This homework requires the use of the following formulas for the density of states and the mode density (S, area; L, length; W, width):

\[ D(E) = \sum_k \delta(E - \varepsilon(\vec{k})) \]

\[ M(E) = \sum_k \delta(E - \varepsilon(\vec{k})) \frac{\pi \hbar |v_x(\vec{k})|}{L} \]

Assume that electrons are confined to a two-dimensional layer having an \( \varepsilon(\vec{k}) \) relation of the form

\[ \varepsilon(\vec{k}) = \frac{\hbar^2}{2m} (k_x^2 + k_y^2) \]

1. Obtain an expression for the (a) density of states D(E) and (b) the mode density, M(E) in terms of the energy E, the area S and constants like m and \( \hbar \). Assume that both L and W large enough that the summations over \( k_x \) and \( k_y \) can both be replaced with appropriate integrals.

2. How would you write the energies of the subbands if the electrons are confined to a narrow channel of width W in the y-direction?

3. Obtain an expression for the density of states D(E) and the mode density M(E), assuming that L is large enough that the summation over \( k_x \) can be replaced with an appropriate integral, but W is NOT large enough to do the same for \( k_y \).