Chapter 7: Mechanical Properties

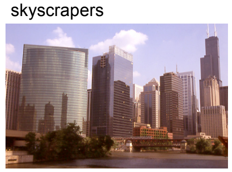
Why mechanical properties?

- **Need to design materials that will withstand applied load and in-service uses for…**

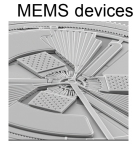
Space exploration

Bridges for autos and people

skyscrapers



MEMS devices



Space elevator?

Chapter 7: Mechanical Properties

ISSUES TO ADDRESS...

- Stress and strain: Normalized force and displacements.

$$\text{Engineering}: \quad \sigma_e = F_i / A_0 \quad \varepsilon_e = \Delta\ell / \ell_0$$
$$\text{True}: \quad \sigma_T = F_i / A_i \quad \varepsilon_T = \ln(\ell_f / \ell_0)$$

- Elastic behavior: When loads are small.

$$\text{Young's Modulus}: \quad E \quad [GPa]$$

- Plastic behavior: *dislocations* and *permanent deformation*

$$\text{Yield Strength}: \quad \sigma_{YS} \quad [MPa] \text{ (permanent deformation)}$$
$$\text{Ultimate Tensile Strength}: \quad \sigma_{TS} \quad [MPa] \text{ (fracture)}$$

- Toughness, ductility, resilience, toughness, and hardness: Define and how do we measure?
- Mechanical behavior of the various classes of materials.

Stress and Strain

Stress: Force per unit area arising from applied load.

Tension, compression, shear, torsion or any combination.

Stress = σ = force/area

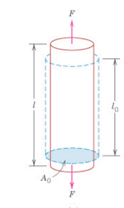
Strain: physical deformation response of a material to stress, e.g., elongation.

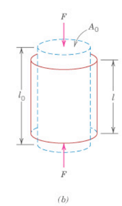
Pure Tension | Pure Compression

stress $\sigma_e = \dfrac{F_{normal}}{A_o}$

strain $\varepsilon_e = \dfrac{\ell - \ell_o}{\ell_o}$

Elastic response $\sigma_e = E\varepsilon$



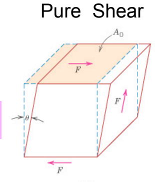


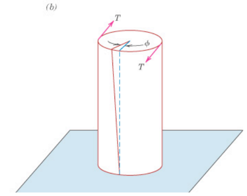
Pure Shear

stress $\tau_e = \dfrac{F_{shear}}{A_o}$

strain $\gamma = \tan\theta$

Elastic response $\tau_e = G\gamma$

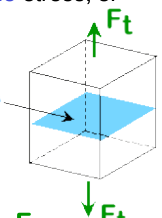




Pure Torsional Shear

Engineering Stress

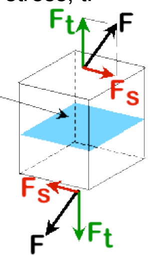
- Tensile stress, s:



$$\sigma = \frac{F_t}{A_o}$$

original area before loading

- Shear stress, t:



$$\tau = \frac{F_s}{A_o}$$

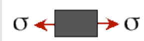
Stress has units: N/m^2 (or lb/in^2)

Common States of Stress

- Simple tension: cable

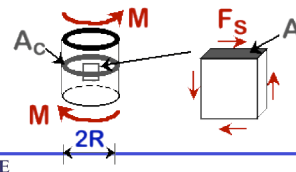
A_o = cross sectional Area (when unloaded)

$$\sigma = \frac{F}{A_o}$$

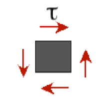


Ski lift (photo courtesy P.M. Anderson)

- Simple shear: drive shaft



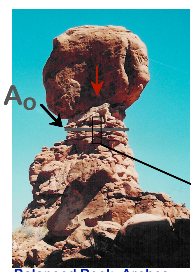
$$\tau = \frac{F_s}{A_o}$$



Note: $\tau = M/A_cR$ here.

Common States of Stress

- Simple compression:



Balanced Rock, Arches National Park (photo courtesy P.M. Anderson)

Canyon Bridge, Los Alamos, NM (photo courtesy P.M. Anderson)

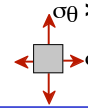
$$\sigma = \frac{F}{A_o}$$

Note: compressive structure member ($\sigma < 0$).

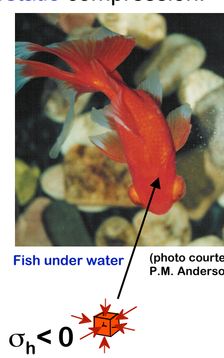
Common States of Stress

- Bi-axial tension:
- Hydrostatic compression:

Pressurized tank (photo courtesy P.M. Anderson)



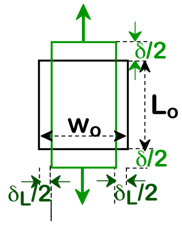
Fish under water (photo courtesy P.M. Anderson)



Engineering Strain

- Tensile strain:

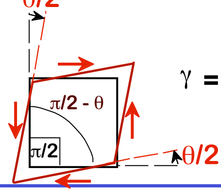
$$\varepsilon = \frac{\delta}{L_o}$$



- Lateral (width) strain:

$$\varepsilon_L = \frac{-\delta_L}{w_o}$$

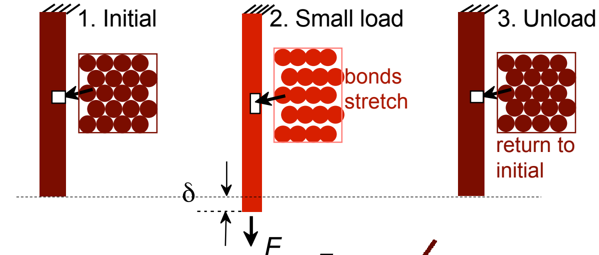
- Shear strain:



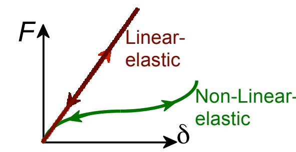
$$\gamma = \tan\theta$$

Strain is always dimensionless.

