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ECE606: Solid State Devices
Lecture 21: p-n Diode I-V Characteristics

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Outline

1) **Derivation of the forward bias formula**
2) Solution in the nonlinear regime
3) I-V in the ambipolar regime
4) Conclusion

Ref. SDF, Chapter 6

Topic Map

	Equilibrium	**DC**	Small signal	Large Signal	Circuits
Diode		◆			
Schottky					
BJT/HBT					
MOSFET					

Continuity Equations for p-n junction Diode

$$\nabla \bullet E = q\left(p - n + N_D^+ - N_A^-\right)$$

$$\frac{\partial n}{\partial t} = \frac{1}{q}\nabla \bullet \mathbf{J}_N - r_N + g_N$$

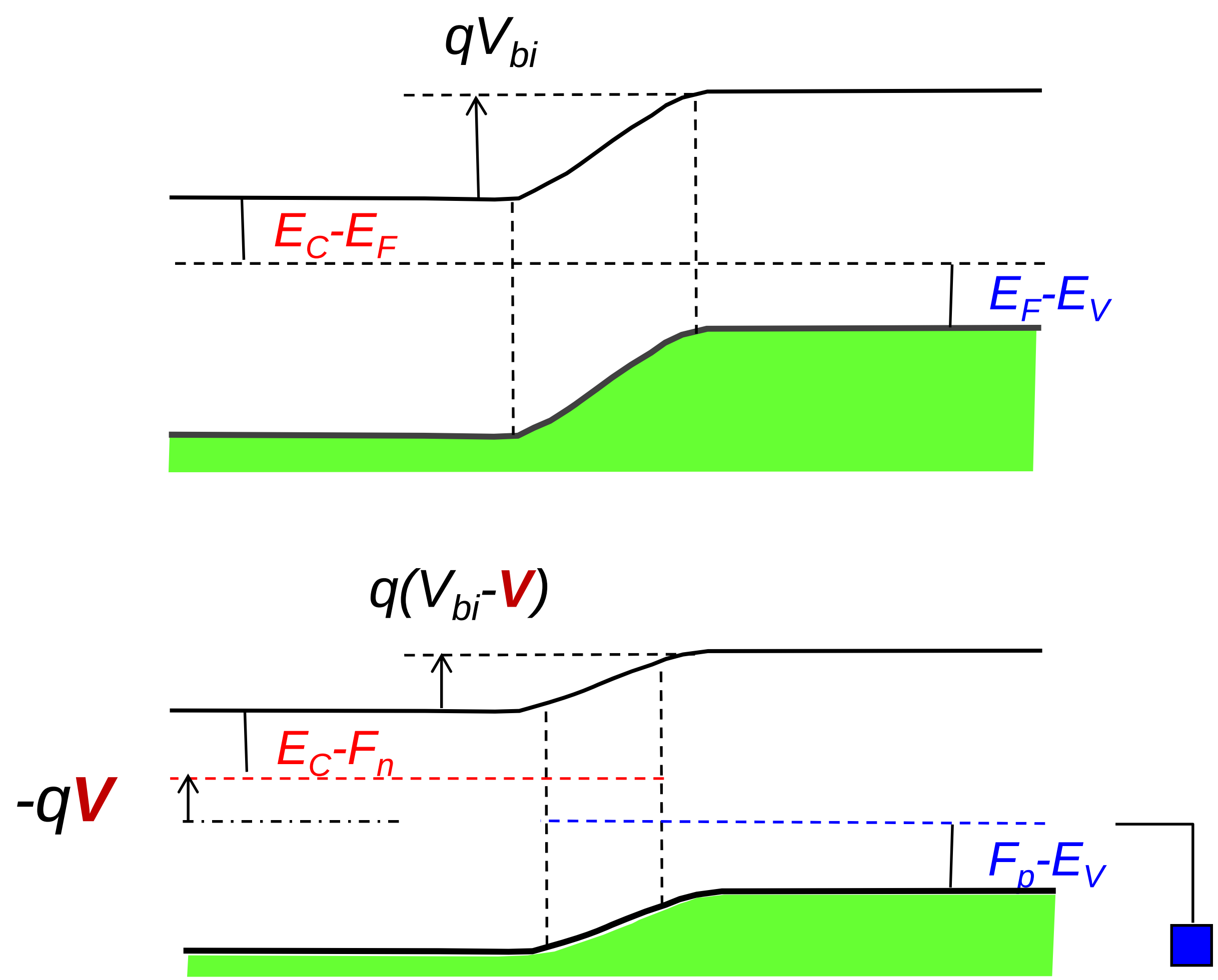
$$\mathbf{J}_N = qn\mu_N E + qD_N \nabla n$$

$$\frac{\partial p}{\partial t} = \frac{1}{q}\nabla \bullet \mathbf{J}_P - r_P + g_P$$

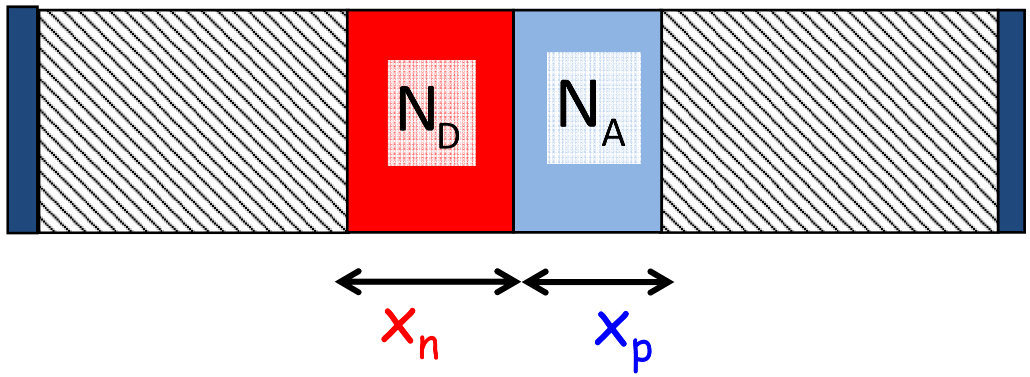
$$\mathbf{J}_P = qp\mu_P E - qD_P \nabla p$$

