Fundamentals of Nanoelectronics

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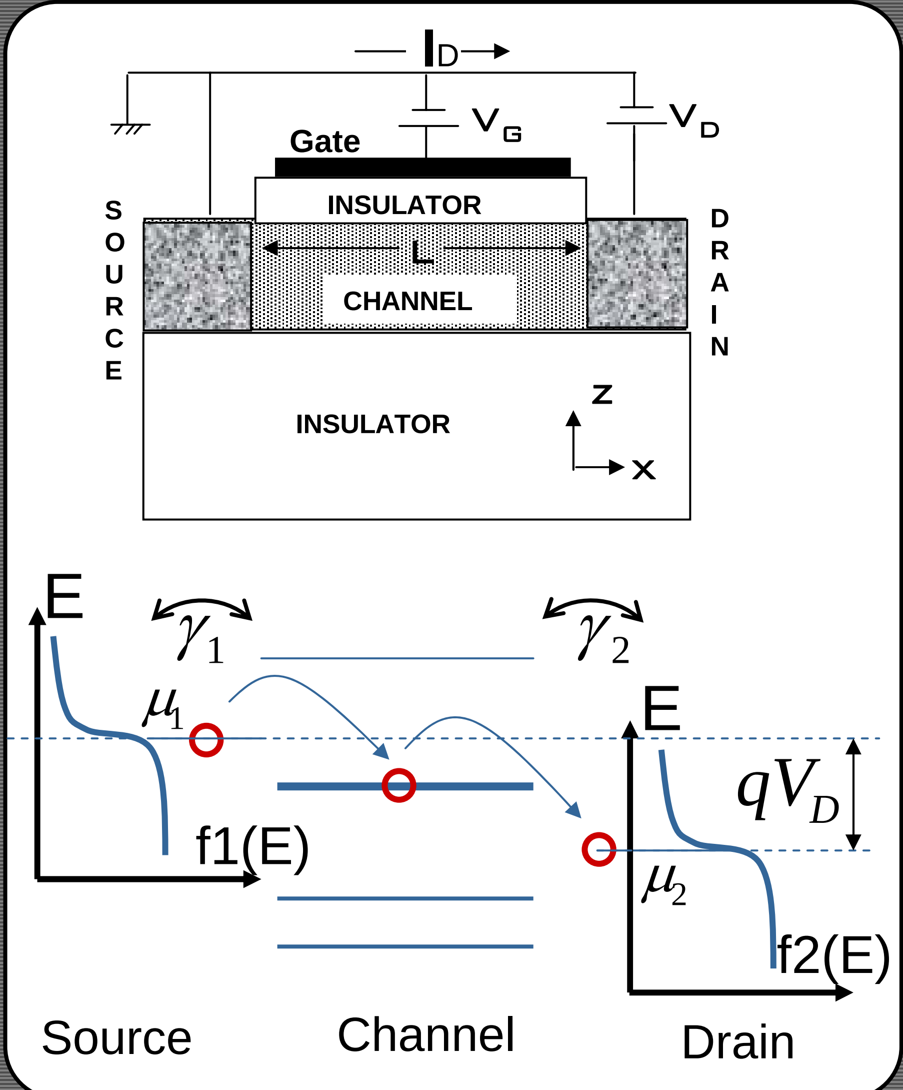
Lecture 6: Charging Effect, Towards Ohm's Law

Ref. Chapter 1.4,1.5 & 1.6

Network for Computational Nanotechnology

NCN nanoHUB online simulations and more

Recap/Self Consistent Solution



- Review of the equations for a small transistor having a broadened level between the chemical potentials of source and drain.
- The equations for current and number of electrons are:

$$I = -\frac{q}{\hbar}\int dE D(E-U)\frac{\gamma_1\gamma_2}{\gamma_1+\gamma_2}(f_1 - f_2)$$

$$N = \int dE D(E-U)\frac{\gamma_1 f_1 + \gamma_2 f_2}{\gamma_1+\gamma_2}$$

- But N has to be solved self consistently with the potential U:

