

ECE 659 Spring'09

Weeks 1-3: Semiclassical Transport

HW#1, Due Friday January 23, 2009 in class

Fermi function: $f(E) = 1/(1 + \exp((E - \mu)/kT))$

Current $I = \frac{q}{h} \int dE \pi \gamma D(E) (f_1(E) - f_2(E))$

Ballistic and diffusive transport: $\gamma = \hbar v_x / L$, $\pi \gamma D \equiv M(E)$

$$I = \frac{q}{h} \int dE M(E) (f^+(E) - f^-(E))$$

Electron density $n(x, E) = \frac{D(x, E)}{2} (f^+(x, E) + f^-(x, E))$

$$\frac{df^+}{dx} = \frac{df^-}{dx} = -\frac{f^+ - f^-}{\lambda} , \quad \lambda \equiv 2 v_x \tau$$

$$f^+(x, E) - f^-(x, E) = \frac{\lambda}{\lambda + L} (f_1(E) - f_2(E))$$

Problem 1: Show that for diffusive transport the conductance can be written as
(A: cross-sectional area)

$$G = \frac{\sigma A}{L + \lambda}, \text{ where } \sigma = q^2 \int_{-\infty}^{+\infty} dE \left(-\frac{\partial f}{\partial E} \right) \frac{D(E)}{AL} v_x^2(E) \tau$$

Noting that the electron density is given by $n = \int_{-\infty}^{+\infty} f(E) \frac{D(E)}{AL}$

show that $\mu_n(\text{mobility}) \equiv \frac{\sigma}{qn} = \frac{q \int_{-\infty}^{+\infty} dE f(E) \frac{d}{dE} (D(E) v_x^2(E) \tau)}{\int_{-\infty}^{+\infty} dE f(E) D(E)}$ (1)

assuming that the density of states $D(E)$ is zero as $E \rightarrow -\infty$.

Problem 2: Typically semiconductors a parabolic $E(k)$ relation, $E = \hbar^2 k^2 / 2m$, the density of states can be written in the form $D(E) = D_0 E^{(d/2)-1}$ where d is the number of dimensions and D_0 is a constant. Also, $v_x^2(E) = v^2(E)/d = 2E/md$. Using these relations in Eq.(1) and assuming τ to be independent of energy, show that $\mu_n = q\tau/m$.

Problem 3: A two-dimensional graphene sheet has a linear $E(k)$ relation, $E = \hbar v_f |k|$, where v_f is a constant. The density of states can be written in the form $D(E) = D_0 E$ where $D_0 = \frac{WL}{2\pi(\hbar v_f)^2}$. Also, $v_x^2(E) = v_f^2/2$. Using these relations in Eq.(1) and assuming τ to be independent of energy, show that $\mu_n = q\tau/m$, if we define the "mass" as

$$m = \frac{2 \langle E \rangle}{v_f^2} \quad \text{where} \quad \langle E \rangle \equiv \frac{\int dE E f(E)}{\int dE f(E)}$$

(ignoring the states below $E=0$)

Finally, please look over Prob.1 of HW#6 and Prob.1 of HW#7 from ECE 495N. Both homeworks and solutions are posted at

<http://sites.google.com/site/ece495n/>

You do not need to submit these, but reviewing them should be helpful in the coming discussions.