Will focus on this part today …

Applying a Bias: Poisson Equation



Depletion Widths



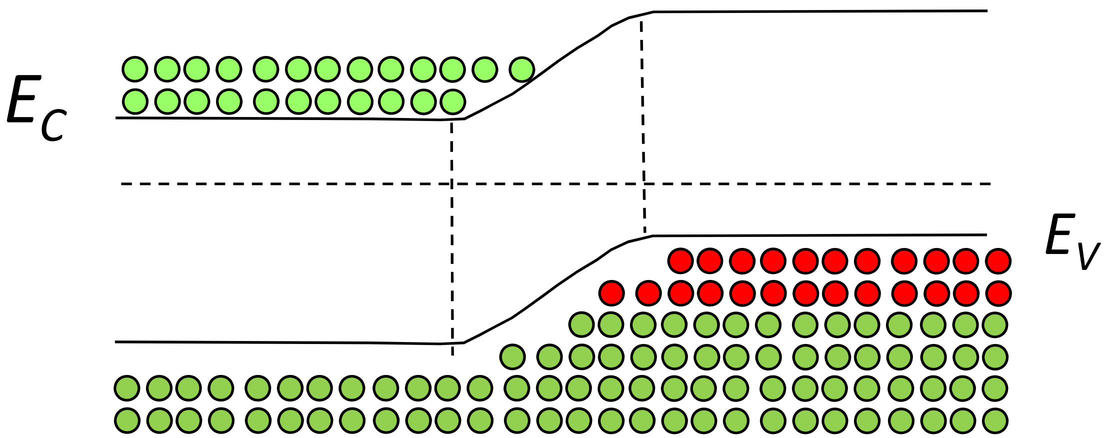
$$N_D x_n = N_A x_p$$

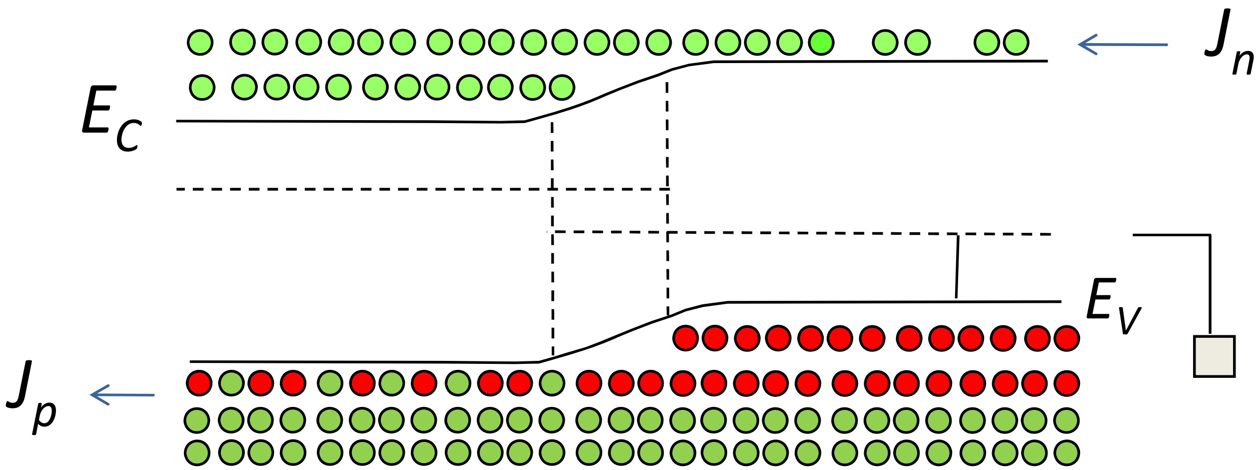
$$q\left(V_{bi} - V\right) = \frac{qN_D x_n^{\ 2}}{2k_s\varepsilon_0} + \frac{qN_A x_p^{\ 2}}{2k_s\varepsilon_0}$$

$$x_n = \sqrt{\frac{2k_s\varepsilon_0}{q}\frac{N_A}{N_D\left(N_A+N_D\right)}(V_{bi}-V)}$$

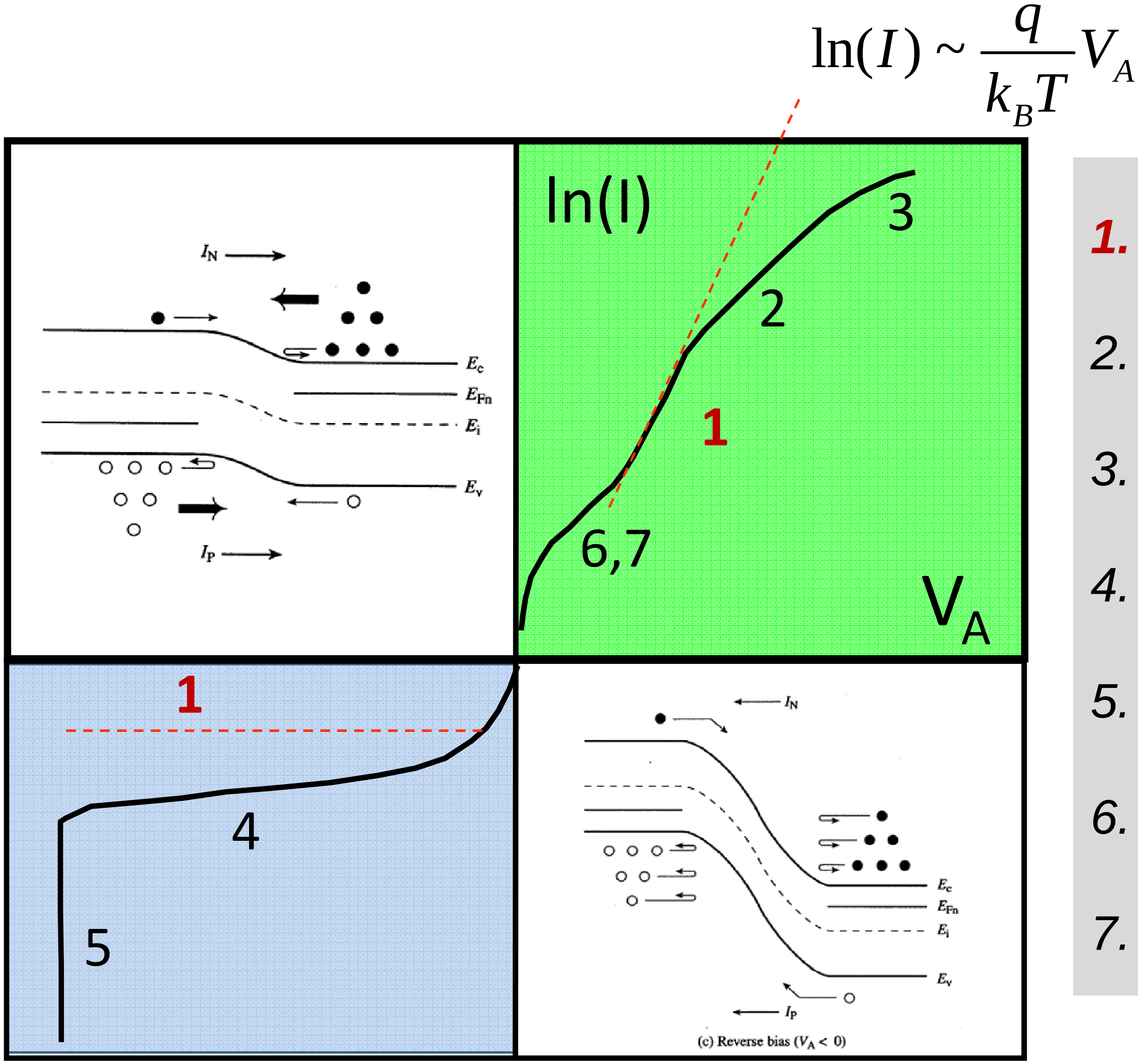
$$x_p = \sqrt{\frac{2k_s\varepsilon_0}{q}\frac{N_D}{N_A\left(N_A+N_D\right)}(V_{bi}-V)}$$