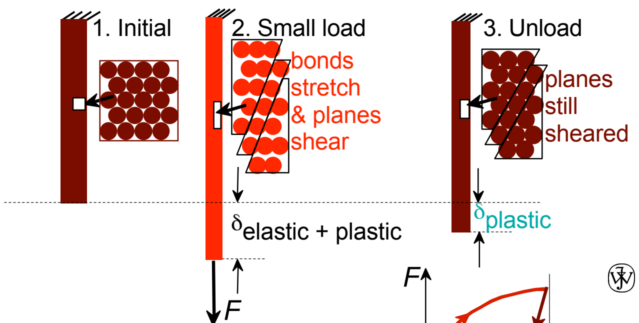
Elastic Deformation



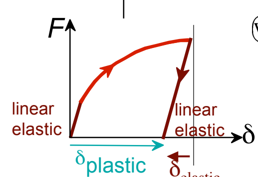
Elastic means reversible!



Plastic Deformation of Metals

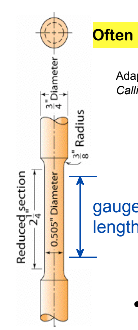


Plastic means permanent!



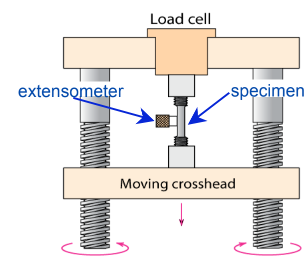
Strain Testing

- Tensile specimen



Adapted from Fig. 7.2, *Callister & Rethwisch 3e.*

- Tensile test machine



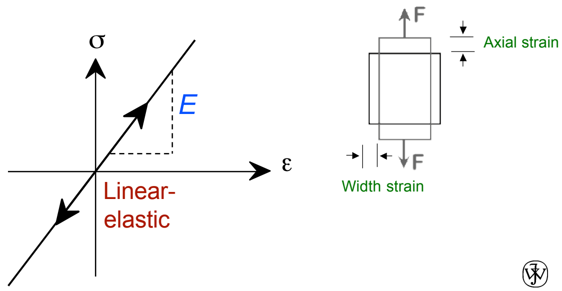
- Other types:
  - -compression: brittle materials (e.g., concrete)
  - -torsion: cylindrical tubes, shafts.

Linear Elasticity

- Modulus of Elasticity, E: (also known as Young's modulus)

Units: E [GPa] or [psi]

- Hooke's Law: $\sigma = E\,\varepsilon$



Example: Hooke's Law

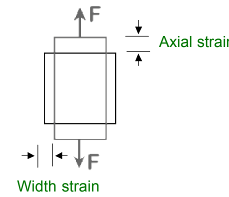
- Hooke's Law: $\sigma = E\,\varepsilon$ (linear elastic behavior)

Copper sample (305 mm long) is pulled in tension with stress of 276 MPa. If deformation is elastic, what is elongation?

For Cu, E = 110 GPa.

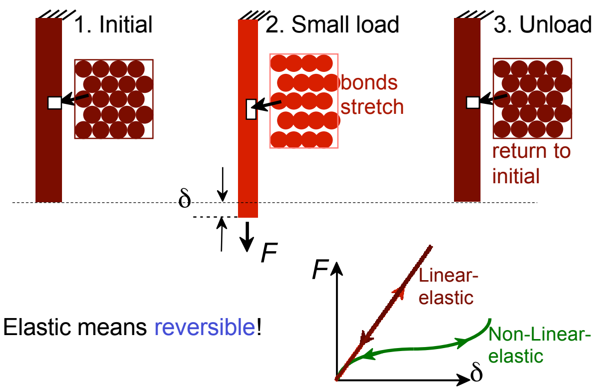
$$\sigma = E\varepsilon = E\left(\frac{\Delta\ell}{\ell_0}\right) \Rightarrow \Delta\ell = \frac{\sigma\ell_0}{E}$$

$$\Delta\ell = \frac{(276MPa)(305mm)}{110x10^3MPa} = 0.77mm$$



Hooke's law involves axial (parallel to applied tensile load) elastic deformation.

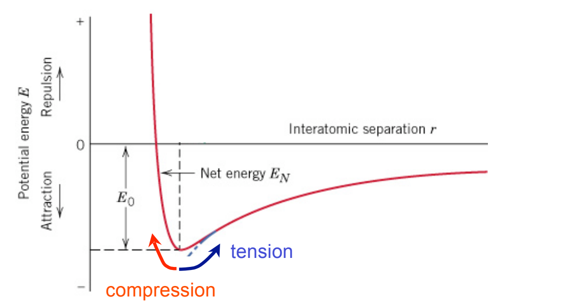
Elastic Deformation



Elastic means reversible!

Mechanical Properties

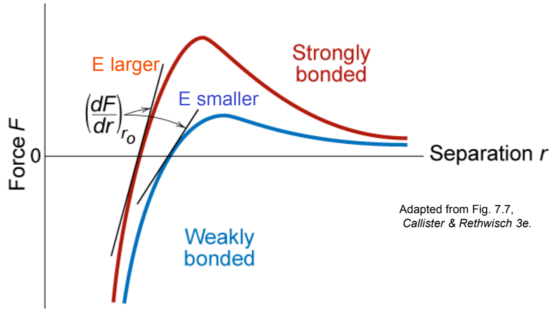
- Recall: Bonding Energy vs distance plots



Adapted from Fig. 2.8 *Callister & Rethwisch 3e.*

Mechanical Properties

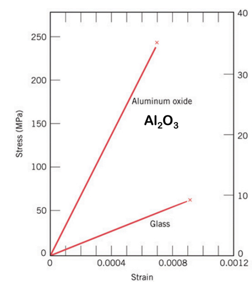
- Recall: Slope of stress strain plot (proportional to the *E*) depends on bond strength of metal



Adapted from Fig. 7.7, *Callister & Rethwisch 3e.*

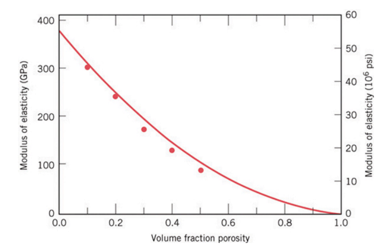
Elasticity of Ceramics

- **Elastic Behavior**



And Effects of Porosity

$E = E_0(1 - 1.9P + 0.9P^2)$



Neither Glass or Alumina experience plastic deformation before fracture!