$$N = \int dE D(E-U)\frac{\gamma_1 f_1 + \gamma_2 f_2}{\gamma_1+\gamma_2}$$

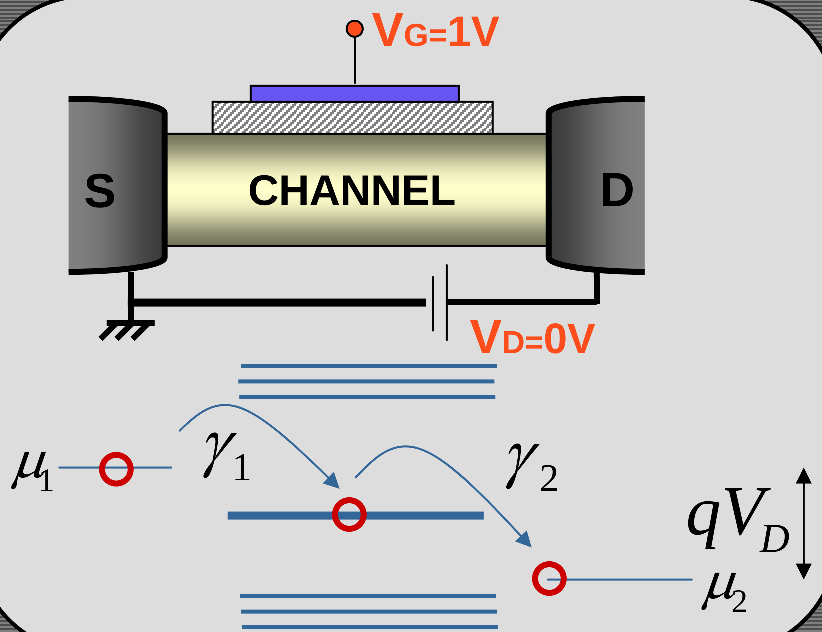
$$U = U_L + U_0(N - N_0)$$

Self Consistent Solution (SCF)

03:24

$$U = U_L + U_0(N - N_0)$$

• The Laplace potential UL is an average potential in the channel due to source, gate and drain electrodes. There is also a second term that depends on charging. To understand what the second term does consider an example



Suppose that the gate is close enough to the channel so that the potential in the channel would be close to U=1V. Is this really right?

• If the potential is 1V, then all levels in the channel move down but as they move down more levels go below Fermi level (μ1) and they get filled, as the number of electrons increases, this increase in the negative charge shifts the levels up. As a result the potential may not be 1V and it is less like 0.1V. This is why we need to solve for U and N self consistently.

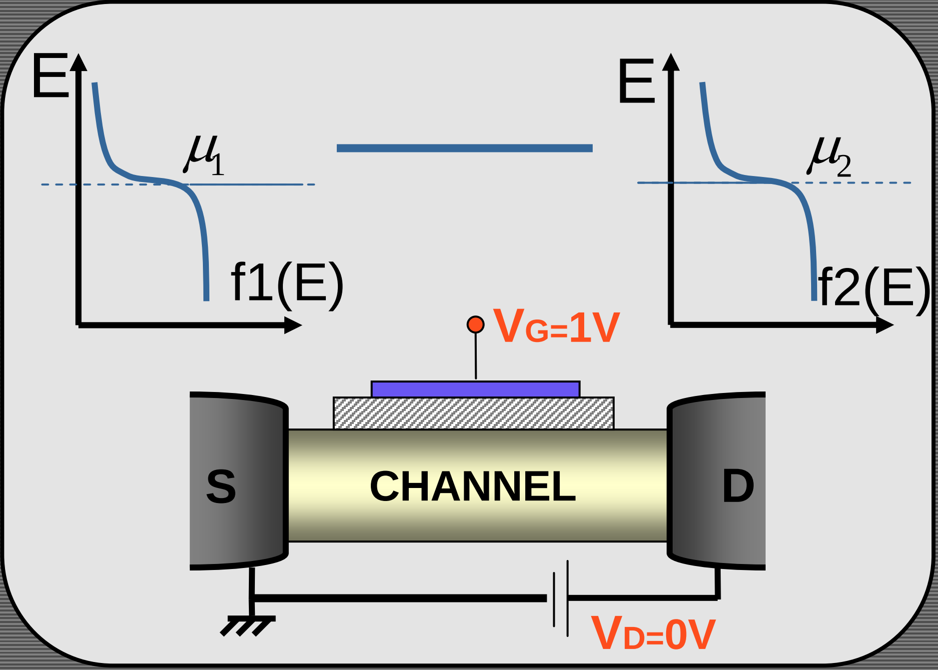
$$N = \int dE D(E - U) \frac{\gamma_1 f_1 + \gamma_2 f_2}{\gamma_1 + \gamma_2}$$