Flat Quasi-Fermi Level up to Junction





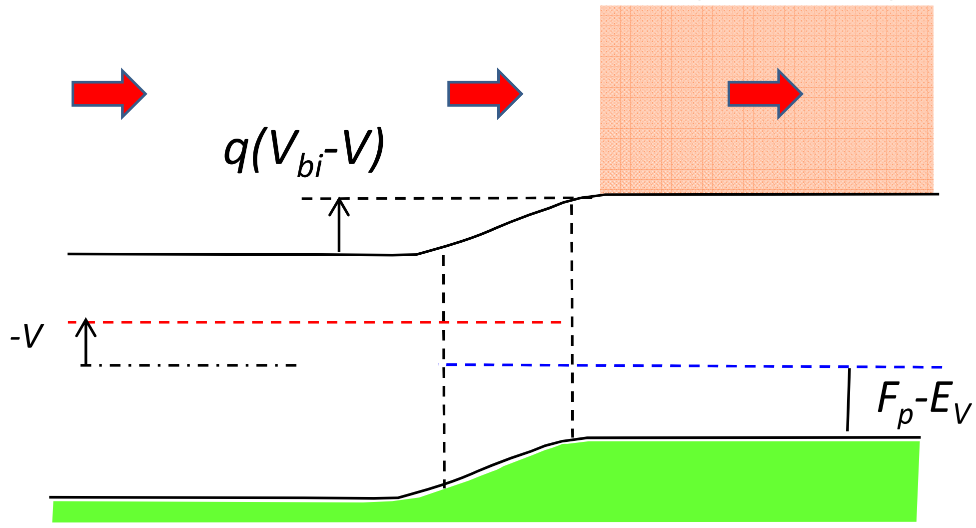
Various Regions of I-V Characteristics



1. **Diffusion limited**
2. *Ambipolar transport*
3. *High injection*
4. *R-G in depletion*
5. *Breakdown*
6. *Trap-assisted R-G*
7. *Esaki Tunneling*

Recall: One Sided Minority Diffusion

Steady state
Acceptor doped



Can calculate current anywhere, let us solve the problem where it is the easiest ...

$$\frac{\partial n}{\partial t} = \frac{1}{q}\frac{dJ_n}{dx} - r_n + g_n$$

$$J_n = qn\mu_n\mathcal{E} + qD_n\frac{dn}{dx}$$

$$0 = D_n\frac{d^2n}{dx^2}$$

Boundary Conditions

$$\Delta n(0^+) = n(0^+)_{V_G} - n(0^+)_{V_G=0}$$

$$= \frac{n_i^2}{N_A}\left(e^{qV_A\beta} - 1\right)$$

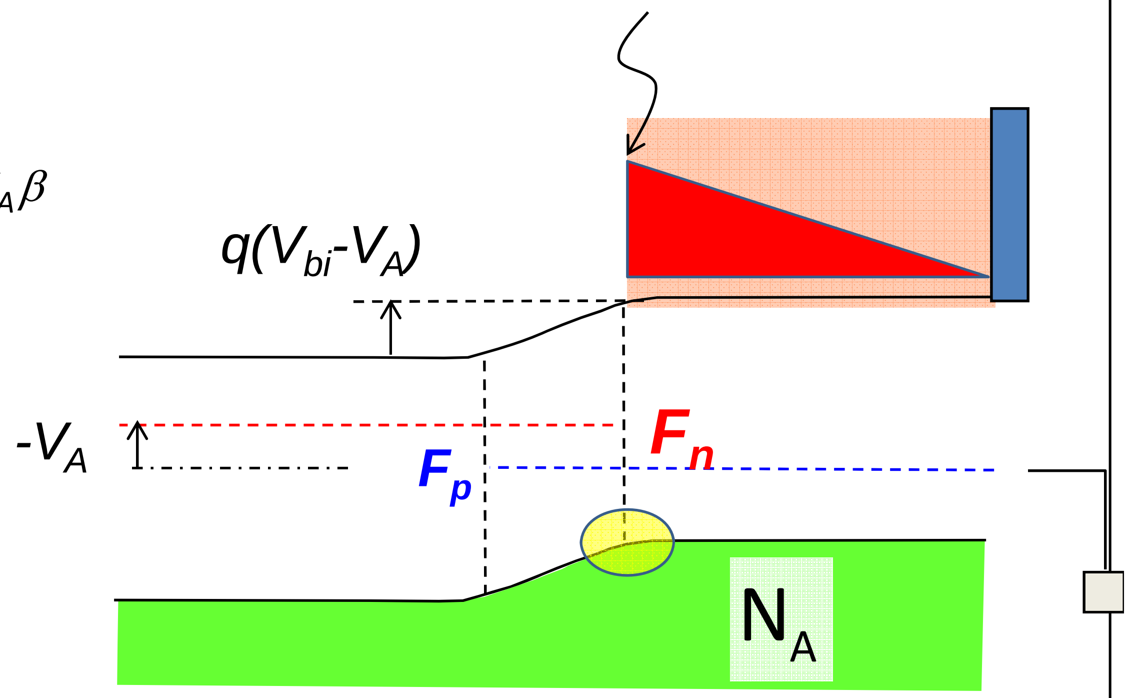
$$n(x=0^+) = n_i\, e^{(F_n - E_i)\beta}$$

$$p(x=0^+) = n_i\, e^{-(F_p - E_i)\beta}$$

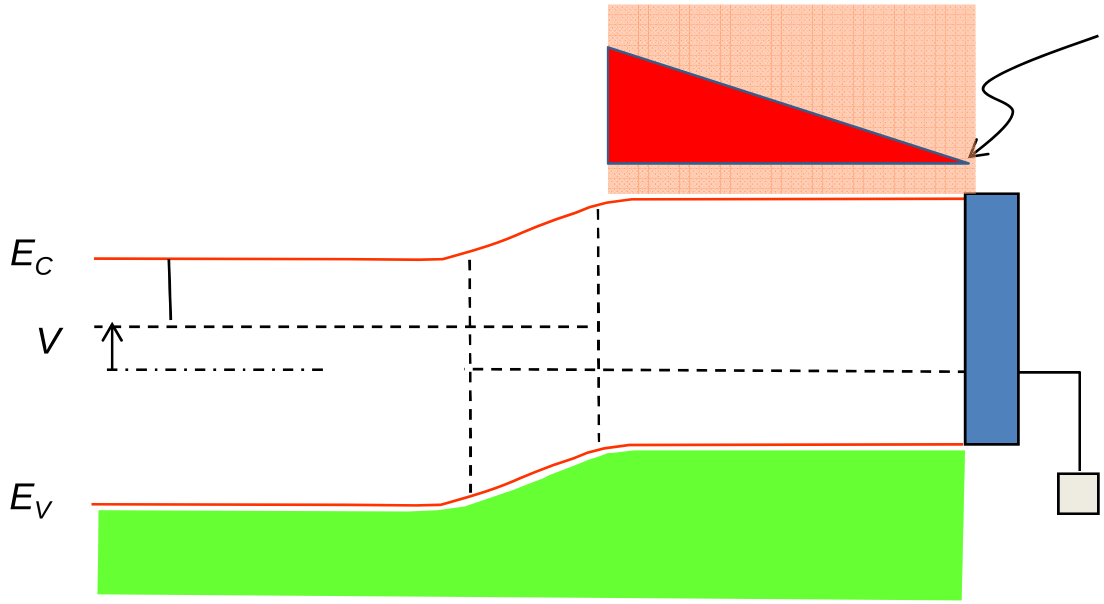
$$np = n_i^2 e^{(F_n - F_p)\beta} = n_i^2 e^{qV_A\beta}$$

