

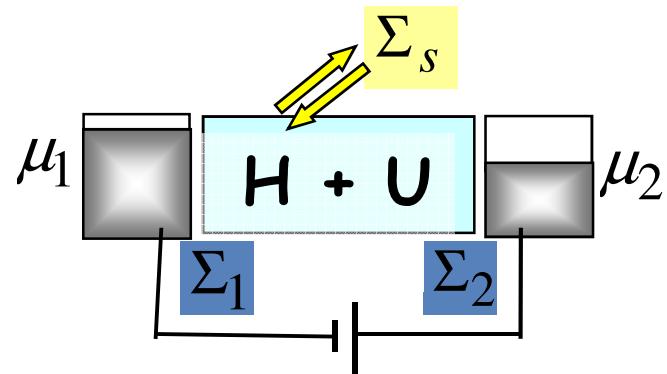
# Spin Transport

A Lecture  
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Notes prepared  
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*Angik Sarkar*

December 2008

NEGF+Landauer model



Further Reading  
*Nanoelectronics: A Unified View*  
Oxford Handbook on  
Nanoscience and Nanotechnology  
Eds. A.V.Narlikar and Y.Y.Fu,  
Volume I, Chapter 1  
<http://arxiv.org/abs/0809.4460>

## Introductory comments

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- Normally discussions of current flow and resistance starts from big and complicated systems : Top down approach
- Reason is historical: Till recently it was not clear whether resistance of small molecule even made sense.
- Experimentalists now can measure the resistance even of a hydrogen molecule
- These developments warrant a Bottom up approach that can provide a complementary (if not clearer) viewpoint.
- General Model: NEGF + Landauer

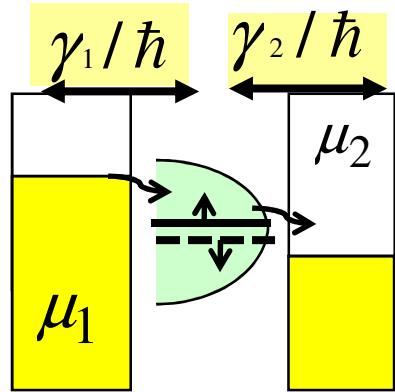
Summer school 2009  
<http://www.nanohub.org/resources/5279/>



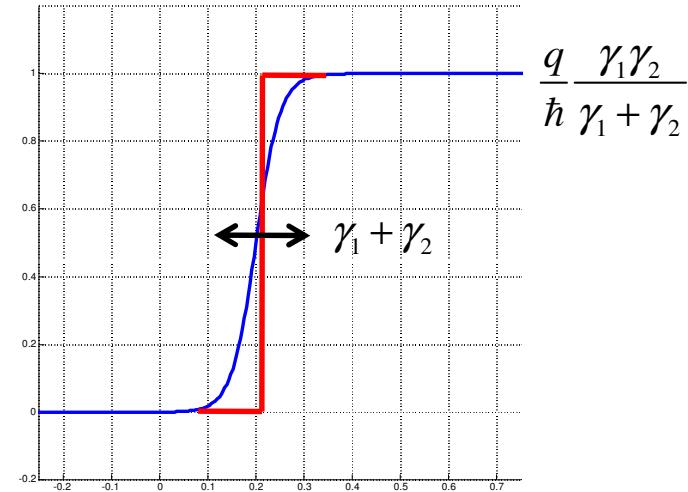
<http://www.nanohub.org/courses/cqt> datta@purdue.edu

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## A 1-level device



$\gamma_1, \gamma_2$  : how easily electrons can get in and out of the channel



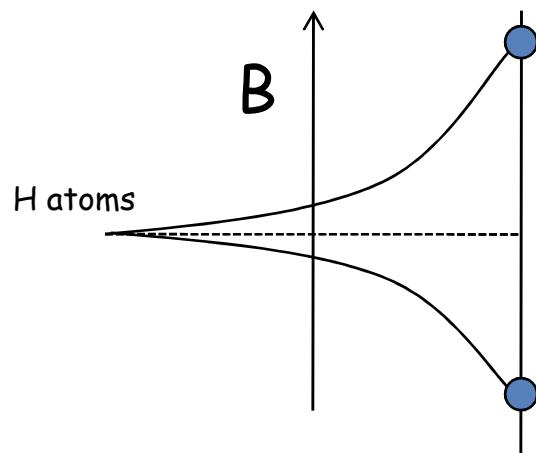
Maximum Conductance for one level:

$$G=q^2/h$$

Levels come in pairs (spins)  
Therefore, maximum conductance:  $G=2q^2/h$

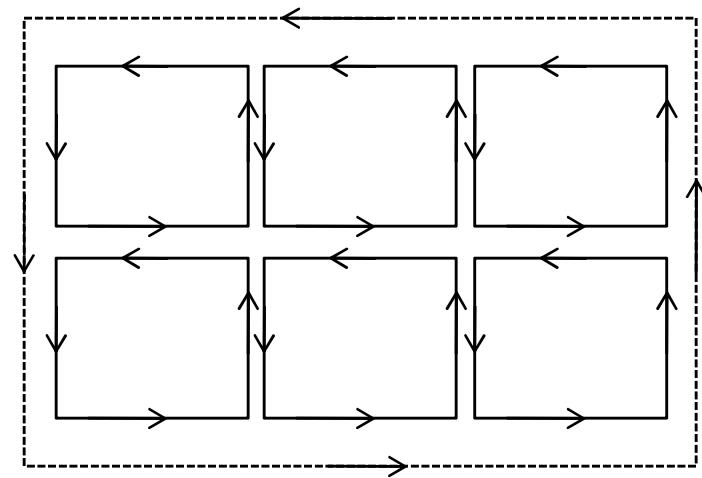
# Spin

## Stern Gerlach experiment: observation of spin

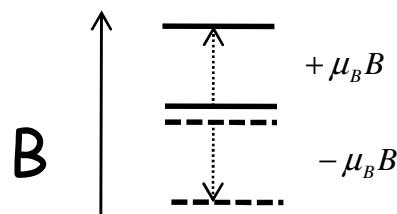


All magnetic moment arises  
due to electrons

$$\mu_B \sim 10^{-23} A \cdot m^2$$
$$= 1mA \cdot 1\text{Ang}^2$$

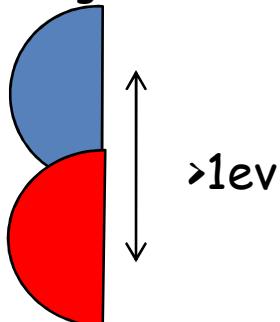


### Zeeman splitting

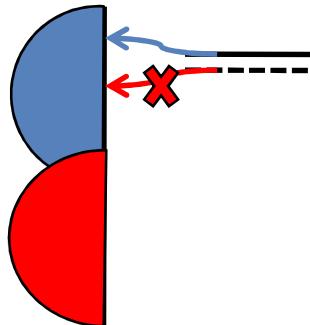


## How to make spintronic device?

Use magnets as contacts



Half metal



Rates ( $\gamma$ ) become different and can be denoted as  $\alpha, \beta$

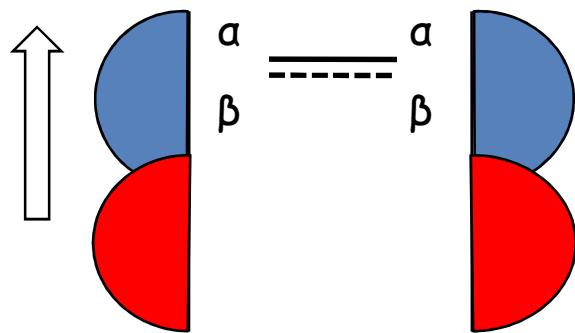
Huge amount of separation

$$\begin{aligned}\mu_B B &= 10^{-23} \times 1T \\ &= 10^{-23} J \\ &= 10^{-4} eV\end{aligned}$$

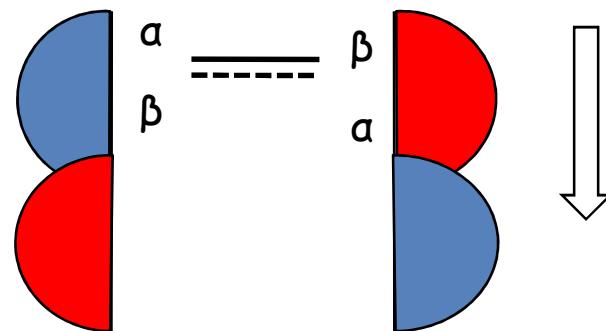
1eV would have required 10000T !!

