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# ECE606: Solid State Devices
## Lecture 36: MOSFET Current-Voltage (II)

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# Topic Map

| | Equilibrium | **DC** | Small signal | Large Signal | **Circuits** |
|---|---|---|---|---|---|
| Diode | | | | | |
| Schottky | | | | | |
| BJT/**HBT** | | | | | |
| MOS | | ◆ | | | |

# Outline

1) **Review of ‘Square law/ simplified bulk charge’ theory**

2) Velocity saturation in simplified theory

3) Few comments about bulk charge theory, small transistors

2) Conclusion

# Square Law Theory

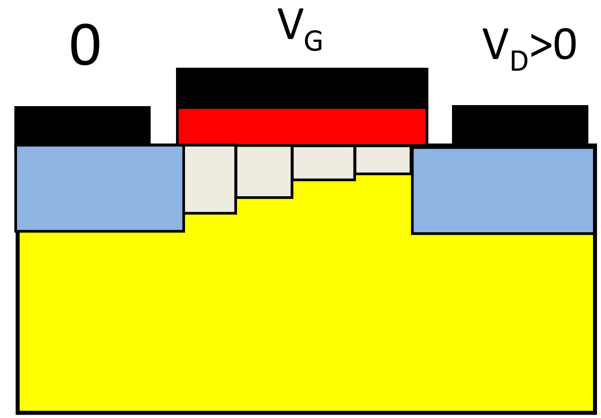
Not true at high fields

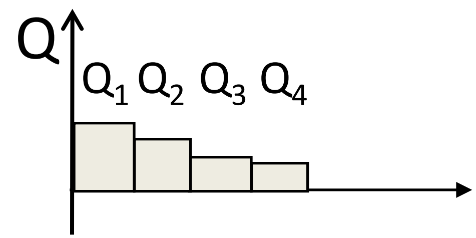
$$J_i = Q_i \upsilon = Q_i \mu \mathcal{E}_1 = Q_i \mu \left.\frac{dV}{dy}\right|_i$$

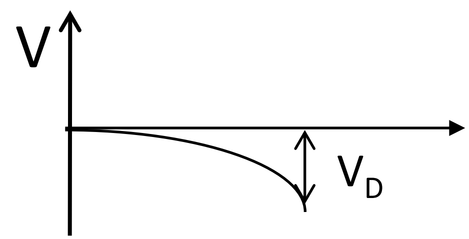
$$\sum_{i=1,N} \frac{J_i dy}{\mu} = \sum_{i=1,N} Q_i \, dV$$

$$\frac{J_D}{\mu_0} \sum_{i=1,N} dy = \int_0^{V_D} C_{ox}(V_G - V_{th} - mV)\, dV$$

$$J_D = \frac{\mu_0 C_{ox}}{L_{ch}} \left[ (V_G - V_{th}) V_D - m \frac{{V_D}^2}{2} \right]$$

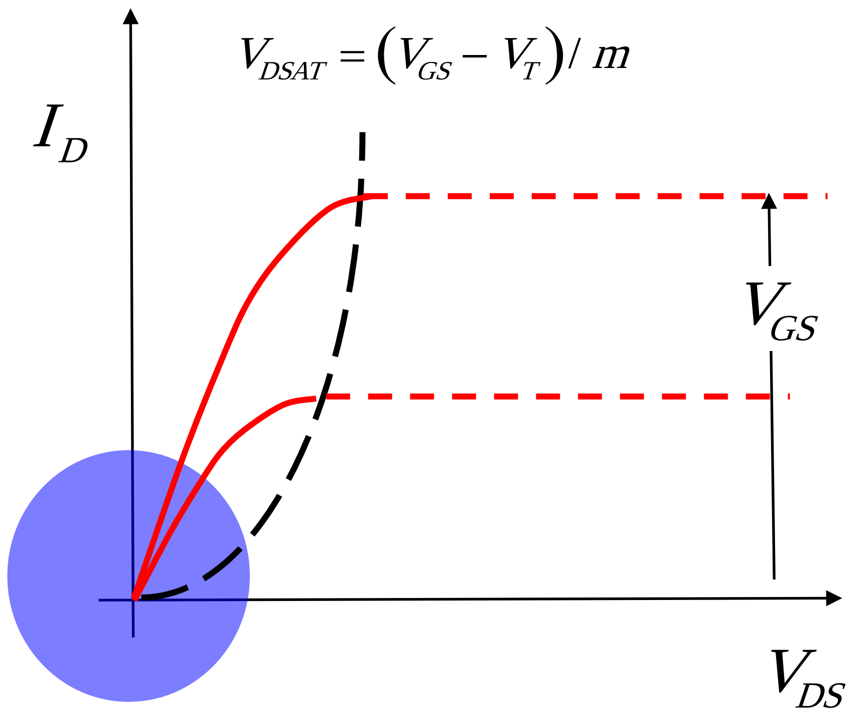


# Square Law or Simplified Bulk Charge Theory

$$I_D = \frac{W\mu_0 C_{ox}}{L_{ch}}\left[\left(V_G - V_{th}\right)V_D - m\frac{V_D^{\,2}}{2}\right]$$




