ECE 659 Quantum Transport: Atom to Transistor

Lecture 27: Spin Orbit Interaction

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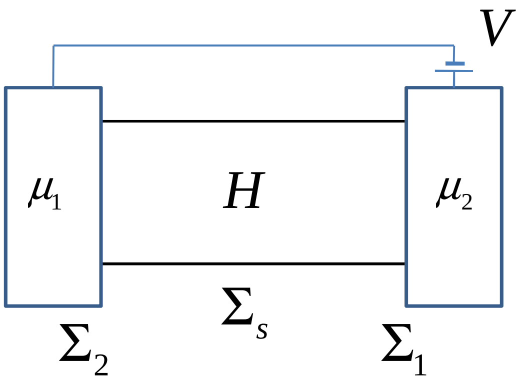
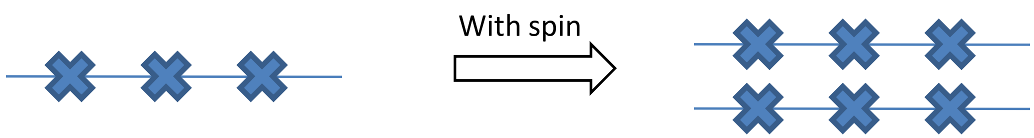


$$i\hbar\frac{\partial\tilde{\psi}}{\partial t}=\left[\frac{\vec{p}\cdot\vec{p}}{2m}-qV\right]\tilde{\psi} \qquad \vec{p}=-i\hbar\nabla$$

$$\sigma_x=\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix} \qquad \sigma_y=\begin{bmatrix}0 & -i\\ i & 0\end{bmatrix} \qquad \sigma_z=\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}$$



If we have a magnetic field in z direction
Schrodinger Equation is given by:

$$i\hbar\frac{\partial}{\partial t}\{\psi\}=\begin{bmatrix}H & 0\\ 0 & H\end{bmatrix}\{\psi\}+\mu_B B_z\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}\{\psi\}$$

$$H=\frac{\vec{p}\cdot\vec{p}}{2m}-qV$$

Magnetic field in x direction introduces a coupling:

$$\mu_B B_x\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}\{\psi\}$$

In a concise way:

$$\mu_B\vec{\sigma}\cdot\vec{B}$$

Where:

$$\vec{\sigma}\cdot\vec{B}=\sigma_x B_x+\sigma_y B_y+\sigma_z B_z$$

$$=\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}B_x+\begin{bmatrix}0 & -i\\ i & 0\end{bmatrix}B_y+\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}B_z$$

Schrodinger Equation becomes:

$$i\hbar\frac{\partial}{\partial t}\{\psi\}=\begin{bmatrix}H & 0\\ 0 & H\end{bmatrix}\{\psi\}+\mu_B\vec{\sigma}\cdot\vec{B}\{\psi\}$$

H in general is given as:

$$H = \left[\frac{(\vec{p} + q\vec{A}).(\vec{p} + q\vec{A})}{2m} - qV \right] I + \mu_B \vec{\sigma} \cdot \vec{B}$$

$$i\hbar \frac{\partial \Psi}{\partial t} = [H]\Psi$$

We can combine the Lorentz force and Zeeman splitting parts together into:

$$\left[\frac{(\vec{p} + q\vec{A}).(\vec{p} + q\vec{A})}{2m} - qV \right] I + \mu_B \vec{\sigma} \cdot \vec{B} = \frac{\left[\vec{\sigma}.(\vec{p} + q\vec{A}) \right]^2}{2m}$$

Schrodinger-Pauli Equation:

In absence of B :

$$[\vec{\sigma}.\vec{p}]^2 = \begin{bmatrix} p_z & p_x - ip_y \\ p_x + ip_y & -p_z \end{bmatrix} . \begin{bmatrix} p_z & p_x - ip_y \\ p_x + ip_y & -p_z \end{bmatrix}$$

$$= \begin{bmatrix} p_z^2 + p_x^2 + p_y^2 & 0 \\ 0 & p_z^2 + p_x^2 + p_y^2 \end{bmatrix}$$

$$= p^2 I$$

So we get back the usual Schrodinger Equation:

$$i\hbar \frac{\partial}{\partial t}\{\psi\} = \frac{1}{2m} \begin{bmatrix} p^2 & 0 \\ 0 & p^2 \end{bmatrix} \{\psi\} - qV\{\psi\}$$

$$= \frac{[\vec{\sigma}.\vec{p}]^2}{2m}\{\psi\} - qV\{\psi\}$$

By introduction of B in previous result:

$$i\hbar \frac{\partial}{\partial t}\{\psi\} = \frac{\left[\vec{\sigma}.(\vec{p} + q\vec{A}) \right]^2}{2m}\{\psi\} - qV\{\psi\}$$

This is known as **Schrodinger-Pauli Equation**

Dirac's Equation:

Schrodinger Equation is a non-relativistic equation:

$$E = \frac{p^2}{2m}$$

To include relativistic effects we need:

$$E^2 = m^2c^4 + p^2c^2$$

This is achieved by Dirac's Equation by turning Schrodinger's equation into a matrix equation using:

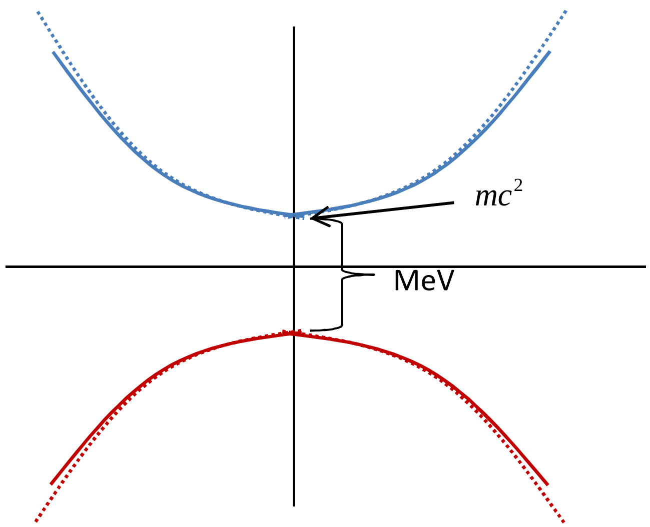
$$E = \pm\sqrt{m^2c^4 + p^2c^2}$$

$$i\hbar\frac{\partial\{\psi\}}{\partial t}=\begin{bmatrix} mc^2 I & c\vec{\sigma}\cdot\vec{p} \\ c\vec{\sigma}\cdot\vec{p} & -mc^2 I \end{bmatrix}\{\psi\}$$

Because:

$$\begin{bmatrix} mc^2 I & c\vec{\sigma}\cdot\vec{p} \\ c\vec{\sigma}\cdot\vec{p} & -mc^2 I \end{bmatrix}\begin{bmatrix} mc^2 I & c\vec{\sigma}\cdot\vec{p} \\ c\vec{\sigma}\cdot\vec{p} & -mc^2 I \end{bmatrix}$$

$$=\begin{bmatrix} \left(m^2c^4+p^2c^2\right)I & 0 \\ 0 & \left(m^2c^4+p^2c^2\right)I \end{bmatrix}$$



$$E=\pm\sqrt{m^2c^4+p^2c^2}$$

$$E=mc^2\sqrt{1+\frac{p^2}{m^2c^2}}$$

$$\simeq mc^2\left(1+\frac{p^2}{2m^2c^2}\right)$$

$$=mc^2+\frac{p^2}{2m}$$

If we take another higher term
we get a correction term:

$$\mu_B\vec{\sigma}\cdot\left[\frac{\vec{E}\times\vec{p}}{2mc^2}\right]$$