PURDUE UNIVERSITY

NCN

www.nanohub.org

ECE606: Solid State Devices
Lecture 33: MOSCAP Electrostatics (II)

Muhammad Ashraful Alam
alam@purdue.edu

Outline

1. **Review**
2. Induced charges in depletion and inversion
3. Exact solution of electrostatic problem
4. Conclusion

REF: Chapters 15-18 from SDF

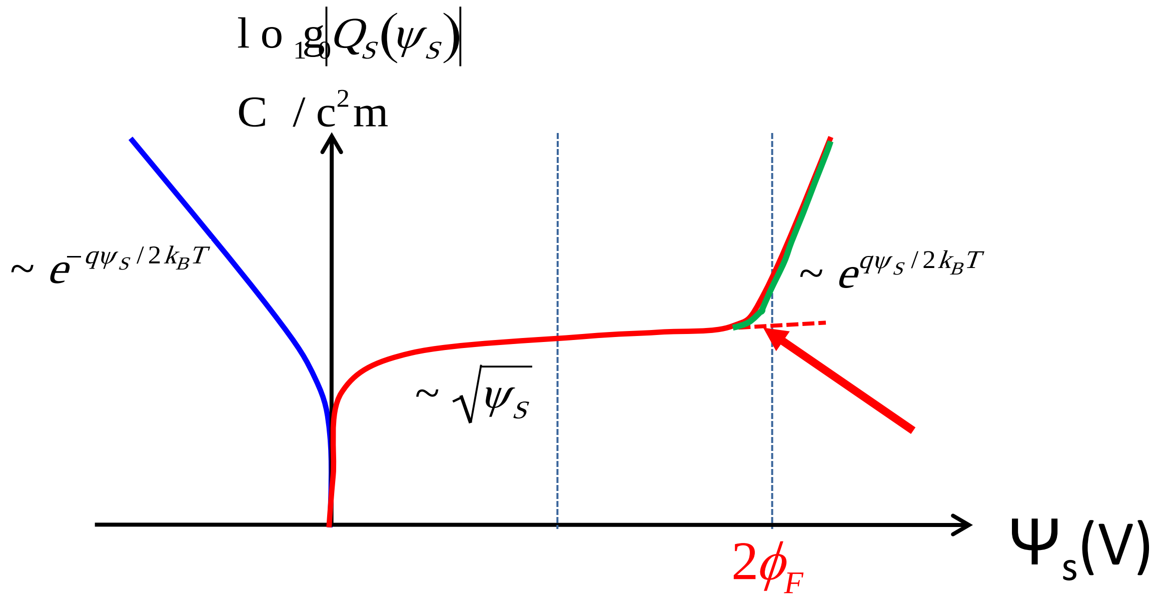
Topic Map

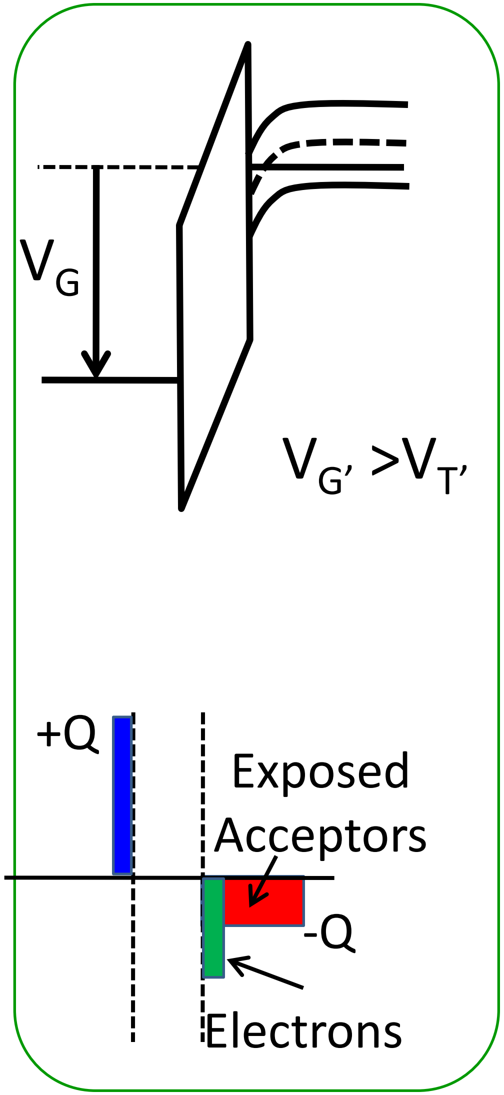
	Equilibrium	**DC**	Small signal	Large Signal	Circuits
Diode					
Schottky					
BJT/HBT					
MOSCAP					

Threshold for Inversion

$$V_G = \frac{qN_A x_0}{\kappa_{ox}\varepsilon_0}\sqrt{\frac{2\kappa_{ox}\varepsilon_0}{qN_A}}\sqrt{\psi_s} + \psi_s$$

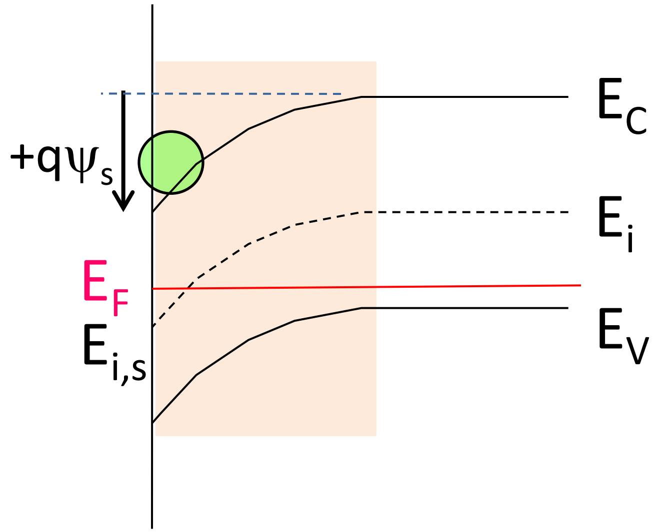
$$V_{th} = \frac{qN_A x_0}{\kappa_{ox}\varepsilon_0}\sqrt{\frac{2\kappa_{ox}\varepsilon_0}{qN_A}}\sqrt{2\phi_F} + 2\phi_F$$





What happens when surface potential is $2\phi_F$?