$$I_D = \frac{W\mu_0 C_o}{2mL_{ch}}\left(V_G - V_T\right)^2$$

$$I_D = \mu_0 C_o \frac{W}{L}\left(V_G - V_T\right)V_D$$

# Outline

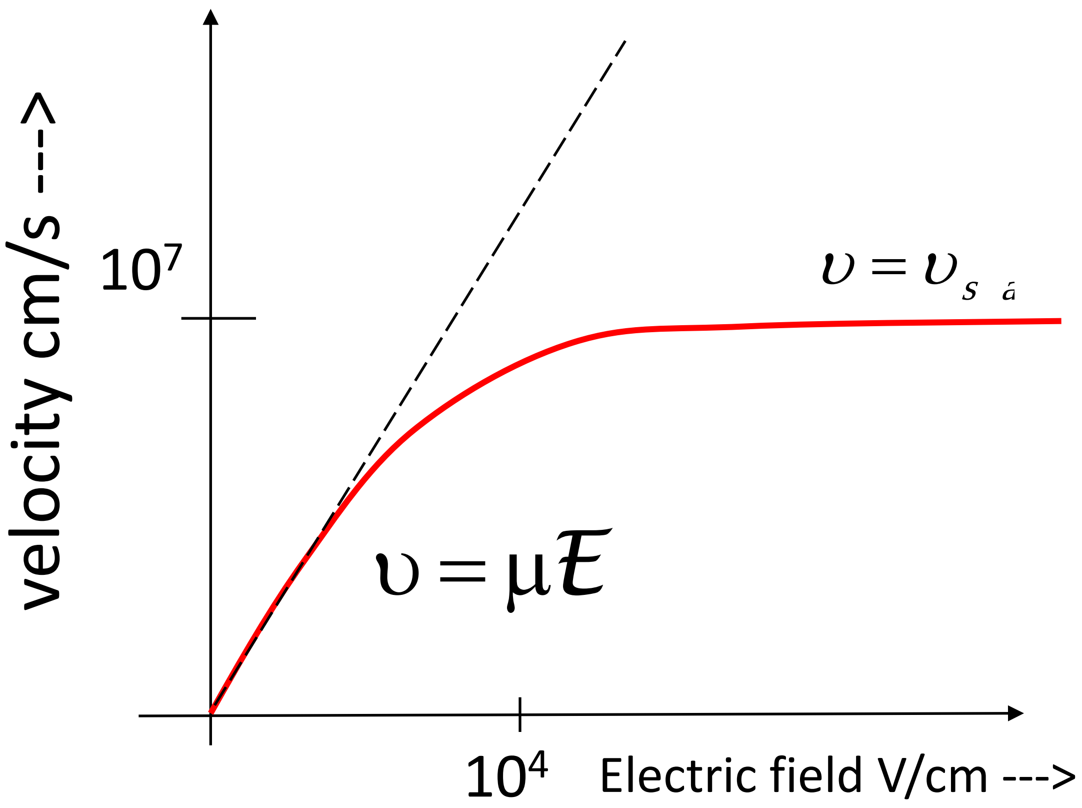
1) Square law/ simplified bulk charge theory

2) **Velocity saturation in simplified theory**

3) Few comments about bulk charge theory, small transistors

2) Conclusion

# Velocity vs. Field Characteristic (electrons)

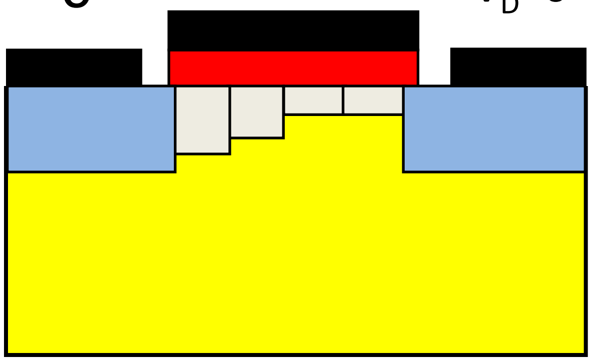



$$\upsilon_d = \frac{-\mu \mathcal{E}}{\left[1 + (\mathcal{E}/\mathcal{E}_c)^2\right]^{1/2}}$$

$$\upsilon_d = \frac{-\mu \mathcal{E}}{1 + (|\mathcal{E}|/\mathcal{E}_c)}$$

$$\upsilon_{d,sat} = \mu \mathcal{E}_c$$

# Velocity Saturation




$$J_1 = Q_1 \mu_1 \mathcal{E}_1 = Q_1 \mu_1 \left.\frac{dV}{dy}\right|_1$$

$$J_2 = Q_2 \mu_2 \mathcal{E}_2 = Q_2 \mu_2 \left.\frac{dV}{dy}\right|_2$$

$$J_3 = Q_3 \mu_3 \mathcal{E}_3 = Q_3 \mu_3 \left.\frac{dV}{dy}\right|_3$$

$$J_4 = Q_4 \mu_4 \mathcal{E}_4 = Q_4 \mu_4 \left.\frac{dV}{dy}\right|_4$$

$$\Rightarrow \sum_{i=1,N} \frac{J_i dy}{\mu(y)} = \sum_{i=1,N} Q_i \, dV$$

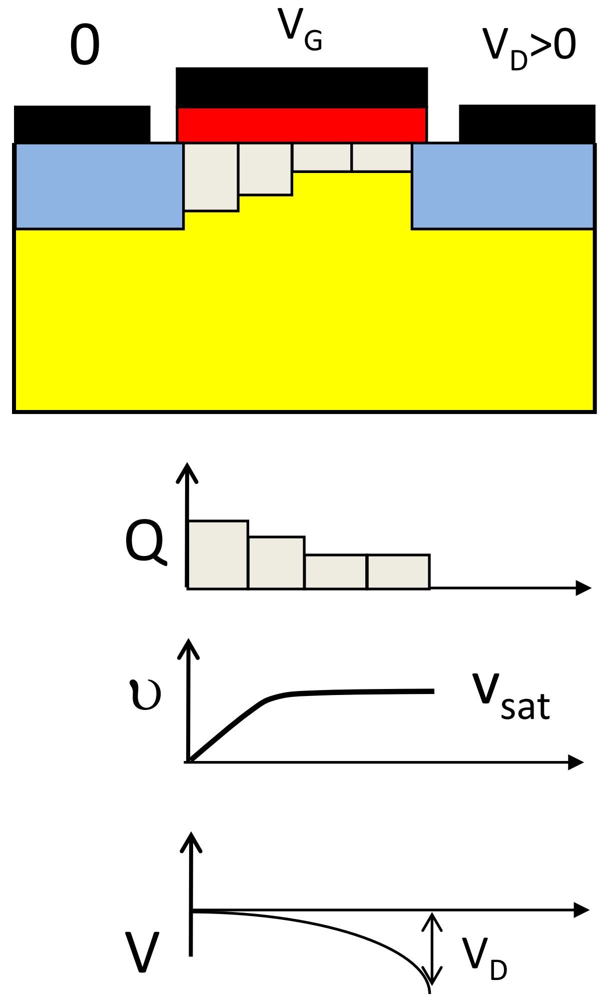
# Velocity Saturation

$$J_D \sum_{i=1,N} \frac{dy}{\mu_0 \Big/ \left[1+\frac{|\mathcal{E}|}{\mathcal{E}_c}\right]} = \int_0^{V_D} C_{ox}(V_G - V_{th} - mV)dV$$

$$\frac{J_D}{\mu_0}\int_0^{L_{ch}} dy\left[1+\frac{1}{\mathcal{E}_c}\frac{dV}{dy}\right] = C_{ox}\left[(V_G - V_{th})V_D - \frac{mV_D^{\;2}}{2}\right]$$

$$\int_0^{L_{ch}} J_D dy + \int_0^{V_{DS}} \frac{J_D}{E_c} dV = C_{ox}\left[(V_G - V_{th})V_D - \frac{mV_D^{\;2}}{2}\right]$$

$$J_D = \frac{\mu_0 C_{ox}}{L_{ch} + \frac{V_D}{\mathcal{E}_c}}\left[(V_G - V_{th})V_D - \frac{mV_D^{\;2}}{2}\right]$$



