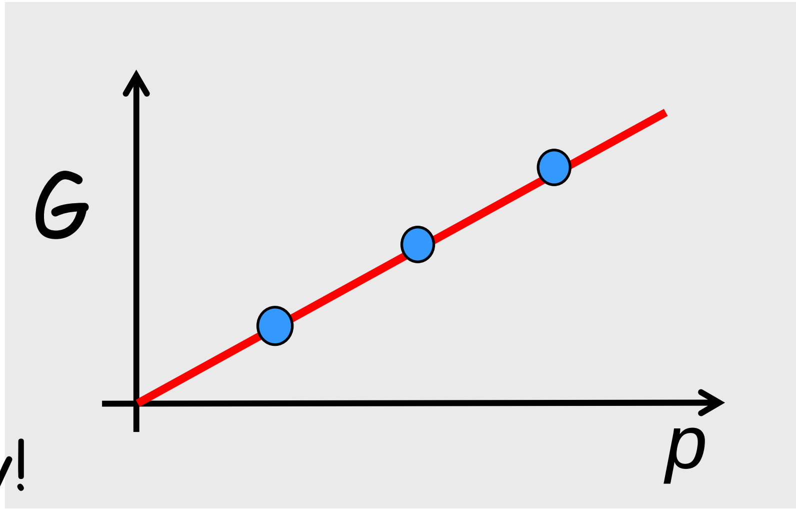


…. wrong on both counts and computer alone can not help

top effective media approach ….

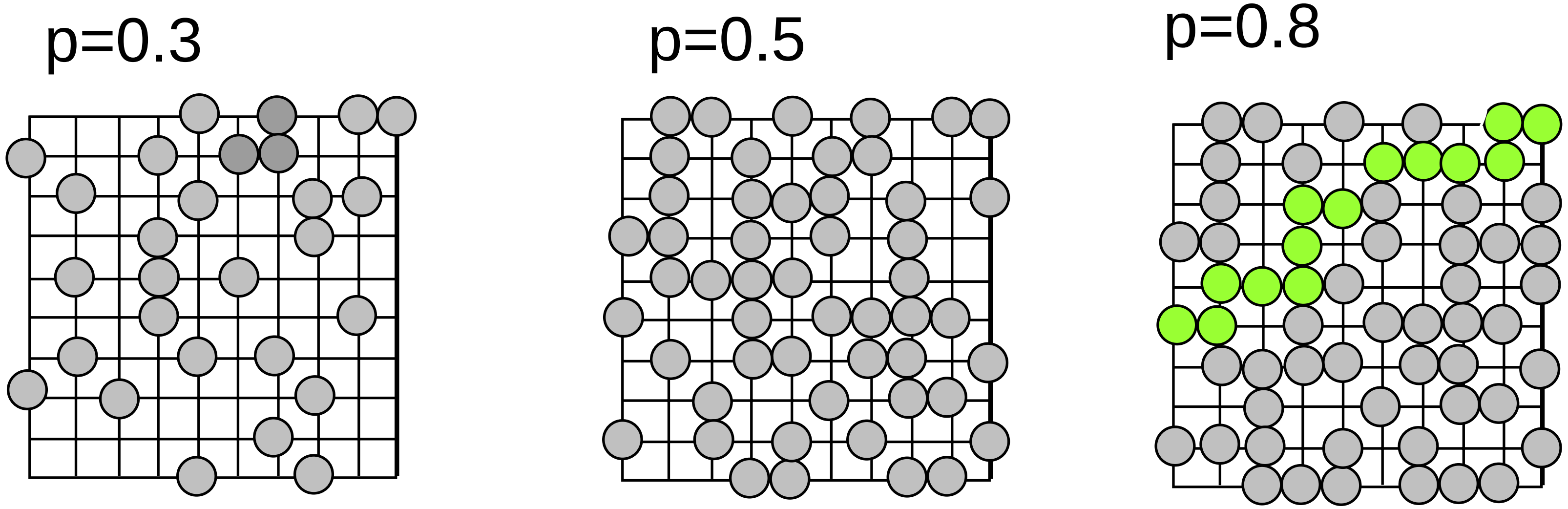


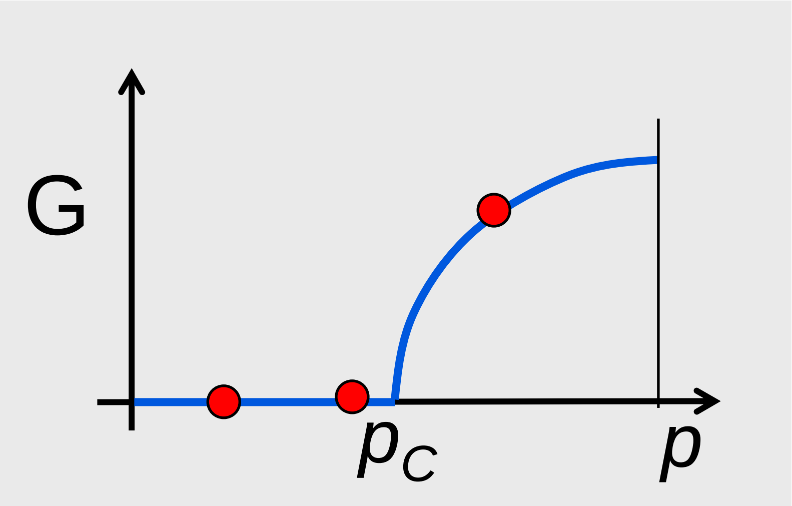




… that's what textbooks say!

basics of percolation: averaging matters

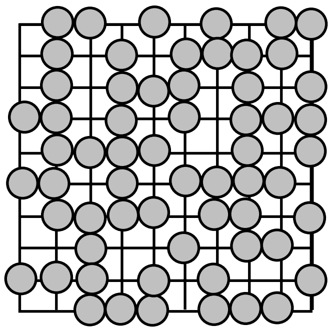




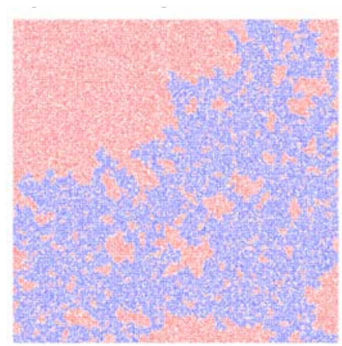
Consequences of adding a new disk depends on existing configuration ...