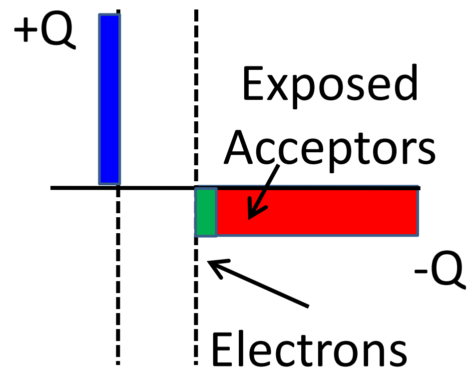
$$V_{th} = \frac{qN_A x_0}{\kappa_{ox}\varepsilon_0}\sqrt{\frac{2\kappa_{ox}\varepsilon_0}{qN_A}}\sqrt{2\phi_F} + 2\phi_F$$



$$n_{1s} = n_i e^{(E_F - E_{is})\beta}$$

$$= n_i e^{(E_F - E_{i(bulk)})\beta} \times e^{(E_{i(bulk)} - E_{is})\beta}$$

$$= n_i e^{-\phi_F \beta} e^{(E_{i(bulk)} - E_{is})\beta}$$



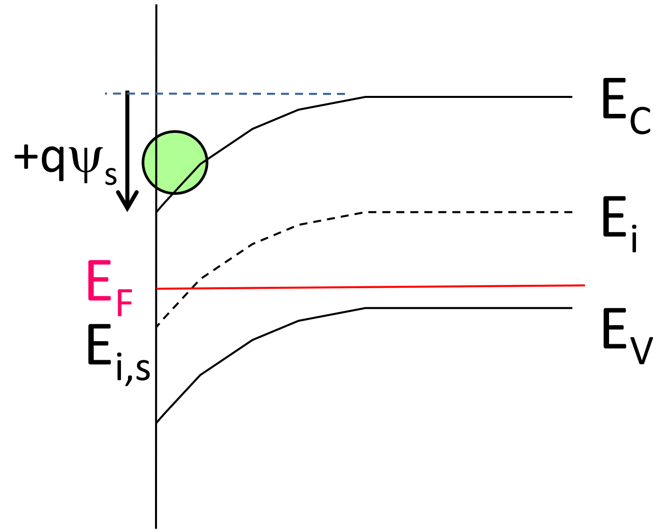
$$n_{1s} = n_i e^{-\phi_F \beta} e^{2\phi_F \beta}$$

$$= n_i e^{\phi_F \beta} = N_A$$

Electron concentration equals background acceptor concentration

A little bit about scaling ....

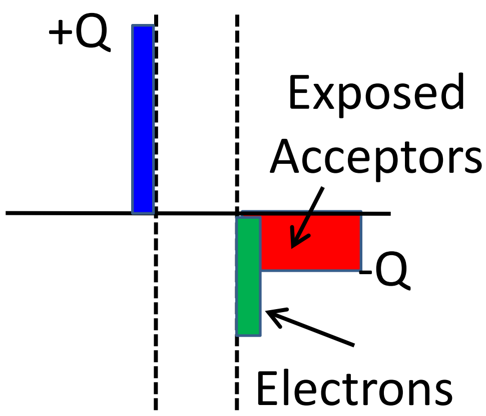
$$V_{th} = \frac{qN_A x_0}{\kappa_{ox}\varepsilon_0}\sqrt{\frac{2\kappa_{ox}\varepsilon_0}{qN_A}}\sqrt{2\phi_F} + 2\phi_F$$



Reduce V_{th} by ...

Reducing oxide thickness (from 1000 A in 1970s to 10 A now)

Increase dielectric constant (SiO$_2$ historically, HfO$_2$ now in Intel Penryn)



Outline

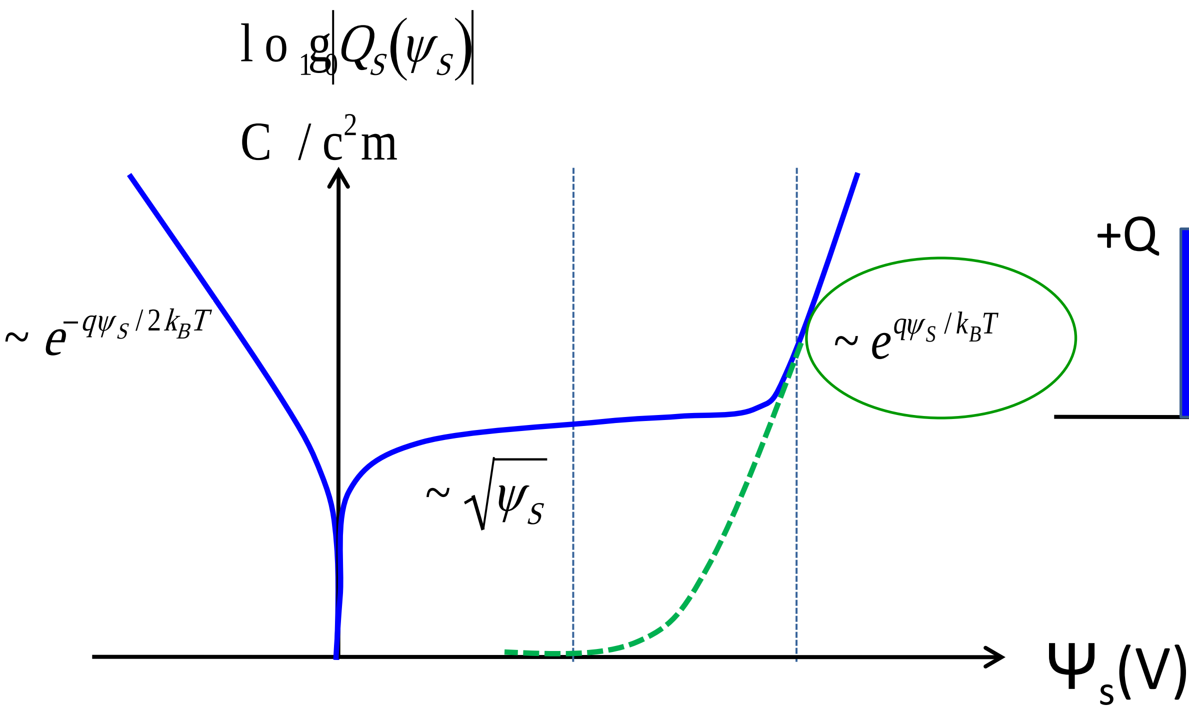
1. Review
2. **Induced charges in depletion and inversion**
3. Exact solution of electrostatic problem
4. Conclusion