# Calculating $V_{DSAT}$

$$\frac{dI_D}{dV_{DS}} = 0$$

$$\frac{I_D}{W} = \frac{\mu_o C_{ox}}{L_{ch} + \dfrac{V_D}{\mathcal{E}_c}} \left[ \left(V_G - V_{th}\right) V_D - m \frac{V_D^2}{2} \right]$$

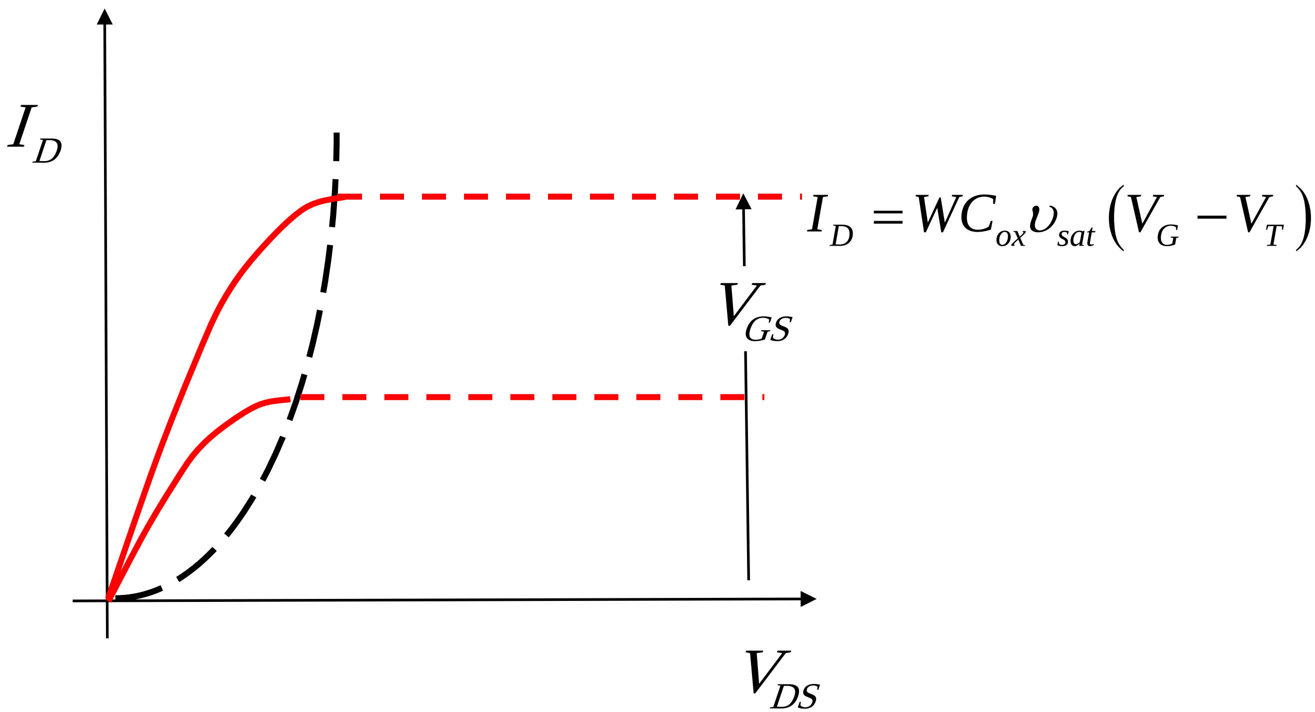
Take log on both sides and then set the derivative to zero ....

$$V_{DSAT} = \frac{2\left(V_G - V_{th}\right)/m}{1 + \sqrt{1 + 2\mu_o \left(V_G - V_{th}\right) / m\upsilon_{sat} L_{ch}}} < \frac{\left(V_{GS} - V_T\right)}{m}$$

# Velocity Saturation

$$J_{D,sat} = \frac{\mu_0 C_{ox}}{L_{ch} + \frac{V_{D,sat}}{\mathcal{E}_C}} \left[ (V_G - V_{th}) V_{D,sat} - \frac{m V_{D,sat}^{\;\;2}}{2} \right]$$

$$\sim \frac{\mu_0 \mathcal{E}_C C_{ox}}{V_{D,sat}} \left[ (V_G - V_{th}) V_{D,sat} - \frac{m V_{D,sat}^{\;\;2}}{2} \right] \sim \upsilon_{sat} C_{ox} (V_G - V_{th})$$



# ‘Linear Law’ Expression at the limit of L --> 0

$$V_{DSAT} = \frac{2\left(V_G - V_{th}\right)/m}{1 + \sqrt{1 + 2\mu_0\left(V_G - V_{th}\right)/m\upsilon_{sat}L_{ch}}}$$