## GMR Effect

Parallel



Anti Parallel



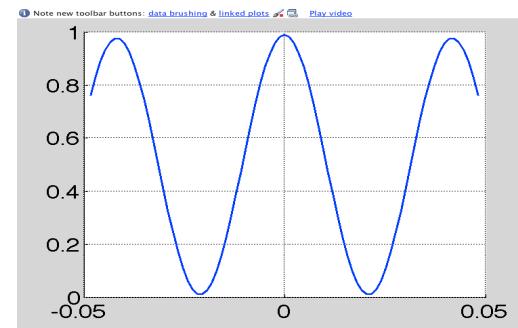
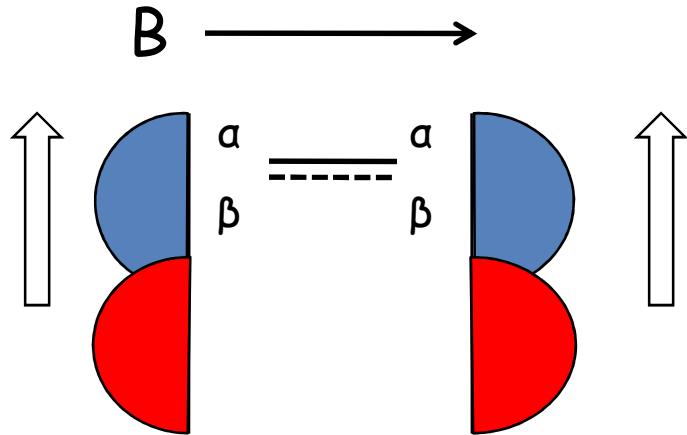
$$I_P = \frac{q}{\hbar} \left[ \frac{\alpha}{2} + \frac{\beta}{2} \right]$$

$$I_{AP} = \frac{q}{\hbar} \frac{2\alpha\beta}{\alpha + \beta}$$

$$\frac{I_P}{I_{AP}} = \frac{(\alpha + \beta)^2}{4\alpha\beta} = \frac{(\alpha + \beta)^2}{(\alpha + \beta)^2 - (\alpha - \beta)^2} > 1$$

GMR Effect

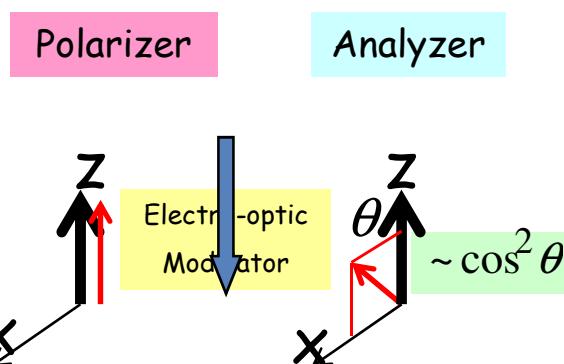
## Hanle Effect



Magnetic field causes precession of  
the spin- polarized electrons : causes oscillations

## Polarization of light : an analogy to spin

Electro optic Modulator



Difference with polarization of light:  
Min. current when rotation is  $\pi$ , not  $\pi/2$

$$\text{spinors} \begin{cases} \text{up} & \cos \theta/2 \\ \text{down} & \sin \theta/2 \end{cases} \quad \frac{1}{\sqrt{2}} \begin{pmatrix} \hat{x} \\ 1 \end{pmatrix}, \frac{1}{\sqrt{2}} \begin{pmatrix} -\hat{x} \\ 1 \end{pmatrix}$$

Vectors:

$$\begin{cases} x : \sin \theta \cos \varphi \\ y : \sin \theta \sin \varphi \\ z : \cos \theta \end{cases}$$

Spinors:

$$\begin{cases} \text{up} : \cos \theta/2 e^{-i\varphi/2} \\ \text{dn} : \sin \theta/2 e^{+i\varphi/2} \end{cases}$$