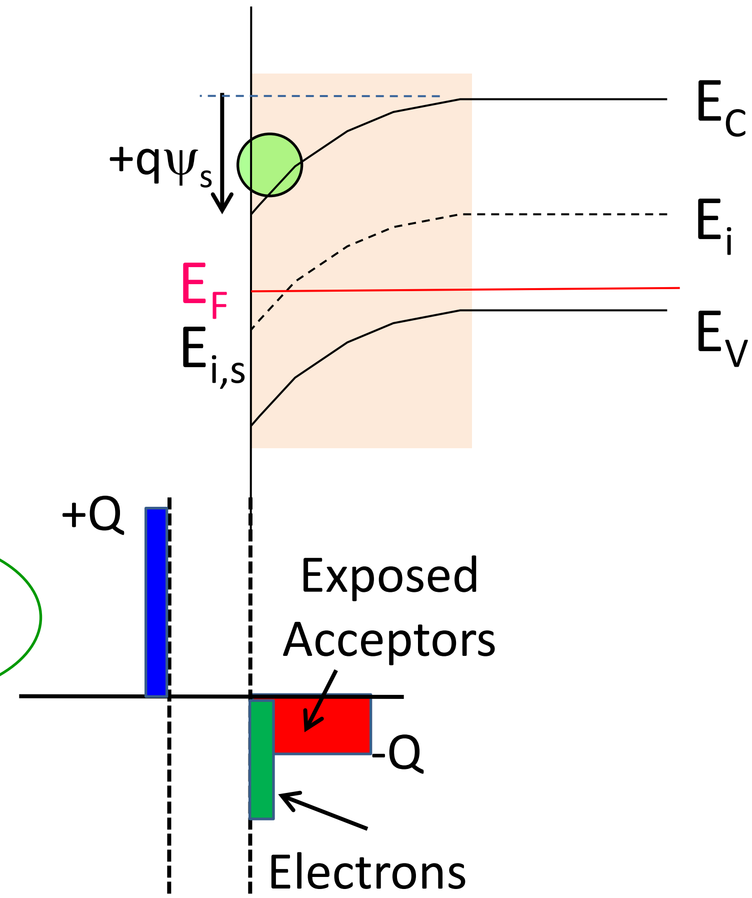
Induced charges below Threshold

$$n_{1s} = n_i e^{-\phi_F \beta} e^{\left(E_{i(bulk)} - E_{is}\right)\beta}$$

$$\equiv B e^{q\psi_s \beta}$$

$$V_G = \frac{qN_A x_0}{\kappa_{ox}\varepsilon_0} \sqrt{\frac{2\kappa_{ox}\varepsilon_0}{qN_A}} \sqrt{\psi_s} + \psi_s$$





Integrated charges below Threshold

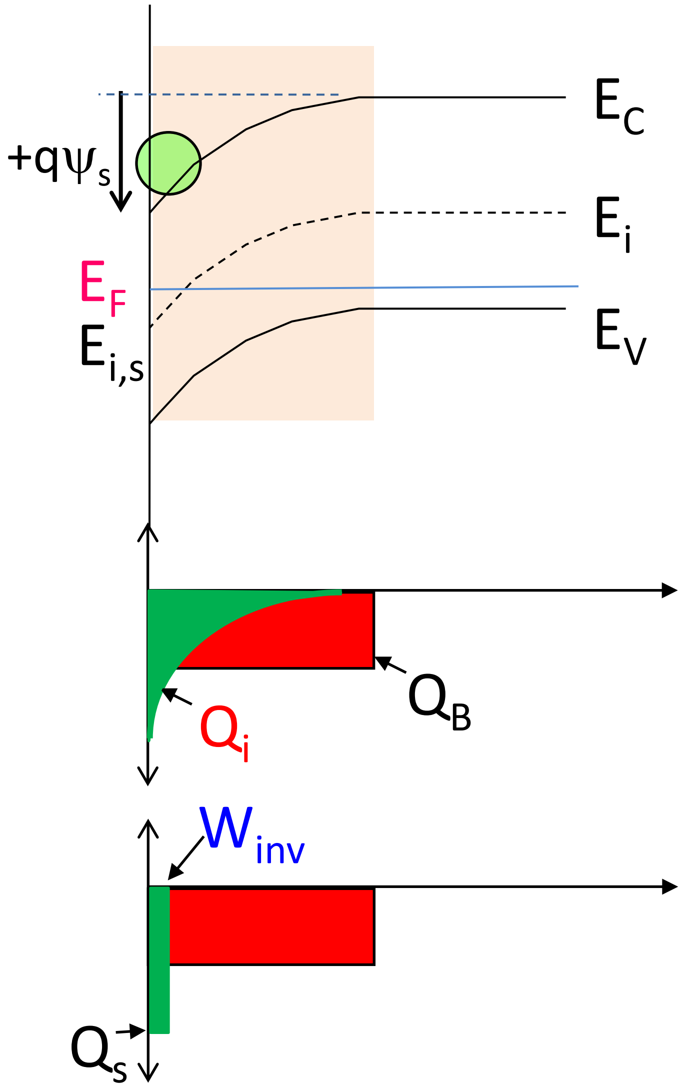
$$\frac{Q_i}{q} = \int_0^\infty n(x)\,dx = \int_0^\infty \frac{n_i^2}{N_B} e^{q\psi(x)\beta}\,dx$$

$$= \frac{n_i^2}{N_B} \int_0^\infty e^{q\psi(x)\beta} \frac{1}{\frac{d\psi}{dx}} d\psi$$

