PURDUE UNIVERSITY

NCN
www.nanohub.org

ECE606: Solid State Devices
Lecture 18: Continuity Equations

Muhammad Ashraful Alam
alam@purdue.edu

Outline

1) **Continuity Equation**
2) Example problems
3) Conclusion

Ref. Advanced Semiconductor Fundamentals , pp. 205-210

Now the Continuity Equations …

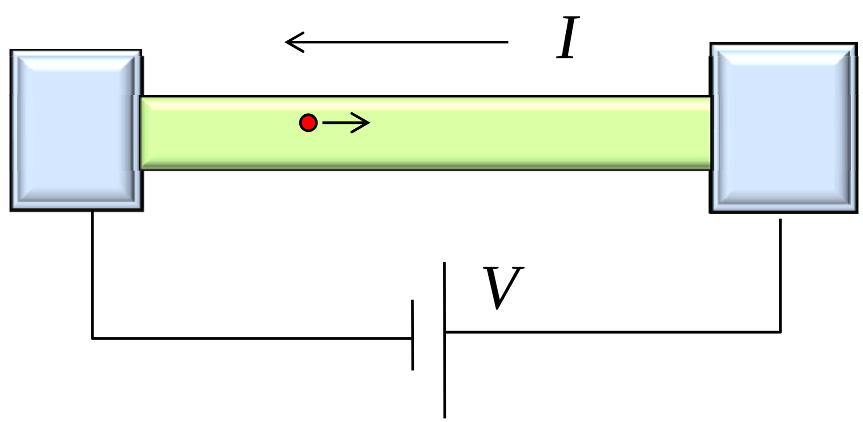
$$\frac{\partial n}{\partial t} = \frac{1}{q}\nabla \bullet \mathbf{J}_N - r_N + g_N$$

$$\mathbf{J}_N = qn\mu_N E + qD_N \nabla n$$

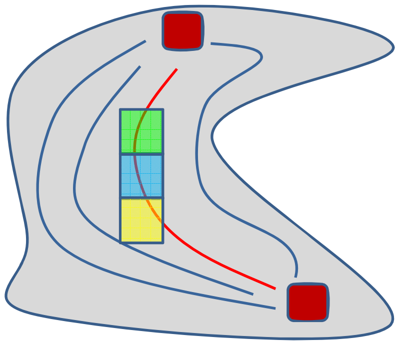
$$\frac{\partial p}{\partial t} = -\frac{1}{q}\nabla \bullet \mathbf{J}_P - r_P + g_P$$

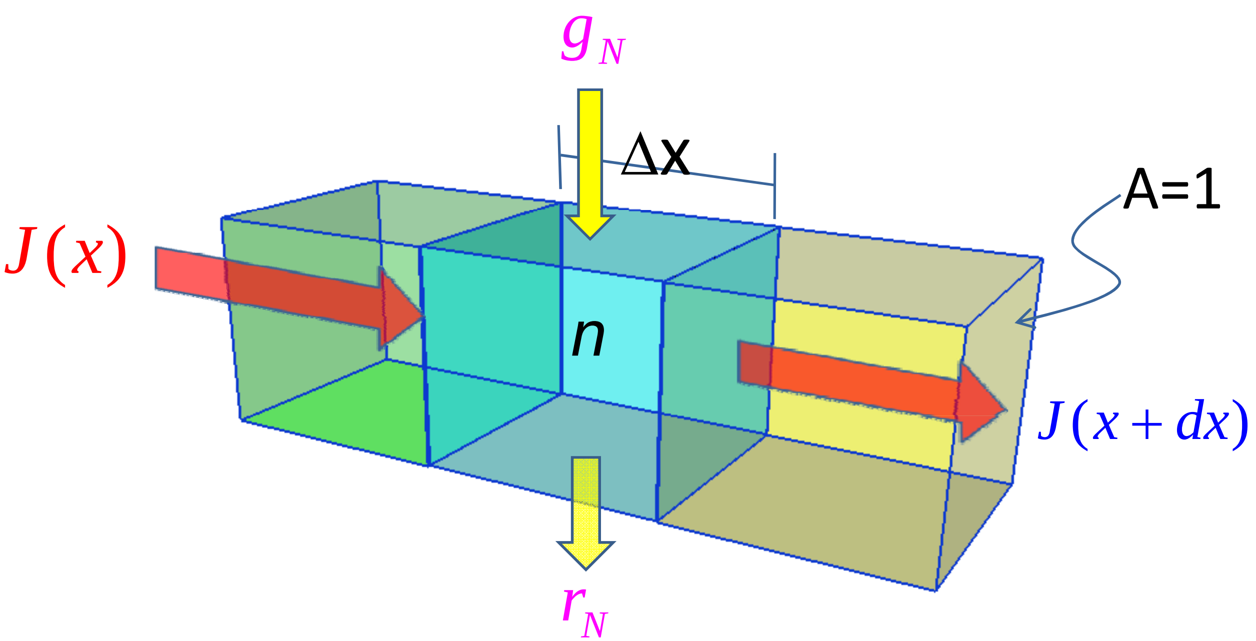
$$\mathbf{J}_P = qp\mu_P E - qD_P \nabla p$$

$$\nabla \bullet D = q\left(p - n + N_D^+ - N_A^-\right)$$



Now the Continuity Equations ...