Comparison of Elastic Moduli

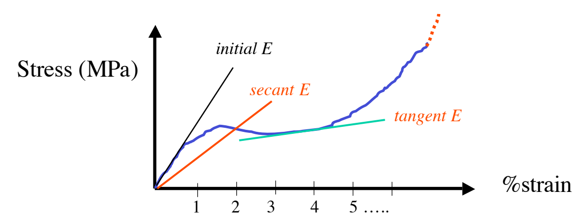
Table 7.1 Room-Temperature Elastic and Shear Moduli, and Poisson's Ratio for Various Materials

	Modulus of Elasticity		Shear Modulus		
Material	*GPa*	*10^6 psi*	*GPa*	*10^6 psi*	*Poisson's Ratio*
	Metal Alloys				
Tungsten	407	59	160	23.2	0.28
Steel	207	30	83	12.0	0.30
Nickel	207	30	76	11.0	0.31
Titanium	107	15.5	45	6.5	0.34
Copper	110	16	46	6.7	0.34
Brass	97	14	37	5.4	0.34
Aluminum	69	10	25	3.6	0.33
Magnesium	45	6.5	17	2.5	0.35

Silicon (single xtal)	120-190	(depends on crystallographic direction)
Glass (pyrex)	70	
SiC (fused or sintered)	207-483	
Graphite (molded)	~12	
High modulus C-fiber	400	
Carbon Nanotubes	**~1000**	Normalize by density, 20x steel wire. strength normalized by density is 56x wire.

Polymers: Tangent and Secant Modulus

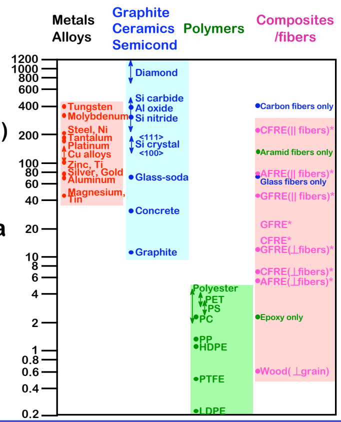
- Tangent Modulus is experienced in service.
- Secant Modulus is effective modulus at 2% strain.
  - grey cast iron is also an example
- Modulus of polymer changes with *time* and *strain-rate*.
  - must report strain-rate $d\varepsilon/dt$ for polymers.
  - must report fracture strain ε_f **before** fracture.



Young's Modulus, E

E(GPa)

10^9 Pa



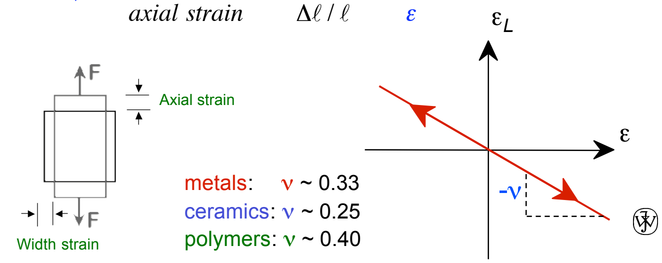
$E_{ceramics}$
$> E_{metals}$
$>> E_{polymers}$

Based on data in Table B2, *Callister 6e*.
Composite data based on reinforced epoxy with 60 vol% of aligned carbon (CFRE), aramid (AFRE), or glass (GFRE) fibers.

Poisson's ratio, ν

- **Poisson's ratio, ν:** Units: ν dimensionless

$$\nu = -\frac{width\ strain}{axial\ strain} = -\frac{\Delta w / w}{\Delta \ell / \ell} = -\frac{\varepsilon_L}{\varepsilon}$$



metals: ν ~ 0.33
ceramics: ν ~ 0.25
polymers: ν ~ 0.40

Why does ν have minus sign?

Limits of the Poisson Ratio

- *Poisson Ratio has a range* $-1 \leq \nu \leq 1/2$

Look at extremes

- No change in aspect ratio: $\Delta w / w = \Delta \ell / \ell$

$$\nu = -\frac{\Delta w / w}{\Delta \ell / \ell} = -1$$

- Volume (V = AL) remains constant: ΔV = 0.

Hence, ΔV = (L ΔA+A ΔL) = 0. So, $\Delta A / A = -\Delta L / L$

In terms of width, A = w², then ΔA/A = 2 w Δw/w² = 2Δw/w = –ΔL/L.

Hence, $$\nu = -\frac{\Delta w / w}{\Delta \ell / \ell} = -\frac{(-\frac{1}{2}\Delta \ell / \ell)}{\Delta \ell / \ell} = 1/2$$ *Incompressible solid. Water (almost).*

Poisson Ratio: materials specific

Metals:	Ir	W	Ni	Cu	Al	Ag	Au
	0.26	0.29	0.31	0.34	0.34	0.38	0.42
						generic value ~ 1/3	
Solid Argon:	0.25						
Covalent Solids:		Si	Ge	Al_2O_3	TiC		
		0.27	0.28	0.23	0.19	generic value ~ 1/4	
Ionic Solids:	MgO	0.19					
Silica Glass:	0.20						

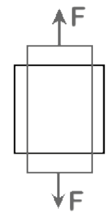
Polymers: Network (Bakelite) 0.49 Chain (PE) 0.40 ~generic value

Elastomer: Hard Rubber (Ebonite) 0.39 (Natural) 0.49

Example: Poisson Effect

Tensile stress is applied along cylindrical brass rod (10 mm diameter). Poisson ratio is ν = 0.34 and E = 97 GPa.

• Determine load needed for $2.5\text{x}10^{-3}$ mm change in diameter if the deformation is entirely elastic?