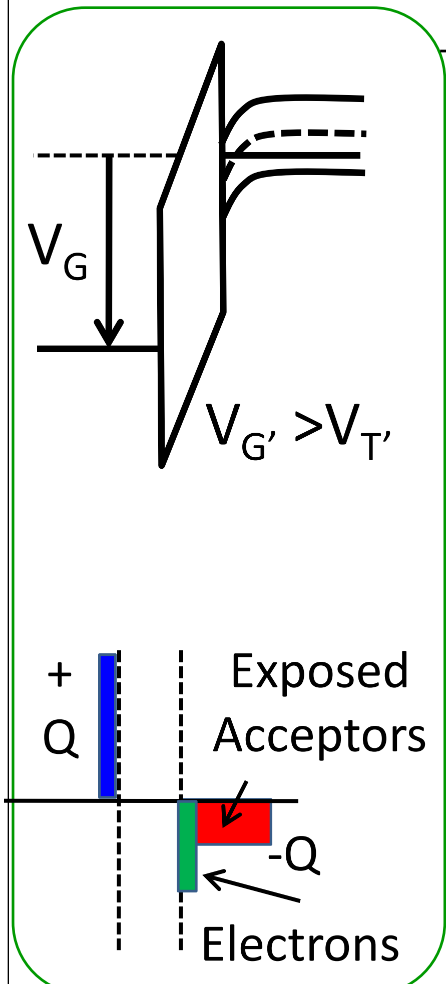
$$= \frac{n_i^2}{N_B} \int_0^\infty e^{q\psi(x)\beta} \frac{1}{\mathcal{E}(x)} d\psi$$

$$\approx \frac{1}{\langle \mathcal{E}(x) \rangle} \frac{n_i^2}{N_B} \int_0^\infty e^{q\psi(x)\beta} d\psi$$

$$= \frac{\left(\frac{k_B T}{q}\right)}{\langle \mathcal{E}(x) \rangle} \times \frac{n_i^2}{N_B} e^{q\psi_s\beta} \equiv W_{inv} \times n_s$$



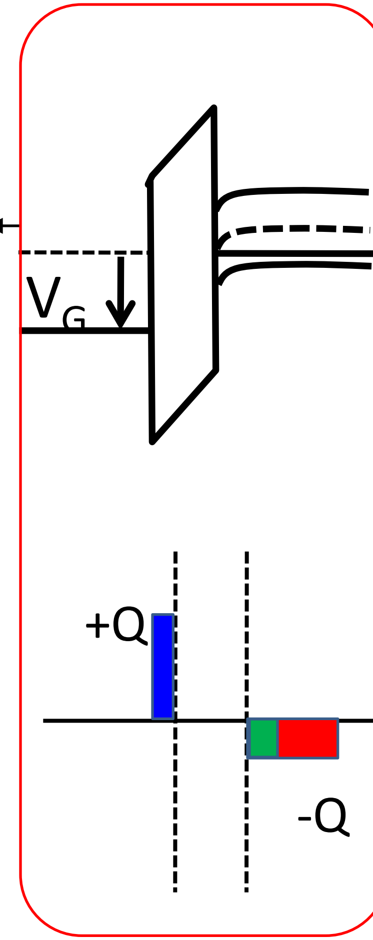
Charges above Threshold



$$V_G = \psi_s + \mathcal{E}_{ox} x_o = \psi_s - \left[\frac{Q_i(\psi_s) + Q_F}{\kappa_{ox}\varepsilon_0} \right] x_o$$

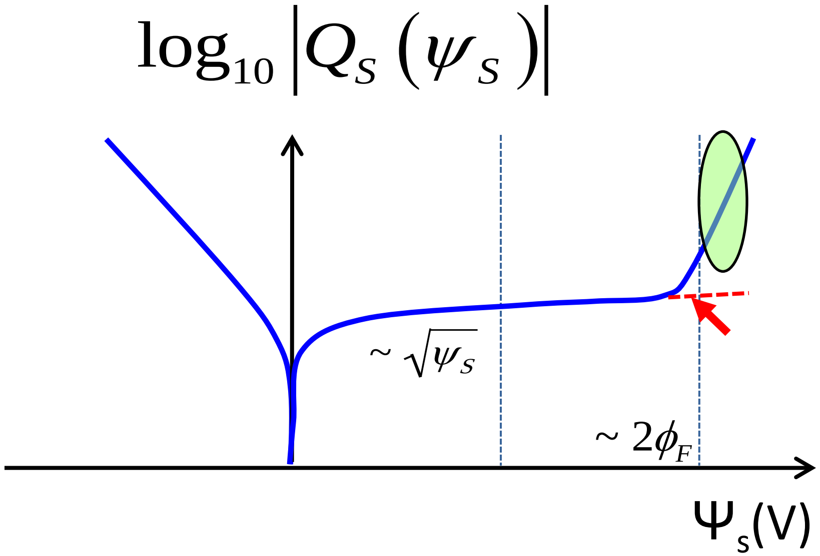
$$V_{th} = 2\phi_F + \mathcal{E}_{ox} x_o = 2\phi_F - \left(\frac{Q_i(2\phi_F) + Q_F}{\kappa_{ox}\varepsilon_0} \right) x_o$$