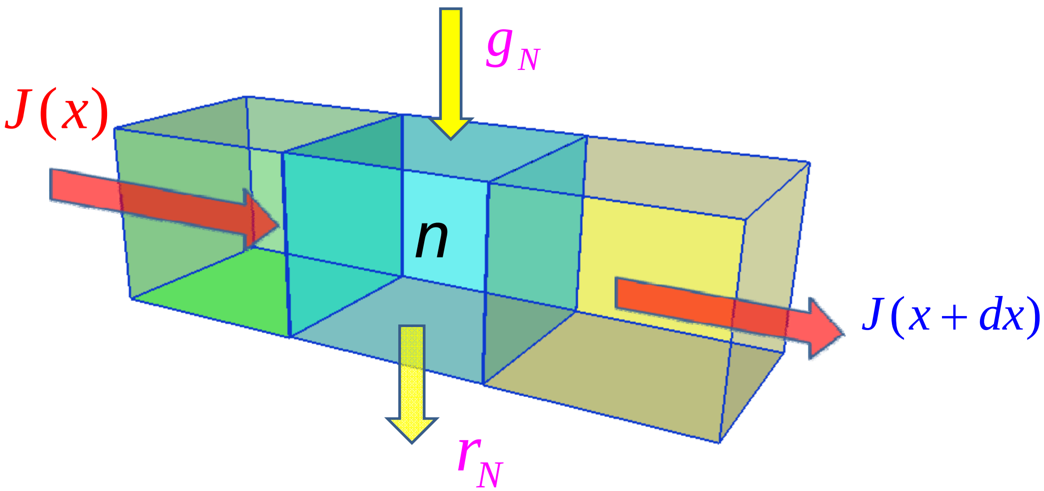




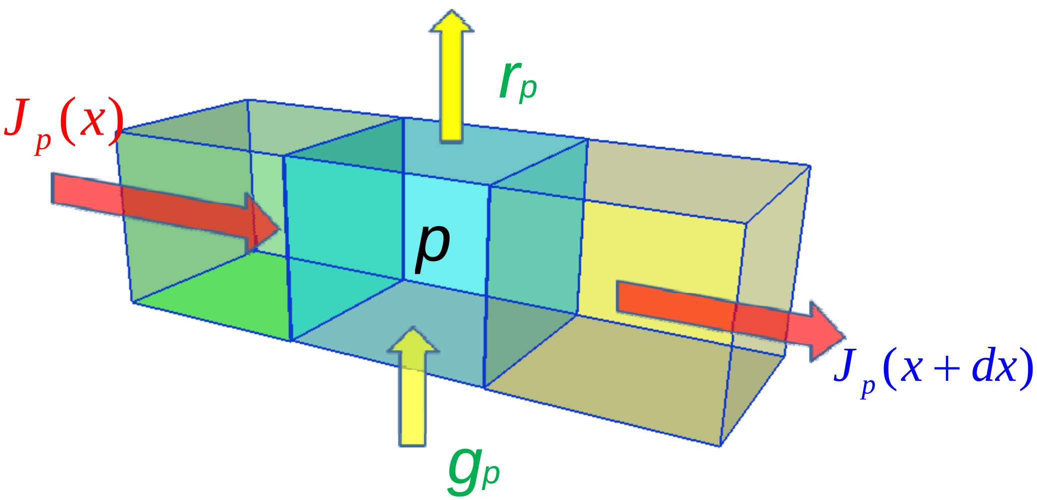
$$\frac{\partial\left(A\times\Delta x\times n\right)}{\partial t}=\frac{A\times J_n(x)-A\times J_n(x+dx)}{-q}+A\times g_N\Delta x-A\times r_N\Delta x$$

$$\frac{\partial n}{\partial t}=\frac{J_n(x)-J_n(x+dx)}{-q\Delta x}+g_N-r_N=\frac{1}{q}\nabla\bullet J_n+g_N-r_N$$

Continuity Equations for Electron/Holes

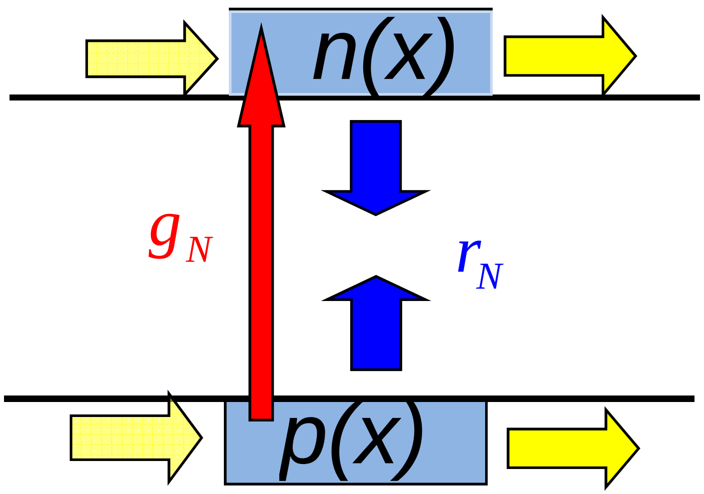


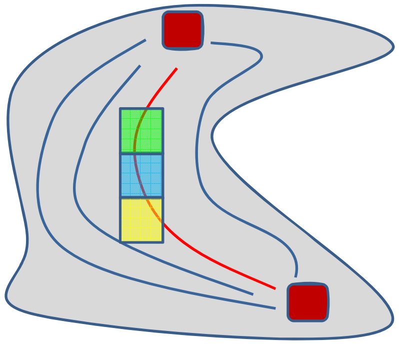
$$\frac{\partial n}{\partial t} = \frac{1}{q}\nabla \bullet J_n + g_N - r_N$$