Width strain: (note reduction in diameter)

$\varepsilon_x = \Delta d/d = -(2.5\text{x}10^{-3}\text{ mm})/(10\text{ mm}) = -2.5\text{x}10^{-4}$

Axial strain: Given Poisson ratio

$\varepsilon_z = -\varepsilon_x/\nu = -(-2.5\text{x}10^{-4})/0.34 = +7.35\text{x}10^{-4}$

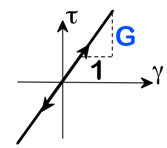
Axial Stress: $\sigma_z = E\varepsilon_z = (97\text{x}10^3\text{ MPa})(7.35\text{x}10^{-4}) = 71.3$ MPa.

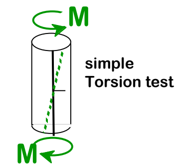
Required Load: $F = \sigma_z A_0 = (71.3\text{ MPa})\pi(5\text{ mm})^2 = 5600$ N.

Other Elastic Properties

• **Elastic Shear modulus, G:**

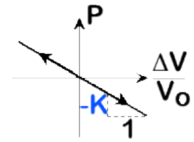
$\tau = G\gamma$

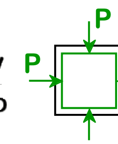




• **Elastic Bulk modulus, K:**

$P = -K\frac{\Delta V}{V_o}$





• **Special relations for isotropic materials:**

$G = \frac{E}{2(1+\nu)}$ $K = \frac{E}{3(1-2\nu)}$

So, only 2 independent elastic constants for isotropic media

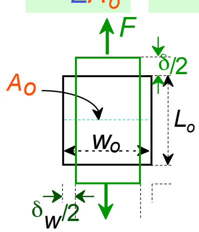
Useful Linear Elastic Relationships

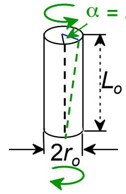
• **Simple tension:**

$\delta = \frac{FL_o}{EA_o}$ $\delta_w = -\nu\frac{Fw_o}{EA_o}$

• **Simple torsion:**

$\alpha = \frac{2ML_o}{\pi r_o^4 G}$



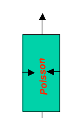


• Material, geometric, and loading parameters all contribute to deflection.
• Larger elastic moduli minimize elastic deflection.

Complex States of Stress in 3D

• There are 3 *principal components* of stress and (small) strain.
• For linear elastic, isotropic case, use "linear superposition".
• Strain || to load by *Hooke's Law*: $\varepsilon_i = \sigma_i/E$, i=1,2,3 (maybe x,y,z).
• Strain ⊥ to load governed by *Poisson effect*: $\varepsilon_{width} = -\nu\varepsilon_{axial}$.

stress / strain	σ_1	σ_2	σ_3	Total Strain
ε_1	σ_1/E	$-\nu\sigma_2/E$	$-\nu\sigma_3/E$	in x
ε_2	$-\nu\sigma_1/E$	σ_2/E	$-\nu\sigma_3/E$	in y
ε_3	$-\nu\sigma_1/E$	$-\nu\sigma_2/E$	σ_3/E	in z



In *x*-direction, total linear strain is:

$$\varepsilon_1 = \frac{1}{E}\{\sigma_1 - \nu(\sigma_2 + \sigma_3)\}$$

$$or \quad = \frac{1}{E}\{(1+\nu)\sigma_1 - \nu(\sigma_1 + \sigma_2 + \sigma_3)\}$$

Complex State of Stress and Strain in 3-D Solid

- Hooke's Law and Poisson effect gives total linear strain:

$$\varepsilon_1 = \frac{1}{E}\{\sigma_1 - \nu(\sigma_2+\sigma_3)\} \quad or \quad \frac{1}{E}\{(1+\nu)\sigma_1 - \nu(\sigma_1+\sigma_2+\sigma_3)\}$$

- For uniaxial tension test $\sigma_1 = \sigma_2 = 0$, so $\varepsilon_3 = \sigma_3/E$ and $\varepsilon_1 = \varepsilon_2 = -\nu\varepsilon_3$.

- Hydrostatic Pressure:

$$P = \sigma_{Hyd} = \frac{\sigma_1+\sigma_2+\sigma_3}{3} = \frac{Tr\sigma}{3} \qquad \varepsilon_1 = \frac{1}{E}\{(1+\nu)\sigma_1 - 3\nu P\}$$

- For volume ($V=l_1l_2l_3$) strain, $\Delta V/V = \varepsilon_1 + \varepsilon_2 + \varepsilon_3 = (1-2\nu)\sigma_3/E$

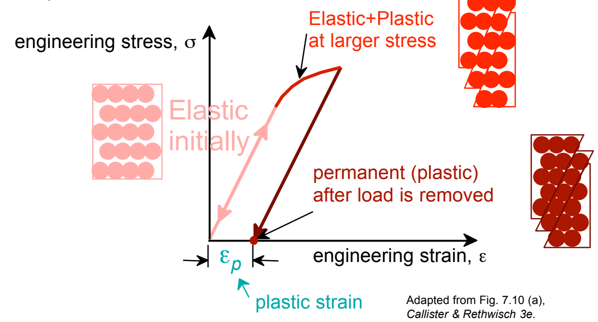
$$\frac{\Delta V}{V} = 3(1-2\nu)\frac{P}{E}$$

Bulk Modulus, B or K: $P = -K\,\Delta V/V$ so $K = E/3(1-2\nu)$ (sec. 7.5)

Plastic (Permanent) Deformation

(at lower temperatures, i.e. $T < T_{melt}/3$)

- Simple tension test:

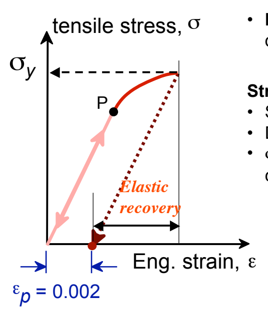


Adapted from Fig. 7.10 (a), *Callister & Rethwisch 3e.*

Yield Stress, σ_Y

- Stress where *noticeable* plastic deformation occurs.

For metals agreed upon 0.2% — when $\varepsilon_p = 0.002$



- **P** is the *proportional limit* where deviation from linear behavior occurs.