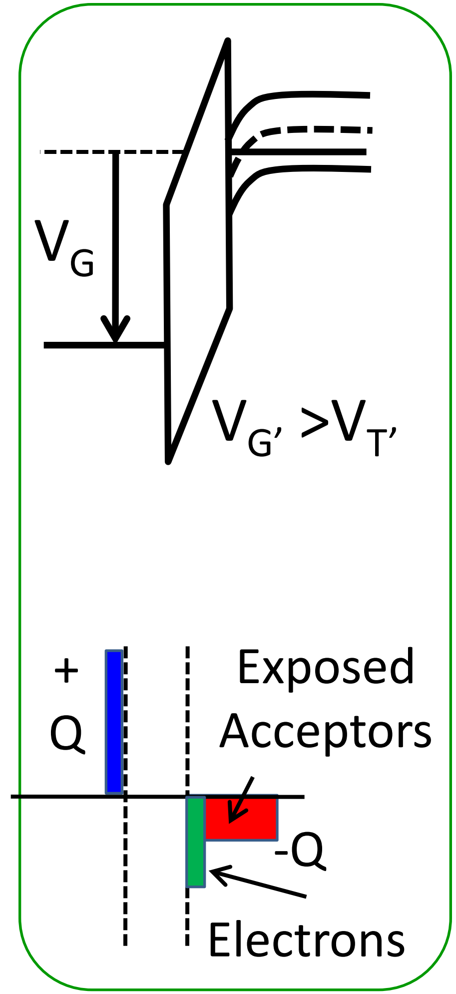
$$V_G - V_{th} = (\psi_s - 2\phi_F) + \frac{Q_i(\psi_s) - Q_i(2\phi_F)}{\kappa_{ox}\varepsilon_0} x_o$$



$$Q_i = C_{ox}(V_G - V_{th})$$

Linear Charge Build-up Above Threshold?





- Small changes ψ_s in changes Q_i a lot ..
- Change in Q_i changes E_{ox}, because $E_{ox}=Q_i/\kappa_0 e_0$
- V_{ox} is large because $V_{ox}=E_{ox}\, x_0$, i.e. most of the drop above $2\psi_F$ goes to V_{ox}.
- Acts like a parallel plate capacitor, hence the inversion equation.

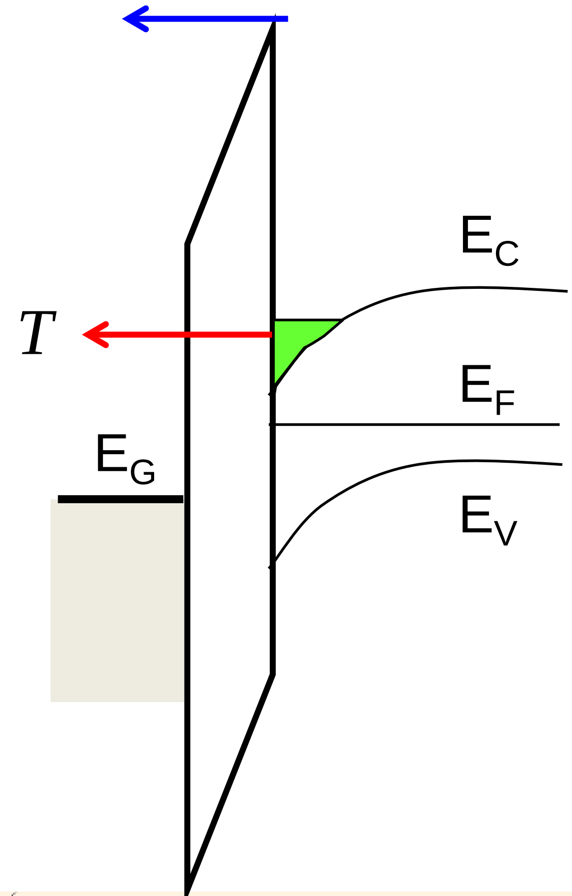
Tunneling Current

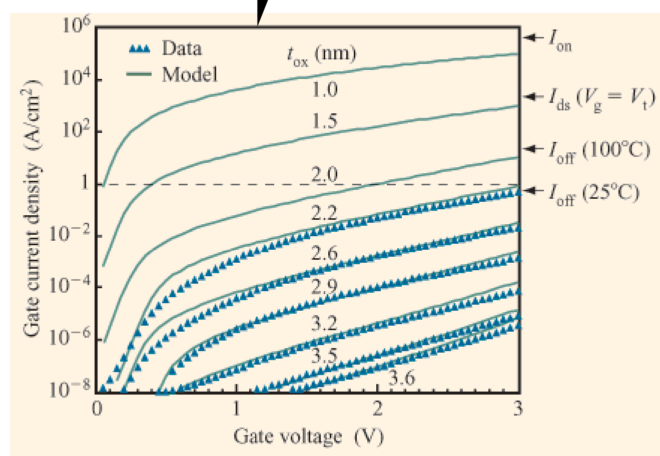
$$J_T = J_{s\to g} - J_{g\to s}$$

$$= \left[Q_i(V_G) e^{-\Delta E_C \beta} - q n_m e^{-\Delta E_C \beta} e^{-qV_{ox}\beta} \right] \upsilon_{th}$$

$$= \left[Q_i(V_G) - q n_m e^{-qV_{ox}\beta} \right] \upsilon_{th} T \qquad T \equiv e^{-\Delta E_C \beta}$$

$$J_T = \left[Q_i(V_G) - q n_m e^{-qV_G\beta} \right] \upsilon_{th} \langle T(E) \rangle$$