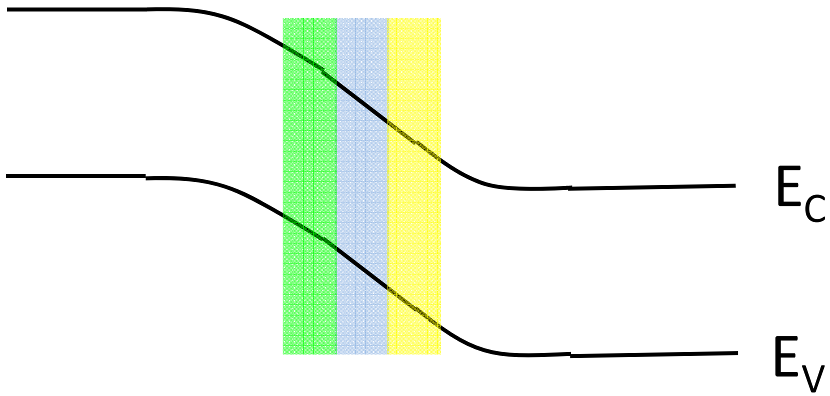


$$\frac{\partial p}{\partial t} = -\frac{1}{q}\nabla \bullet J_P + g_P - r_P$$

Continuity Equations via Energy Bands



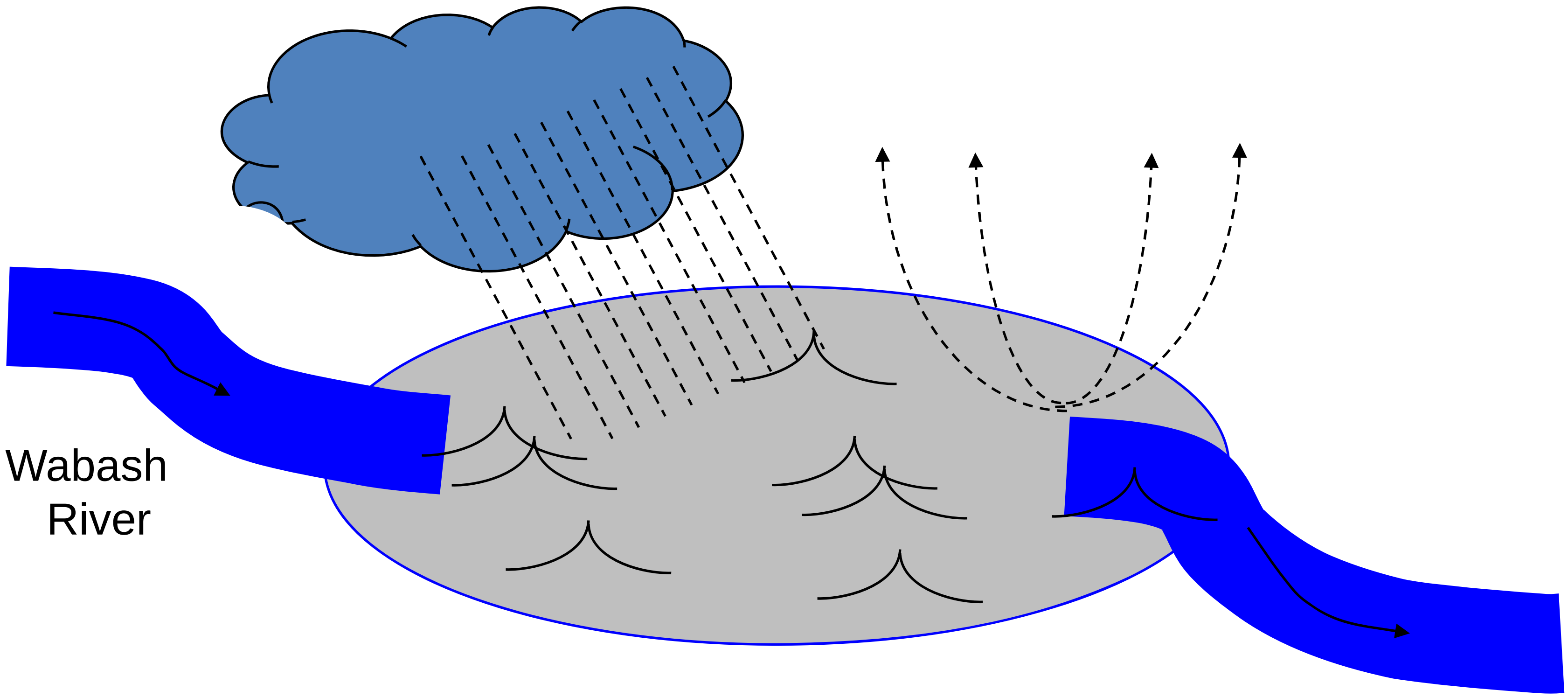




$$\frac{\partial n}{\partial t} = \frac{1}{q} \nabla \bullet J_n + g_N - r_N$$

$$\frac{\partial p}{\partial t} = -\frac{1}{q} \nabla \bullet J_P + g_P - r_P$$

Continuity Equation: A Good Analogy



Rate of increase of water level in lake = (in flow - outflow) + rain - evaporation