$$V_{DSAT} \to \sqrt{2\upsilon_{sat}L_{ch}\left(V_G - V_{th}\right)/m\mu_0}$$

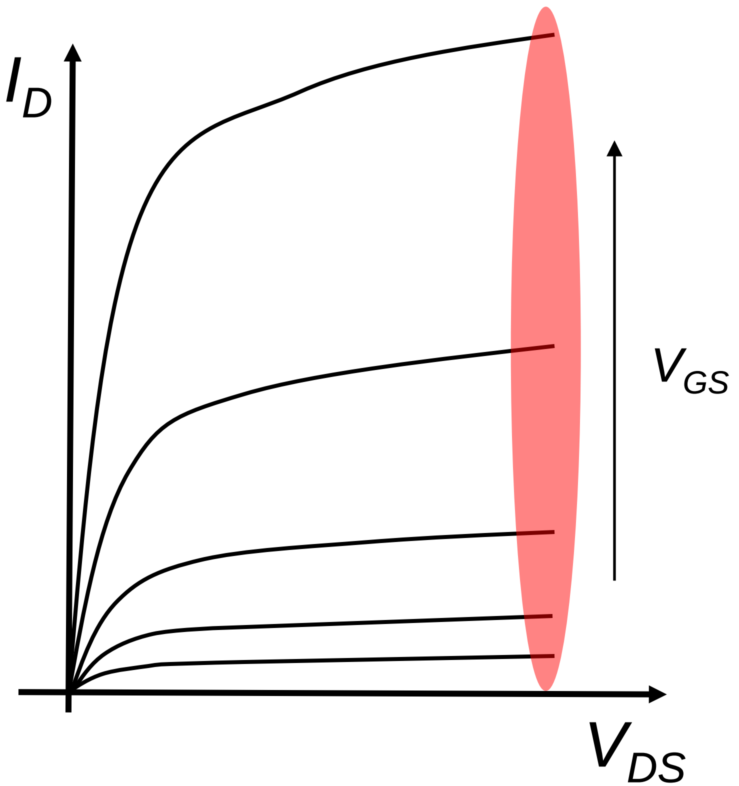
$$I_{DSAT} = W\,C_{ox}\upsilon_{sat}\left(V_G - V_{th}\right)\frac{\sqrt{1 + 2\mu_0\left(V_G - V_{th}\right)/m\upsilon_{sat}L_{ch}} - 1}{\sqrt{1 + 2\mu_0\left(V_G - V_{th}\right)/m\upsilon_{sat}L_{ch}} + 1}$$

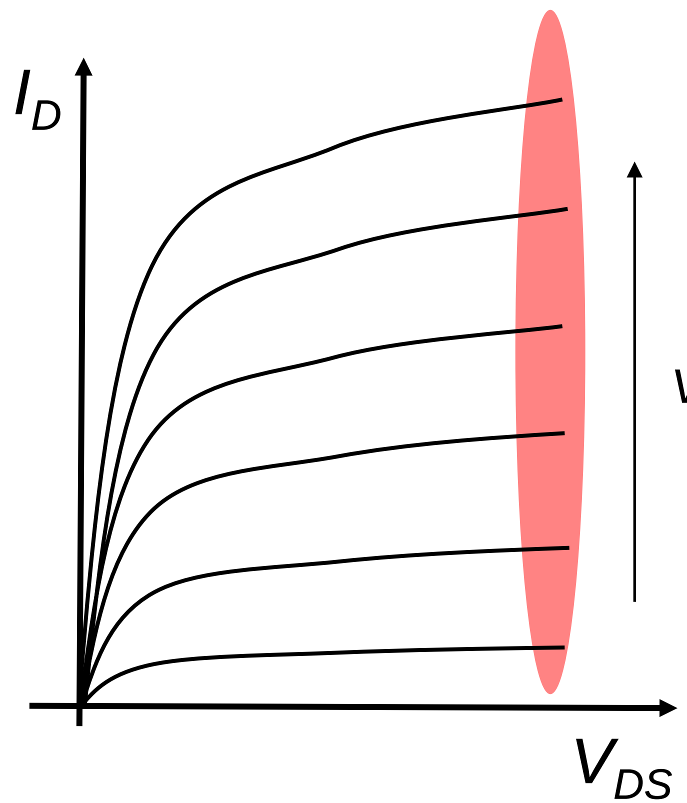
$$I_{DSAT} = W\,C_{ox}\upsilon_{sat}\left(V_G - V_{th}\right)$$

**Complete velocity saturation**

**Current independent of *L***

# ‘Signature’ of Velocity Saturation

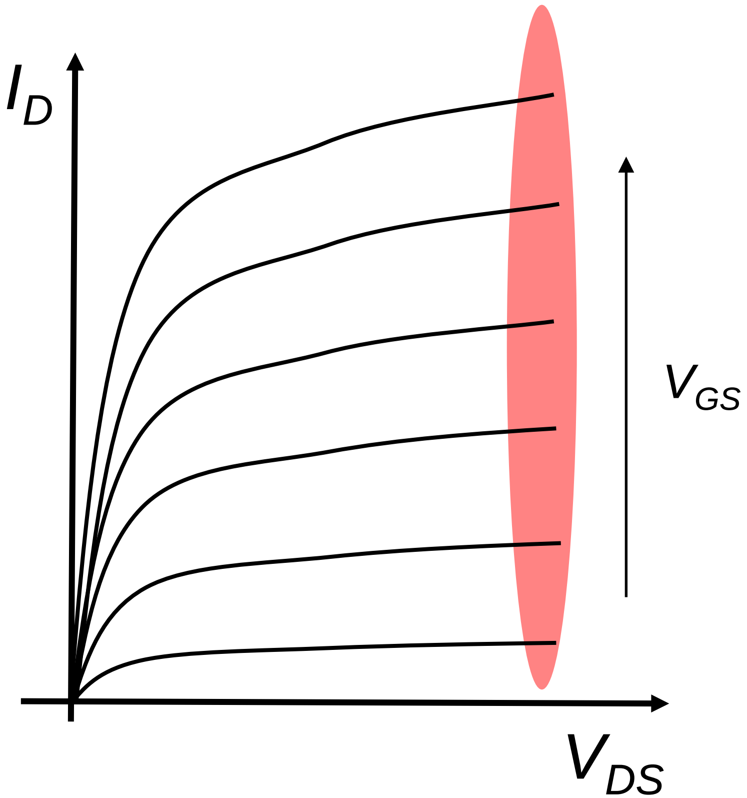



$$I_D = \frac{W}{2L_{ch}}\mu_0 C_{ox}\frac{\left(V_G - V_{th}\right)^2}{m}$$

$$I_D = W\upsilon_{sat}C_{ox}\left(V_G - V_{th}\right)$$

# $I_D$ and $(V_{GS} - V_T)$: In practice .....




$$I_D(V_D = V_{DD}) \sim (V_G - V_{th})^{\alpha}$$

$$1 < \alpha < 2$$

Complete velocity saturation

Long channel

# Outline

1) Square law/ simplified bulk charge theory

2) Velocity saturation in simplified theory

3) **Few comments about bulk charge theory, small transistors, etc.**

2) Conclusion

# Approximations for Inversion Charge

$$Q_i = -C_O(V_G - V_{th} - V) + qN_A\left(W_T(V) - W_T(V=0)\right)$$

$$= -C_O(V_G - V_{th} - V) + \sqrt{2q\kappa_S\varepsilon_o N_A\left(2\phi_B + V\right)} - \sqrt{2q\kappa_S\varepsilon_o N_A\left(2\phi_B\right)}$$