Outline

1. Review
2. Induced charges in depletion and inversion
3. **Exact solution of electrostatic problem**
4. Conclusion

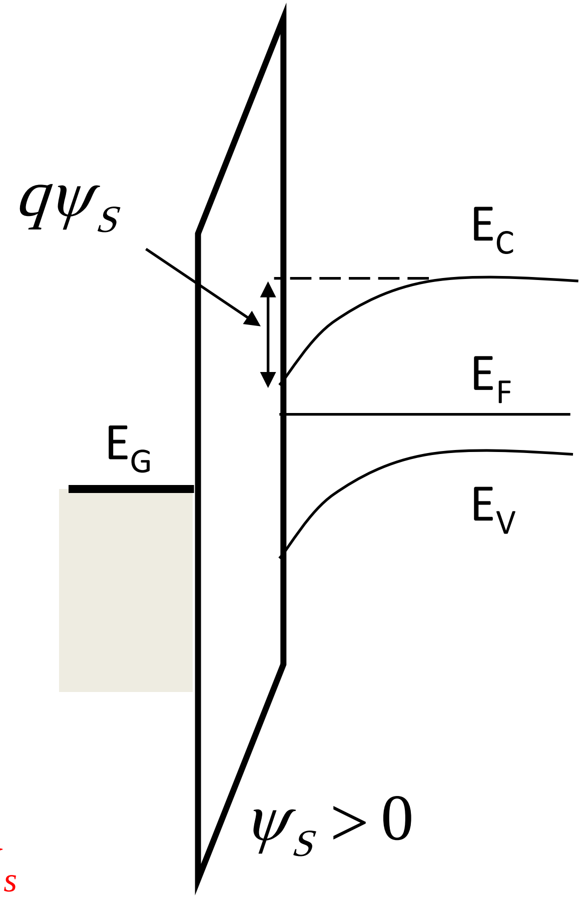
A step back: 'Exact' Solution of $Q_S(\psi_S)$

$$\nabla \bullet \vec{D} = \rho$$

$$\nabla \bullet \left(\vec{J}_n / -q \right) = (G - R)$$

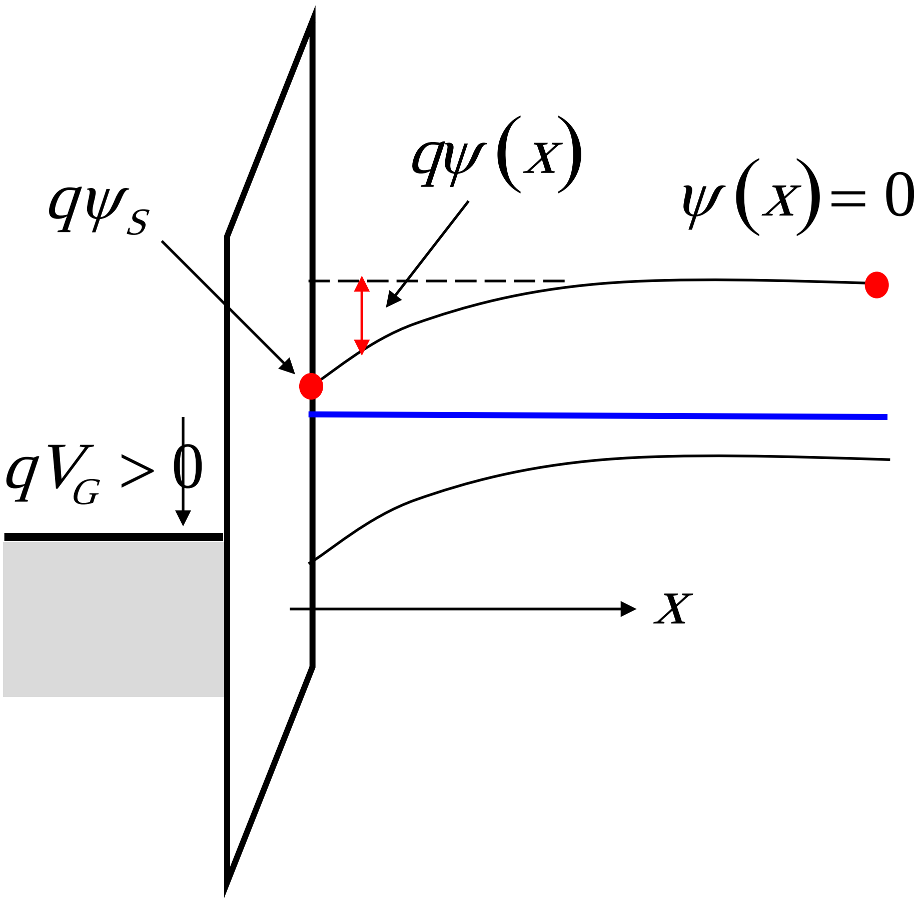
$$\nabla \bullet \left(\vec{J}_p / \; q \right) = (G - R)$$

$$\frac{d^2\psi}{dx^2} = \frac{-q}{\kappa_{si}\varepsilon_0}\left[p_0(x) - n_0(x) + N_D^+ - N_A^- \right]$$



Approximate … $$V_G = \frac{qN_A x_0}{\kappa_{ox}\varepsilon_0}\sqrt{\frac{2\kappa_{ox}\varepsilon_0}{qN_A}}\sqrt{\psi_s} + \psi_s$$

