

# ECE 659 Quantum Transport: Atom to Transistor

Lecture 10: Two probe/Four probe

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Spring 2009

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$$\{i^-\} = [S]\{i^+\}$$

$$i^+ = \frac{q^2}{h} M \frac{\mu^+}{q}$$

$$\{i^-\} = \underbrace{[S][M]}_{\bar{S}} \{i^+\}$$

For 2 terminal :

$$\{i^-\} = [S]\{i^+\}$$

$$\{i\} = \{i^+\} - \{i^-\}$$

$$= [I - S]\{i^+\}$$

$$= [I - S]M\{\mu^+\}$$

$$= [M - \bar{S}]\{\mu^+\}$$

$$Y = [I - S][M]$$

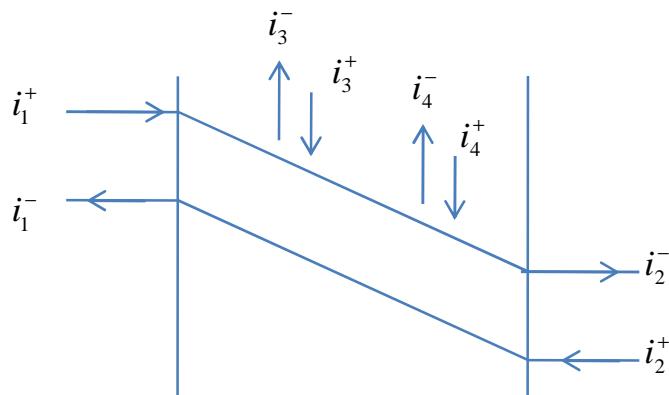
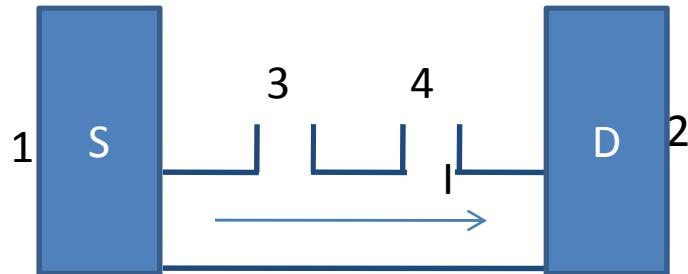
$$= M - \bar{S}$$

$$\frac{i^+ + i^-}{2} = \frac{1}{2}[I + S]i^+$$

$$i^+ = 2[I + S]^{-1} \frac{i^+ + i^-}{2}$$

a=(1,2)

b=(3,4)



$$i^+ - i^- = \underbrace{2[I - S][I + S]^{-1} M}_{\text{conductance of channel}} \frac{\mu^+ + \mu^-}{2}$$

$$Y_{channel} = [M - \bar{S}]M^{-1}[I + S]^{-1}M$$

$$= [M - \bar{S}][M + \bar{S}]^{-1}M$$

Four probe :

$$i_a^- = Ai_a^+ + Ci_b^+$$

$$i_b^- = Di_a^+ + Bi_b^+$$

$$i_b^+ = i_b^-$$

$$[I - B]i_b^+ = Di_a^+$$

$$i_b^+ = [I - B]^{-1}Di_a^+$$

$$i_a^+ = D^{-1}[I - B]i_b^+$$

$$i_a^- = \underbrace{A + C[I - B]^{-1}D}_{\equiv P} i_a^+$$

$$Y_{2-pt} = [I - P][M_A]$$

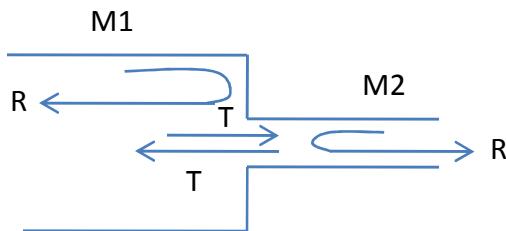
$$i_a^- = Pi_a^+$$

$$\{i\} = \{i_a^+\} - \{i_a^-\} = [I - P]\{i_a^+\} = [I - P][M_A]\mu^+$$

$$\{i_a^+\} - \{i_a^-\} = \{i\} = [I - P]D^{-1}[I - B]\{i_b^+\}$$

$$\{i\} = [I - P]D^{-1}[I - B][M_B]\{\mu_b^+\}$$

$$Y_{4-pt} = [I - P][M_A]D^{-1}[I - B][M_B]$$



$$S = \begin{bmatrix} R_1 & T_2 \\ T_1 & R_2 \end{bmatrix}$$

Row need not add to 1

$$\bar{S} = \begin{bmatrix} R_1 & T_2 \\ T_1 & R_2 \end{bmatrix} \begin{bmatrix} M_1 & 0 \\ 0 & M_2 \end{bmatrix} = \begin{bmatrix} M_1 R_1 & M_2 T_2 \\ M_1 T_1 & M_2 R_2 \end{bmatrix}$$

Now, rows also add to M1 and M2

$$M_1 T_1 = M_2 T_2$$

$$\{i^-\} = \underbrace{[S]}_{\bar{S}} [M] \{\mu^+\}$$

$$i_1^- = M_1 R_1 \mu_1^+ + M_2 T_2 \mu_2^+$$

$$i_2^- = M_1 T_1 \mu_1^+ + M_2 R_2 \mu_2^+$$

In equilibrium:  $\mu_1^+ = \mu_2^+ = \mu_0$

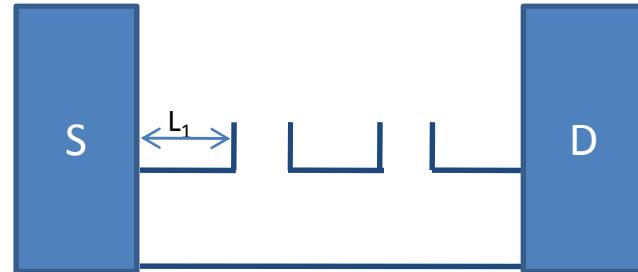
$$i_1^- = i_1^+$$

$$i_1^- = M_1 \mu_0$$

$$M_1 R_1 + M_2 T_2 = M_1$$

$M_1 T_1 = M_2 R_2$  Sum rule

$$\bar{S} = SM = \begin{bmatrix} \times & t' & a' & c' \\ t & \times & b' & d' \\ a & b & \times & 0 \\ c & d & 0 & \times \end{bmatrix}$$



$$a = \frac{k\lambda}{\lambda + L_1}$$

If a magnetic field is present  $a$  and  $a'$  are unequal

In a two terminal device they are equal even in presence of magnetic field due to sum rule