… and so does the fluctuation

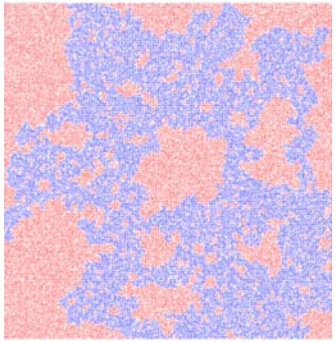
p=0.8

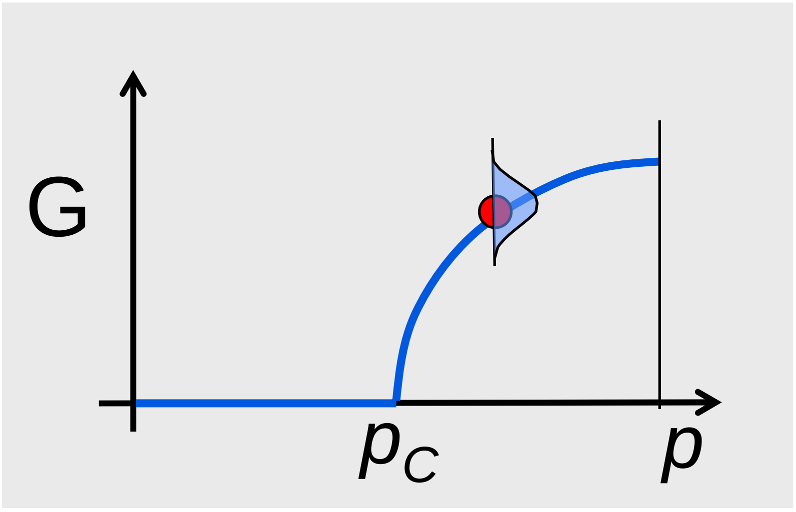


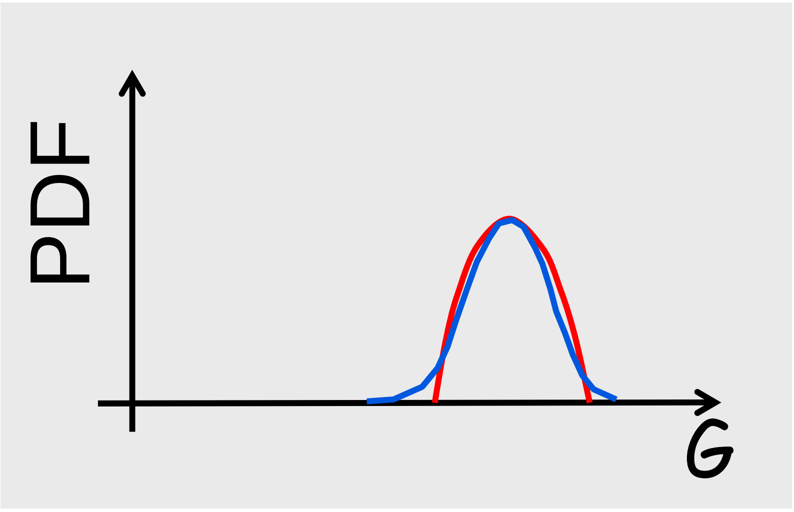
p=0.8



p=0.8

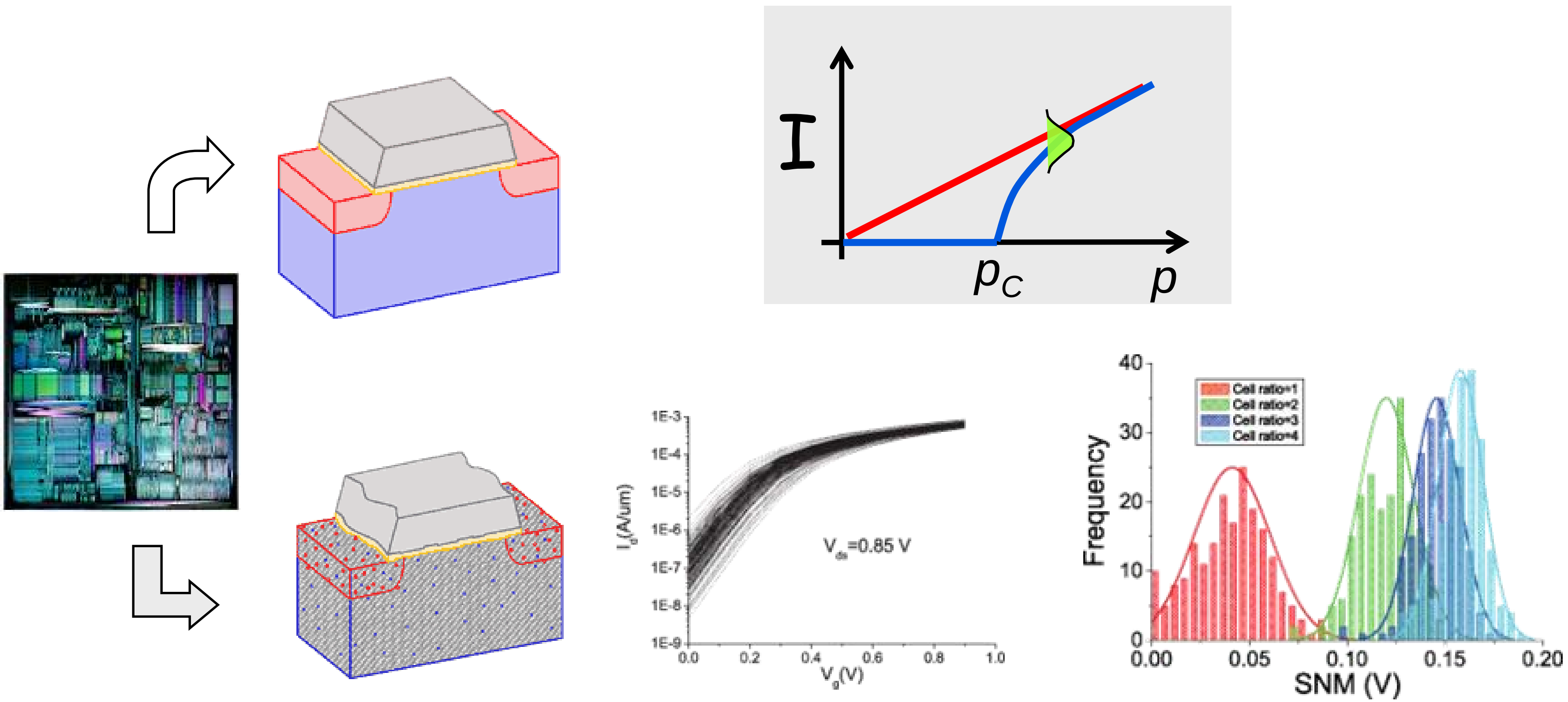