Spinors have 2 complex components

## Why does spin precess: Schrodinger equation

$$i\hbar \frac{d}{dt} \begin{pmatrix} u \\ d \end{pmatrix} = \begin{bmatrix} \mu_B B & 0 \\ 0 & -\mu_B B \end{bmatrix} \begin{pmatrix} u \\ d \end{pmatrix}$$

$$i\hbar \frac{du}{dt} = \mu_B B u$$

$$u(t) = u(0) e^{\frac{\mu_B B}{i\hbar} t}$$

$$d(t) = d(0) e^{-\frac{\mu_B B}{i\hbar} t}$$

$$\phi(t) = \frac{2\mu_B B t}{i\hbar}$$

Vector components:

$$\frac{d}{dt} S_x = -S_y \frac{2\mu_B B}{\hbar}$$

$$\frac{d}{dt} S_y = S_y \frac{2\mu_B B}{\hbar}$$

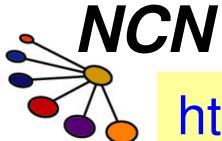
$$\frac{d}{dt} S_z = 0$$

In Matrix form:

$$\frac{d}{dt} \begin{pmatrix} S_x \\ S_y \\ S_z \end{pmatrix} = \frac{2\mu_B B_z}{\hbar} \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{pmatrix} S_x \\ S_y \\ S_z \end{pmatrix}$$

Spinor:

$$\frac{d}{dt} \begin{pmatrix} u \\ d \end{pmatrix} = \frac{\mu_B B_z}{i\hbar} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{pmatrix} u \\ d \end{pmatrix}$$



## Spin precession: contd.

For  $B$  in  $x$  direction:

$$\frac{d}{dt} \begin{pmatrix} S_x \\ S_y \\ S_z \end{pmatrix} = \frac{2\mu_B B_x}{\hbar} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \begin{pmatrix} S_x \\ S_y \\ S_z \end{pmatrix} \quad \longrightarrow \quad \frac{d}{dt} \vec{S} = -\frac{2\mu_B}{\hbar} (\vec{S} \times \vec{B})$$

In spinor components:

$$\frac{d}{dt} \begin{pmatrix} u \\ d \end{pmatrix} = \frac{\mu_B B_x}{i\hbar} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{pmatrix} u \\ d \end{pmatrix}$$

Rotation matrices for vectors:

$$L_z L_x - L_x L_z = L_y$$

For spinors

$$\sigma_z \sigma_x - \sigma_x \sigma_z = 2i\sigma_y$$

$\sigma_x, \sigma_y, \sigma_z \quad \longrightarrow \quad$  Pauli spin matrices



## Pauli spin matrices using Basis transformation

$$\sigma_z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

If we use  $+x$  and  $-x$  as basis:

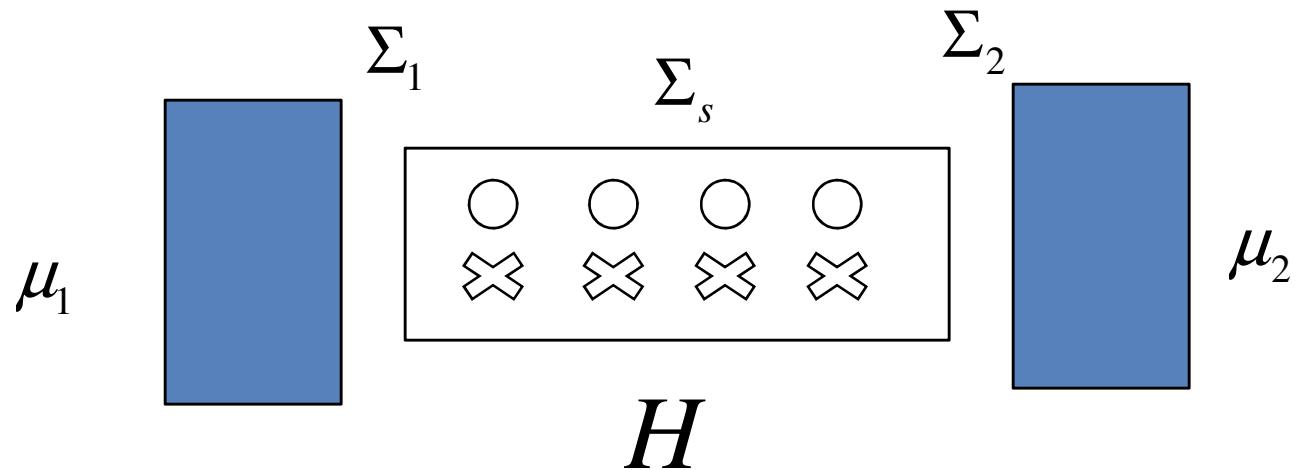
$$\sigma_x = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