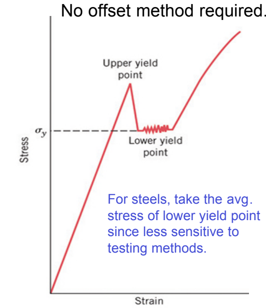
Strain off-set method for Yield Stress
- Start at 0.2% strain (for most metals).
- Draw line parallel to elastic curve (slope of E).
- σ_Y is value of stress where dotted line crosses stress-strain curve (dashed line).

Note: for 2 in. sample
$\varepsilon = 0.002 = \Delta z/z$
$\therefore \Delta z = 0.004$ in

Yield Points and σ_{YS}

- Yield-point phenomenon occurs when elastic plastic transition is abrupt.

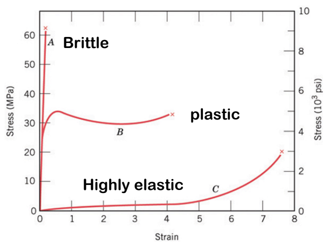
No offset method required.



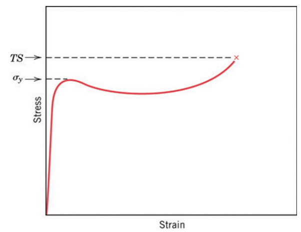
- In steels, this effect is seen when dislocations start to move and unbind for interstitial solute.
- Lower yield point taken as σ_Y.
- Jagged curve at lower yield point occurs when solute binds dislocation and dislocation unbinding again, until work-hardening begins to occur.

Stress-Strain in Polymers

- 3 different types of behavior

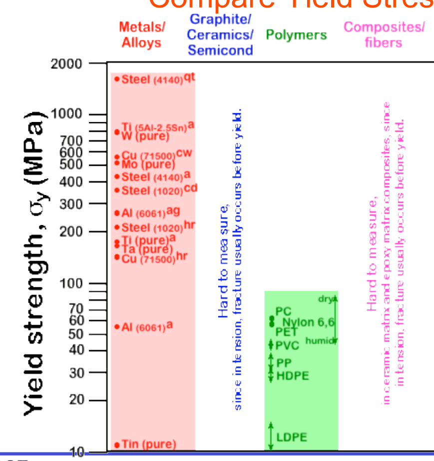


For plastic polymers:
- YS at maximum stress just after elastic region.
- TS is stress at fracture!



- Highly elastic polymers:
  - Elongate to as much as 1000% (e.g. silly putty).
  - 7 MPa < E < 4 GPa   3 order of magnitude!
  - TS(max) ~ 100 MPa   some metal alloys up to 4 GPa

Compare Yield Stress, σ_{YS}



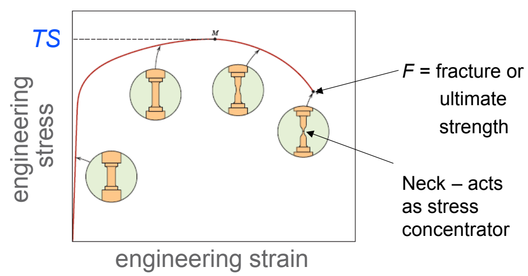
σ_y(ceramics) >> σ_y(metals) >> σ_y(polymers)

Room T values

Based on data in Table B4, *Callister 6e*.
- a = annealed
- hr = hot rolled
- ag = aged
- cd = cold drawn
- cw = cold worked
- qt = quenched & tempered

(Ultimate) Tensile Strength, σ_{TS}

- **Maximum possible engineering stress in tension.**



- Metals: occurs when necking starts.
- Ceramics: occurs when crack propagation starts.
- Polymers: occurs when polymer backbones are aligned and about to break.

Metals: Tensile Strength, σ_{TS}

For Metals: max. stress in tension when necking starts, which is the metals work-hardening tendencies vis-à-vis those that initiate instabilities.

$dF = 0$ **Maximum eng. Stress (at necking)**

$$dF = 0 = \sigma_T dA_i + A_i d\sigma_T \rightarrow \frac{d\sigma_T}{\sigma_T} = -\frac{dA_i}{A_i}$$

- $\sigma_T dA_i$: decreased force due to *decrease in gage diameter*
- $A_i d\sigma_T$: Increased force due to *increase in applied stress*
- $\frac{d\sigma_T}{\sigma_T}$: Fractional Increase in Flow stress
- $-\frac{dA_i}{A_i}$: fractional decrease in load-bearing area

At the point where these two competing changes in force equal, there is permanent neck.

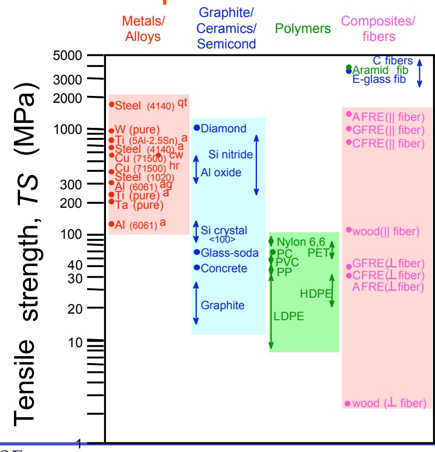
Determined by slope of "true stress" - "true strain" curve

$$\frac{d\sigma_T}{\sigma_T} = -\frac{dA_i}{A_i} = \frac{d\ell_i}{\ell_i} = d\varepsilon_T \Rightarrow \frac{d\sigma_T}{d\varepsilon_T} = \sigma_T$$

If $\sigma_T = K(\varepsilon_T)^n$, then $n = \varepsilon_T$

n = strain-hardening coefficient

Compare Tensile Strength, σ_{TS}



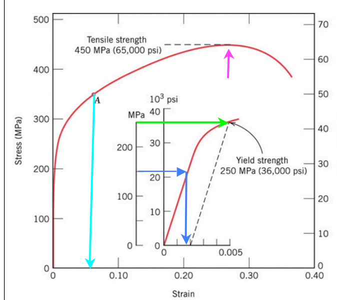
Room T values

TS(ceram)
~TS(met)
~ TS(comp)
>> TS(poly)