Approximations:

$Q_i \approx -C_{ox}(V_G - V_{th} - V)$ Square law approximation …

$Q_i \approx -C_{ox}(V_G - V_{th} - mV)$ Simplified bulk charge approximation …

# Complete Bulk-charge Theory

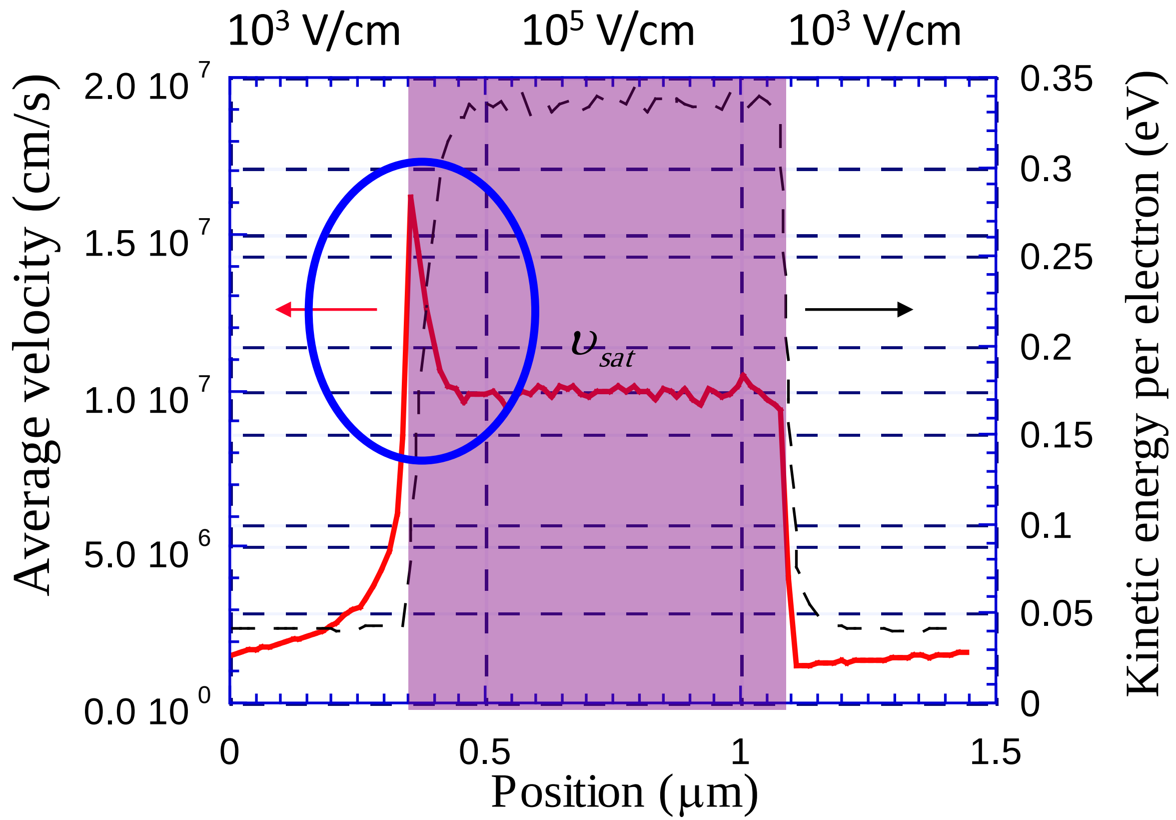
$$\frac{J_D}{\mu_0}\sum_{i=1,N} dy = \int_0^{V_D} C_O(V_G - V_{th} - V)dV + \int_0^{V_D} [.........]dV$$

$$\frac{J_D}{\mu_0}\int_0^{L_{ch}} dy = \int_0^{V_D} C_O(V_G - V_{th} - V)dV + \int_0^{V_D} [.........]dV$$

$$J_D = \frac{\mu_0 C_{ox}}{L_{ch}}\left[(V_G - V_{th})V_D - \frac{V_D{}^2}{2} - \frac{4}{3}\frac{qN_A W_T}{C_O}\phi_F\left\{\left(1+\frac{V_D}{2\phi_F}\right)^{3/2} - \left(1+\frac{3V_D}{4\phi_F}\right)\right\}\right]$$

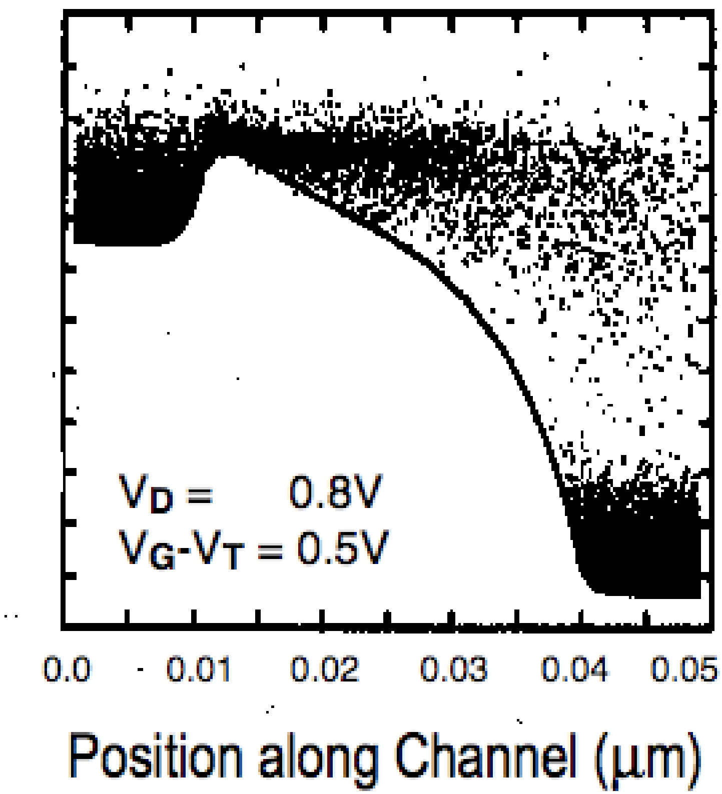
(Eq. 17.28 in SDF) .... Explicit dependence on bulk doping