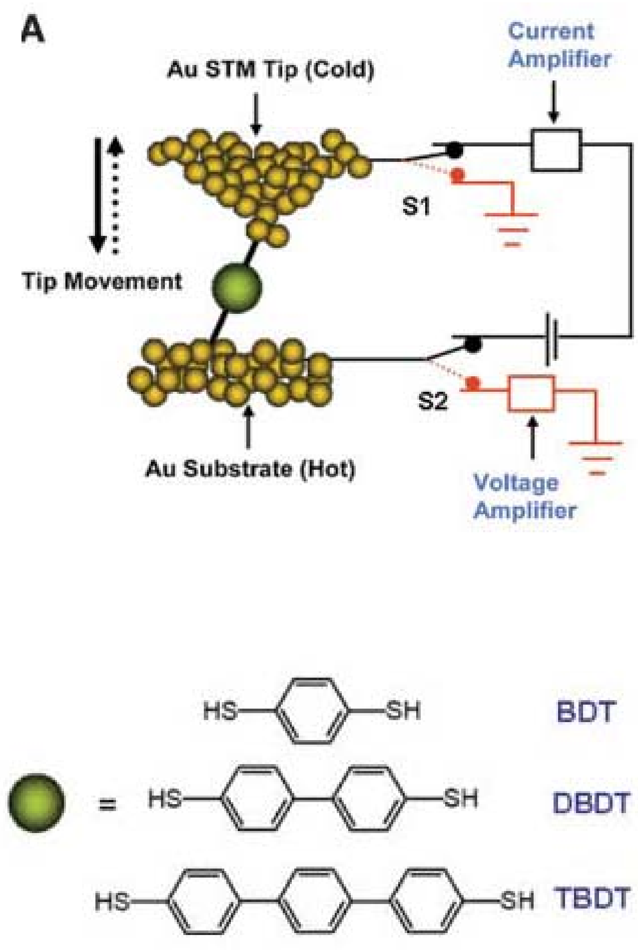
May look the same, but have very different implications

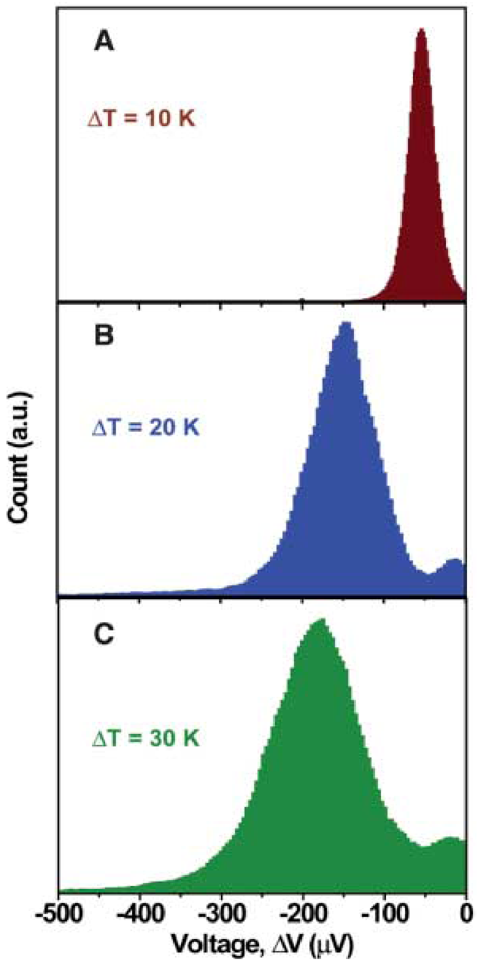
current approach: transistor design



Is Gaussian distribution appropriate ?!!

