ECE 495N EXAM II

CLOSED BOOK

Wednesday, Nov.5, 2008

NAME : SOLUTION

PUID # : ______________________

Please show all work and write your answers clearly.
This exam should have seven pages.

Problem 1	**[p. 2, 3]**	**8 points**
Problem 2	**[p. 4, 5]**	**9 points**
Problem 3	**[p. 6, 7]**	**8 points**
Total		**25 points**

Useful Relations:

$$f(E) = \frac{1}{1+\exp(E-\mu)/kT} \quad \textit{Fermi function}$$

$$P_\alpha = \frac{1}{Z}\exp(-(E_\alpha - \mu N_\alpha)/kT) \quad \textit{Law of equilibrium}$$

$$[h(\vec{k})] = \sum_m [H_{nm}]\exp(i\vec{k}.(\vec{d}_m - \vec{d}_n)) \quad \textit{Bandstructure}$$

$$D(E) = \sum_k \delta(E-\varepsilon(\vec{k})) \quad \textit{Density of states}$$

$$M(E) = \sum_k \delta(E-\varepsilon(\vec{k}))\frac{\pi\hbar\left|v_x(\vec{k})\right|}{L} \quad \textit{Density of modes}$$

$$v_x(\vec{k}) = \frac{1}{\hbar}\frac{\partial\varepsilon(\vec{k})}{\partial k_x}$$

Problem 1: A channel has two energy levels with the same energy ε, but the interaction energy is so high that no more than one of these levels can be occupied at the same time. What is the average number of electrons in the channel if it is in equilibrium with chemical potential μ and temperature T? Your answer should be in terms of ε, μ and T.

01   10

00

$$P_{01} = \frac{1}{Z} e^{-\underbrace{(\varepsilon-\mu)/kT}_{\equiv x}}$$

$$= P_{10}$$

$$P_{00} = \frac{1}{Z}$$

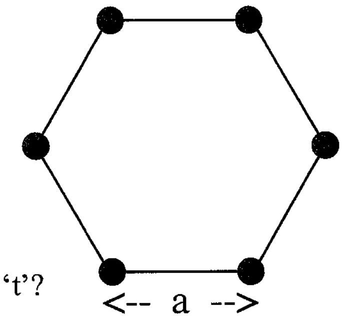
$$P_{00} + P_{01} + P_{10} = \frac{1 + 2e^{-x}}{Z} = 1$$

$$Z = 1 + 2e^{-x}$$

$$\langle N \rangle = P_{01} \cdot 1 + P_{10} \cdot 1 + P_{00} \cdot 0$$

$$= \frac{2e^{-x}}{1 + 2e^{-x}} = \frac{1}{1 + \frac{1}{2} e^{x}}$$

Problem 2: Benzene molecule consists of six carbon atoms arranged at the corners of a regular hexagon of side 'a'. Assume (1) one orbital per carbon atom as basis function ; (2) the overlap matrix [S] is a (6x6) identity matrix; and (3) the Hamiltonian matrix is given by



$H_{n,n} = \varepsilon$ (site energy)

$H_{n,m} = t$ if n, m are neighboring atoms

$H_{n,m} = 0$ if n, m are NOT nearest neighbors

(a) What are the six energy eigenvalues in terms of 'ε' and 't'?

(b) What are the corresponding eigenvectors?

$$E = \varepsilon + 2t\cos\phi, \qquad \phi = \frac{2\pi}{6}\left[0, \pm 1, \pm 2, 3\right]$$

$$= \varepsilon + 2t, \quad \varepsilon + t, \quad \varepsilon - t, \quad \varepsilon - 2t$$

$$(1) \qquad (2) \qquad (2) \qquad (1)$$

$$\begin{Bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{Bmatrix} \quad \begin{Bmatrix} 1 \\ e^{\pm i\pi/3} \\ e^{\pm i2\pi/3} \\ -1 \\ e^{\pm i4\pi/3} \\ e^{\pm i5\pi/3} \end{Bmatrix} \quad \begin{Bmatrix} 1 \\ e^{\pm i2\pi/3} \\ e^{\pm i4\pi/3} \\ 1 \\ e^{\pm i2\pi/3} \\ e^{\pm i4\pi/3} \end{Bmatrix} \quad \begin{Bmatrix} 1 \\ -1 \\ 1 \\ -1 \\ 1 \\ -1 \end{Bmatrix}$$

Problem 3: Suppose a *large* two-dimensional conductor has an $\varepsilon(\vec{k})$ relationship given by:

$$\varepsilon(\vec{k}) = \quad A k^4$$

where A is a constant and $k^2 = k_x^2 + k_y^2$.

Derive an expression for the density of states, D(E).

Your answer should be in terms of the energy E, A, width W and length L.

$$D(E) = \sum_{\vec{k}} \delta\left(\mathcal{E}(\vec{k}) - E\right)$$

$$= \frac{LW}{4\pi^2} \int_{-\pi}^{+\pi} d\theta \int_0^\infty dk \; k \; \delta(\mathcal{E} - E)$$

$$= \frac{LW}{2\pi} \int_0^\infty d\mathcal{E} \; \delta(\mathcal{E} - E) \frac{k}{\underbrace{d\mathcal{E}/dk}_{4Ak^3}}$$

$$= \frac{LW}{2\pi} \int_0^\infty d\mathcal{E} \; \delta(\mathcal{E} - E) \frac{1}{4A} \sqrt{\frac{A}{\mathcal{E}}}$$

$$= \frac{LW}{8\pi\sqrt{A}} E^{-1/2}$$