$$U = U_L + U_0(N - N_0)$$

Overview

- In the next few weeks, we'll understand where the energy levels are for a given material. We'll be able to derive density of states (DOS). What we've been doing thus far has been to calculate current and average number of electrons in the channel given that we knew what the functional form of DOS was.
- Today how ever, we'll talk about single electron charging effect. This is something that can arise in small structures under certain conditions. Under such situations, what we've done thus far will not capture the right physics. One has to consider what's called Coulomb Blockade regime of operation. At this stage, this would be a diversion for us and we'll not get into it. We want to talk about single electron charging effect…

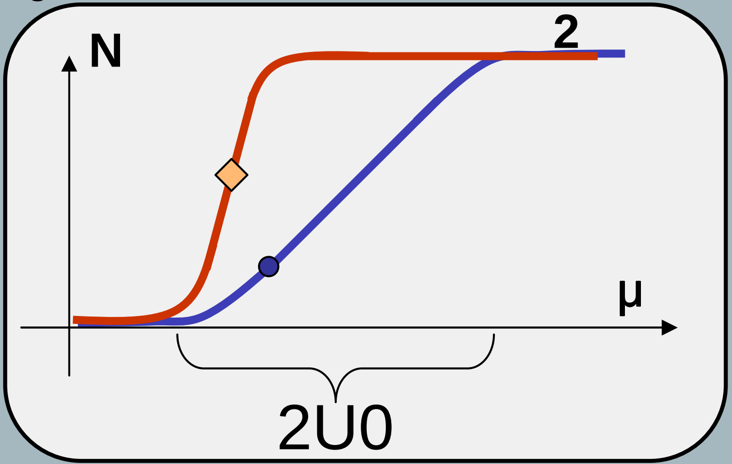
- Suppose we have device at equilibrium with only one level:



- As the gate voltage is increased this level slides down and gets filled after it passes the Fermi level.

Single Electron Charging 1

- As the gate voltage is increased the level slides down, but since the Fermi level is fixed, we can hold the level constant and raise the Fermi level. The two approaches are equivalent. The question is how does the number of electrons inside the channel change as the Fermi level is raised?



- As the Fermi level is increased the level gets filled and we'll have 2 electrons in it because the levels is composed of two degenerate levels for up spin and down spin electrons. This is the red curve.

- If the effect of single electron charging is taken into account, we'll find that the dependence between N and μ is more like the blue curve. This means that μ has to be raised quit a bit before N reaches 2. Why? The reason is that as the level is filled, it floats up and Fermi level has to be raised more to fill it up. The amount by which the level floats up depends on how big U0 is.

$$U = U_L + U_0(N - N_0)$$

- It is important how big U0 is. Its is called single electron charging energy and it tells us how much channel's potential changes when one electron is added to it.

Order of Magnitude Estimation

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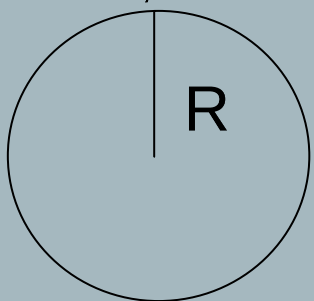
- To get an order of magnitude estimation for U0, consider a sphere of charge.



$$U = \frac{q^2}{4\pi\varepsilon R}$$

$$U = \frac{1.6\times10^{-19}\,coul}{4\times3.14\times8.85\times10^{-12}\,\frac{F}{m}\times10^{-7}\,m}$$

