# ECE 495N PRACTICE EXAM I

# CLOSED BOOK, Sept. 25, 2007

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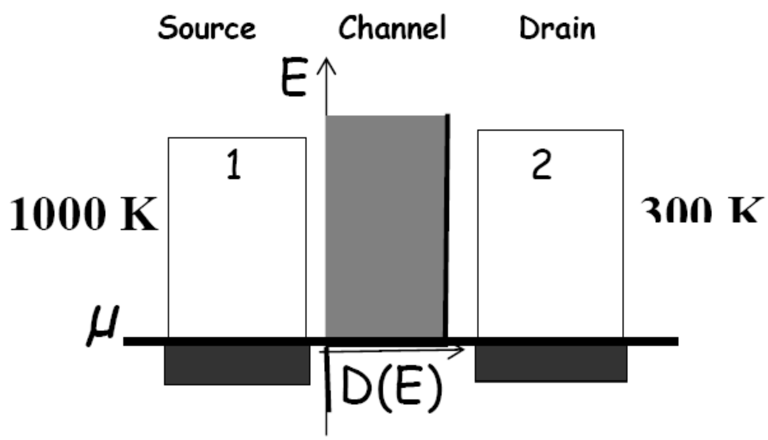
|---|---|---|
| **Problem 1** | **[p. 2,]** | **8 points** |
| **Problem 2** | **[p. 3]** | **8 points** |
| **Problem 3** | **[p. 4]** | **9 points** |
| **Total** | | **25 points** |

**Problem 1**

**(a) 5 points**




A channel has a density of states as shown, namely a constant non-zero value for $E \geq 0$ and zero for $E < 0$, that is, $D(E) = D_0\ \vartheta(E)$, where $\vartheta$ represents the unit step function. It is connected to two contacts with the same electrochemical potential μ whjich is ~ 0. Is there a current in the external circuit ? If so, is it in the direction shown (from the hot to the cold contact) or opposite to the direction shown (from the cold to the hot contact)? Explain your reasoning.

**(b) 3 points**
Which side (the hot side or the cold side) will be positive if we leave the terminals open?

**Problem 2**

We have seen in class that free electrons in the absence of any external potential are described by (in one dimension)

$$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2} \qquad (1)$$

whose solutions can be written in the form

$$\psi(x,t) = \underbrace{A}_{constant} \; e^{+ikx} \; e^{-iEt/\hbar} \qquad (2)$$

with E and k related by the dispersion relation:

$$E = \hbar^2 k^2 / 2m \qquad (3)$$

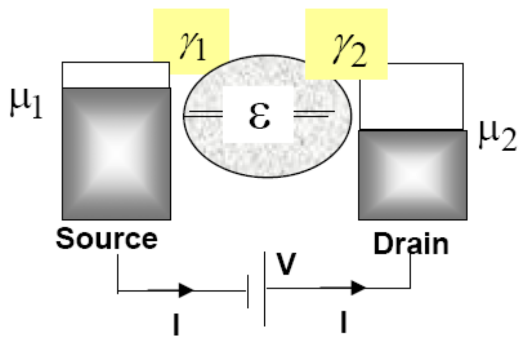
**(a) 3 points**
Can solutions be written in the form of (2) if a sinusoidal external potential $U = U_0 \cos(x/a)$ is present (a is a constant)?

**(b) 5 points**
Can you suggest a suitable differential equation to replace (1) if you wanted the dispersion relation to look like $E^2 = (mc^2)^2 + \hbar^2 c^2 k^2$ (3')
(m, c, $\hbar$ are all constants) instead of (3) ?

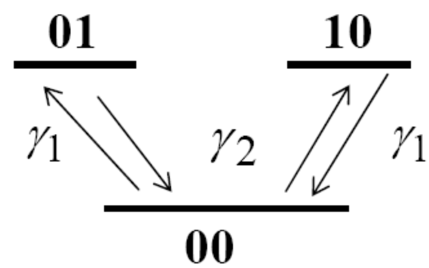
**Problem 3**

We wish to calculate the current, I through a channel having two discrete energy levels with energy $\varepsilon$, but with a very high interaction energy such that both levels CANNOT be simultaneously occupied in our voltage range of interest.

***One-electron picture***




***Multi-electron picture***




***(a) 6 points***

Write rate equations in the multielectron picture to obtain an expression for the maximum current I that flows when a voltage V is applied ***with the polarity as shown***. Your answer should be in terms of the couplings $\gamma_1$ and $\gamma_2$ for the two contacts (and fundamental constants like q and $\hbar$).

***(b) 3 points***

Assuming $\gamma_1 = 10\,\gamma_2$, would you get more current or less current (in magnitude) if you reversed the polarity of the applied voltage? Please explain your reasoning.