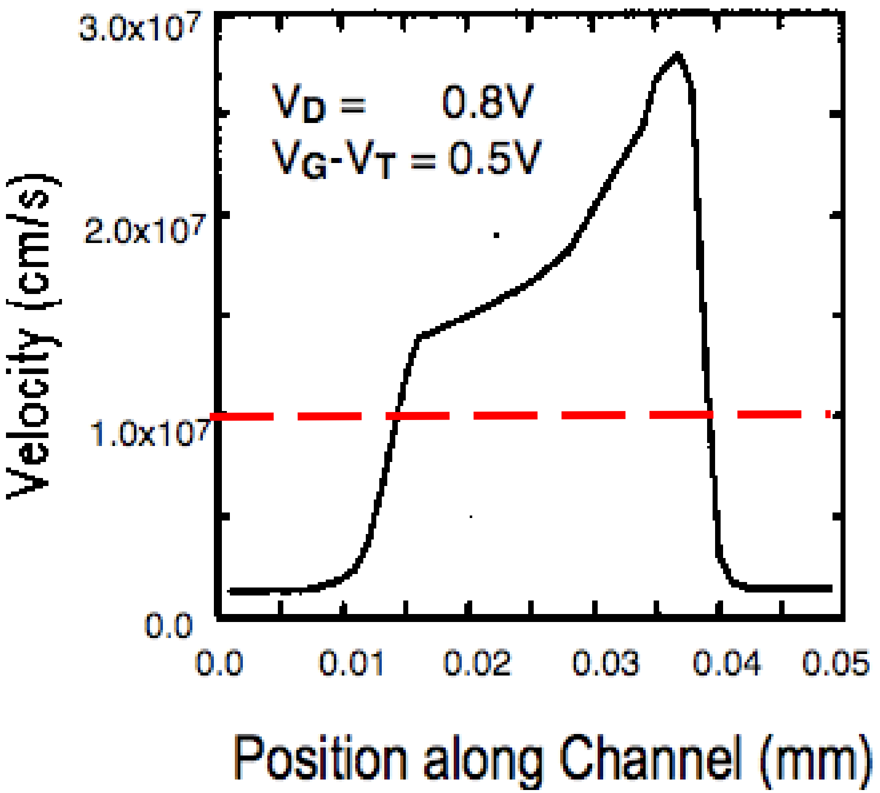
# Velocity Overshoot




$$\upsilon \neq \mu_n(E)E$$

# Velocity Overshoot in a MOSFET



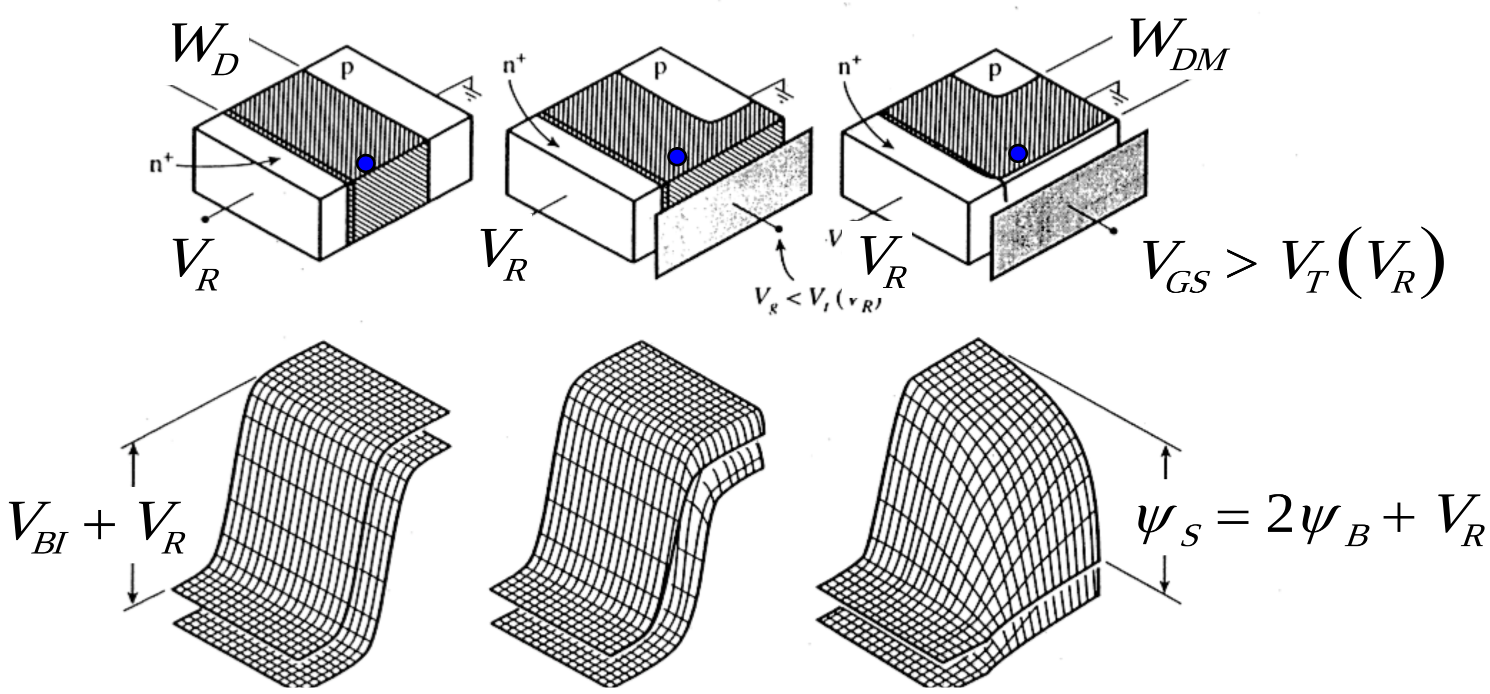

Frank, Laux, and Fischetti, IEDM Tech. Dig., p. 553, 1992

# Summary

1) Velocity saturation is an important consideration for short channel transistors (e.g., $V_D$=1V, $L_{ch}$=20nm). Therefore, $\alpha \sim 1$ for most modern transistors.
2) Bulk charge theory explains why MOSFET current depends on substrate (bulk) doping. In the simplified bulk charge theory, doping dependence is encapsulated in ***m***.
3) Additional considerations of velocity overshoot could complicate calculation of current.
4) Good news is that for very short channel transistors, electrons travel from source to drain without scattering. A considerably simpler 'Lundstrom theory of MOSFET' applies.