$$p(0^+) = N_A$$

$$n(0^+) = \frac{n_i^2}{N_A} e^{qV_A\beta}$$



Right Boundary Condition



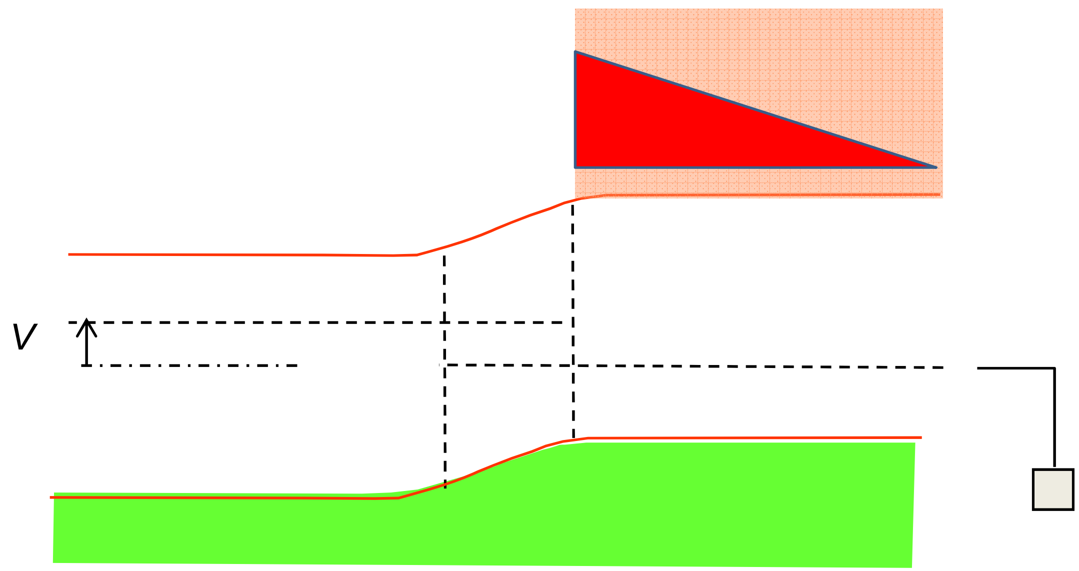
$$n(x = W_p) \approx \frac{n_i^2}{N_A}$$

$$\Delta n(x = W_p) = 0$$

Example: One Sided Minority Diffusion

$$D_N \frac{d^2 n}{dx^2} = 0$$

$$\Delta n(x,t) = C + Dx$$



$$x = W_p, \quad \Delta n(x = W_p) = 0 \Rightarrow C = -DW_p$$

$$x = 0', \quad \Delta n(x = 0) = \frac{n_i^2}{N_A}\left(e^{qV_A\beta} - 1\right) = C$$

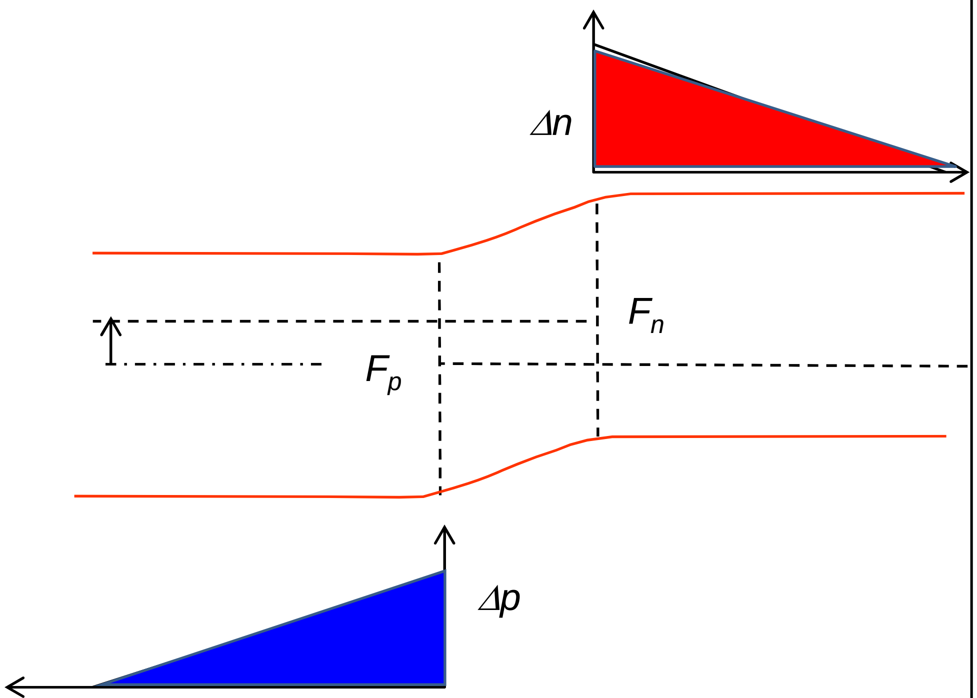
$$\Delta n(x,t) = \frac{n_i^2}{N_A}\left(e^{qV_A\beta} - 1\right)\left(1 - \frac{x}{W_p}\right)$$

Electron & Hole Fluxes

$$\Delta n(x) = \frac{n_i^2}{N_A}\left(e^{qV_A\beta} - 1\right)\left(1 - \frac{x}{W_p}\right)$$

$$\mathbf{J}_N = qn\mu_N \boldsymbol{\mathcal{E}} + qD_N \nabla n$$

$$J_n = qD_n \left.\frac{dn}{dx}\right|_{x=0} = -\frac{qD_n}{W_p}\frac{n_i^2}{N_A}\left(e^{qV_A\beta} - 1\right)$$



$$J_p = -qD_p \left.\frac{dp}{dx}\right|_{x=0'} = -\frac{qD_p}{W_n}\frac{n_i^2}{N_D}\left(e^{qV_A\beta} - 1\right)$$

