

ECE 495N

Fundamentals of Nanoelectronics

Fall 2008

**Instructor: Supriyo Datta
Purdue University**

Lecture: 4

**Title: Quantitative Model for Nanodevices I
Date: September 3, 2008**

**Video Lectures posted at:
<https://www.nanohub.org/resources/5346/>**

**Class notes taken by: Panagopoulos Georgios
Purdue University**

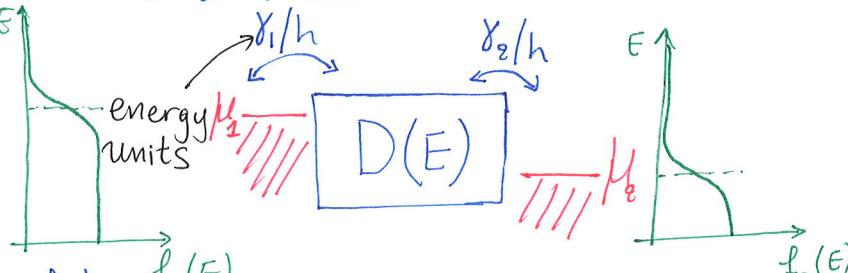


Quantitative Model for Nanodevices I

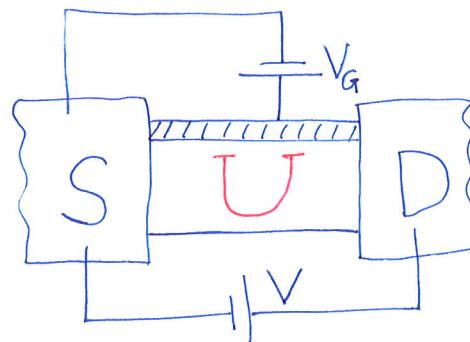
Lecture 4

Sept. 03, 2008

The basic quantities that we want to know are:



Now we will derive the basic equations for the general case (for example $\gamma_1 \neq \gamma_2$)



$$R = \frac{V}{I}$$

The previous week we wrote:

$$I = \frac{q}{h} dE \cdot D(E) \frac{\gamma}{2} \sim \begin{bmatrix} \text{i.e. 1 ps to get in} \\ \text{and 1 ps to get out} \end{bmatrix}$$

BUT the correct expression is

$$I = \int \frac{q}{h} dE \cdot D(E) \frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2} (f_1 - f_2) \quad \text{in which}$$

$$\left[\text{if } \gamma_1 = \gamma_2 = \gamma \Rightarrow \frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2} = \frac{\gamma^2}{2\gamma} = \frac{\gamma}{2} \right]$$

$$I = \int \frac{q}{h} dE \cdot D(E-U) \frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2} (f_1(E) - f_2(E)) \quad (\text{I})$$

$$U = U_L + U_0 (N - N_0) \quad (\text{II})$$

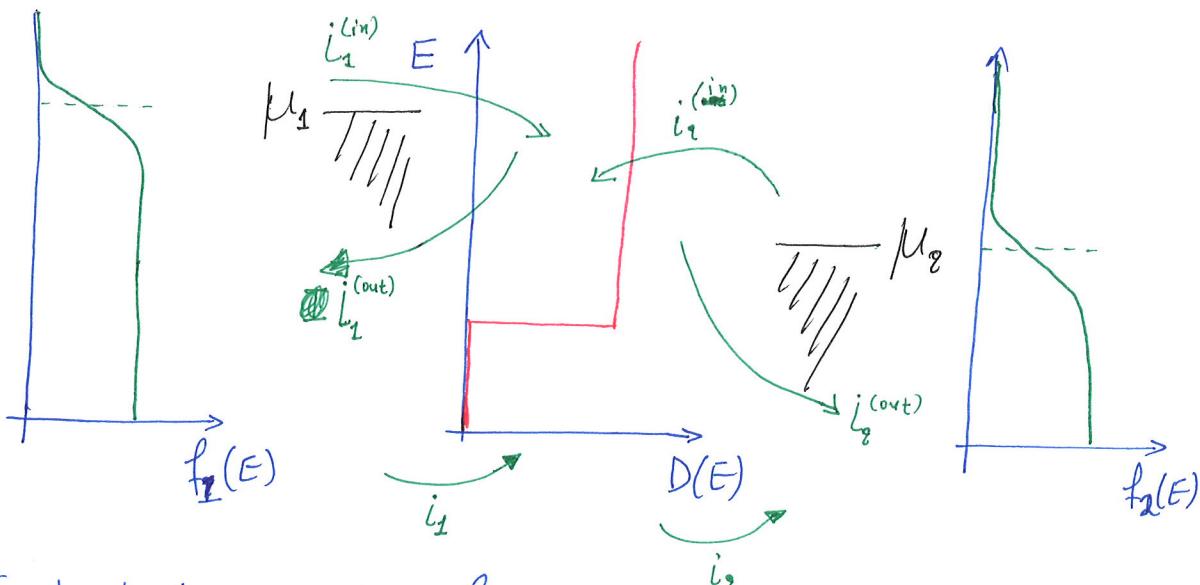
↑ reflects the change of
the potential due to electrons