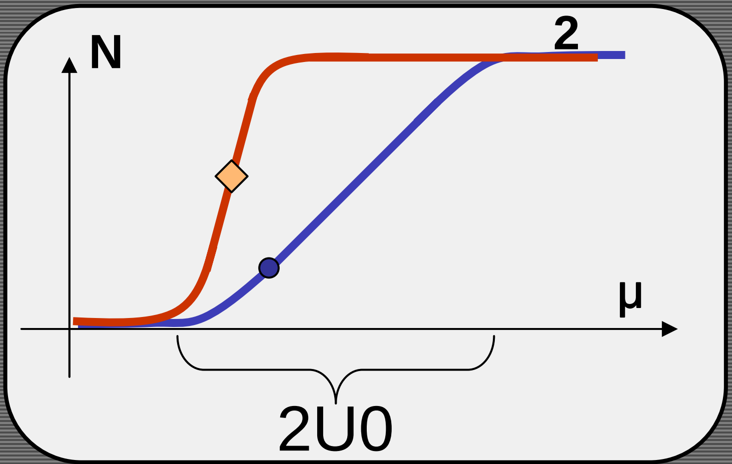
$$U \approx 14meV$$

- This is in the order of kT which is 25 meV at room temperature.
- R was taken to be 1000 Å. If larger, then U0 will even be smaller. This means that anytime an electron is added to the channel the amount of potential doesn't change much.

- We can see that for small devices, U0 will be larger and can affect the physical picture.

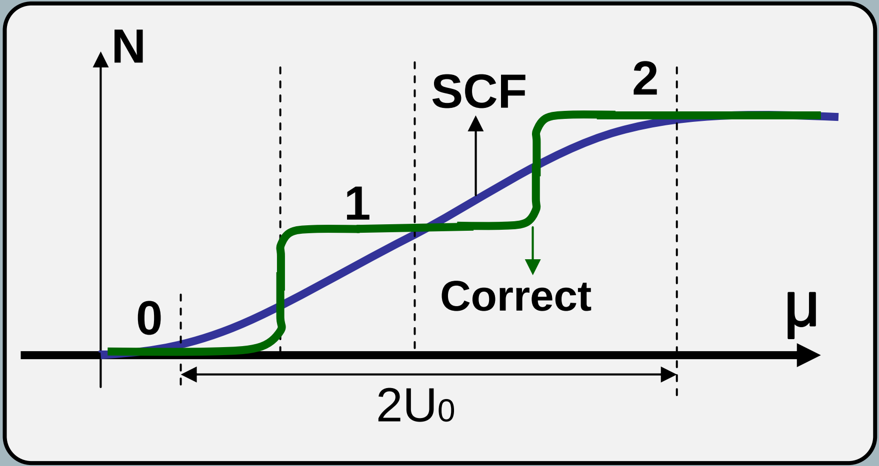
$$U = U_L + U_0(N - N_0)$$

- In big devices U0 is smaller but N-N0 is in millions of electrons so the second term for U cannot be neglected.

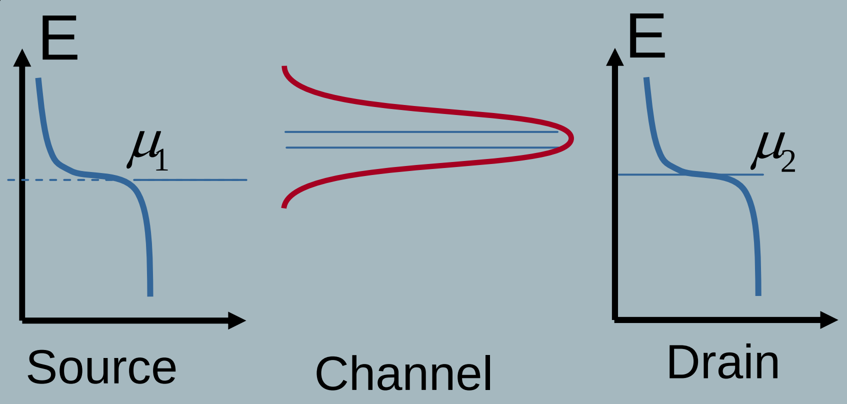


Single Electron Charging Effect

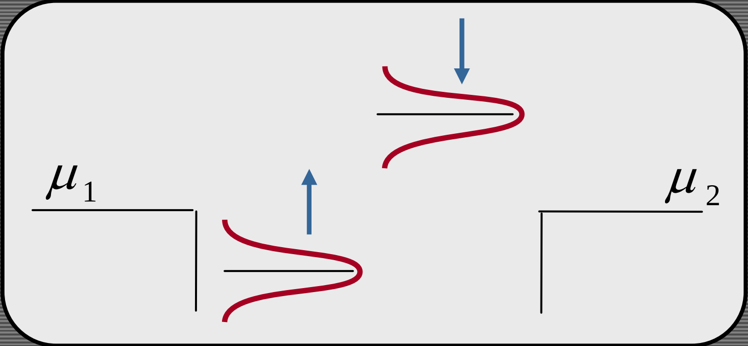
- So what is the single electron charging effect?