current approach: thermal conduction



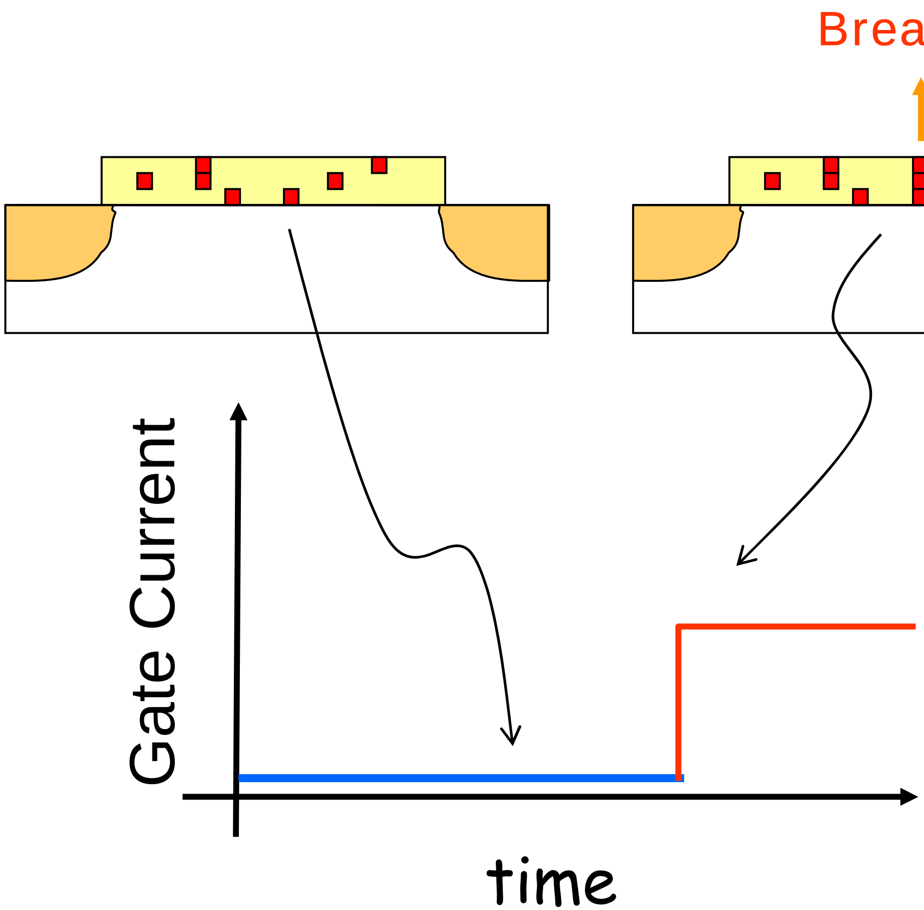


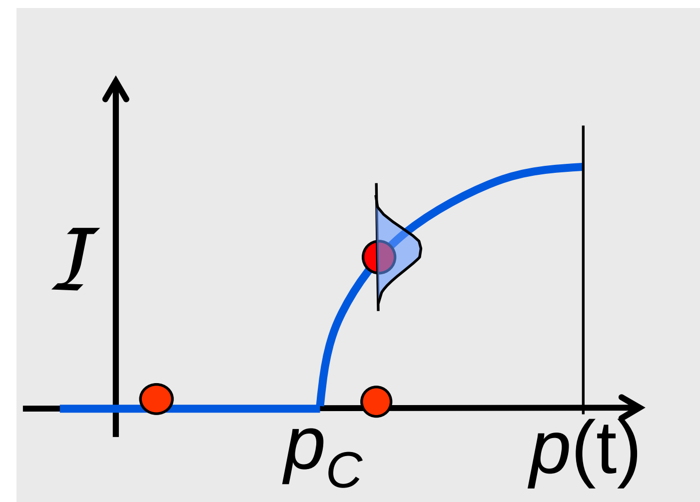
A. Majumdar, Nature, 2007

outline of lecture 1

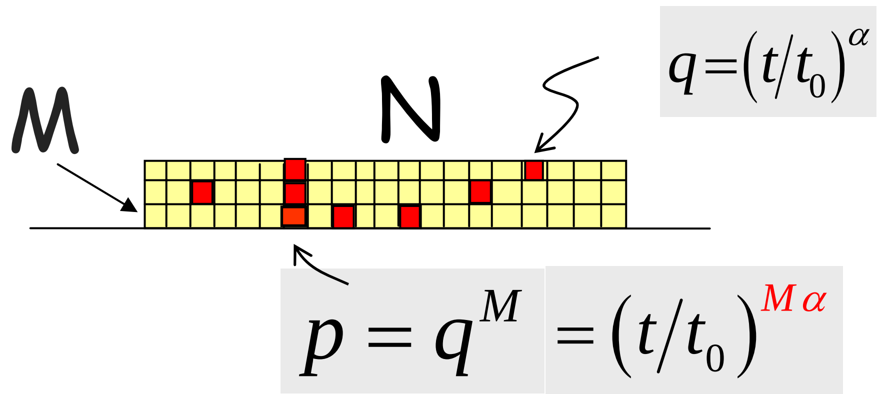
1) Order is an anomaly … randomness is the rule
2) Why did we not hear about it
3) Approximate randomness at your own peril
4) Conclusions

oxide degradation and breakdown





(simple) theory of breakdown



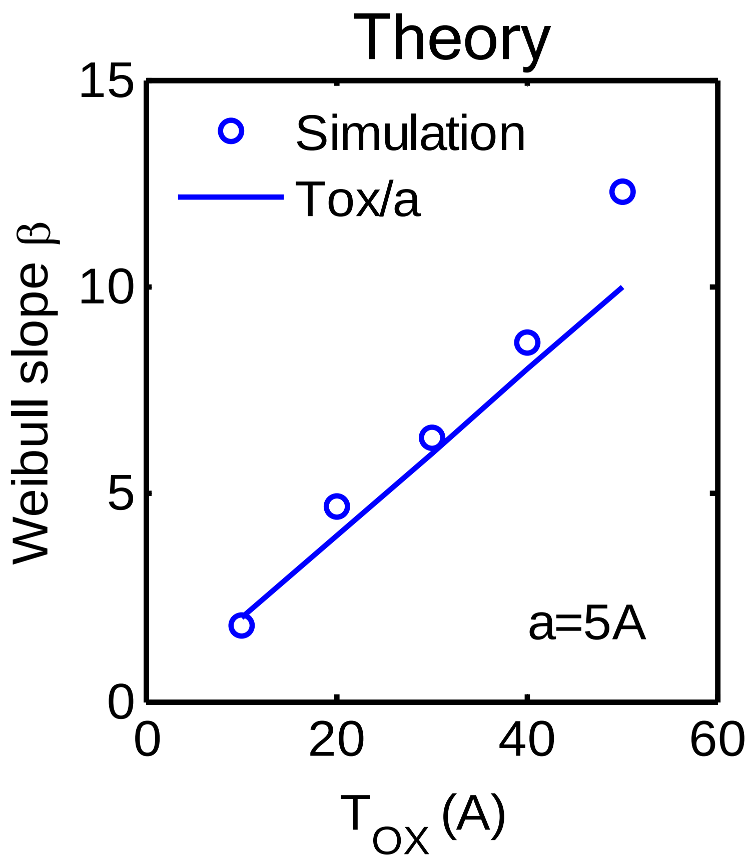
$$P_0 = (1 - p)^N = (1 - Np/N)^N = \exp(-Np)$$

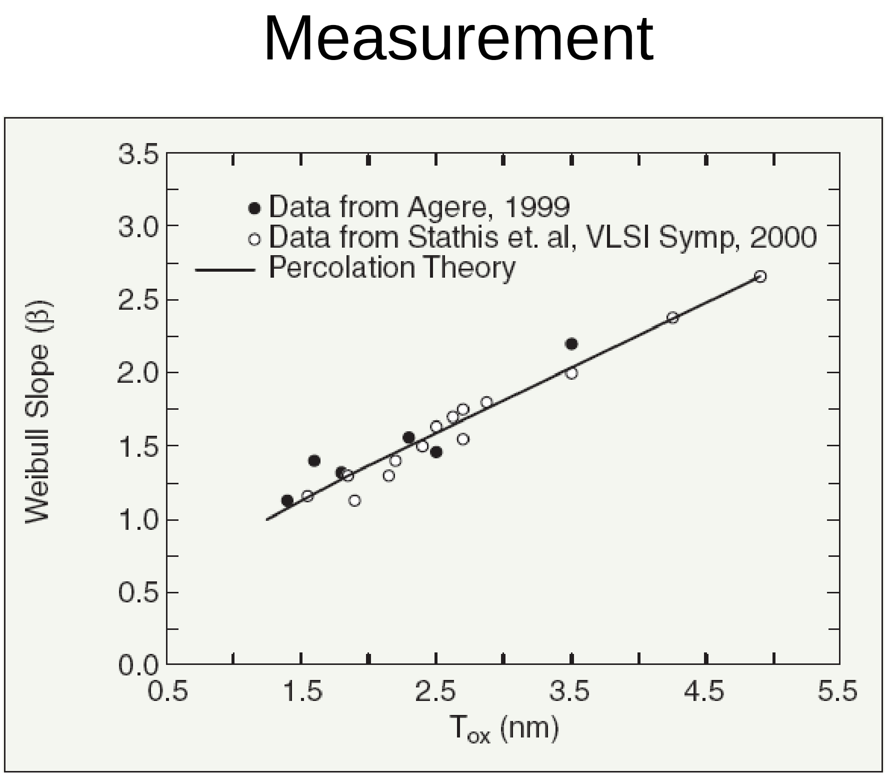
$$1 - F(p) = P_0 = \exp(-Np)$$

$$W \equiv \ln(-\ln(1 - F)) = M\alpha \ \ln(t) - M\alpha \ \ln(t_0) + \ln(N)$$

If the bottom up view is correct, then we will have a straight-line in a Weibull plot and slope proportional to thickness

