ECE 495N   EXAM I

CLOSED BOOK

Wednesday, Oct.1, 2008

NAME: SOLUTION

PUID #: 

Please show all work and write your answers clearly.

This exam should have seven pages.

<table>
<thead>
<tr>
<th>Problem</th>
<th>Page(s)</th>
<th>Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>Problem 1</td>
<td>[p. 2, 3]</td>
<td>8 points</td>
</tr>
<tr>
<td>Problem 2</td>
<td>[p. 4, 5]</td>
<td>8 points</td>
</tr>
<tr>
<td>Problem 3</td>
<td>[p. 6, 7]</td>
<td>9 points</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td></td>
<td><strong>25 points</strong></td>
</tr>
</tbody>
</table>
**Problem 1:** We have seen in class that the current-voltage (I-V) characteristics of a nanoscale device can be calculated from

\[ I = \frac{2q}{h} \int dE \, D(E-U) \frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2} \left[ f_1(E) - f_2(E) \right] \]

where \( f_1(E) = \frac{1}{e^{(E-\mu_1)/kT} + 1} \) and \( f_2(E) = \frac{1}{e^{(E-\mu_2)/kT} + 1} \).

Also \( U = U_L + U_0(N-N_0) \). Assume \( U_0 = 0 \) and the Laplace potential \( U_L \) to be a fraction \( \alpha \) of the drain potential \( V_D \) (the source potential is assumed zero):

\[ U_L = -q \, \alpha V_D \], \( \alpha \) being a constant between 0 and 1.

A channel has a density of states as shown, namely a constant non-zero value for \( E \geq E_c \) and zero for \( E < E_c \). Assume that the equilibrium electrochemical potential \( \mu \) is located above \( E_c \) as shown. Sketch the current versus drain voltage assuming that the electrostatic potential of the channel (a) remains fixed with respect to the source (\( \alpha = 0 \)) and (b) assumes a value halfway between the source and drain potentials (\( \alpha = 0.5 \)).

*Explain your reasoning clearly.*
(a) Channel potential fixed with respect to source: \( \alpha = 0 \).

\[ \frac{qV_D}{\mu - E_c} \]

\( \mu_2 \) drops below \( E_c \),
no further increase in \( I \).

(b) Channel potential halfway between source and drain: \( \alpha = 0.5 \)

\[ \frac{qV_D}{\mu - E_c} \]

\( \mu_2 \) drops below \( E_c \),
but current keeps increasing at half the rate because \( \mu_1 \) keeps rising relative to channel.
Problem 1-(b)

Diagram:

1. $E$ vs $T$ with $M_S$ and $M_D$, $D(E)$ indicated.
2. $E$ vs $T$ with $M_S$ and $M_D$, $D(E)$ indicated.
3. $E$ vs $T$ with $M_S$ and $M_D$, $D(E)$ indicated.
4. $E$ vs $T$ with $M_S$ and $M_D$, $D(E)$ indicated.
5. $E$ vs $T$ with $M_S$ and $M_D$, $D(E)$ indicated.

Additional current: $\uparrow$
Equilibrium chemical potential: $\downarrow$
**Problem 2:** We have seen in class that free electrons in the absence of any external potential are described by (in one dimension)

\[ \frac{i\hbar}{\partial t} \psi = -\frac{\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} \]  

(1)

whose solutions can be written in the form 

\[ \psi(x,t) = C e^{ikx} e^{-iE_i t} \]  

(2)

with E and k related by the dispersion relation: 

\[ E = \frac{\hbar^2 k^2}{2m} \]  

(3)

We have also seen that if the electrons are confined in a box of length L, the energy levels become discrete with the lowest energy given by 

\[ E_1 = \frac{\hbar^2 \pi^2}{2mL^2} \]  

(4)

(a) Can you suggest a suitable differential equation to replace (1) if you wanted the dispersion relation to look like 

\[ E = Ak^4 \]  

(3')

(A being a constant) instead of (3) ?

(b) If a system of electrons with a dispersion relation given by (3') were confined in a box of length L, how would the expression for the lowest energy given in (4) be modified?

(a) \[ i\hbar \frac{\partial \psi}{\partial t} = A \frac{\partial^4 \psi}{\partial x^4} \]

(b) \[ E_1 = A \left( \frac{\pi}{L} \right)^4 \]
Problem 3:

\[ \frac{\gamma_1}{\hbar} \quad \frac{\gamma_2}{\hbar} \]

\[ \mu_1 \quad \varepsilon \quad \mu_2 \]

A box has four degenerate energy levels all having energy \( \varepsilon \). We know that for non-interacting electrons the maximum current under bias is

\[ I = \frac{q}{h} \frac{4\gamma_1\gamma_2}{\gamma_1 + \gamma_2} \]

Use the multielectron picture to derive the correct expression for the maximum current if the electron-electron interaction energy is so high that no more than one electron can be inside the box at the same time.

\[ \gamma_2 \quad \overline{0000} \quad \gamma_1 \]

\[ \frac{P_{0000}}{P_0} \cdot 4\gamma_2 = (P_{0001} + P_{0010} + P_{0100} + P_{1000}) \cdot \gamma_1 \equiv P_1 \]

\[ \frac{P_1}{P_0} = \frac{4\gamma_2}{\gamma_1} \Rightarrow \frac{P_1}{P_0 + P_1} = \frac{4\gamma_2}{\gamma_1 + 4\gamma_2} \]

\[ I = \frac{q}{h} \gamma_1 P_1 = \frac{q}{h} \frac{4\gamma_1\gamma_2}{\gamma_1 + 4\gamma_2} \]