Based on data in Table B4, *Callister & Rethwisch 3e.*

Example for Metals: Determine E, YS, and TS

Stress-Strain for Brass



- Gage is 250 mm (10 in) in length and 12.8 mm (0.505 in) in diameter.
- Subject to tensile stress of 345 MPa (50 ksi)

- **Young's Modulus, E** (bond stretch)

$$E = \frac{\sigma_2 - \sigma_1}{\varepsilon_2 - \varepsilon_1} = \frac{(150-0)MPa}{0.0016-0} = 93.8GPa$$

- **Offset Yield-Stress, YS** (plastic deformation)

$$YS = 250\ MPa$$

- **Max. Load from Tensile Strength TS**

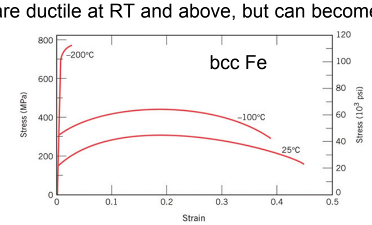
$$F_{max} = \sigma_{TS} A_0 = \sigma_{TS}\pi\left[\frac{d_0}{2}\right]^2$$
$$= 450MPa\left(\frac{12.8x10^{-3}m}{2}\right)^2 \pi = 57{,}900N$$

- **Change in length at Point A, $\Delta l = \varepsilon l_0$**

$$\Delta l = \varepsilon l_0 = (0.06)250\ mm = 15\ mm$$

Temperature matters (see Failure)

Most metals are ductile at RT and above, but can become brittle at low T



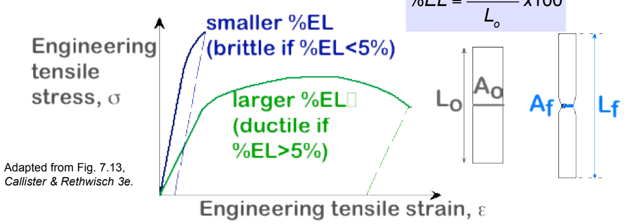
cup-and-cone fracture in Al

brittle fracture in mild steel

Ductility (%EL and %RA)

- Plastic tensile strain at failure:

$$\%EL = \frac{L_f - L_o}{L_o} x100$$



Adapted from Fig. 7.13, *Callister & Rethwisch 3e.*

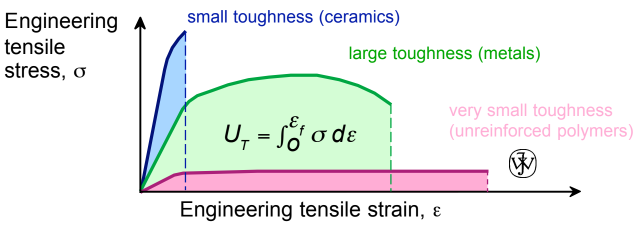
- Another ductility measure:

$$\%RA = \frac{A_o - A_f}{A_o} x100$$

- Note: %RA and %EL are often comparable.
  - Reason: crystal slip does not change material volume.
  - %RA > %EL possible if internal voids form in neck.

Toughness

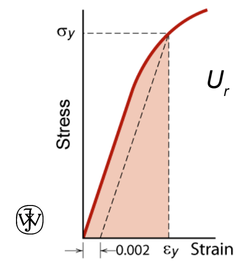
- Energy to break a unit volume of material, *or absorb energy to fracture.*
- Approximate as area under the stress-strain curve.



Brittle fracture: elastic energy
Ductile fracture: elastic + plastic energy

Resilience, U_r

- Resilience is capacity to absorb energy when deformed *elastically* and *recover* all energy when unloaded ($=\sigma^2_{YS}/2E$).
- Approximate as area under the elastic stress-strain curve.



$$U_r = \int_0^{\varepsilon_Y} \sigma \, d\varepsilon$$

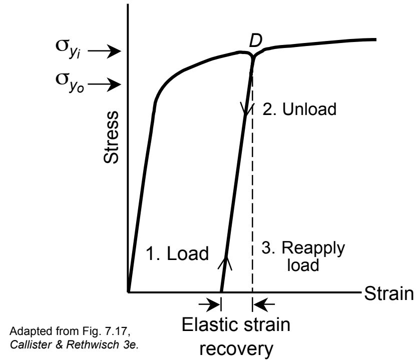
$$= \int_0^{\varepsilon_Y} E\varepsilon \, d\varepsilon \sim E\frac{\varepsilon_Y^2}{2} = \frac{\sigma_Y \varepsilon_Y}{2} = \frac{\sigma_Y^2}{2E}$$

If linear elastic

Area up to 0.2% strain

Elastic Strain Recovery

- Unloading in step 2 allows elastic strain to be recovered from bonds.
- Reloading leads to higher YS, due to work-hardening already done.

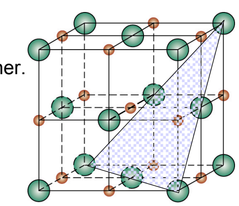


Adapted from Fig. 7.17, *Callister & Rethwisch 3e.*

Ceramics Mechanical Properties

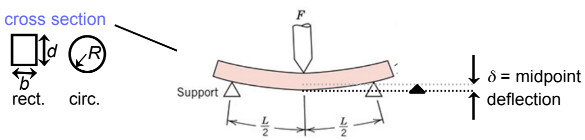
Ceramic materials are more brittle than metals.
Why?

- Consider mechanism of deformation
  - In crystalline materials, by dislocation motion
  - In highly ionic solids, dislocation motion is difficult
    - few slip systems
    - resistance to motion of ions of like charge (e.g., anions) past one another.

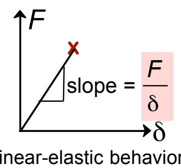


Strength of Ceramics - Elastic Modulus

- RT behavior is usually elastic with brittle failure.
- 3-point bend test employed (tensile test not best for brittle materials).