- The right curve is what people observe in experiments and that is the green curve. The blue curve is what we would get based on the equations we've seen thus far. But how can we understand the green curve?
- Consider the two levels of the next figure. While an electron in a level doesn't feel a potential due to itself, it does feel a potential due an electron in the other level.



- What happens as the result is that as one of the levels gets filled, the other one floats up because it feels a potential due the electron in the other level.



Coulomb Blockade, Self Consistent Field

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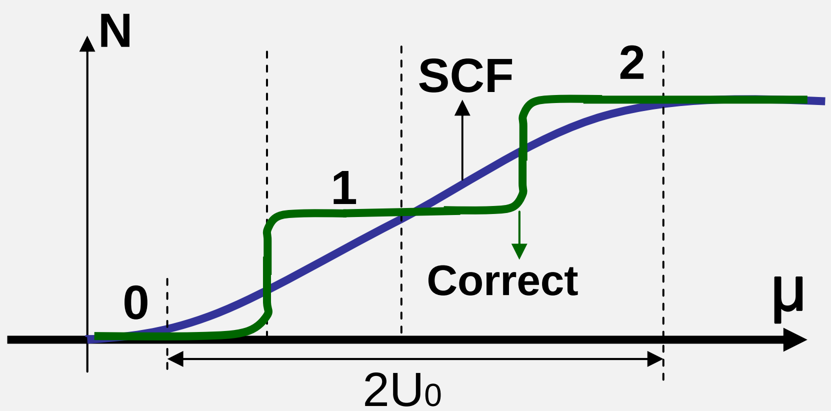
- When does U0 become important?



- The single electron charging effect is important whenever

$$U_0 >> k_B T + \gamma$$

- If this is not the case then the broadening in the green curve will smear it to look like the blue curve and we can use the Self Consistent Solution (SCF)…

- $$U_0 << k_B T + \gamma \Rightarrow$$

$$N = \int dE D(E-U) \frac{\gamma_1 f_1 + \gamma_2 f_2}{\gamma_1 + \gamma_2}$$

$$U = U_L + U_0 (N - N_0)$$

- In the event of

$$U_0 >> k_B T + \gamma$$

We have to use Coulomb Blockade regime of operation to get the right answers. We'll talk more about CB later on.

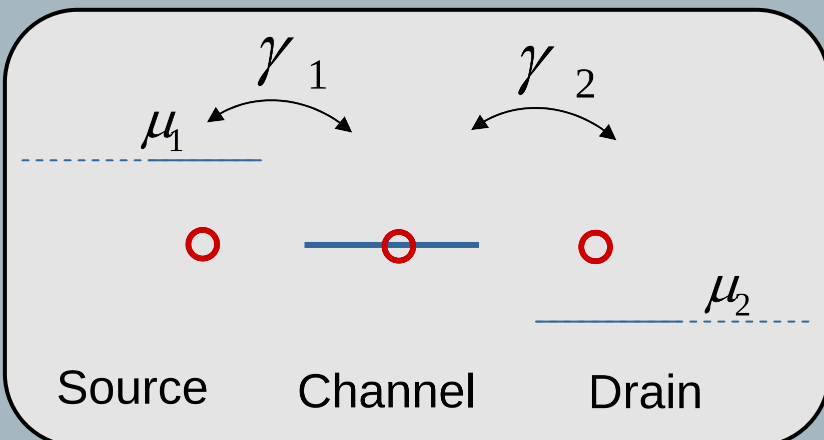
Where is the heat dissipation?

35:00

• A comment regarding the dissipated heat:

• For small devices with very short channels, the electron current does not give rise to any dissipation in the channel. In 1980's people thought that if there is no dissipation there would be no resistance.

$$p = I^2 R$$



• But there is a measured resistance for small devices and its value is 12.9kOhms. So what happens to the dissipated power?

• That heating happens in the contacts. The way to think about this is that as an electron from gets in from source, it leaves an empty level in the contact at the energy of the level in the contact. Other electrons in the source loose to fill up this level. As the result of this action there is heat dissipated. On the other hand as the electron gets in the drain looses its energy and slides down to the levels below the Fermi level. This action also dissipates heat.