$$N = \int dE \cdot N(E) = \int dE \cdot D(E-U) \frac{\gamma_1 f_1 + \gamma_2 f_2}{\gamma_1 + \gamma_2} \quad (\text{III})$$

How to solve these equations?

We will solve these equations self-consistently, that is:

- assume or U (guess)
- find N , using III
- find a new U , using II
- do step (ii) until converge
- using (I) find current



Contact 1 occupation: f_1

Contact 2 occupation: f_2

Channel occupation: we do not know, suppose f , which is between f_1 and f_2

$$i_1^{(out)} = q \frac{\gamma_1}{\hbar} dE \cdot D(E) \cdot (f)$$

like lifetime number of states
 per unit time number of electrons

If $f_1 = f$ there is no current (we are in equilibrium)

So the current gets in is

$$i_1^{(in)} = q \frac{\gamma_1}{\hbar} dE \cdot D(E) (f_1), \text{ so that there difference to be zero}$$

if $f_1 = f$

Hence $i_1 = i_1^{(in)} - i_1^{(out)} = q \frac{\gamma_1}{\hbar} dE D(E) (f_1 - f)$

with the same way:

$i_2 = i_2^{(out)} - i_2^{(in)} = q \frac{\gamma_2}{\hbar} dE D(E) (f - f_2)$

and when we equate i_1 with the i_2 we have:

$$i_1 = i_2 \Rightarrow \gamma_1 (f_1 - f) = \gamma_2 (f - f_2) \Rightarrow (\gamma_1 + \gamma_2) f = \gamma_1 f_1 + \gamma_2 f_2$$

$\Rightarrow f = \frac{\gamma_1 f_1 + \gamma_2 f_2}{\gamma_1 + \gamma_2}$ ← weighted average determined (with weights) the rates γ_1 and γ_2 .

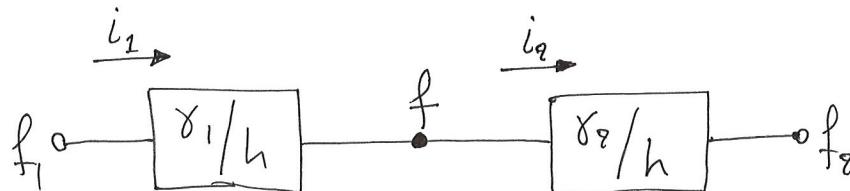
How we will calculate the expression for current?

$I = i_1 = i_2$ and substituting "f" to either i_1 or i_2 we have:

$$I = i_1 = q \frac{\gamma_1}{\hbar} dE \cdot D(E) \left(f_1 - \frac{\gamma_1 f_1 + \gamma_2 f_2}{\gamma_1 + \gamma_2} \right)$$

$$\Rightarrow I = \frac{q}{\hbar} dE \cdot D(E) \frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2} (f_1 - f_2)$$

And finally if we integrate over all energies we are taking the formula we written in the beginning.



$$I = \frac{q}{\hbar} \int dE \cdot D(E-U) \frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2} (f_1(E) - f_2(E))$$

$$N = \int dE \cdot D(E-U) \frac{\gamma_1 f_1(E) + \gamma_2 f_2(E)}{\gamma_1 + \gamma_2}$$

$$\Rightarrow N = \int_{-\infty}^{+\infty} dE \cdot D(E) \frac{\gamma_1 f_1(E-U) + \gamma_2 f_2(E-U)}{\gamma_1 + \gamma_2}$$

Usually we don't have analytic expressions for DOS like $D(E) = \frac{1}{(E-a)^2 + b^2}$, so it's preferable to move the fermi functions instead the $D(E)$.

$$E' = E - U$$

$$dE' = dE$$

We also need N_{eq} . In the above formula for N we are setting $f_1 = f_2 = f_{eq}$ then $N = \int dE \cdot D(E) \cdot f_{eq}$.