- Determine elastic modulus according to:



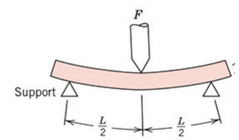
slope $= \frac{F}{\delta}$

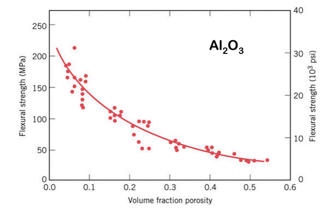
$$E = \frac{F}{\delta}\frac{L^3}{4bd^3} \quad \text{(rect. cross section)}$$

$$E = \frac{F}{\delta}\frac{L^3}{12\pi R^4} \quad \text{(circ. cross section)}$$

Strength of Ceramics - Flexural Strength

- 3-point bend test employed for RT Flexural strength.





Rectangular cross-section

$$\sigma_{fs} = \frac{3F_f L}{2bd^2}$$

Circular cross-section

$$\sigma_{fs} = \frac{8F_f L}{\pi d^3}$$

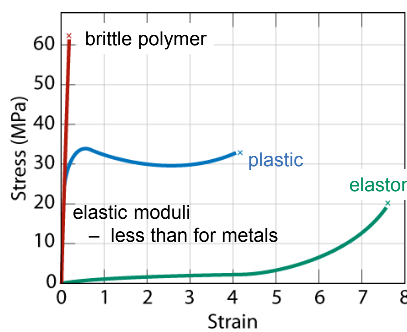
L= length between load pts
b = width
d = *height* or *diameter*

- Typical values:

Material	σ_{fs}(MPa)	E(GPa)
Si nitride	250-1000	304
Si carbide	100-820	345
Al oxide	275-700	393
glass (soda-lime)	69	69

Data from Table 7.2, *Callister & Rethwisch 3e.*

Stress-Strain in Polymers

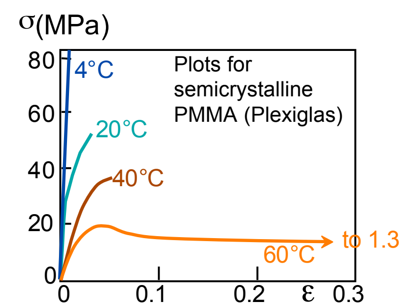


Adapted from Fig. 7.22, *Callister & Rethwisch 3e.*

- Fracture strengths of polymers ~ 10% of those for metals.
- Deformation strains for polymers > 1000%.
  - for most metals, deformation strains < 10%.

Influence of *T* and Strain Rate on Thermoplastics

- Decreasing *T*...
  - -- increases *E*
  - -- increases *TS*
  - -- decreases %*EL*
- Increasing strain rate...
  - -- same effects as decreasing *T*.

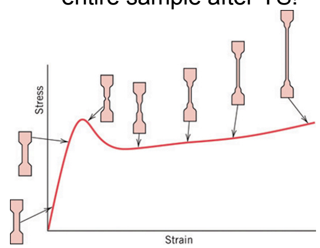


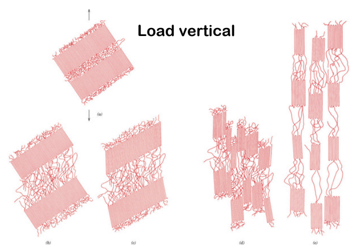
Adapted from Fig. 7.24, *Callister & Rethwisch 3e.* (Fig. 7.24 is from T.S. Carswell and J.K. Nason, 'Effect of Environmental Conditions on the Mechanical Properties of Organic Plastics", *Symposium on Plastics*, American Society for Testing and Materials, Philadelphia, PA, 1944.)

Stress-Strain in Polymers

- Necking appears along entire sample after YS!

- Mechanism unlike metals, necking due to alignment of crystallites.





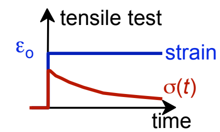
•Align crystalline sections by straightening chains in the amorphous sections

See Chpt 8

•After YS, necking proceeds by unraveling; hence, neck propagates, unlike in metals!

Time-dependent deformation in Polymers

- Stress relaxation test:
  - -strain in tension to ε_o and hold.
  - - observe decrease in stress with time.



- Relaxation modulus:

$$E_r(t) = \frac{\sigma(t)}{\varepsilon_o}$$

- Large decrease in E_r for $T > T_g$.

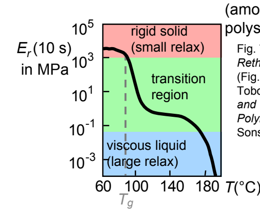


Fig. 7.28, *Callister & Rethwisch 3e*. (Fig. 7.28 from A.V. Tobolsky, *Properties and Structures of Polymers*, Wiley and Sons, Inc., 1960.)

- Representative T_g values (°C):

PE (low density)	- 110
PE (high density)	- 90
PVC	+ 87
PS	+100
PC	+150

Selected values from Table 11.3, *Callister & Rethwisch 3e*.

True Stress and Strain

Engineering stress $\sigma = \frac{F}{A_0}$ Initial area always

True stress $\sigma_t = \frac{F}{A_i}$ instantaneous area

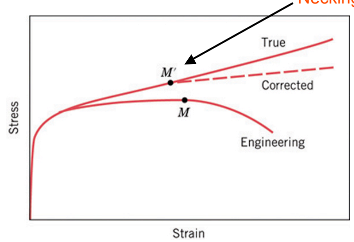
True strain $\varepsilon_t = \ln\frac{\ell_i}{\ell_0}$ Relative change

Relation before necking

$$\sigma_T = \sigma(1+\varepsilon)$$

$$\varepsilon_T = \ln(1+\varepsilon)$$

Necking: 3D state of stress!



Why use True Strain?

- Up to YS, there is volume change due to Poisson Effect!

- In a metal, from YS and TS, there is plastic deformation, as dislocations move atoms by slip, but ΔV=0 (volume is constant).

$$A_0\ell_0 = A_i\ell_i$$

$$\varepsilon_t = \ln\frac{\ell_i}{\ell_0} \longrightarrow \ln\frac{\ell_i - \ell_0 + \ell_0}{\ell_0} = \ln(1+\varepsilon)$$

Eng. Strain

Test	length	Eng. 0-1-2-3	Eng. 0-3
0	2.00		
1	2.20	0.1	
2	2.42	0.1	
3	2.662	0.1	0.662/2.0
TOTAL		0.3	0.331

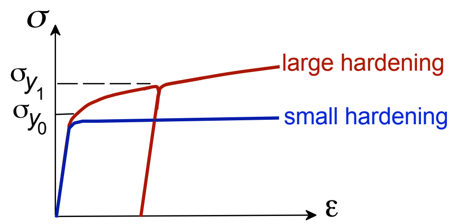
Sum of incremental strain does NOT equal total strain!