Normalized Variable (to save some writing)…



$$E_C(x) = \text{constant} - q\psi(x)$$

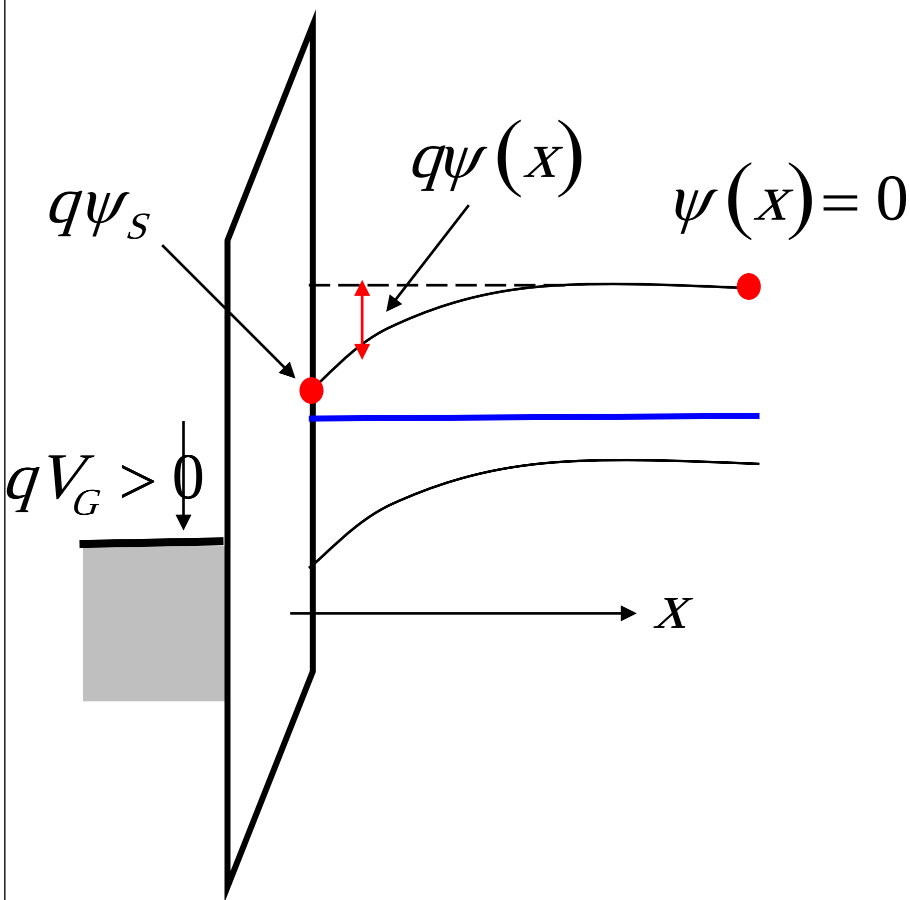
$$\psi(x) = \frac{E_{C,bulk} - E_C(x)}{q}$$

$$u = \frac{\psi(x)}{k_B T/q} = \frac{E_{i(bulk)} - E_{i(x)}}{k_B T}$$

$$u_S = \frac{\psi_S}{k_B T/q} = \frac{E_{i(bulk)} - E_{i(surface)}}{k_B T}$$

$$u_F = \frac{\phi_F}{k_B T/q} = \frac{E_{i(bulk)} - E_F}{k_B T}$$

Normalized Variable (to save some writing!)



$$p(x) = n_i e^{[E_i(x) - E_F]\beta} = n_i e^{+(U_F - U)}$$

$$n(x) = n_i e^{-[E_i(x) - E_F]\beta} = n_i e^{-(U_F - U)}$$

$$N_D^+ = n_i e^{[E_F - E_{i,bulk}]\beta} = n_i e^{-(U_F)}$$

$$N_A^- = n_i e^{-[E_F - E_{i,bulk}]\beta} = n_i e^{(U_F)}$$

Poisson-Boltzmann Equation

$$\left.\frac{d^2\psi}{dx^2}\right| = \frac{-q}{\kappa_s\varepsilon_0}\left[p(x) - n(x) + N_D^+ - N_A^-\right]$$

$$\frac{q}{k_BT}\frac{d^2U}{dx^2} = \frac{-qn_i}{\kappa_s\varepsilon_0}\left[e^{+(U_F-U)} - e^{-(U_F-U)} + n_ie^{-U_F} - n_ie^{U_F}\right] \equiv g(U,U_F)$$

Can be evaluated at any U

$$\left(2\frac{dU}{dx}\right)\times\frac{d^2U}{dx^2} = -\left(\frac{n_ik_BT}{\kappa_s\varepsilon_0}\right)g(U,U_F)\times\left(2\frac{dU}{dx}\right)$$

$$\frac{d}{dx}\left(\frac{dU}{dx}\right)^2 dx = -\frac{1}{2L_D^2}g(U,U_F)\left(2\frac{dU}{dx}\right)dx$$

$$\int_0^{-q\mathcal{E}(x)/kT} d\left(\frac{dU}{dx}\right)^2 = -\frac{1}{L_D^2}\int_0^{U(x)} g(U,U_F)\,dU$$

Exact Solution (continued)

$$\int_0^{-q\mathcal{E}(x)/kT} d\left(\frac{dU}{dx}\right)^2 = -\frac{1}{L_D^2}\int_0^{U(x)} g(U,U_F)dU$$

$$\left[\frac{q\mathcal{E}(x)}{kT}\right]^2 = \frac{1}{L_D^2}\int_0^{U(x)} g(U,U_F)dU \equiv \frac{F^2(U,U_F)}{L_D^2}$$

$$\mathcal{E}_s = \frac{k_B T}{qL_D} F(U_S,U_F)$$

Compare ...

$$V_G = \frac{qN_A x_0}{\kappa_{ox}\varepsilon_0}\sqrt{\frac{2\kappa_{ox}\varepsilon_0}{qN_A}}\sqrt{\psi_s} + \psi_s$$

V_{ox}

$$V_G = \psi_s + \left[\frac{\kappa_s}{\kappa_{ox}}\mathcal{E}_s\right]x_0 = \psi_s + \frac{\kappa_s}{\kappa_{ox}}\frac{k_B T}{qL_D}F(U_s,U_F)x_0$$

How does the calculation go …

$$\left[\frac{q\mathcal{E}(x)}{kT}\right]^2 = \frac{1}{L_D^2}\int_0^{U(x)} g(U,U_F)\,dU \equiv \frac{F^2(U,U_F)}{L_D^2}$$

$$V_G = \psi_s + \frac{\kappa_s}{\kappa_{ox}}\mathcal{E}_s x_0 = \psi_s + \frac{\kappa_s}{\kappa_{ox}}\frac{k_B T}{qL_D}F(U_s,U_F)x_0$$