Total Current

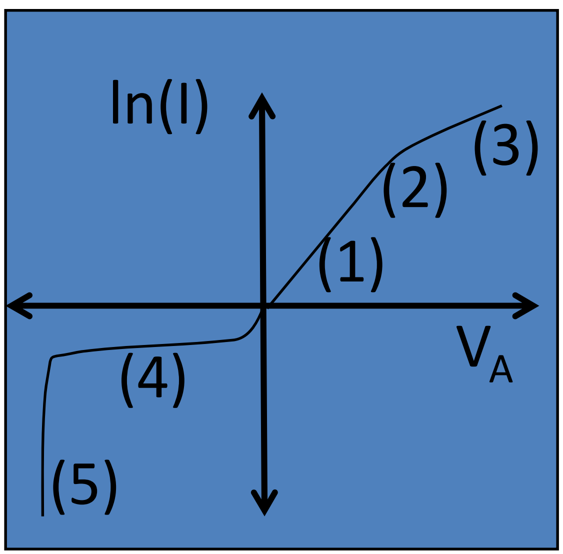
$$J_T = -q\left[\frac{D_n}{W_p}\frac{n_i^2}{N_A} + \frac{D_p}{W_n}\frac{n_i^2}{N_D}\right]\left(e^{qV_A\beta} - 1\right)$$

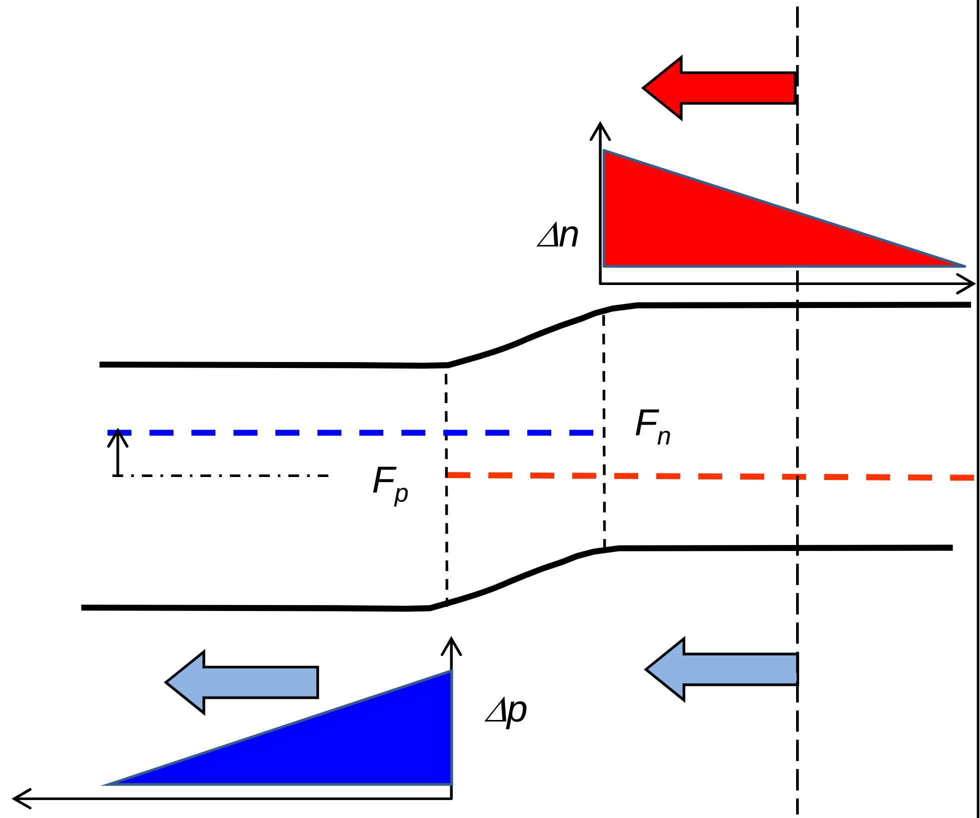
Forward Bias

$$\ln J_T \approx qV_A/k_BT + \ln(const.)$$

Reverse Bias

$$J_T \approx const.$$





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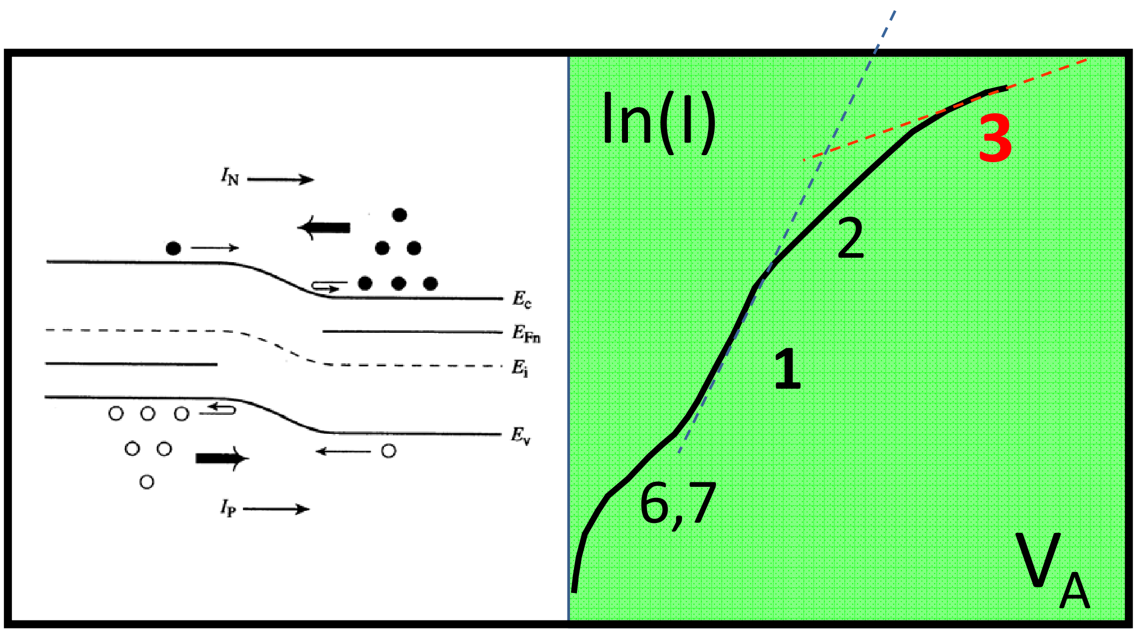
Nonlinear Regime (3) ...

$$J_T = -q\left[\frac{D_n}{W_p}\frac{n_i^2}{N_A} + \frac{D_p}{W_n}\frac{n_i^2}{N_D}\right]\left(e^{\left(qV_A - \Delta F_n - \Delta F_p\right)\beta} - 1\right) = I_0\left(e^{q\left(V_A - aJ_n - bJ_p\right)\beta} - 1\right)$$



Flat Quasi-Fermi Level up to Junction ?