We use basis transformation to get back to z basis

$$\sigma_x = \frac{1}{2} \hat{z} \begin{bmatrix} \hat{x} & -\hat{x} \\ 1 & -1 \end{bmatrix} \times \hat{x} \begin{bmatrix} \hat{x} & -\hat{x} \\ 1 & 0 \end{bmatrix} \times \hat{x} \begin{bmatrix} \hat{z} & -\hat{z} \\ 1 & 1 \end{bmatrix} = -\hat{z} \begin{bmatrix} \hat{z} & -\hat{z} \\ 0 & 1 \end{bmatrix}$$

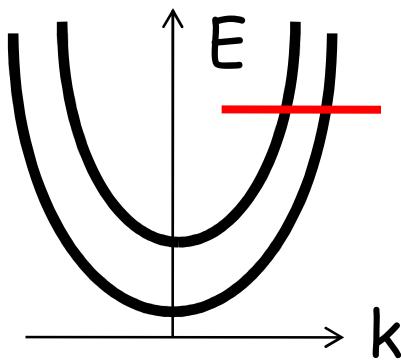
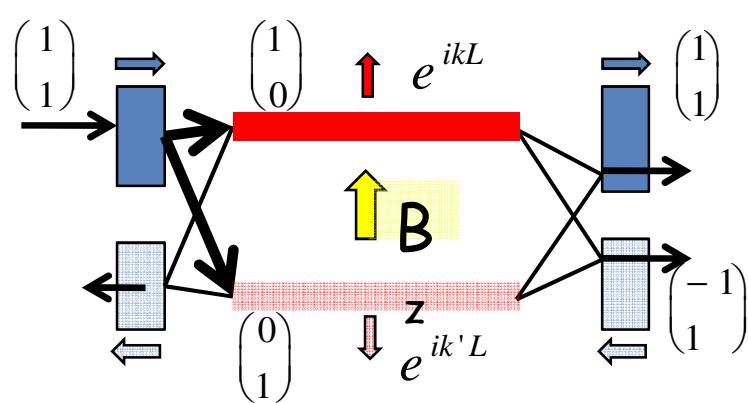


## General NEGF-Landauer Model and inclusion of spin



Size of  $[H]$  depends on basis functions used  
If we include spin, all matrices become double in size