# Additional Notes …

# Effect of Drain Bias




Gated doped or p-MOS with adjacent, reverse-biased $n^+$ region

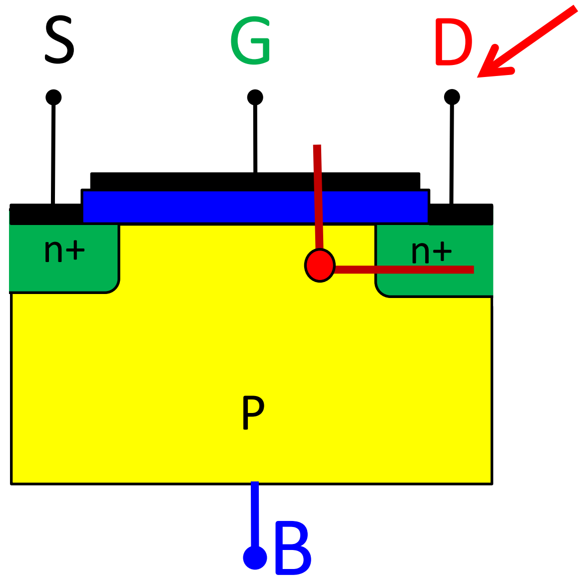
a) gate biased at flat-band

b) gate biased in depletion

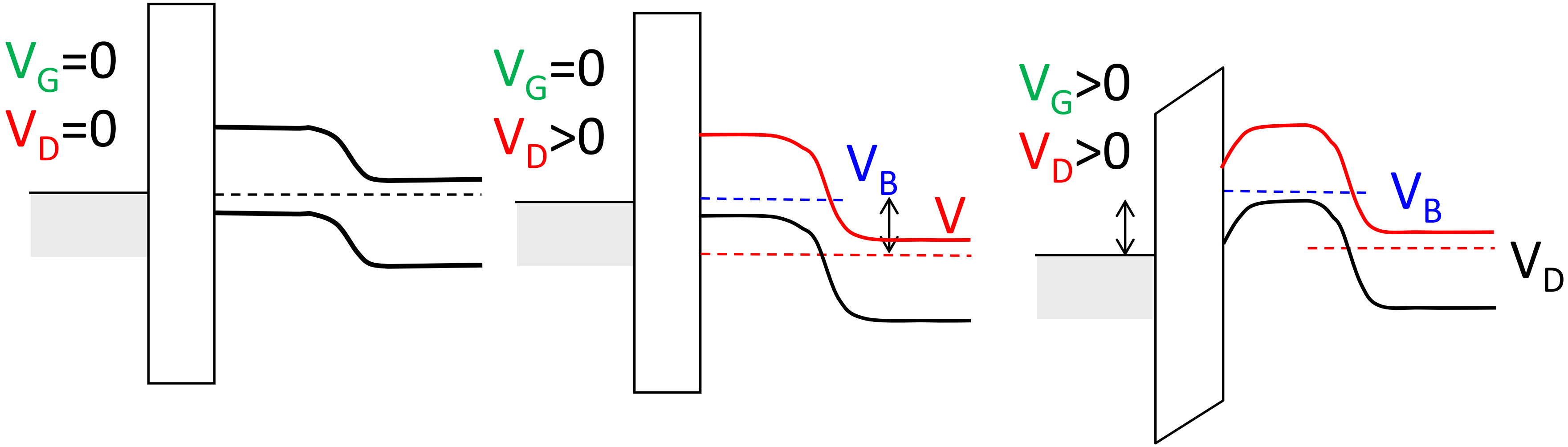
b) gate biased in inversion

A. Grove, *Physics of Semiconductor Devices*, 1967.

# Inversion Charge in the Channel

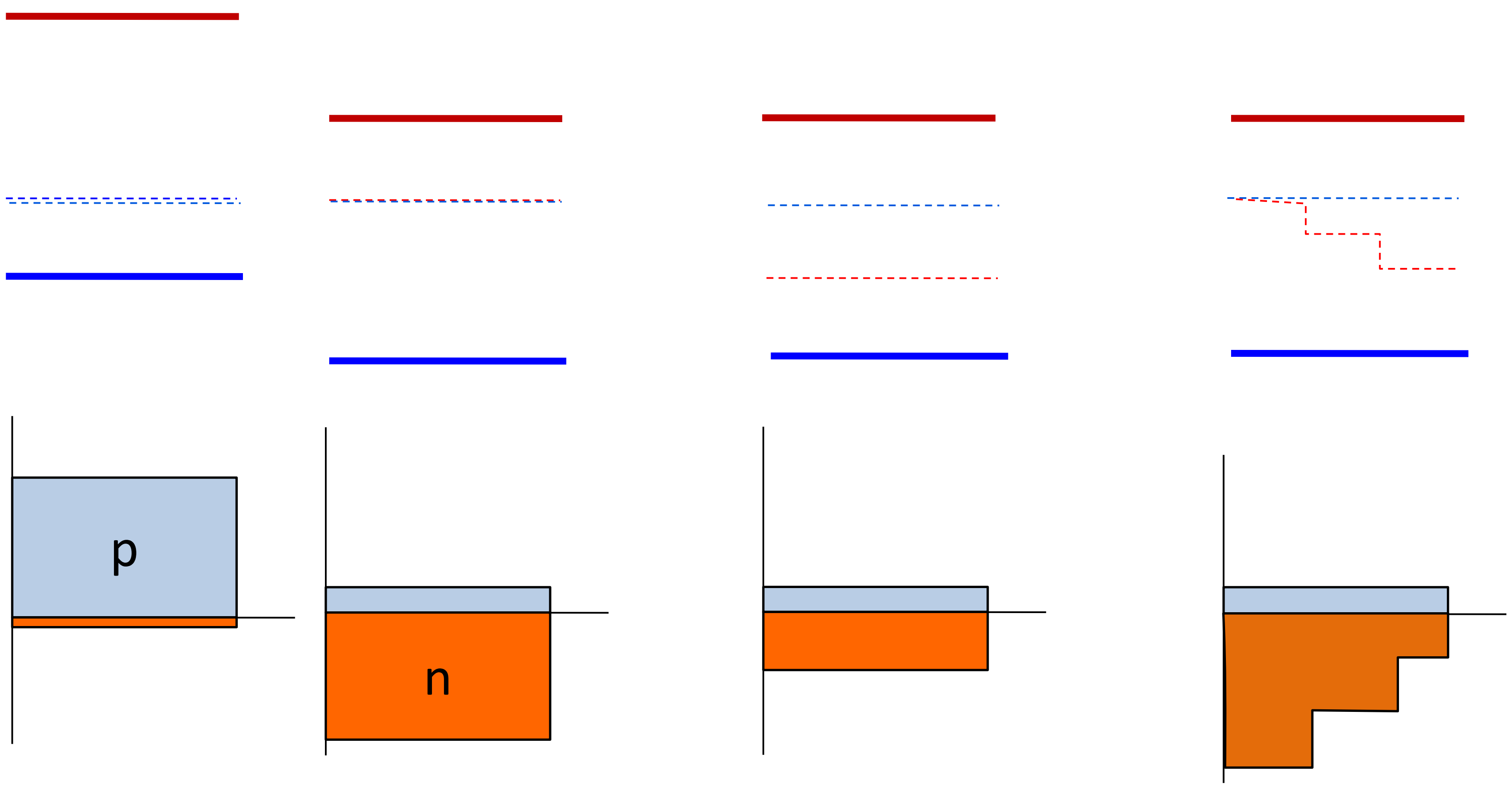