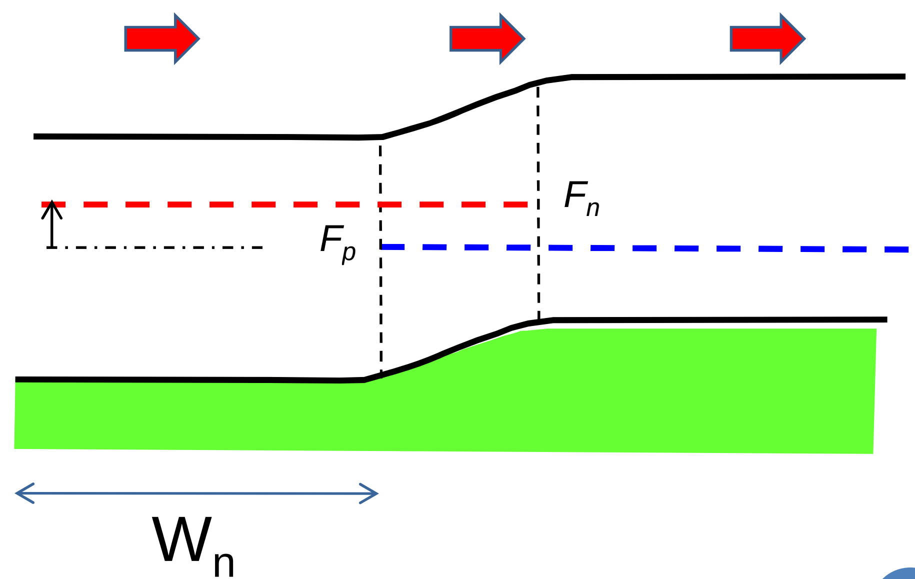
$$\mathbf{J}_N = qn\mu_N\mathcal{E} + qD_N\frac{dn}{dx}$$

$$J_n = n\mu_n\frac{dF_n}{dx} \Rightarrow \Delta F_n = \frac{J_n W_n}{\mu_n N_D}$$

$$n = n_i e^{\beta(F_n - E_i)} \qquad qD_N\frac{dn}{dx} = qD_N\beta\left[\frac{dF_n}{dx} - \mathcal{E}\right]\left[n_i e^{\beta(F_n - E_i)}\right]$$

$$qD_N\frac{dn}{dx} = qD_N n\beta\left[\frac{dF_n}{dx} - \mathcal{E}\right]$$

$$= q\mu_N n\left[\frac{dF_n}{dx} - \mathcal{E}\right] \qquad \because \frac{D_N}{\mu_n} = \frac{k_B T}{q}$$



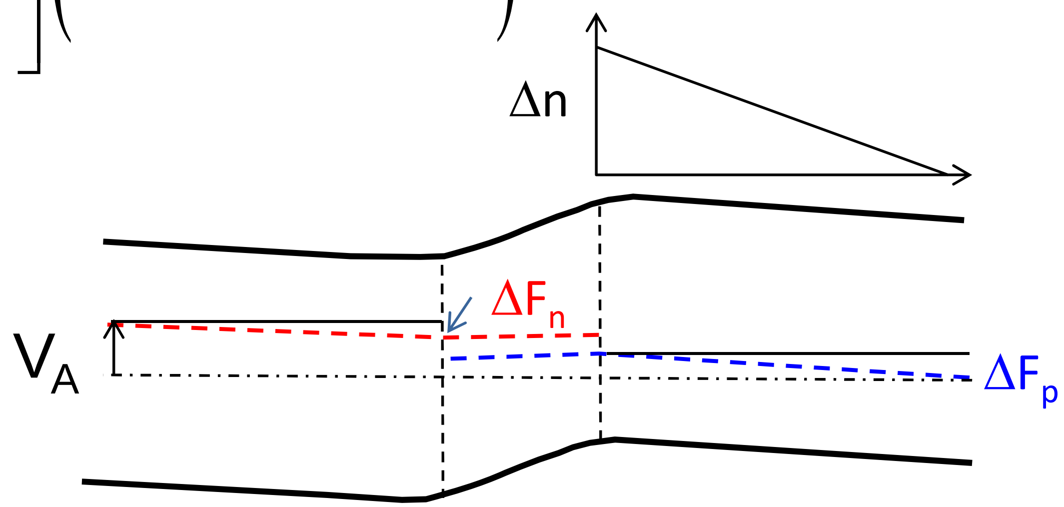
Forward Bias: Nonlinear Regime …

$$n(0^+) = \frac{n_i^2}{N_A} e^{(F_n - F_p)\beta}\Big|_{junction} = \frac{n_i^2}{N_A} e^{(qV_A - \Delta F_n - \Delta F_p)\beta} \Rightarrow \Delta n(0^+) = \frac{n_i^2}{N_A}\left(e^{(qV_A - \Delta F_n - \Delta F_p)\beta} - 1\right)$$

$$J_T = -q\left[\frac{D_n}{W_p}\frac{n_i^2}{N_A} + \frac{D_p}{W_n}\frac{n_i^2}{N_D}\right]\left(e^{(qV_A - \Delta F_n - \Delta F_p)\beta} - 1\right)$$

$$\Delta F_n = \frac{J_n W_n}{\mu_n N_D}$$

$$\Delta F_p = \frac{J_p W_n}{\mu_n N_D}$$



Approx: Still diffusion dominated transport?

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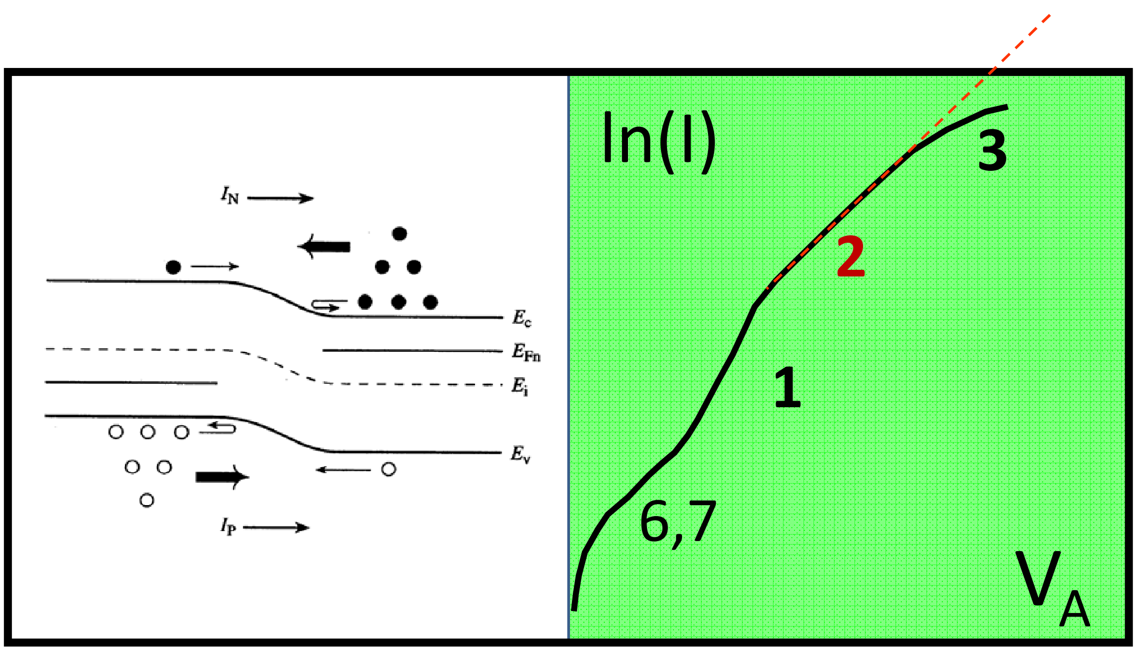
Region (2): Ambipolar Transport