• The point is that all the heating associated with $I^2 R$ happens in the contacts. So the resistance is coming from the contacts.

Conductance/ Towards Ohm's Law

41:21

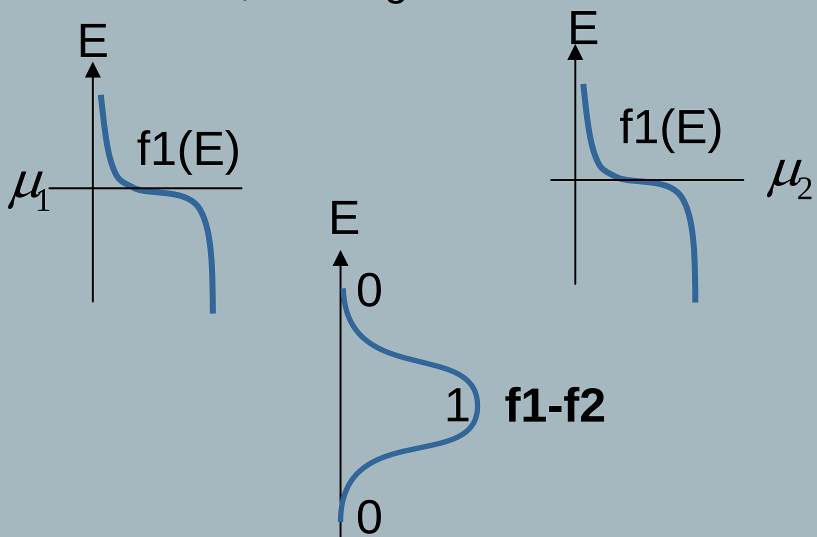
- Let's calculate the conductance using the equation:

$$I = -\frac{q}{\hbar}\int dED(E-U)\frac{\gamma_1\gamma_2}{\gamma_1+\gamma_2}(f_1 - f_2)$$

- For a small applied voltage we can write:

$$I = -\frac{q}{\hbar}D(E)\frac{\gamma_1\gamma_2}{\gamma_1+\gamma_2}qV_D$$

- To see this, investigate:



- Another question one could ask is about the conductance as the device gets bigger.
- What is the relationship between the conductance of the device and its width and length?
- Ohm's law tells us that as the device gets longer the conductance should decrease and as it gets wider, the conductance should increase:

$$G \propto \frac{A}{L}$$

- What if we use the theory we've learned? Will the conductance change in the same way as described by the ohm's law?
- What we'll show is that the proportionality to W is obtainable but the conductance's relation to the length L needs more work.
- Let's explain all these…

Towards Ohm's Law

$$I = -\frac{q}{\hbar} D(E) \frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2} qV_D$$

- We know that more states will give us more current.
- It is also reasonable to think that larger devices have more states. For instance if you have a 2 dimensional solid, then you'd expect that D would increase as the area increases: $D \propto WL$ Or in the case of a 3-D solid we'd have: $D \propto AL$
- Wait a minute. We've just obtained something that contradicts Ohm's law. So what's wrong?
- We can easily explain why the L goes away but for the inverse proportionality of conductance and length we cannot yet describe it with the theory we have learned thus far.
- So how does the L go away?
- This is explained on the next page…

Towards Ohm's Law (Continued)

- As you make the device bigger and bigger, what happens is that γ decreases as 1/L. Why?
- Answer: Gamma represents the escape rate of the electron in the channel. The point is that the bigger the box, the lower is the chance of electron hitting the wall and getting out of the channel.
- Looking back at the equation or the current we have:

$$I = -\frac{q}{\hbar}\int dE D_{\varepsilon}(E-U)\frac{\gamma_1\gamma_2}{\gamma_1+\gamma_2}(f_1-f_2)$$

$$D \propto WL \quad \text{and} \quad \frac{\gamma_1\gamma_2}{\gamma_1+\gamma_2} \propto 1/L$$

- When multiplied, L cancels out.
- So far our theory gives us a conductance that is independent of length L which is true for a ballistic device. Ballistic device is one for which electron transport does not have to confront scattering which is what makes electron flow more difficult and causes resistance. So the resistance that is observed is actually the contact resistance. It does not matter how long the channel is.