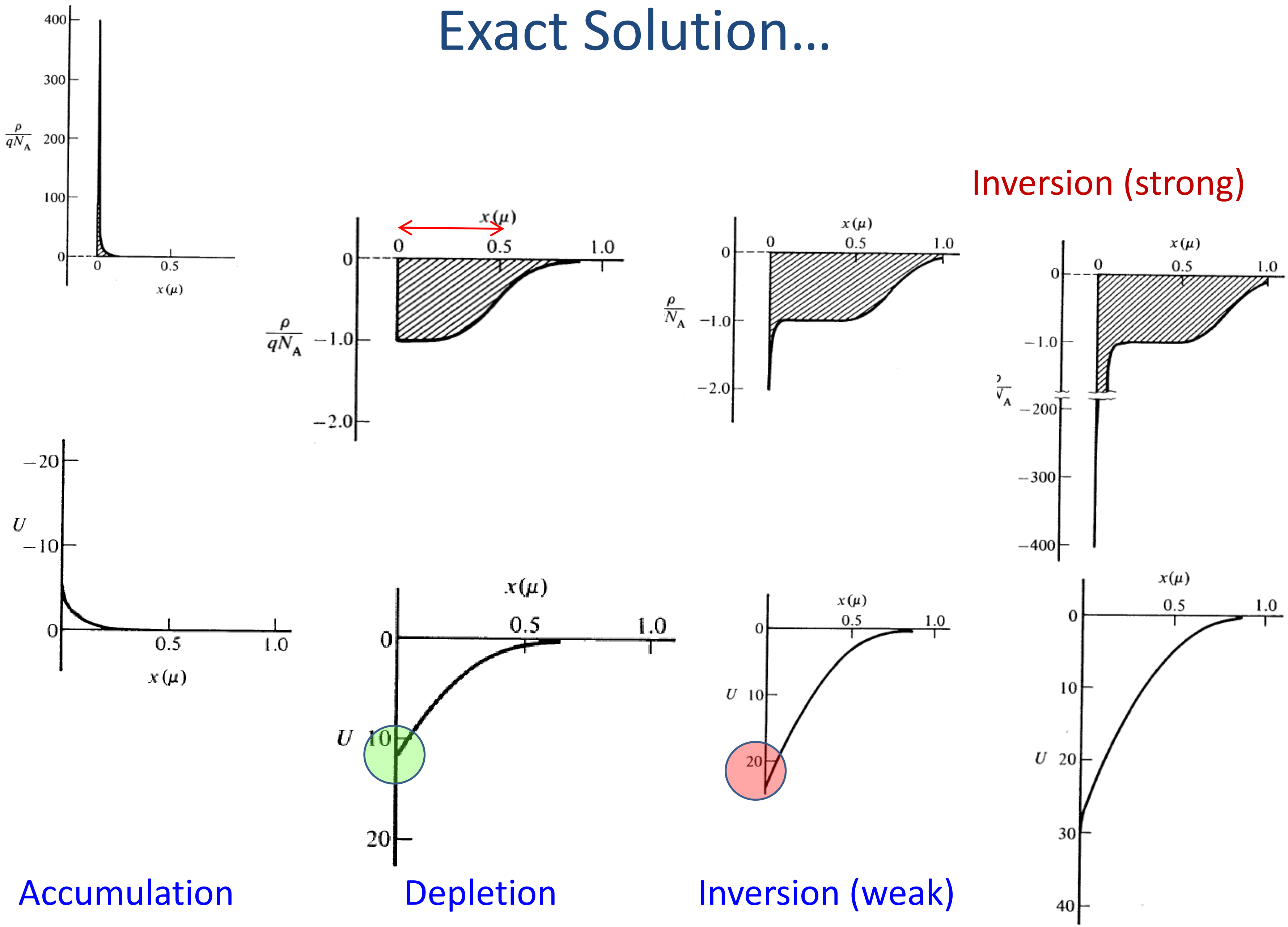
Begin with a surface potential

Calculate U_s and then divide U_s by N points.

Calculate $g(U, U_F)$ at those points
and integrate to find $F(U_s,U_F)$

Find V_G.

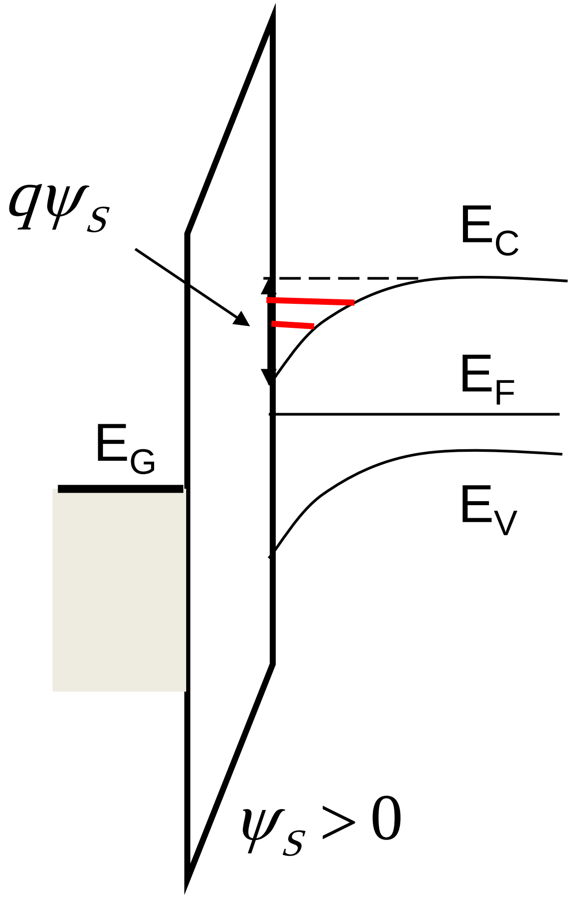
Exact Solution…



“Exact” solution is not really exact …

$$\left.\frac{d^2\psi}{dx^2}\right| = \frac{-q}{\varepsilon}\left[p(x) - n(x)\left|\psi(x)\right|^2 + N_D^+ - N_A^- \right]$$

wavefunction, not potential !



Wave function should be accounted for

Bandgap widening near the interface must also should be accounted for.

Assumption of nondegeneracy may not always be valid

Conclusion

Our discussion today was focused on calculating the induced charge in the depletion and inversion region as a function of gate bias.

We found that we could calculate the tunneling current from the inversion changes by using the thermionic emission theory.

We also discussed the "exact" solution of the MOS-capacitor electrostatics. The "exact" solution is mathematically exact, but not necessarily physically exact solution of the electrostatic problem.