$$J_T \approx -q\left[\frac{D_n}{W_p}+\frac{D_p}{W_n}\right]n_i e^{\left(qV_A-\Delta F_n-\Delta F_p\right)\beta/2}$$

$$\ln(J_T) \approx \frac{qV_A}{2k_BT}$$



Nonlinear Regime: Ambipolar Transport

$$np = n_i^2 e^{(F_n - F_p)\beta}$$

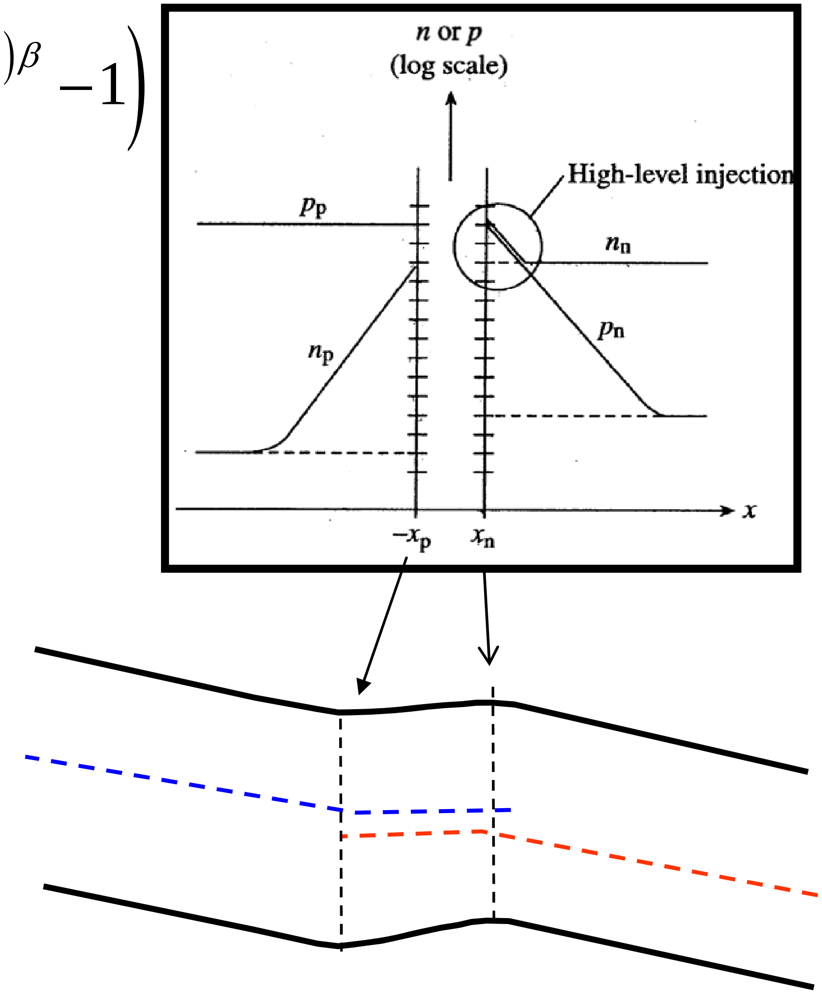
$$\left(\frac{n_i^2}{N_A} + \Delta n\right)(N_A + \Delta p) = n_i^2 \left(e^{q(V_A - \Delta F_n - \Delta F_p)\beta} - 1\right)$$

$$\Delta n \approx \Delta p = n_i \sqrt{\left(e^{q(V_A - \Delta F_n - \Delta F_p)\beta} - 1\right)}$$

$$\approx n_i\, e^{q(V_A - \Delta F_n - \Delta F_p)\beta/2}$$

$$J_n = -qD_n \frac{\Delta n}{W_p} = \frac{qD_n n_i}{W_p} e^{(qV_A - \Delta F_n - \Delta F_p)\beta/2}$$

$$J_p = -qD_p \frac{\Delta n}{W_n} = \frac{qD_p n_i}{W_n} e^{(qV_A - \Delta F_n - \Delta F_p)\beta/2}$$



Note: junction never disappears!