bottom-up prediction for oxide scaling

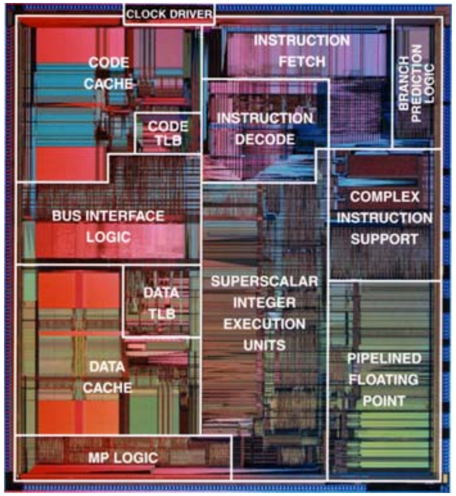




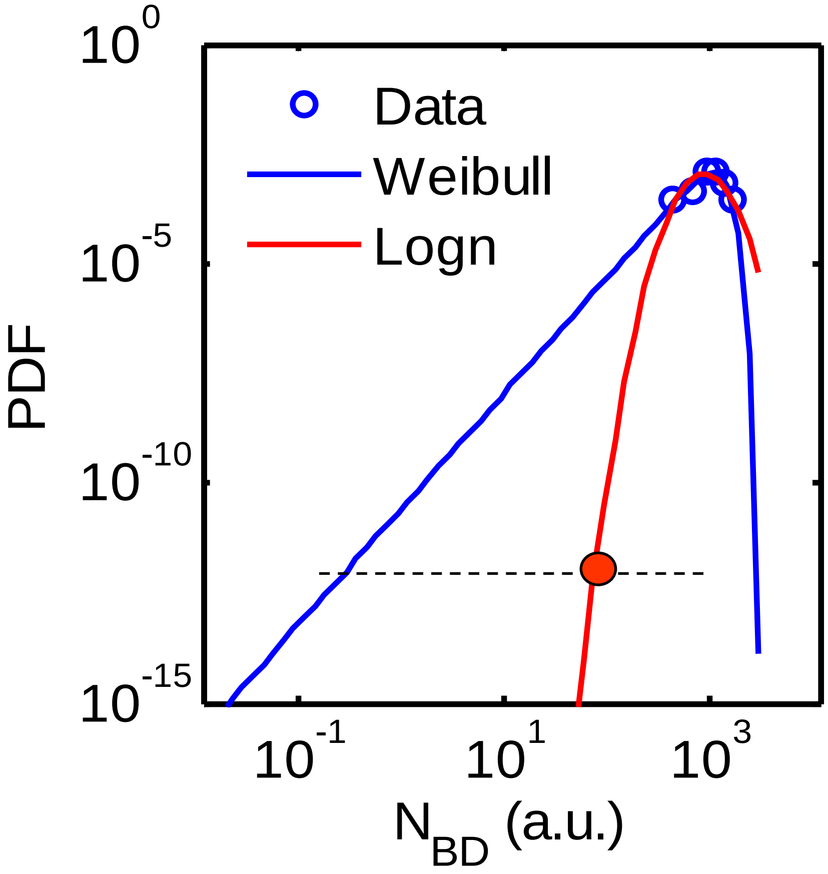
Thin oxide breaks much faster than thick oxide due to percolation, process-improvement can not solve this problem

very different lifetime projection …

1 CPU ~ 10^9 Transistors

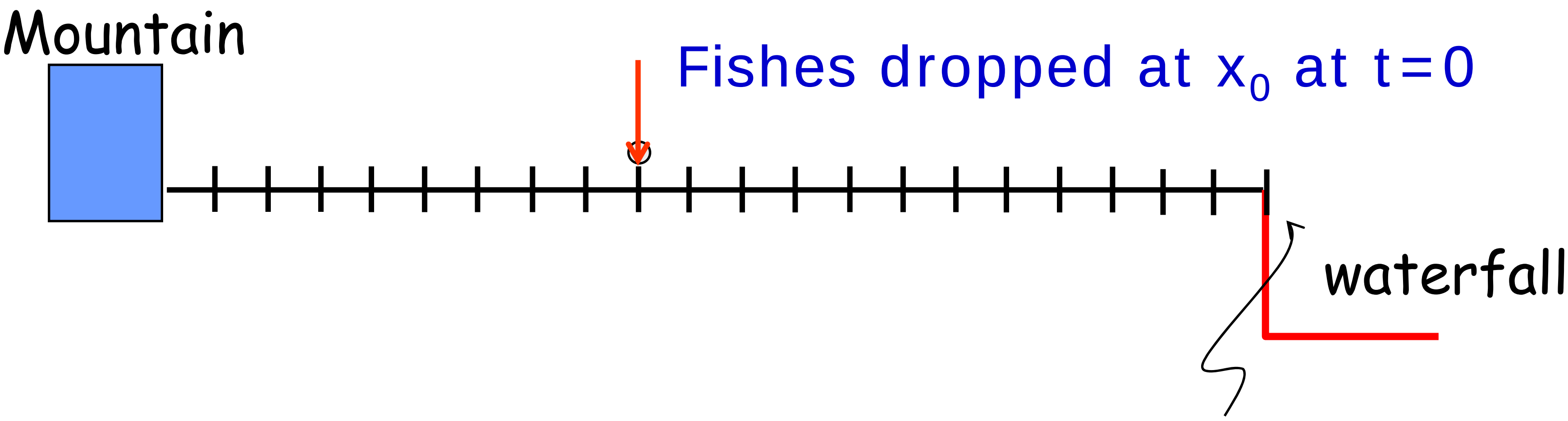


When one fails, so does the whole. Mean is not enough ….



Statistical distributions are physical ….