$$Q_i = -C_{ox}(V_G - V_{th} - V) + qN_A(W_T(V) - W_T(V=0))$$



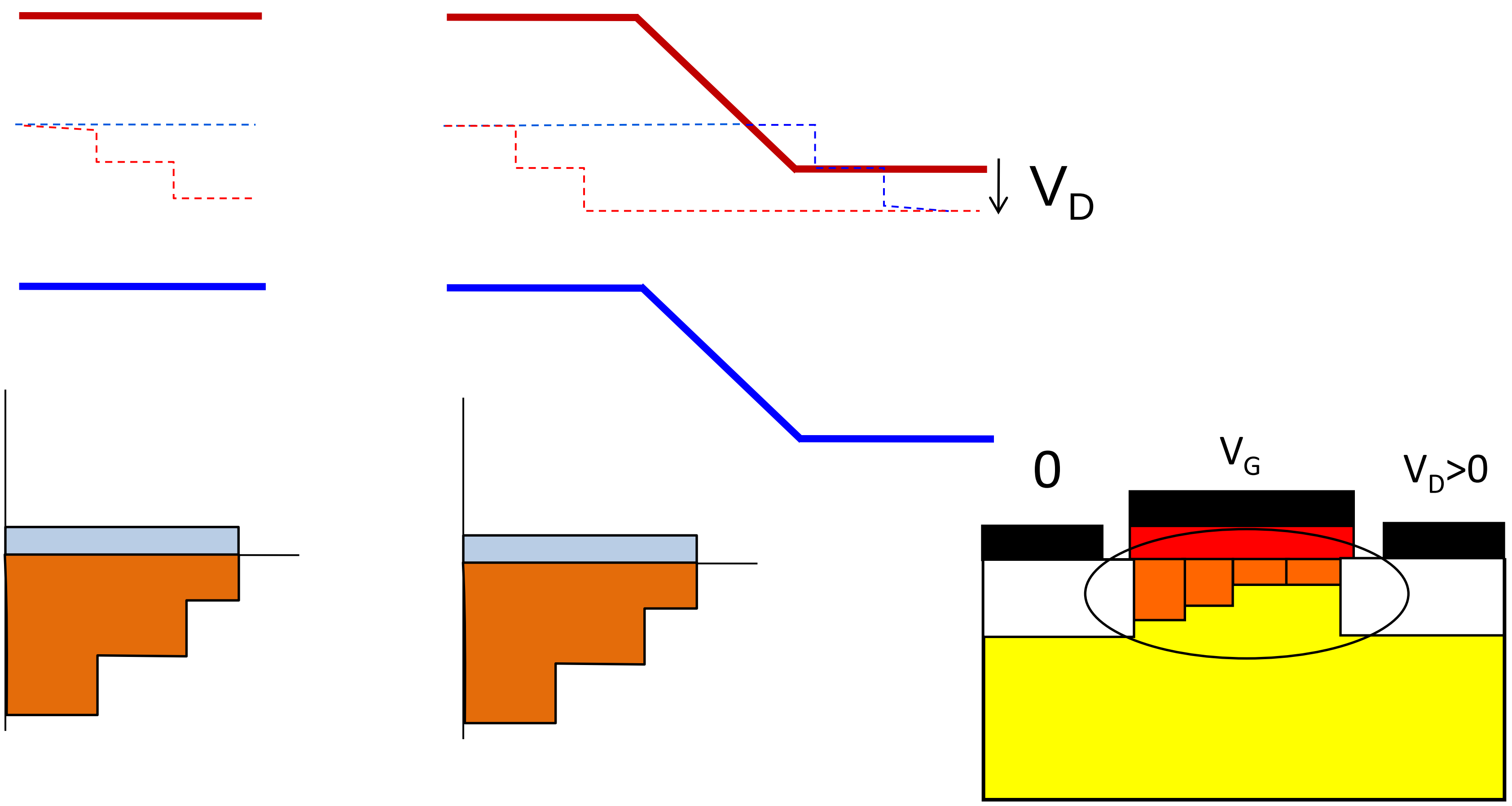
# Charge along the channel ....

$$n = N_C e^{-(E_C - F_n)\beta} \qquad p = N_C e^{(E_V - F_p)\beta}$$



# Charge along the channel …

$$n = N_C e^{-(E_C - F_n)\beta} \qquad p = N_C e^{(E_V - F_p)\beta}$$



# Depletion into the channel ....

$$n = N_C e^{-(E_C - F_n)\beta} \qquad p = N_C e^{(E_V - F_p)\beta}$$