True Strain $\varepsilon_t = \ln\frac{2.2}{2.0} + \ln\frac{2.42}{2.20} + \ln\frac{2.662}{2.42} = \ln\frac{2.662}{2.00}$

Sum of incremental strain does equal total strain.

Hardening

- An increase in σ_y due to plastic deformation.



- Curve fit to the stress-strain response after YS:

$$\sigma_T = C(\varepsilon_T)^n$$

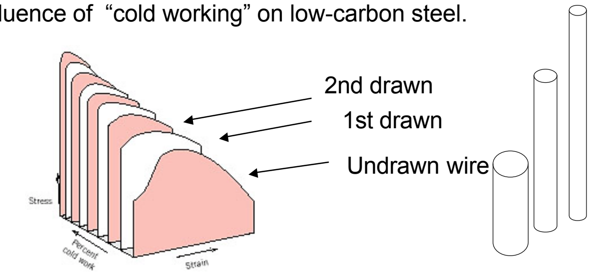
hardening exponent:
n=0.15 (some steels)
to n=0.5 (some copper)

"true" stress (F/A)

"true" strain: $\ln(L/L_o)$

Using Work-Hardening

Influence of "cold working" on low-carbon steel.

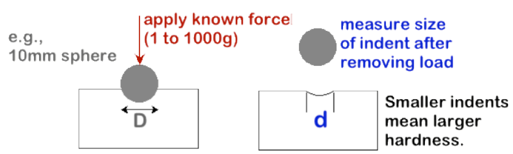


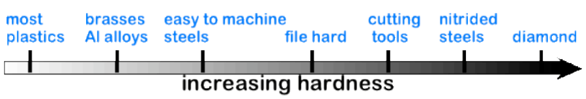
Processing: Forging, Rolling, Extrusion, Drawing,…

- Each draw of the wire *decreases* ductility, *increases* YS.
- Use drawing to strengthen and thin "aluminum" soda can.

Hardness

- Resistance to permanently indenting the surface.
- Large hardness means:
  --resistance to plastic deformation or cracking in compression.
  --better wear properties.





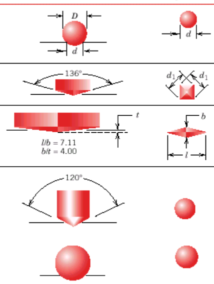
Adapted from Fig. 7.18.

Hardness: Measurement

- Rockwell
  - No major sample damage
  - Each scale runs to 130 (useful in range 20-100).
  - Minor load 10 kg
  - Major load 60 (A), 100 (B) & 150 (C) kg
    - A = diamond, B = 1/16 in. ball, C = diamond
- HB = Brinell Hardness
  - *TS* (psia) = 500 x HB
  - *TS* (MPa) = 3.45 x HB

Hardness: Measurement

Table 6.4 Hardness Testing Techniques



Test	Indenter	Shape of Indentation: Side View	Shape of Indentation: Top View	Load	Formula for Hardness Number[a]
Brinell	10-mm sphere of steel or tungsten carbide			P	$HB = \frac{2P}{\pi D[D - \sqrt{D^2 - d^2}]}$
Vickers microhardness	Diamond pyramid			P	$HV = 1.854P/d_1^2$
Knoop microhardness	Diamond pyramid			P	$HK = 14.2P/l^2$
Rockwell and Superficial Rockwell	Diamond cone; $\frac{1}{16}, \frac{1}{8}, \frac{1}{4}, \frac{1}{2}$ in. diameter steel spheres			60 kg, 100 kg, 150 kg: Rockwell; 15 kg, 30 kg, 45 kg: Superficial Rockwell	

[a] For the hardness formulas given, P (the applied load) is in kg, while D, d, d_1, and l are all in mm.

Source: Adapted from H. W. Hayden, W. G. Moffatt, and J. Wulff, *The Structure and Properties of Materials,* Vol. III, *Mechanical Behavior.* Copyright © 1965 by John Wiley & Sons, New York. Reprinted by permission of John Wiley & Sons, Inc.

Account for Variability in Material Properties

- Elastic modulus is material property
- Critical properties depend largely on sample flaws (defects, etc.). Large sample to sample variability.
- Statistics
  - Mean $$\overline{x} = \frac{\sum^{n} x_n}{n}$$
  - Standard Deviation $$s = \left[\frac{\sum^{n}\left(x_i - \overline{x}\right)^2}{n-1}\right]^{\frac{1}{2}}$$

where n is the number of data points

Design Safety Factors

- Design uncertainties mean we do not push the limit.
- Factor of safety, N (sometime given as S)

$$\sigma_{working} = \frac{\sigma_y}{N}$$

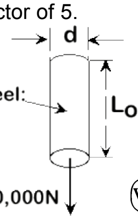
Often N is between 1.2 and 4

- Ex: Calculate diameter, d, to ensure that no yielding occurs in the 1045 carbon steel rod. Use safety factor of 5.

$$\sigma_{working} = \frac{\sigma_y}{N}$$

$$\frac{220{,}000N}{\pi\left(d^2/4\right)} \qquad 5$$

1045 plain carbon steel:
σ_y=310MPa
TS=565MPa



d = 0.067 m = 6.7 cm

Summary

- Stress and strain: These are size-independent measures of load and displacement, respectively.
- Elastic behavior: This reversible behavior often shows a linear relation between stress and strain. To minimize deformation, select a material with a large elastic modulus (E or G).
- Plastic behavior: This permanent deformation behavior occurs when the tensile (or compressive) uniaxial stress reaches sy.
- Toughness: The energy needed to break a unit volume of material.
- Ductility: The plastic strain at failure.