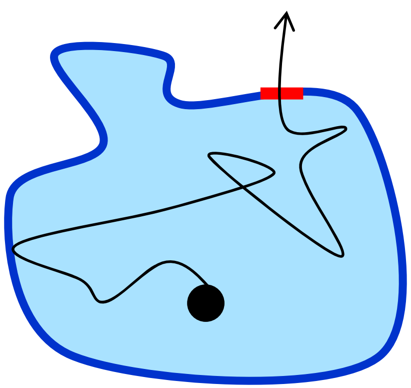
example 2: arrival time distribution



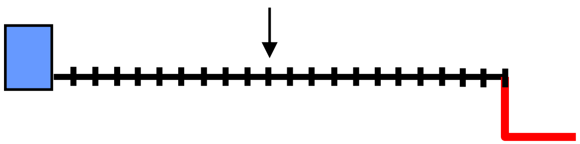
Find the arrival time distribution at the waterfall.

1D model for .....

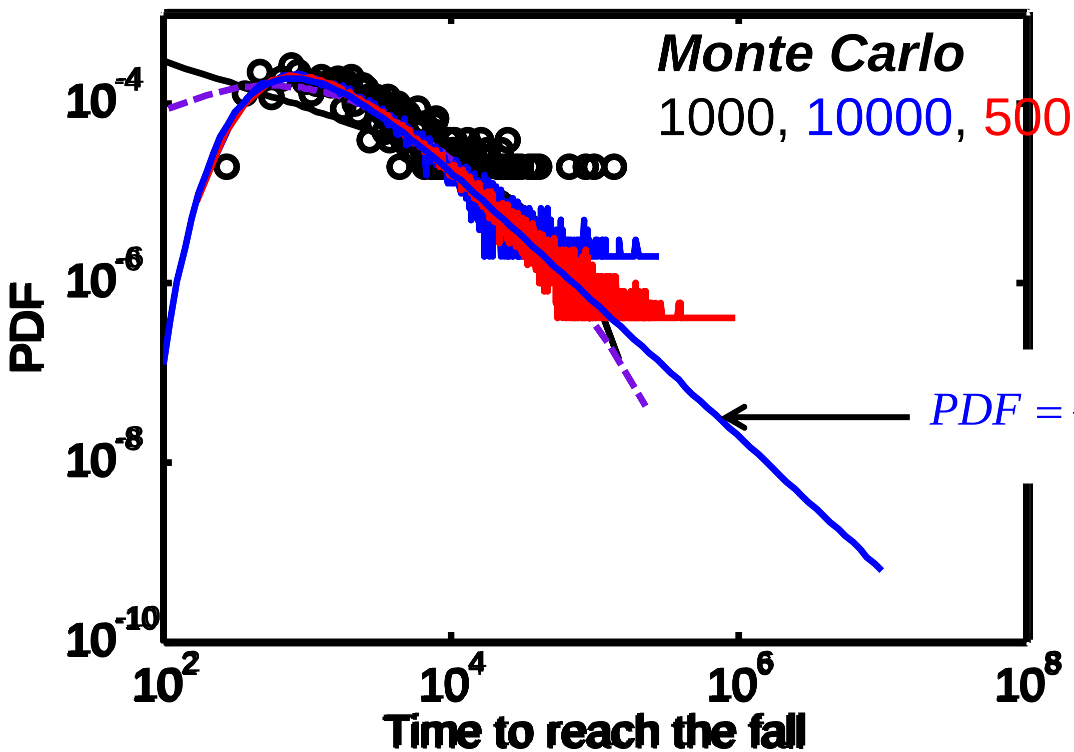
- field-return of components
- charge loss in Nanocrystal Flash
- release of proteins from inside the cells, etc.
- Drug release from capsules, etc.



approximated by classical distributions



$$f_G(t) = \frac{t^{k-1} e^{-t/\theta}}{\Gamma(k)\theta^k} \qquad T_{avg} = k\theta$$



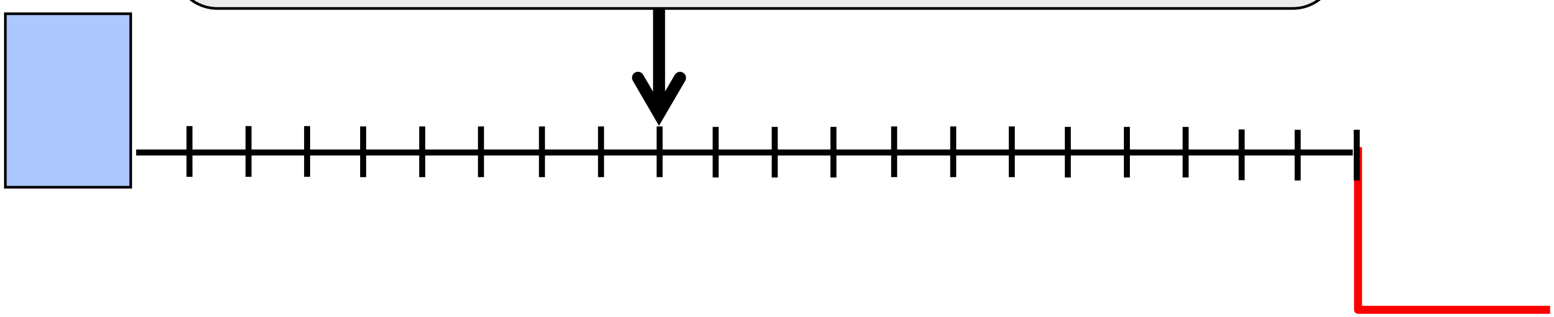
$$PDF = \frac{x_0}{\sqrt{4\pi D t^3}} e^{-x_0^2/4Dt}$$