$$\frac{\partial n}{\partial t} = \frac{1}{q}\nabla \bullet J_n + g_N - r_N$$

Equations in place, learning to solve them…

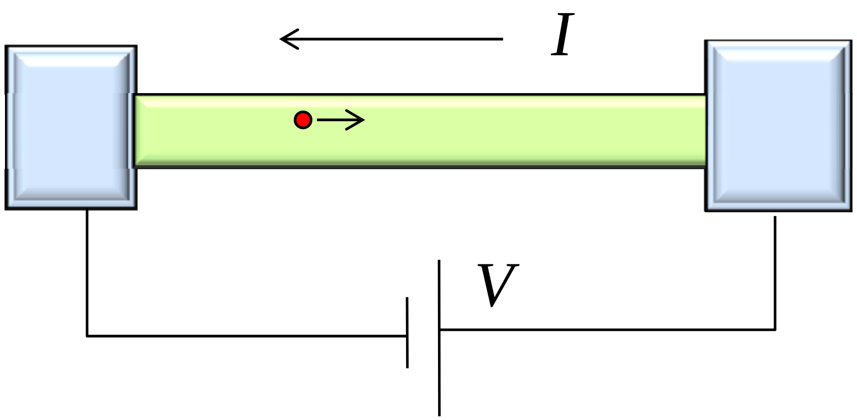
$$\frac{\partial n}{\partial t} = \frac{1}{q}\nabla \bullet \mathbf{J}_N - r_N + g_N$$

$$\mathbf{J}_N = qn\mu_N E + qD_N \nabla n$$

$$\frac{\partial p}{\partial t} = \frac{-1}{q}\nabla \bullet \mathbf{J}_P - r_P + g_P$$

$$\mathbf{J}_P = qp\mu_P E - qD_P \nabla p$$

$$\nabla \bullet E = q\left(p - n + N_D^+ - N_A^-\right)$$



Two methods of solution:

Numerical and Analytical

Analytical Solutions

$$\nabla \bullet D = q\left(p - n + N_D^+ - N_A^-\right)$$ ← Band-diagram

$$\frac{\partial n}{\partial t} = \frac{1}{q}\nabla \bullet \mathbf{J}_N - r_N + g_N$$

$$\mathbf{J}_N = qn\mu_N E + qD_N \nabla n$$

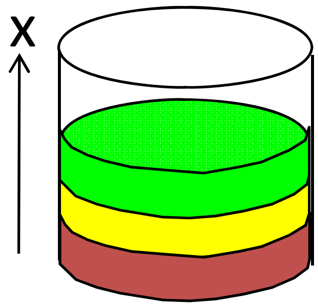
$$\frac{\partial p}{\partial t} = \frac{-1}{q}\nabla \bullet \mathbf{J}_P - r_P + g_P$$ ← Diffusion approximation, Minority carrier transport, Ambipolar transport

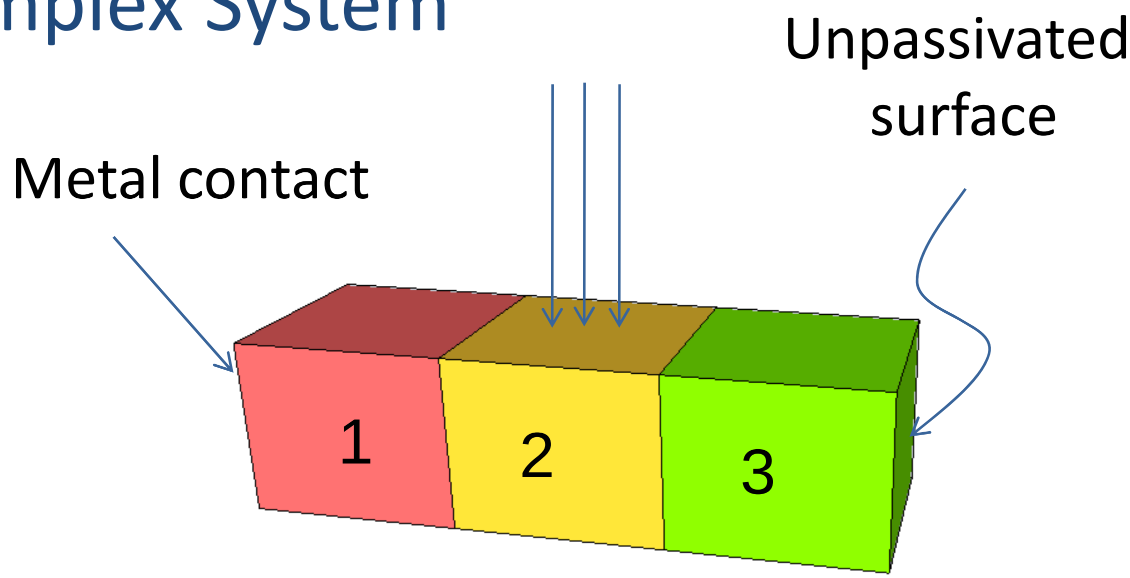
$$\mathbf{J}_P = qp\mu_P E - qD_P \nabla p$$

Outline

1) Continuity Equation
2) **Example problems**
3) Conclusion

Consider a Complex System





- Acceptor doped
- Light turned on in the middle section.
- The right region is full of mid-gap traps
- Interface traps at the end of the right region.
- The left region is trap free.
- The left/right regions contacted by metal electrode.

