

# ECE 659 Quantum Transport: Atom to Transistor

Lecture 3: Mobility

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Spring 2009

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$$I = \frac{q}{h} \int dE \bar{T}(E) (f_1 - f_2)$$

Ballistic Transport:

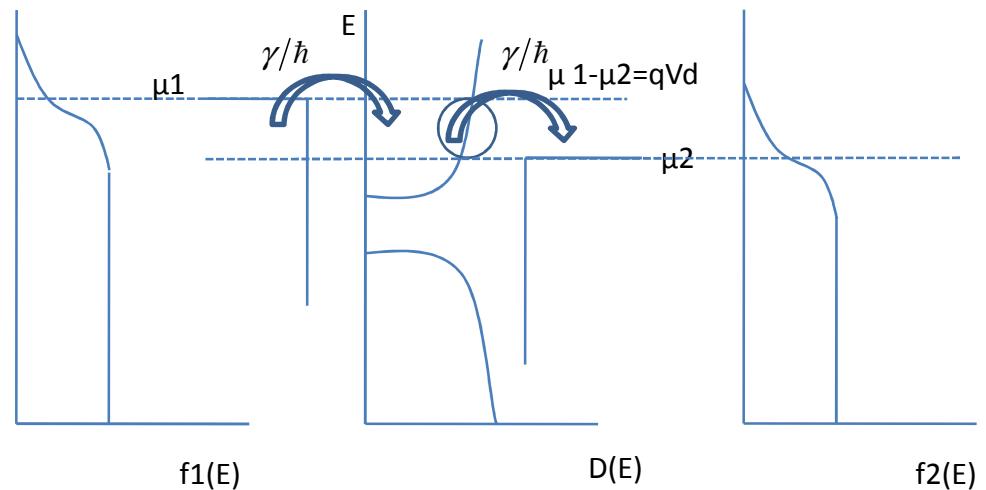
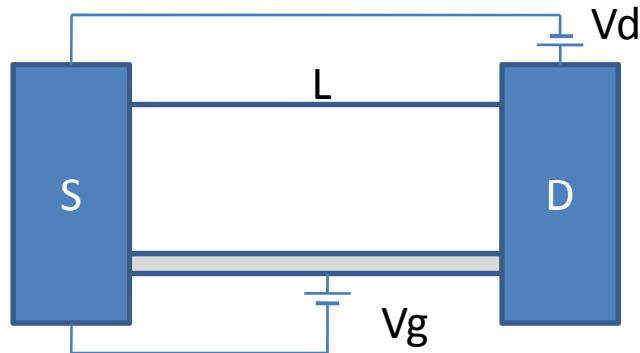
$$\begin{array}{c} f_1 \xrightarrow{\frac{D}{2} f_1} \\ f_2 \xleftarrow{\frac{D}{2} f_2} \end{array}$$

$$I_{L \rightarrow R} = q \frac{D}{2L} f_1 v_x$$

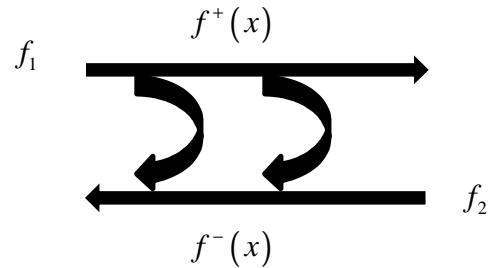
$$\bar{T}(E) = \frac{hD}{2L} v_x$$

$$M(E) = \frac{\pi \hbar D v_x}{L}$$

$$I = \frac{q}{h} \int dE M(E) (f_1 - f_2)$$

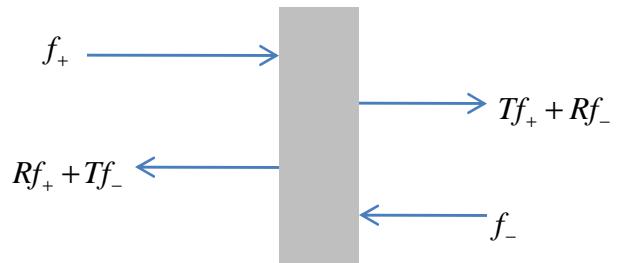


Diffusive Transport:



$$I = \frac{q}{h} \int dE M(E) (f^+ - f^-)$$

We shall see :  $(f^+ - f^-) = (f_1 - f_2) \frac{\lambda}{\lambda + L}$



$$\Delta f^+ = (T - 1) f^+ + R f^-$$

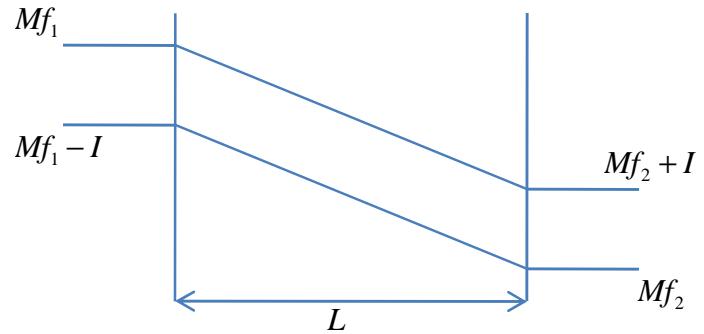
$$\Delta f^+ = -R(f^+ - f^-)$$

$$\Delta f^- = -R(f^+ - f^-)$$

$$\Delta f^+ = \Delta f^- = -\frac{\Delta x}{\lambda} (f^+ - f^-)$$

$$\frac{df^+}{dx} = \frac{df^-}{dx} = -\frac{(f^+ - f^-)}{\lambda}$$

Boundary Conditions:  $f^+(0) = f_1, f^-(L) = f_2$



$$(f^+ - f^-) \sim I$$

$$I \left( \frac{1}{\lambda} + \frac{1}{L} \right) = \frac{M(f_1 - f_2)}{L}$$

$$\frac{I}{M} = (f_1 - f_2) \frac{\lambda}{\lambda + L}$$

How do we obtain Conductance?

$$\frac{I}{V} = G = \sigma \frac{A}{L + \lambda}$$

We do a Taylor's series expansion of  $f_1 - f_2$ , assuming low biases

$$f(E) = \frac{1}{e^{(E-\mu)/kT} + 1}$$

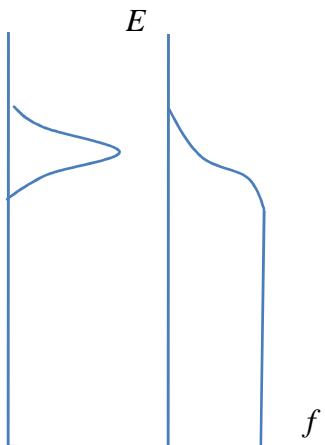
$$f_1 - f_2 = \frac{\partial f}{\partial \mu} (\mu_1 - \mu_2)$$

$$\frac{\partial f}{\partial \mu} \rightarrow -\frac{\partial f}{\partial E}$$

$$f_1 - f_2 = -\frac{\partial f}{\partial E} qV$$

$$\frac{I}{V} = \frac{q^2}{h} \int dE \left( -\frac{\partial f}{\partial E} \right) \bar{T}(E)$$

$$\frac{I}{V} = q^2 \int dE \left( -\frac{\partial f}{\partial E} \right) \frac{D v_x \lambda}{2L} \frac{1}{L + \lambda}$$



$$G = \underbrace{q^2 \int dE \left( -\frac{\partial f}{\partial E} \right)}_{\sigma} \frac{D}{AL} \frac{v_x \lambda}{2} \frac{A}{L + \lambda}$$

$$G = \sigma \frac{A}{L + \lambda} = \sigma' \frac{A}{L}$$

$$\sigma' = \sigma \frac{L}{L + \lambda}$$

$$\sigma = qn\mu_n$$

$$\mu_n = \frac{\sigma}{qn} = \frac{q\tau}{m}$$

$$\sigma = q^2 \int dE \left( -\frac{\partial f}{\partial E} \right) \frac{D}{AL} \frac{v_x \lambda}{2}$$

$$n = \int dE D(E) f(E)$$

We can solve for  $\sigma$  by taking  $D(E) \rightarrow 0$  far below  $\mu$

$$\sigma = q^2 \int dE f(E) \frac{d}{dE} \left( \frac{D}{AL} \frac{v_x \lambda}{2} \right)$$