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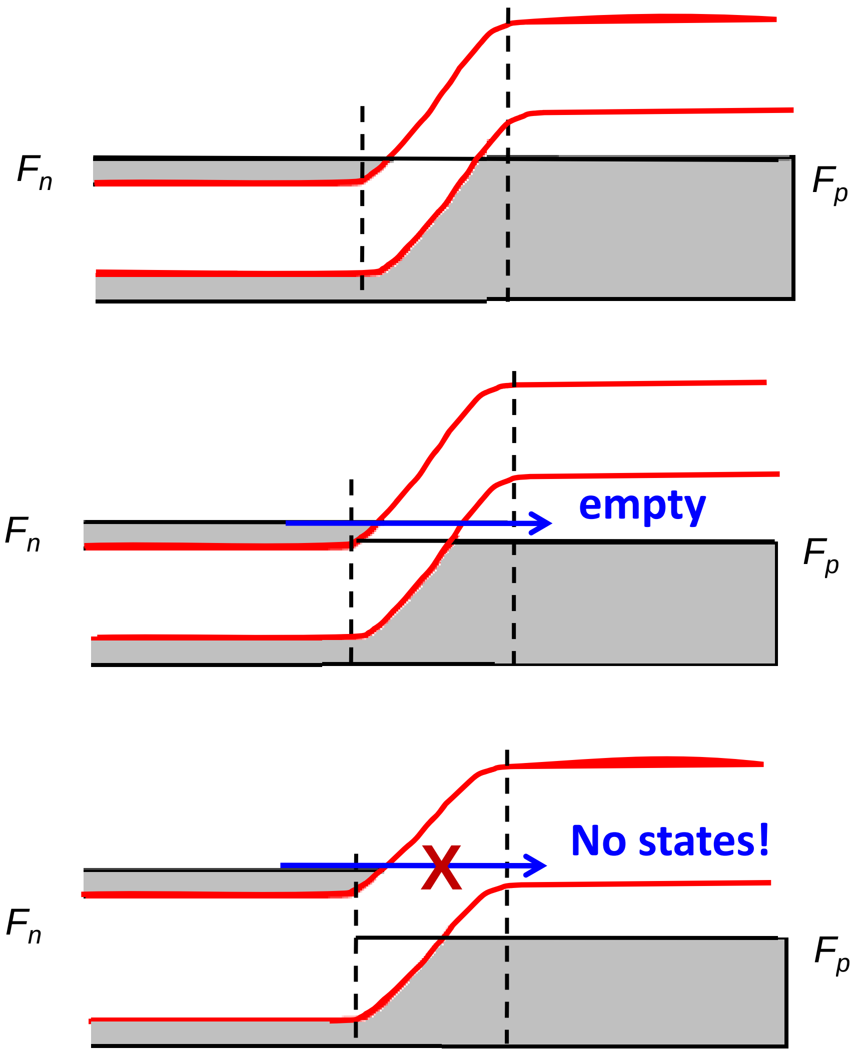
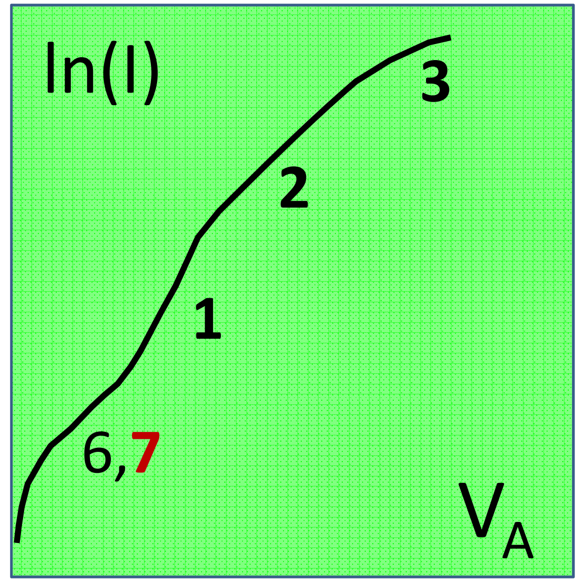
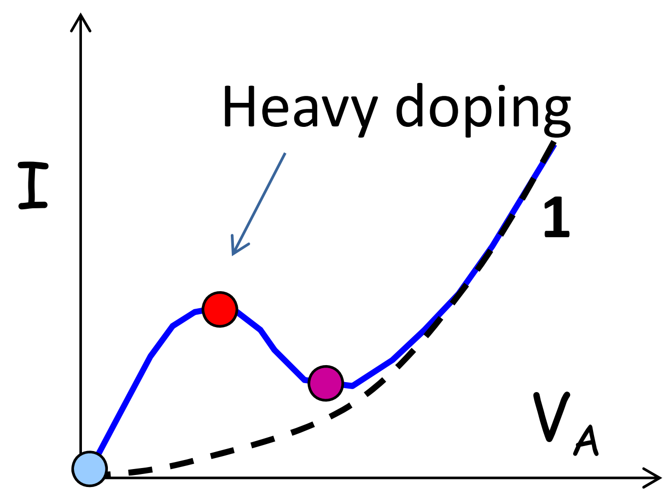
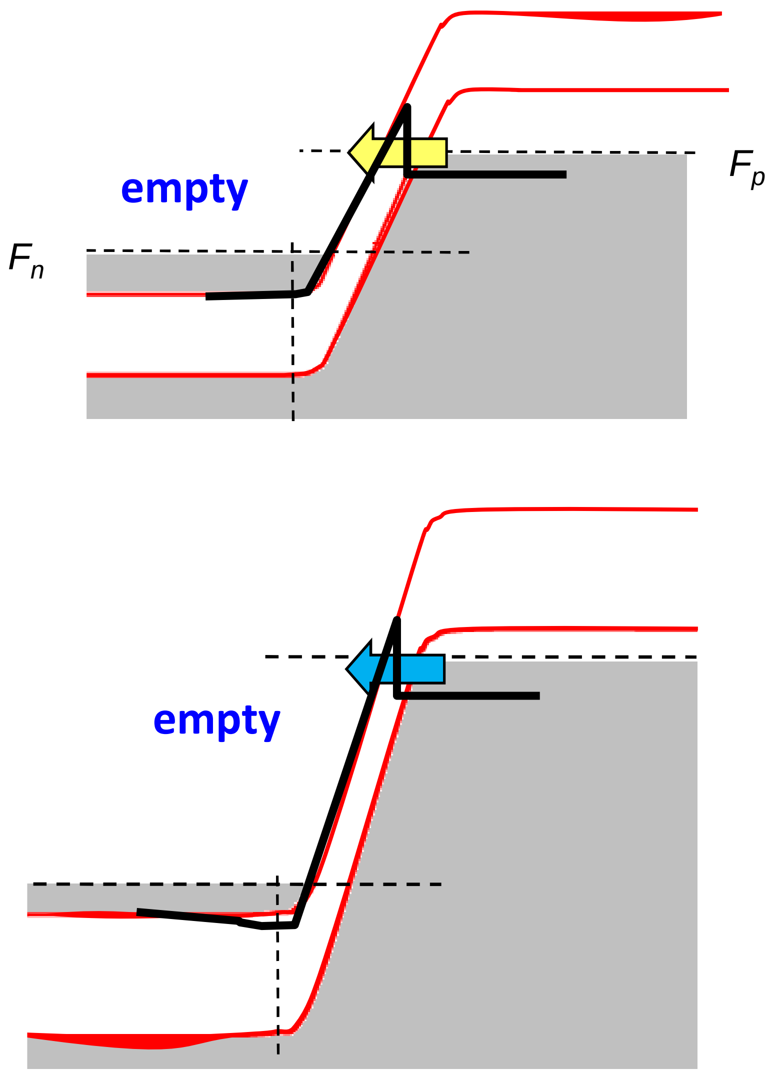
Forward Bias Nonlinearity (7): Esaki Diode

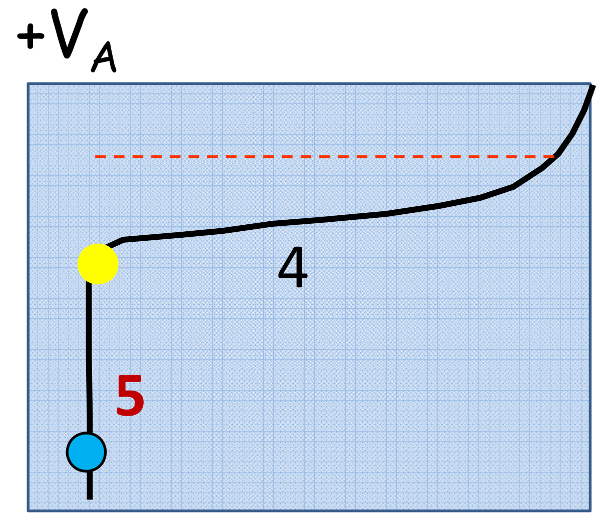






Reverse Bias (5): Zener Tunneling





$$I = qpT\upsilon$$

$$T = \frac{4}{4\cosh^2 \alpha d + \left(\frac{\alpha}{k} - \frac{k}{\alpha}\right)\sinh^2 \alpha d}$$

(p.49 ADF)

Conclusion

1) I-V characteristics of a p-n junction is defined by many interesting phenomena including diffusion, ambipolar transport, tunneling etc.
2) The separate regions are identified by specific features. Once we learn to identify them, we can see if one or the other mechanism is dominated for a given technology.
3) In the next class, we will discuss a few more non-ideal effects.