Recall: Analytical Solution of Schrodinger Equation

1) $$\frac{d^2\psi}{dx^2} + k^2\psi = 0$$ → 2N unknowns for N regions

2) $$\psi(x = -\infty) = 0$$
$$\psi(x = +\infty) = 0$$ → Reduces 2 unknowns

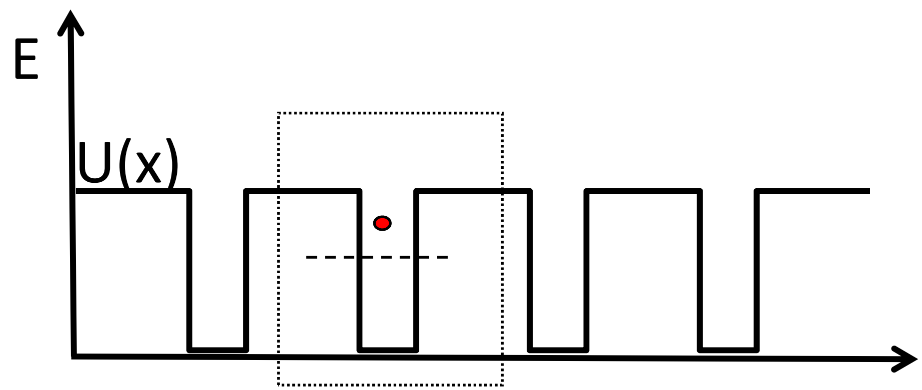
3) $$\psi\Big|_{x=x_B^-} = \psi\Big|_{x=x_B^+}$$
$$\left.\frac{d\psi}{dx}\right|_{x=x_B^-} = \left.\frac{d\psi}{dx}\right|_{x=x_B^+}$$ → Set 2N-2 equations for 2N-2 unknowns (for continuous U)

4) Det(coefficient matix)=0
And find E by graphical or numerical solution

5) $$\int_{-\infty}^{\infty} |\psi(x,E)|^2 \, dx = 1$$

for wave function

Recall: Bound-levels in Finite well

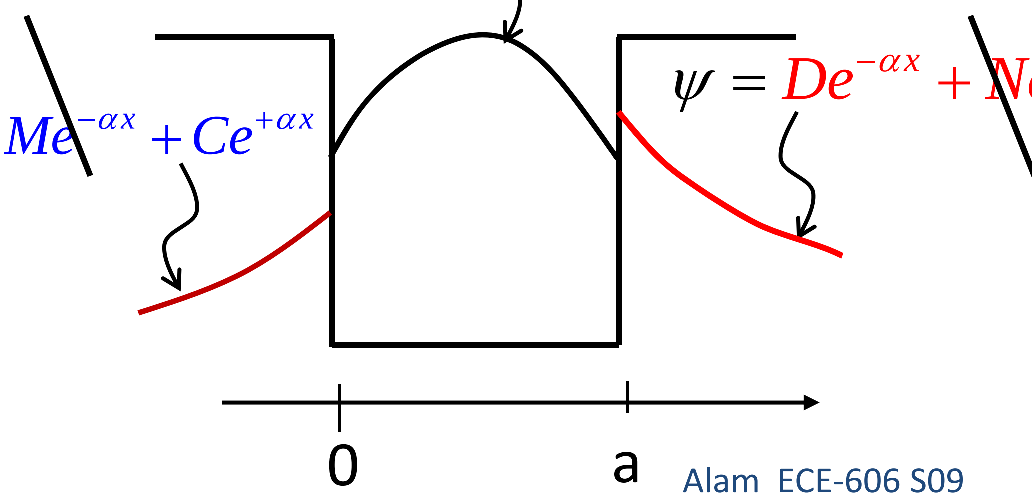


1)

$$\psi = A\sin kx + B\cos kx$$

$$\psi = Me^{-\alpha x} + Ce^{+\alpha x}$$

$$\psi = De^{-\alpha x} + Ne^{+\alpha x}$$



2) Boundary Conditions …

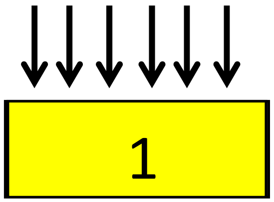
$$\psi(x = -\infty) = 0$$

$$\psi(x = +\infty) = 0$$

Example: Transient, Uniform Illumination

(uniform)

$$\frac{\partial n}{\partial t} = \frac{1}{q} \cancel{\nabla} \bullet \mathbf{J}_N - r_N + g_N \qquad \mathbf{J}_N = qn\mu_N E + qD_N \nabla n$$



$$\frac{\partial(\cancel{n_0} + \Delta n)}{\partial t} = -\frac{\Delta n}{\tau_n} + G$$

Acceptor doped

(uniform)

$$\frac{\partial p}{\partial t} = \frac{-1}{q} \cancel{\nabla} \bullet \mathbf{J}_p - r_P + g_p \qquad \mathbf{J}_p = qp\mu_p E - qD_P \nabla p$$

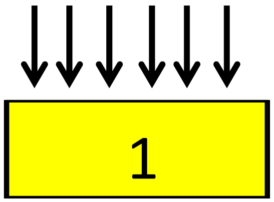
$$\frac{\partial(\cancel{p_0} + \Delta p)}{\partial t} = -\frac{\Delta p}{\tau_p} + G$$

