NEGF equations for elastic transport
(D=0 yields coherent transport)
\[ \Gamma_1 = i[\Sigma_1 - \Sigma_1^+] \]
\[ \Gamma_2 = i[\Sigma_2 - \Sigma_2^+] \]
\[ \Gamma_s = i[\Sigma_s - \Sigma_s^+] \]
1. \( G(E) = [EI - H - \Sigma_1 - \Sigma_2 - \Sigma_s]^{-1} \),
   \[ \text{where} \quad [\Sigma_s] = D [G] \]
2. \( A(E) = i[G - G^+] \)
3. \[ [G^n(E)] = [G \Gamma_1 G^+] f_1 + [G \Gamma_2 G^+] f_2 + [G \Sigma_s G^+] \]
   \[ \text{where} \quad [\Sigma_s] = D [G^n] \]
4. \( i\hbar I_{\text{op}} = [HG^n - G^n H] + [\Sigma G^n - G^n \Sigma^+] + [\Sigma^+ G^n - G \Sigma^+ G^n] \)
4a. \( I_{a \rightarrow b}(E) = \frac{q}{\hbar} i[H_{ab} G^n_{ba} - G^n_{ab} H_{ba}] \quad a, b: \text{Internal Points} \)
4b. \( I_i(E) = \frac{q}{\hbar} (\text{Trace}[\Sigma_{i} A - \Gamma_{j} G^n]) \quad \text{Current/energy at terminal 'i'} \)
4c. \( I_i(E) = \frac{q}{\hbar} \sum_j \text{Trace}[\Gamma_i \Gamma_j G^+] (f_i(E) - f_j(E)) \quad (\text{used only if } D \text{ is zero}) \)

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Pauli spin matrices:
\[ \sigma_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \sigma_y = \begin{bmatrix} 0 & -i \\ +i & 0 \end{bmatrix}, \sigma_z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \]
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Consider the same 1-D wire as in Problem 1 of HW6

\[ \Sigma_1 \begin{bmatrix} 1 \\ H \end{bmatrix} \Sigma_2 \]

modeled with a Hamiltonian having \( H_{n,n} = 2t_0 \), \( H_{n,n+1} = -t_0 = H_{n,n-1} \) (all other elements are zero), such that the dispersion relation is given by \( E = 2t_0(1 - \cos ka) \). \( \Sigma_1 \) is a matrix with all zeroes except for \( \Sigma_1(1,1) = \Sigma \), while \( \Sigma_2 \) has all zeroes except for \( \Sigma_2(N_p,N_p) = \Sigma \), where \( \Sigma(E) = -t_0 e^{+ika} \). Assume \( D = 0 \). The overall channel length = 150nm and a magnetic field is present in the middle one-third of the channel giving rise to and additional \( H_B = \varepsilon \sigma_z \) where \( \varepsilon = 0.03 \) eV.

We will assume that we have separate contacts for up and down-spin channels (along +x and -x respectively) described by \( \Sigma_{1,u}, \Sigma_{1,d}, \Sigma_{2,u}, \Sigma_{2,d} \).

In HW6 we calculated the transmission coefficients from one terminal to another. Here the objective is to calculate internal quantities inside the channel.

(a) Plot numerically the electron density and the x, y and z-components of the spin density along the channel assuming that only the up-spin contact 1 injects spins along +x. \( E = 0.2 \) eV, \( a = 1 \) nm, \( m = 0.03 \times \) free electron mass, \( t_0 = \hbar^2 / 2ma^2 \).

(b) Plot numerically the current density and the x, y and z-components of the spin current density along the channel again assuming that only the up-spin contact 1 injects spins along +x. Hint: Use Eq.(4a) for the current operator inside the channel, with
(i) \( b=a+1 \) for right current,
(ii) \( b=a-1 \) for left current,
(iii) \( b=a \) for “spin-torque”. The three should add up to zero.

(c) Compare the result in (b) from (iii) with \( \frac{2\varepsilon}{\hbar} (z \times S) \).

Please remember to turn in a copy of your MATLAB codes.