derivation of “Fishy” (or BFRW) distribution

$$\frac{\partial P}{\partial t} = D\frac{\partial^2 P}{\partial x^2}$$

$$P(x, t=0) = \delta(x - x_0)$$

$$P(x=0, t) = 0$$

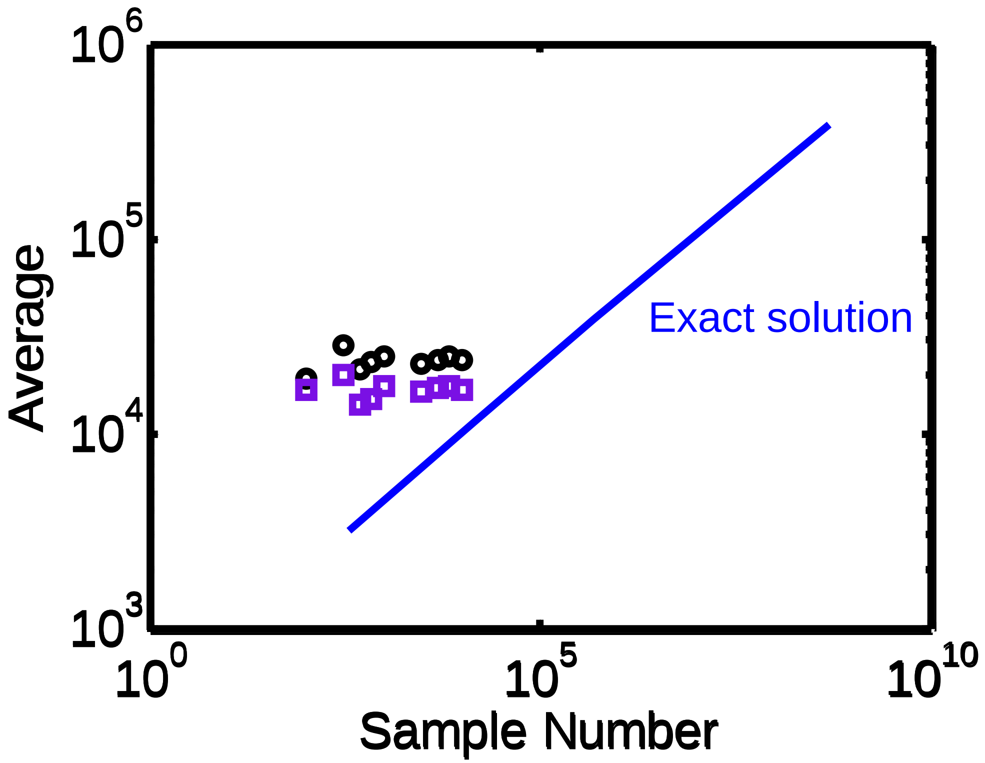


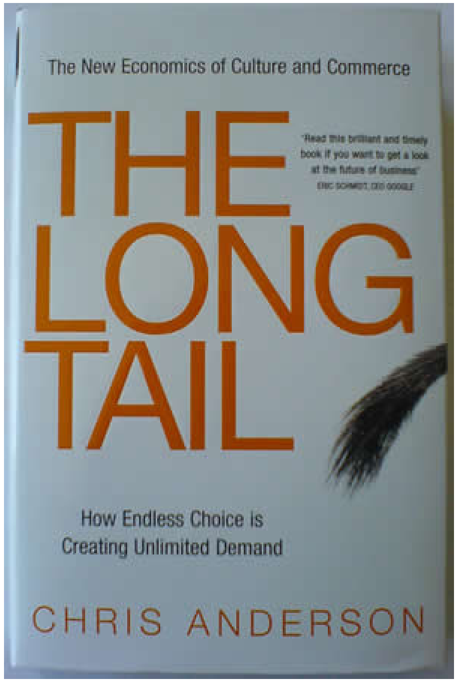
$$P(x,t) = (4\pi Dt)^{-1/2}\left[e^{-(x-x_0)^2/4Dt} - e^{-(x+x_0)^2/4Dt} \right]$$

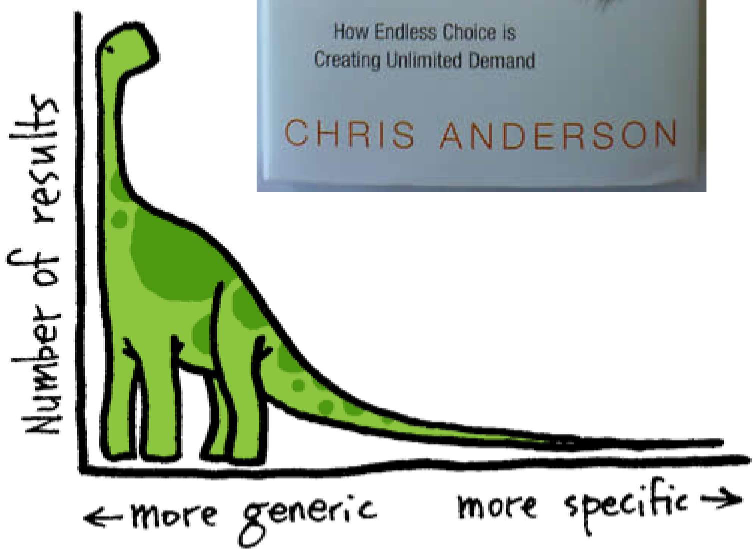
$$\int_0^t f(\tau)d\tau + \int_0^L P(x,t)dx = 1 \Rightarrow \quad f(t) = \frac{x_0}{\sqrt{4\pi Dt^3}} e^{-x_0^2/4Dt}$$

long tail of a distribution

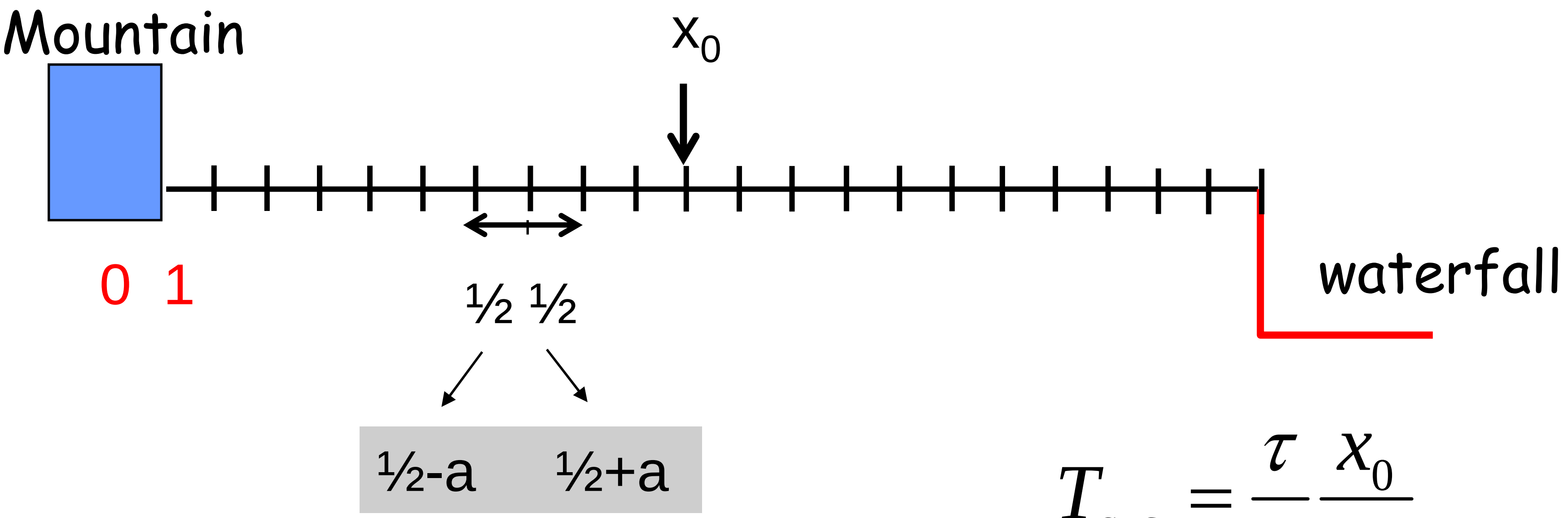
$$T_{avg} = \int_0^\infty t\, f(t)dt = \int_0^\infty \frac{x_0 t}{\sqrt{4\pi D t^3}} e^{-x_0{}^2/4Dt}\, dt \to \infty$$