Majority carrier

$$\nabla \bullet D = q\left(p - n + N_D^+ - N_A^-\right) = q\left(p_0 + \Delta n - n_0 - \Delta p + N_D^+ - N_A^-\right) = 0$$

Example: Transient, Uniform Illumination

$$\frac{\partial(\Delta n)}{\partial t} = -\frac{\Delta n}{\tau_n} + G$$



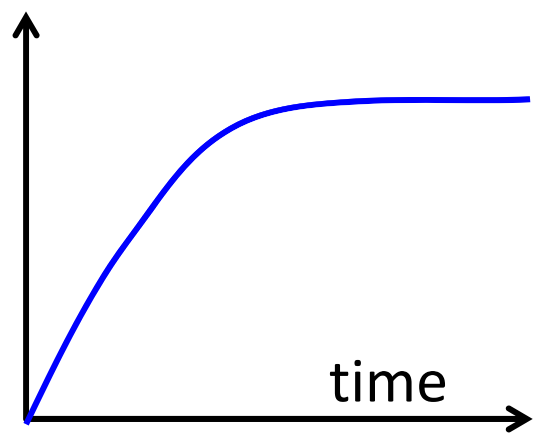
Acceptor doped

$$\Delta n(x,t) = A + Be^{-t/\tau_n}$$

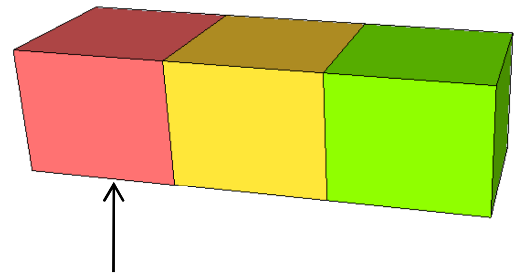
$$t = 0, \quad \Delta n(x,0) = 0 \Rightarrow A = -B$$

$$t \to \infty, \quad \Delta n(x,\infty) = G\tau_n = A$$

$$\Delta n(x,t) = G\tau_n\left(1 - e^{-t/\tau_n}\right)$$



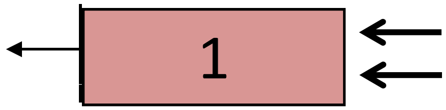
Example: One sided Minority Diffusion



Trap-free

Steady state

Acceptor doped



$$\cancel{\frac{\partial n}{\partial t}} = \frac{1}{q}\frac{dJ_n}{dx} - \cancel{r_N} + \cancel{g_N}$$

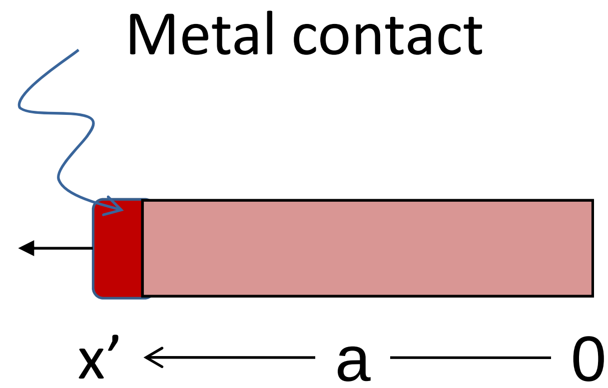
$$\mathbf{J}_N = qn\cancel{\mu_N}E + qD_N\frac{dn}{dx}$$

$$0 = D_N\frac{d^2 n}{dx^2}$$

Example: One sided Minority Diffusion

$$0 = D_N \frac{d^2 n}{dx^2}$$

$$J = q\, n_m \upsilon_m$$

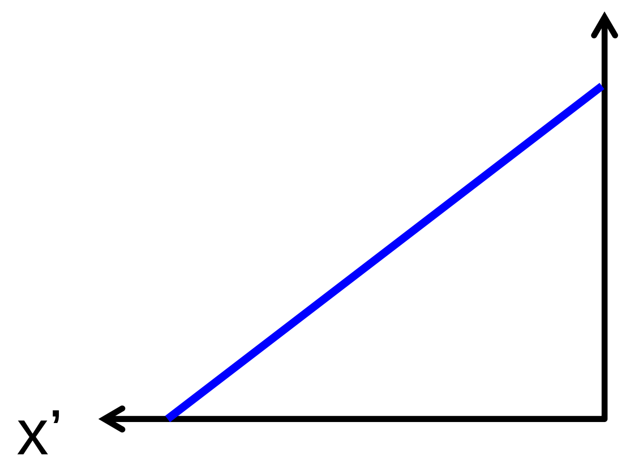


$$\Delta n(x,t) = C + Dx'$$

$$x = a, \quad \Delta n(x' = a) = 0$$

$$x = 0', \quad \Delta n(x' = 0') = C$$

$$\Delta n(x,t) = \Delta n(x = 0')\left(1 - \frac{x'}{a}\right)$$



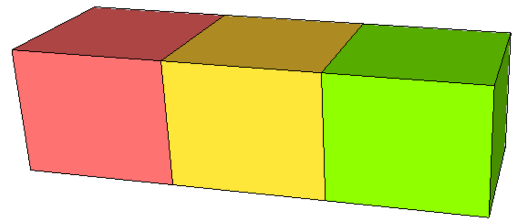
Example: Minority Diffusion with RG

$$\cancel{\frac{\partial n}{\partial t}} = \frac{1}{q}\frac{dJ_n}{dx} - r_N + \cancel{g_N}$$