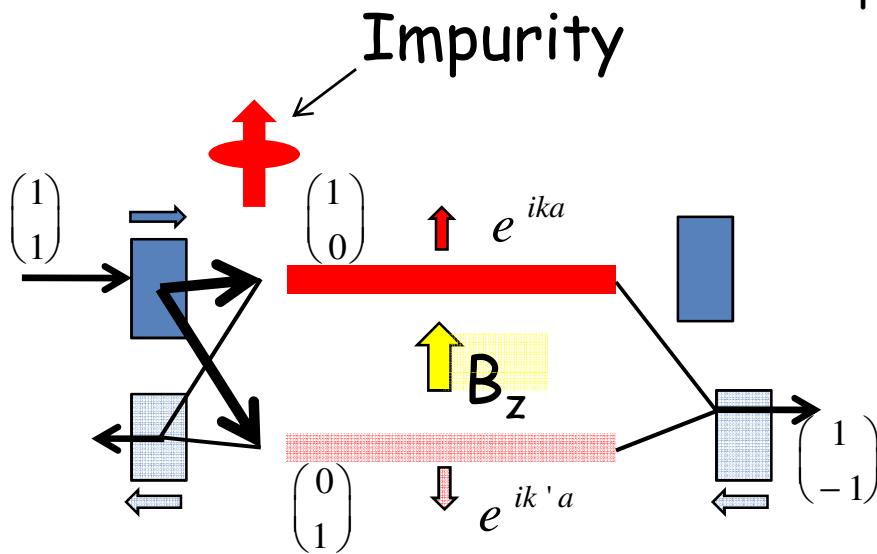
## Hanle Effect as interference effect



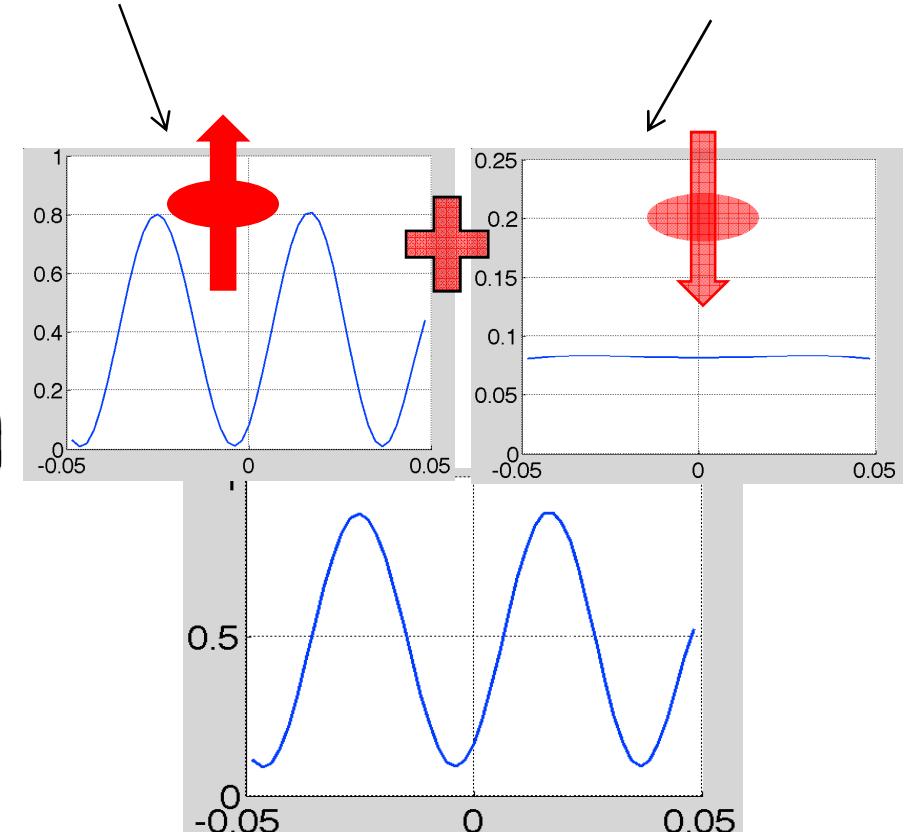
Interference due to phase difference of  $(k-k')L$

Interference effect is destroyed by differential scattering

## Spin interference



Impurity does not flip



Impurity flips

# Spin Torque

A thin magnet can be flipped by spins

How ? A simple view

The magnet exerts a torque on the spin (d electrons  $\rightarrow$  s electrons)

The conduction electrons apply an equal and opposite torque on d electrons (s $\rightarrow$ d)

$$\vec{\mu} \times \vec{B}_{eff} = -\vec{m} \times \vec{B}$$

Diagram illustrating the Spin Torque effect:

- Left side:  $\vec{\mu} \times \vec{B}_{eff}$  (indicated by a double-headed arrow between  $\vec{\mu}$  and  $\vec{B}_{eff}$ )
- Middle:  $= -\vec{m} \times \vec{B}$  (indicated by a double-headed arrow between  $-\vec{m}$  and  $\vec{B}$ )
- Labels:
  - $s$  (spin)
  - $d \rightarrow s$  (transition from d to s electrons)
  - $d$  (d electrons)
  - $s \rightarrow d$  (transition from s to d electrons)
  - $10^{-6} \text{ spins}$
  - $10^4 T$
  - $1 \text{ spin}$
  - $0.01T$

Spin torque effect  
gives rise to  
possibility of using  
magnets as "Spin  
capacitors"

Two distinct fields  
Spintronics and Magnetoelectronics  
are merging into a single field  
with interesting possibilities



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