Lecture 27: Minimum Resistance of a Quantum Wire
Ref. Chapter 6.3
Today we want to talk about the maximum current that can run through a quantum wire. Is there a limit? If there is, what determines that limit?

The idea of Q. Wire comes from confinement of a large solid. This confinement is categorized in the box based on the number of dimensions that are confined.

For electronic properties we are interested in the levels close to the Fermi level. Near the Fermi level we can approximate the dispersion relation.

\[
E = E_C + \frac{\hbar^2}{2m_C} \left( k_x^2 + k_y^2 + k_z^2 \right)
\]

Bulk Solid: \( E(k_x, k_y, k_z) \)

Q. Well: \( E_v(k_x, k_y) \)

Q. Wire: \( E_{v,v'}(k_x) \)

Q. Dot: \( E_{v,v',v''} \)

Instrumental in making semiconductor lasers and field effect transistors.
• The wavefunction can be written as:
  \[ \psi \propto \sin k_z z \] where \( k_z = \nu \pi / L_z \)
• Which gives us the subbands:
  \[ E_v(k_x, k_y) = E_C + \frac{\nu^2 \hbar^2 \pi^2}{2m_c L_z^2} + \frac{\hbar^2}{2m_c} (k_x^2 + k_y^2) \]
If we take a Q. well which is confined in the z direction and very large in the x, y plane and confine it yet in another direction, take y for instance, then what we have is a Q. wire. The subbands are then given by:

\[ E_{v,v'}(k_x) = E_C + \frac{v^2 \hbar^2 \pi^2}{2m_CL_z^2} + \frac{v'^2 \hbar^2 \pi^2}{2m_CL_y^2} + \frac{\hbar^2}{2m_C}k_x^2 \]

For the quantum wire we get various subbands corresponding to different values of \( v, v' \)

The quantized wave functions in the y and z directions look like \( \rightarrow \)
• Now, what does the density of states look like for a quantum wire? Clearly, as with the carbon nanotube, the density of states peaks where each subband is crossed.

\[
D_{1D}(E) = \frac{1}{\pi dE / dk}
\]
Boundary Conditions: Two Different Cases

- Last day we discussed what happens to DOS when one makes a nanotube out of Graphene.

    Bulk Solid: \(E(k_x, k_y, k_z)\)
    Q. Well: \(E_{\nu}(k_x, k_y) \Rightarrow \text{Graphene}\)
    Q. Wire: \(E_{\nu,\nu'}(k_x) \Rightarrow \text{Nanotube}\)
    Q. Dot: \(E_{\nu,\nu',\nu''}\)

- In that problem our assumption of periodic boundary conditions was actually exact for Carbon Nanotubes.

    \(k_y = \left(\frac{2\pi}{L_y}\right)\nu: \text{EXACT}\)

- However for today’s problem if we assume the potential walls are very large, we have:

\[
k_z = \left(\frac{\pi}{L_z}\right)\nu: \text{Approximation}
\]
At equilibrium the current carried by red states is equal in magnitude and opposite in sign to blue states → no net current at equilibrium

\[
\mu = \frac{\hbar^2 k^2}{2m_c}
\]

\[
f(E) = \frac{1}{1 + e^{(E-\mu)/kT}}
\]

\[
N = \sum_k f(E_k) = 2\int \frac{dk L}{2\pi} f(E_k) \Rightarrow \frac{N}{L} = 2\int \frac{dk}{2\pi} f(E_k) = 2\int \frac{dE_k}{2\pi \frac{dE_k}{dk}} f(E_k)
\]
• Under certain conditions we can have a picture like this where the amount states carrying positive current is more than the ones carrying negative current. Their algebraic sum we’ll give us the net current which is due to extra positive k states shaded in the figure:

\[
I_{net} = q \frac{N}{L} v = q \int_{0}^{\infty} \frac{dk}{2\pi} \nu_k f(E_k) = \frac{q}{\hbar} \int_{0}^{\infty} \frac{dE_k}{2\pi} \frac{dE_k}{dk} f(E_k) = \frac{q}{\hbar} \int_{0}^{\infty} dE_k f(E_k) = \frac{40\mu A}{eV}
\]

• In general we have many subbands. So we have to multiply by the number of subbands that are available in the energy range of interest.
Quantum of Conductance

• Let’s put some numbers in…

\[ \frac{q}{h} = \frac{1.6 \times 10^{-19} \text{coul}}{6.63 \times 10^{-34} \text{J} \cdot \text{sec}} 1.6 \times 10^{-19} \frac{J}{eV} = \frac{2.56}{6.63} \times 10^{-4} \frac{A}{eV} = \frac{256}{6.63} \mu A \]

\[ q \approx 40 \frac{\mu A}{eV} \]

• What this quantity means is that the current carried by a quantum wire having one mode is this value multiplied by the energy range of interest.

Positive current carrying states