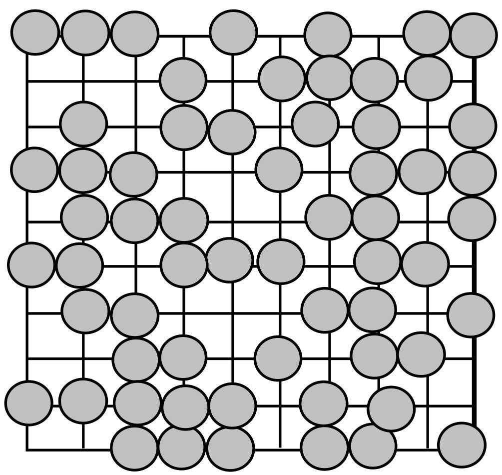
aside: averageless distribution

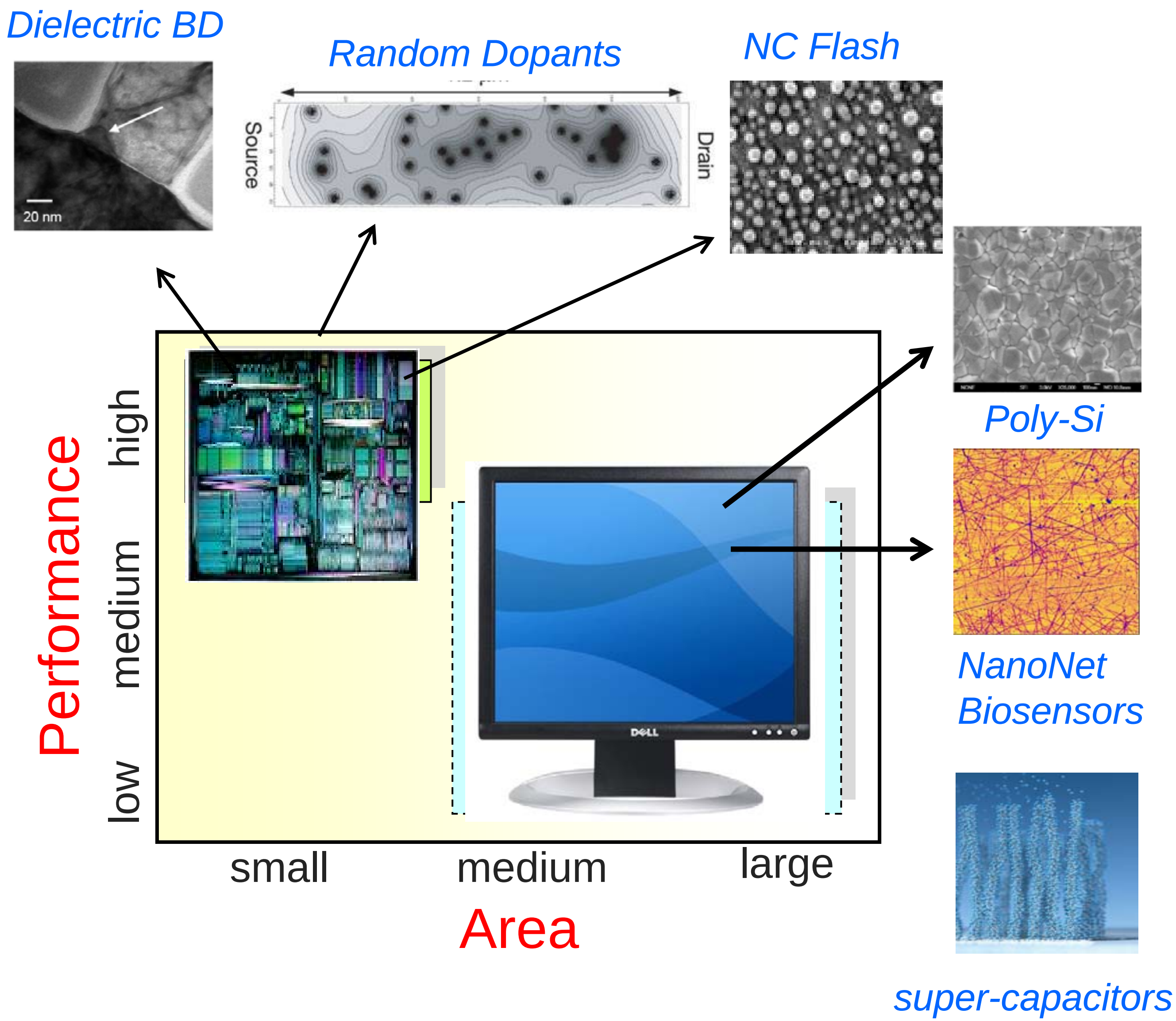


$$T_{avg} = \frac{\tau}{\delta}\frac{x_0}{2a}$$

… bottom line is that computer simulation alone would not do

so percolation theory is indeed necessary …

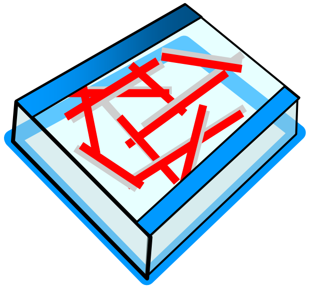


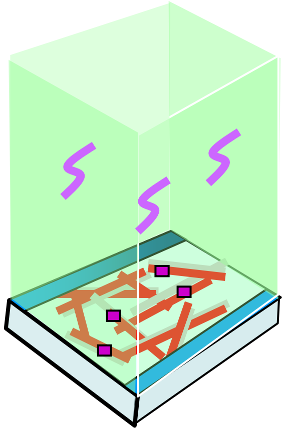


… but classical percolation is not enough

- ❑ Large system in thermodynamic limit

  Not explicitly concerned with fluctuation
- ❑ Disk percolation as a central paradigm
- ❑ Linear response theory
- ❑ Heterogeneous percolation is seldom used
- ❑ Transport on plane or a volume
- ❑ Primary interest in steady state systems





See you in lecture 2 then!