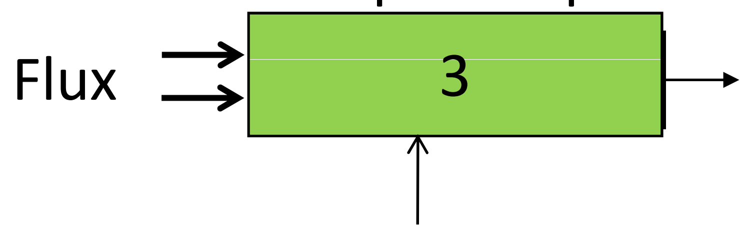
$$\mathbf{J}_N = \cancel{qn\mu_N E} + qD_N\frac{dn}{dx}$$

$$0 = D_N\frac{d^2(n_0+\Delta n)}{dx^2} - \frac{\Delta n}{\tau_n}$$

$$0 = D_N\frac{d^2\Delta n}{dx^2} - \frac{\Delta n}{\tau_n}$$



Steady state
Acceptor doped



Trap-filled

Diffusion with Recombination ...

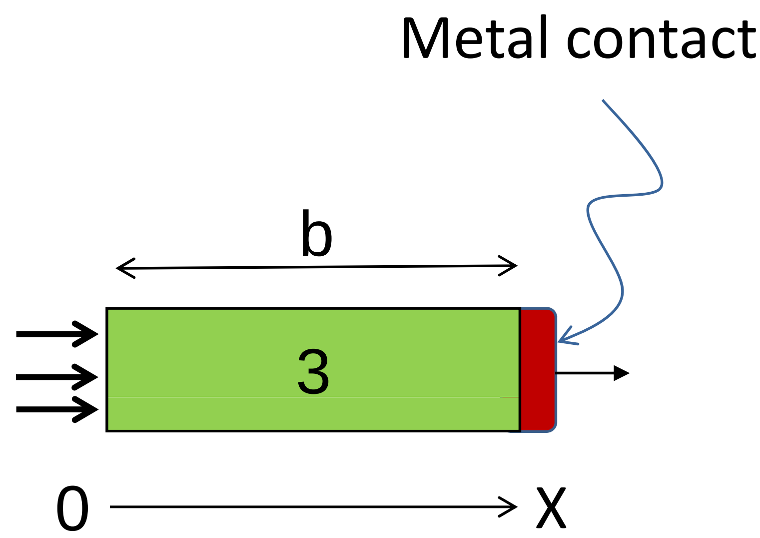
$$D_N \frac{d^2 \Delta n}{dx^2} - \frac{\Delta n}{\tau_n} = 0$$

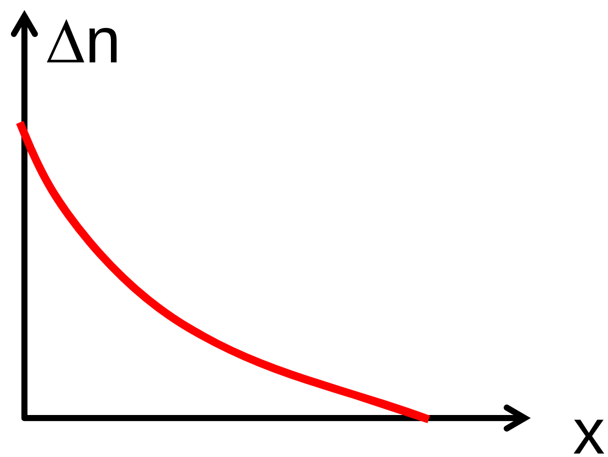
$$\Delta n(x,t) = E e^{x/L_n} + F e^{-x/L_n}$$

$$x = b, \quad \Delta n(x = b) = 0 \quad \Rightarrow F = -E e^{2b/L_n}$$

$$x = 0, \quad \Delta n(x = 0) = E + F = \Delta n(x = 0)$$

$$\Delta n(x,t) = \frac{\Delta n(0)}{(1 - e^{2b/L_n})} (e^{x/L_n} - e^{2b/L_n} e^{-x/L_n})$$

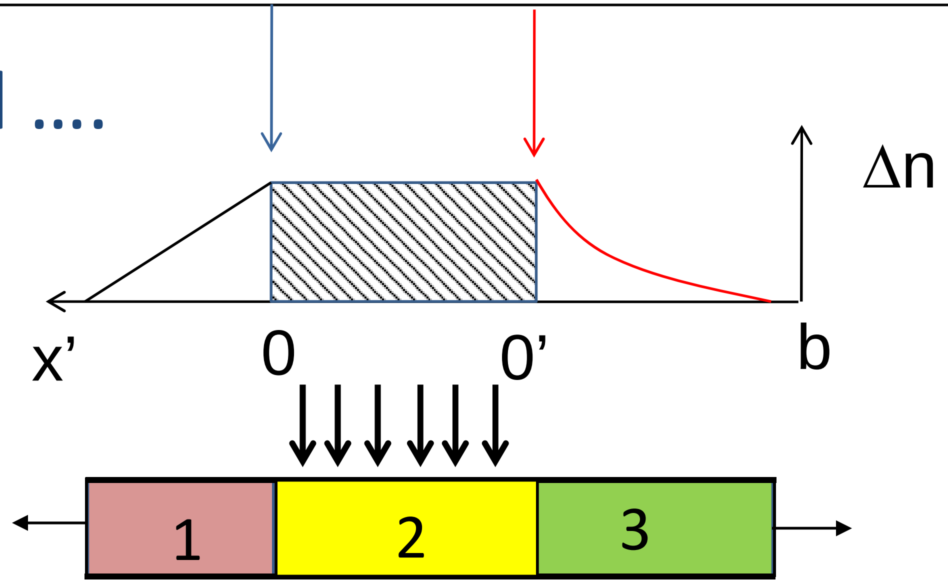




Combining them all ....

