ECE 495N EXAM II:
Wednesday, Nov.5, 2008, ME118, 1130A-1220P
CLOSED BOOK

PRACTICE EXAM II

Problem 1	**[p. 2, 3]**	**8 points**
Problem 2	**[p. 4, 5]**	**8 points**
Problem 3	**[p. 6, 7]**	**9 points**
Total		**25 points**

Useful Relations: These will be provided in the exam.

$$f(E) = \frac{1}{1+\exp(E-\mu)/kT}$$ Fermi function

$$P_\alpha = \frac{1}{Z}\exp(-(E_\alpha - \mu N_\alpha)/kT)$$ Law of equilibrium

$$[h(\vec{k})] = \sum_m [H_{nm}]\exp(i\vec{k}.(\vec{d}_m - \vec{d}_n))$$ Bandstructure

$$D(E) = \sum_{\vec{k}} \delta(E-\varepsilon(\vec{k}))$$ Density of states

$$M(E) = \sum_{\vec{k}} \delta(E-\varepsilon(\vec{k}))\frac{\pi\hbar\left|v_x(\vec{k})\right|}{L}$$ Density of modes

Problem 1: A channel has four energy levels all with the same energy ε, but the interaction energy is so high that no more than one of these levels can be occupied at the same time. What is the average number of electrons in the channel if it is in equilibrium with chemical potential μ and temperature T? Your answer should be in terms of ε, μ and T.

Problem 2: ***Use the principles of bandstructure*** to write down the eigenvalues and eigenvectors of the matrix $\begin{bmatrix} 0 & b & 0 & b \\ b & 0 & b & 0 \\ 0 & b & 0 & b \\ b & 0 & b & 0 \end{bmatrix}$. Please do not diagonalize directly.

Problem 3: Suppose a ***large*** two-dimensional conductor has an $\varepsilon(\vec{k})$ relationship given by:

$$\varepsilon(\vec{k}) = +\alpha\sqrt{k_x^{\,2} + k_y^{\,2}}$$

where α is a constant. Derive an expression for the density of states, D(E) and the density of modes, M(E). Your answer should be in terms of the energy E, α, width W and length L.