ECE 659 Spring '09 Weeks 9-11: Spin Transport

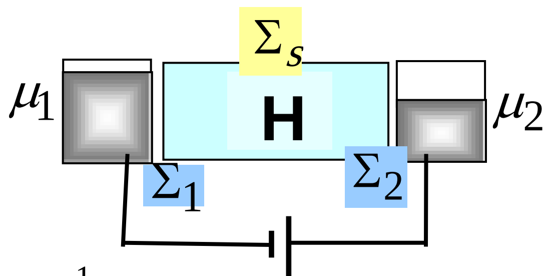
Take-home Exam: Due Monday April 20, 2009 in class

Please do not discuss with <u>anyone</u>

other than the instructor (Datta) and the TA (Hong).

NEGF equations for elastic transport

(D=0 yields coherent transport)



$\Gamma_1 = i[\Sigma_1 - \Sigma_1^+], \Gamma_2 = i[\Sigma_2 - \Sigma_2^+],$

$\Gamma_s = i[\Sigma_s - \Sigma_s^+]$

1. $G(E) = [EI - H - \Sigma_1 - \Sigma_2 - \Sigma_s]^{-1},$

   where $[\Sigma_s] = D[G]$

2. $A(E) = i[G - G^+]$

3. $[G^n(E)] = \ [G\Gamma_1 G^+]\ f_1 + [G\Gamma_2 G^+]\ f_2 + [G\Sigma_s^{in} G^+]$

   where $\left[\Sigma_s^{in}\right] = D\left[G^n\right]$

4. $i\hbar I_{op} = [HG^n - G^n H] + [\Sigma G^n - G^n \Sigma^+] + [\Sigma^{in} G^+ - G\Sigma^{in}]$

4a. $I_{a\to b}(E) = \dfrac{q}{h}\, i[H_{ab} G^n_{ba} - G^n_{ab} H_{ba}]$ ***a, b: Internal Points***

4b. $I_i(E) = \dfrac{q}{h}\,((Trace[\Sigma_i^{in} A - \Gamma_i G^n])$ ***Current/energy at terminal 'i'***

4c. $I_i(E) = \dfrac{q}{h} \sum_j Trace[\Gamma_i G \Gamma_j G^+](f_i(E) - f_j(E))$ ***(used only if D is zero)***

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Pauli spin matrices: $\sigma_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \ \sigma_y = \begin{bmatrix} 0 & -i \\ +i & 0 \end{bmatrix}, \ \sigma_z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

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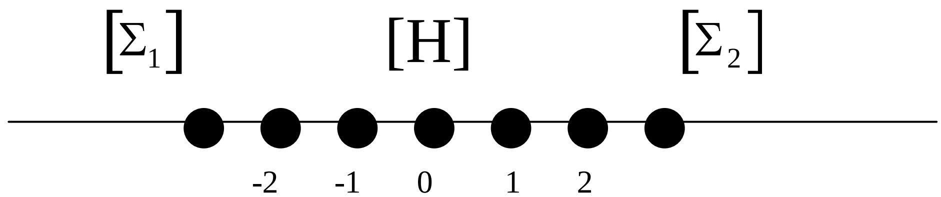
Consider the same 1-D wire as in Problem 1 of HW6



modeled with a Hamiltonian having $H_{n,n} = +2t_0$, $H_{n,n+1} = -t_0 = H_{n,n-1}$ (all other elements are zero), such that the dispersion relation is given by $E = 2t_0(1-\cos ka)$. Σ_1 is a matrix with all zeroes except for $\Sigma_1(1,1) = \Sigma$, while Σ_2 has all zeroes except for $\Sigma_2(Np, Np) = \Sigma$, where $\Sigma(E) = -t_0\, e^{+ika}$. Assume D = 0. The overall channel length = 150nm and a magnetic field is present in the middle one-third of the channel giving rise to and additional $H_B = \varepsilon\sigma_z$ where $\varepsilon = 0.03$ eV .

We will assume that we have separate contacts for up and down-spin channels (along +x and -x respectively) described by $\Sigma_{1,u}, \Sigma_{1,d}, \Sigma_{2,u}, \ \Sigma_{2,d}$.

In HW6 we calculated the transmission coefficients from one terminal to another. Here the objective is to calculate ***internal*** quantities inside the channel.

(a) Plot numerically the electron density and the x, y and z-components of the spin density along the channel assuming that only the up-spin contact 1 injects spins along +x. E = 0.2 eV, a = 1 nm, m = 0.03 * free electron mass, $t_0 = \hbar^2 / 2ma^2$.

(b) Plot numerically the current density and the x, y and z-components of the spin current density along the channel again assuming that only the up-spin contact 1 injects spins along +x. Hint: Use Eq.(4a) for the current operator inside the channel, with

(i) b=a+1 for right current,

(ii) b=a-1 for left current,

(iii) b=a for “spin-torque”. The three should add up to zero.

(c) Compare the result in (b) from (iii) with $\dfrac{2\varepsilon}{\hbar}(\vec{z} \times \vec{S})$.

Please remember to turn in a copy of your MATLAB codes.