$$\Delta n_2(x) = G\tau_n =$$

$$\Delta n_2(0) = \Delta n_2(0')$$



Match boundary condition

$$\Delta n_1(x') = \Delta n(x=0)\left(1-\frac{x'}{a}\right) = G\tau_n\left(1-\frac{x'}{a}\right)$$

$$\Delta n(x) = \frac{\Delta n(0')}{(1-e^{2b/L_n})}(e^{x/L_n} - e^{2b/L_n}e^{-x/L_n}) = \frac{G\tau_n(e^{x/L_n} - e^{2b/L_n}e^{-x/L_n})}{(1-e^{2b/L_n})}$$

Calculating current $\quad \mathbf{J}_N = qn\mu_N E + qD_N \frac{dn}{dx}$

Why is the E-field for minority carriers negligible ?

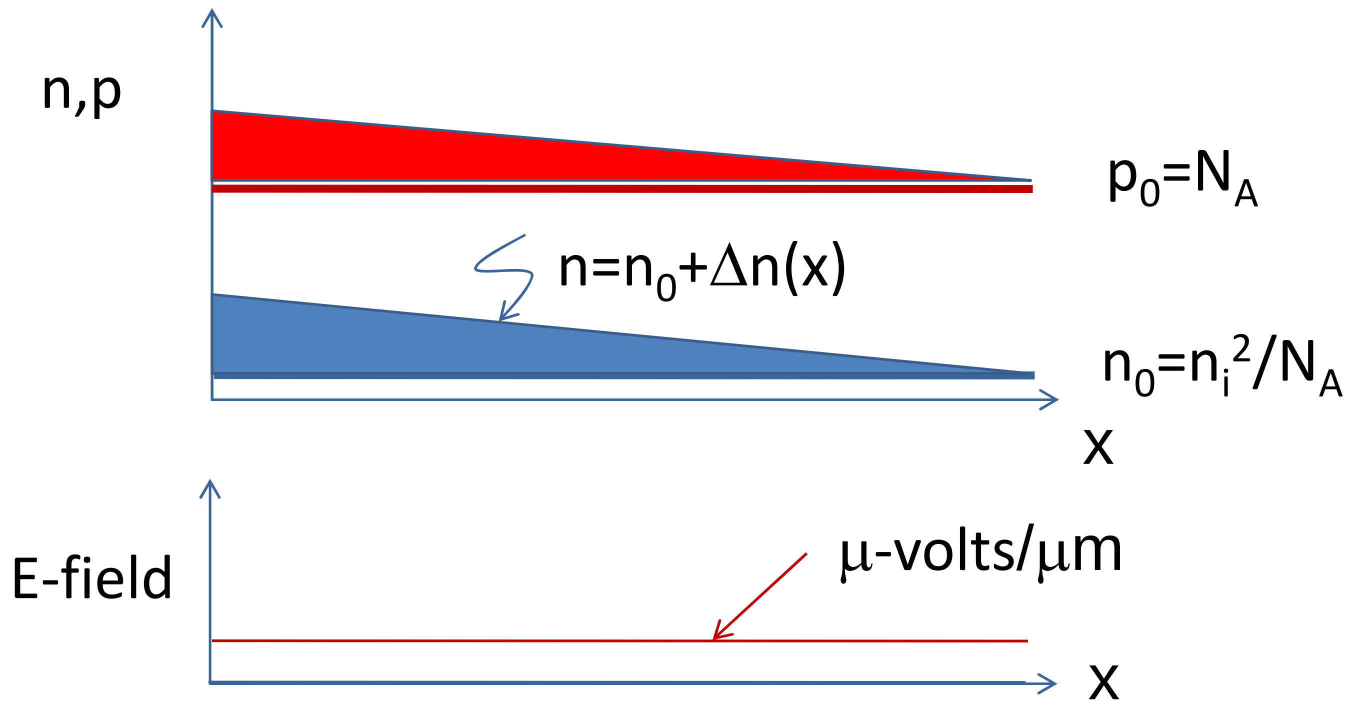
$$\nabla \bullet D_0 = q\left(p_0 - n_0 + N_D^+ - N_A^-\right) = 0$$

$$\mathbf{J}_N = qn\cancel{\mu_N \mathcal{E}} + qD_N \frac{dn}{dx}$$

$$\nabla \bullet D = q\left(p - n + N_D^+ - N_A^-\right)$$

$$= q\left(p_0 + \Delta n - n_0 + N_D^+ - N_A^-\right) \neq 0$$

$$\nabla \bullet D = q\left(p_0 + \Delta n - n_0 - \Delta p + N_D^+ - N_A^-\right) \approx 0$$



Conclusion

1) Continuity Equations form the basis of analysis of all the devices we will study in this course.

2) Full numerical solution of the equations are possible and many commercial software are available to do so.

3) Analytical solutions however provide a great deal of insight into the key physical mechanism involved in the operation of a device.