ECE 495N PRACTICE EXAM I

CLOSED BOOK, Sept. 25, 2007

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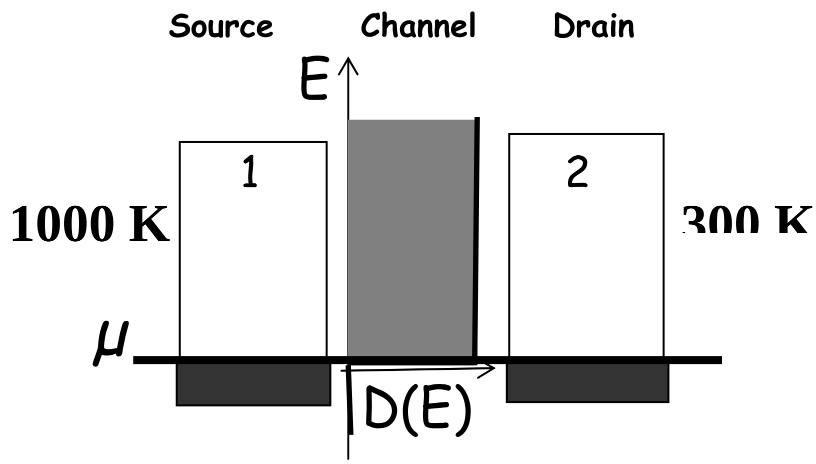
Total		**25 points**

Problem 1

(a) 5 points



A channel has a density of states as shown, namely a constant non-zero value for $E \geq 0$ and zero for $E < 0$, that is, $D(E) = D_0\ \vartheta(E)$, where ϑ represents the unit step function. It is connected to two contacts with the same electrochemical potential μ whjich is ~ 0. Is there a current in the external circuit ? If so, is it in the direction shown (from the hot to the cold contact) or opposite to the direction shown (from the cold to the hot contact)? Explain your reasoning.

Inside the channel

Electrons flow from hot to cold above μ, and from cold to hot below μ.
D(E) is **higher** for energies **above μ** than for energies below μ.
Hence net flow of electrons is from hot to cold.

In external circuit

Electrons flow from cold to hot.
Current flows from hot to cold.

External current will be in the direction shown.

(b) 3 points

Which side (the hot side or the cold side) will be positive if we leave the terminals open?

Inside the channel

Electrons will flow initially from hot to cold, making hot side positive and cold side negative. This will raise the electrochemical potential on the cold side relative to the cold side.

Hot side will be positive.

Problem 2

We have seen in class that free electrons in the absence of any external potential are

described by (in one dimension) $$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2} \qquad (1)$$

whose solutions can be written in the form $$\psi(x,t) = \underbrace{A}_{constant}\; e^{+i\,kx}\; e^{-i\,Et/\hbar} \qquad (2)$$

with E and k related by the dispersion relation: $$E = \hbar^2 k^2/2m \qquad (3)$$

(a) 3 points

Can solutions be written in the form of (2) if a sinusoidal external potential $U = U_0 \cos(x/a)$ is present (a is a constant)?

No, for solutions like (2) to work, differential equation must have constant coefficients.

(b) 5 points

Can you suggest a suitable differential equation to replace (1) if you wanted the dispersion relation to look like $E^2 = (mc^2)^2 + \hbar^2 c^2 k^2$ (3')
(m, c, $\hbar$ are all constants) instead of (3) ?

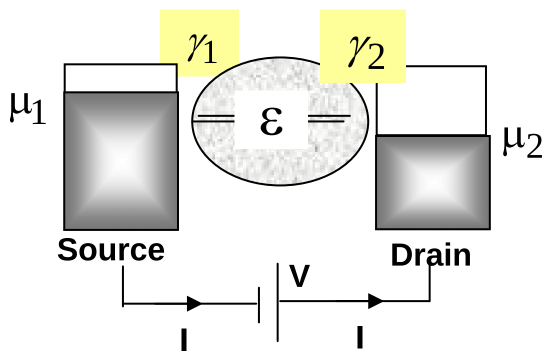
$$-\hbar^2 \frac{\partial^2 \psi}{\partial t^2} = \left(mc^2\right)^2 \psi - \hbar^2 c^2 \frac{\partial^2 \psi}{\partial x^2}$$

that is, $$\frac{\partial^2\psi}{\partial t^2} + \left(\frac{mc^2}{\hbar}\right)^2 \psi = c^2 \frac{\partial^2\psi}{\partial x^2}$$

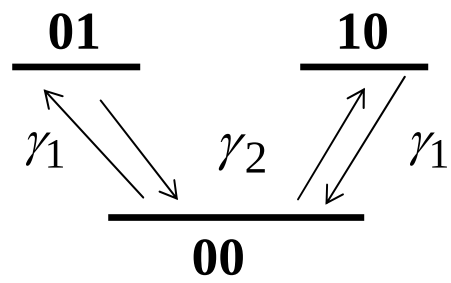
Problem 3

We wish to calculate the current, I through a channel having two discrete energy levels with energy ε, but with a very high interaction energy such that both levels CANNOT be simultaneously occupied in our voltage range of interest.

One-electron picture



Multi-electron picture



(a) 6 points

Write rate equations in the multielectron picture to obtain an expression for the maximum current I that flows when a voltage V is applied ***with the polarity as shown***. Your answer should be in terms of the couplings γ_1 and γ_2 for the two contacts (and fundamental constants like q and $\hbar$).

$$\frac{P_{01}+P_{10}}{P_{00}} = \frac{2\gamma_1}{\gamma_2} \qquad \Rightarrow \quad P_{00} = K, \quad P_{01}+P_{10} = \frac{2\gamma_1}{\gamma_2}K \qquad \text{where} \quad K = \frac{1}{1+2\dfrac{\gamma_1}{\gamma_2}}$$

$$I = \frac{q}{\hbar}2\gamma_1 P_{00} = \frac{q}{\hbar}\frac{2\gamma_1}{1+2\dfrac{\gamma_1}{\gamma_2}} = \frac{q}{\hbar}\frac{2\gamma_1\gamma_2}{2\gamma_1+\gamma_2}$$

(b) 3 points

Assuming $\gamma_1 = 10\,\gamma_2$, would you get more current or less current (in magnitude) if you reversed the polarity of the applied voltage? Please explain your reasoning.

$$I = \frac{q}{\hbar}\frac{2\gamma_1\gamma_2}{2\gamma_1+\gamma_2} = \frac{q}{\hbar}\frac{20\gamma_2}{21}$$

When polarity is reversed, the roles of the couplings γ_1 and γ_2 are reversed.

$$I \;=\; \frac{q}{\hbar}\,\frac{2\gamma_1\gamma_2}{2\gamma_2+\gamma_1} \;=\; \frac{q}{\hbar}\,\frac{20\gamma_2}{12}$$

which is larger than for the polarity indicated. **We get more current on reversing polarity.**